An Experiment to Determine the Accuracy of Squeeze-Film Damping Models in the Free-Molecule Regime

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Gas Damping Is Important in MEMS.

Motivation:

- Electrostatic, parallel-plate actuation is very important in many microsystems applications.
- Squeezed-film damping determines the dynamics of plates moving a few microns above the substrate. Examples abound in
  - MEMS accelerometers.
  - MEMS switches.
  - MEMS gyroscopes.

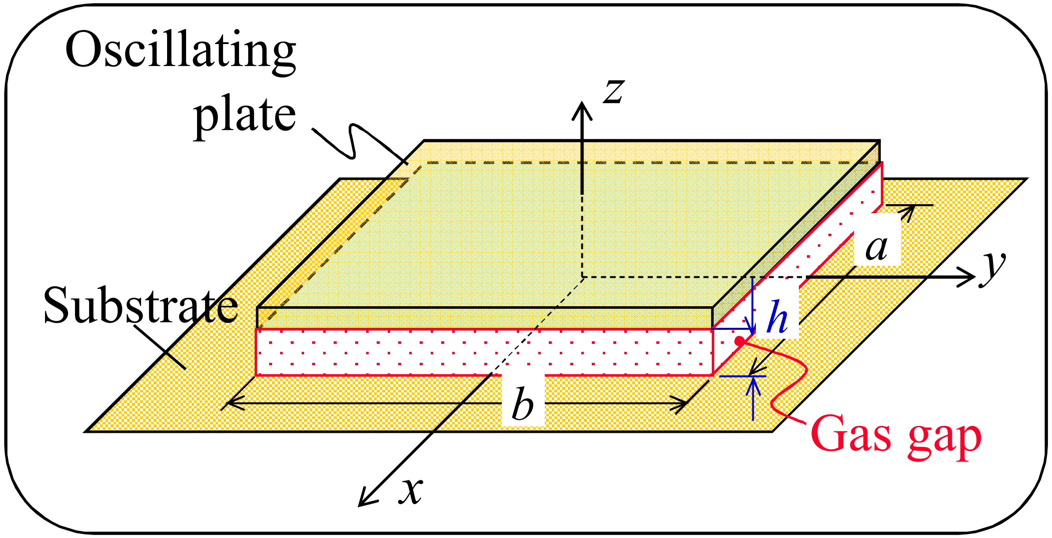
- In high-frequency or low-pressure regime, models for predicting squeezed-film damping give different results.
  - So which model to use?
  - Need experimental validation.

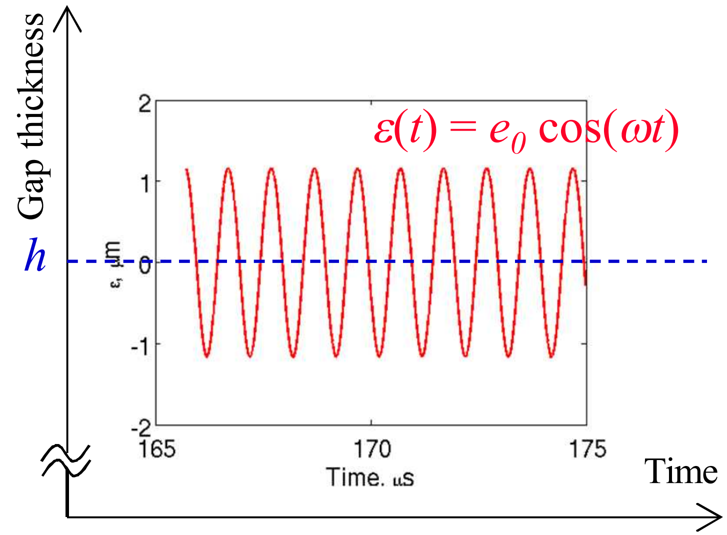
Objective:

- Provide experimental validation to widely used squeezed-film damping models for plates.

Squeezed Fluid Damps Oscillation.

Plate oscillates at fequency ω.





The squeezed fluid between the plate and the substrate creates damping forces on the plate.

Models Abound.
But Experimental Validation is Needed

- Forces on moving plate from gas layer can be obtained from the linearized **Reynolds equation**

$$\frac{Ph^2}{12\mu}\nabla^2\left(\frac{p}{P}\right)-\frac{\partial}{\partial t}\left(\frac{p}{P}\right)=\frac{\partial}{\partial t}\left(\frac{z}{h}\right)$$

P = ambient pressure, Pa
h = gap size, m
μ = viscosity, Pa s
p = pressure at (x,y), Pa
t = time, s

Assumptions:

1. Rigid plate
2. Small gap
3. Small displacement
4. Small pressure variation
5. Isothermal process
6. Pressure on edges = ambient pressure
7. Small molecular mean free path

- Models will be compared with measurement:

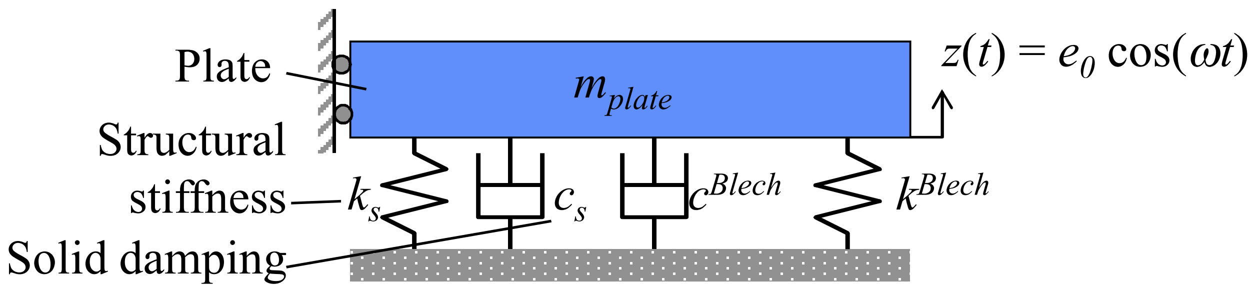
•Blech's model ← (8. Inertia of fluid neglected)

•Andrews et al.'s model ← (9. High-pressure limit of Blech's)

•Veijola's model ← ~~(10. Inertia~~ of fluid included)

Blech's Model Adds Damping And Stiffness to the Structure.

- Oscillating plate has stiffness and intrinsic (non-squeeze-film) damping.



$$m_{plate}\ddot{z} + c\dot{z} + kz = \tilde{f}$$

$$c_s + c^{Blech} \qquad k_s + j\omega\kappa^{Blech}$$

- Squeeze film causes an extra damping coefficient

- and an extra spring stiffness

e_0 = amplitude, m
m_{plate} = plate mass, kg
t = time, s
z = gap displacement, m
ω = frequency, rad/s

Blech, J.J., 1983, "On Isothermal Squeeze Films", *Journal of Lubrication Technology*, 105, p 615-620.

For Low-Squeeze Numbers, Blech's Model Reduces to Andrew's et al.'s Model.

- Blech's damping coefficient

$$c^{Blech}(\omega) = \frac{768}{\pi^6} \frac{a^3 b}{h^3} \mu \sum_{m,n \text{ odd}} \frac{m^2 + n^2 (a/b)^2}{m^2 n^2 \left[m^2 + n^2 (a/b)^2\right]^2 + \sigma^2 / \pi^4}$$

- Depends on the **squeeze number**

$$\sigma = 12\mu \left(\frac{a}{h_m}\right)^2 \left(\frac{\omega}{P}\right)$$

- For low squeeze numbers, $\sigma << \pi^2$

$$c^{Andrews} = 0.42(ab)^2 \mu / h^3$$

a	= plate width, m
b	= plate length, m
h	= gap height, m
P	= ambient pressure, Pa
μ	= viscosity, Pa s
ω	= frequency, rad/s

- Sample applicable range: $\mu = 1.82(10)^{-5}$Pa.s; $a = 144\mu$m; $h = 4.5\mu$m. in atmosphere P=9.3(10)^{4}Pa: $\sigma < 1$ for

$$\omega/(2\pi) < 70\text{kHz}$$

Andrews, M., Harris, I., Turner, G., 1993, "A comparison of squeeze-film theory with measurements on a microstructure", *Sensors and Actuators A*, 36, p 79-87.

Veijola's Model Accounts for Fluid Inertia.

- Taking into account the inertia of the gas flowing in and out of the gap, Veijola (2004) modified Reynolds equation into

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\mu} Q_{pr} \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\mu} Q_{pr} \frac{\partial p}{\partial y}\right) = \frac{\partial \rho h}{\partial t}$$

h = gap size, m
p = pressure at (x,y), Pa
t = time, s
μ = viscosity, Pa s
ρ = density, kg/m^3

- If the gap oscillation is $\varepsilon(t) = e_0 \exp(j\omega t)$, then the damping force complex amplitude is

$$F^{Veij}(\omega) = j\omega e_0 \Phi(\omega)$$

$$\Phi(\omega) = \sum_{m=1,3,\ldots}^{M} \sum_{n=1,3,\ldots}^{N} \frac{1}{Q_{pr} G_{mn} + j\omega C_{mn}}$$

$$C_{mn} = \frac{\pi^4 h (mn)^2}{64 ab n_\gamma P}$$

$$G_{mn} = \frac{\pi^6 h^3 (mn)^2}{768 \mu ab}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)$$

$$Q_{pr} = \sum_{k=1,3,\ldots} \frac{1 + 6K_s}{\frac{k^4 \pi^4}{96} + j\omega \frac{k^2 \pi^2 \rho h^2 (1 + 10K_s + 30K_s^2)}{96 \mu (1 + 6K_s)}}$$

Knudsen number
$K_s = 1.016\ \lambda/h$
λ = mean free path, m

a = width, m
b = length, m
e_0 = amplitude, m
j = $\sqrt{-1}$
n_γ = 1 for isothermal, (= c_p/c_v for adiabatic).
P = ambient pressure, Pa
μ = viscosity, Pa s
ω = frequency, rad/s
ρ = gas mass density, kg/m^3

Veijola, T., 2004, "Compact models for squeezed-film dampers with inertial and rarefied gas effects", *Journal of Micromechanics and Microengineering*, 14, p 1109-1118.

Gallis and Torczynski's Model Is a Direct Simulation Monte Carlo (DSMC) Method

Instead of the trivial boundary conditions at the plate edges, introduce

$$P - p = \eta G(\hat{\mathbf{n}} \cdot \nabla p) + \zeta \left(\frac{12\mu U}{G}\right)\left(1 + \chi \frac{6\Lambda}{G}\right)^{-1}$$

DSMC simulations were used to determine correlations for the gas-damping parameters

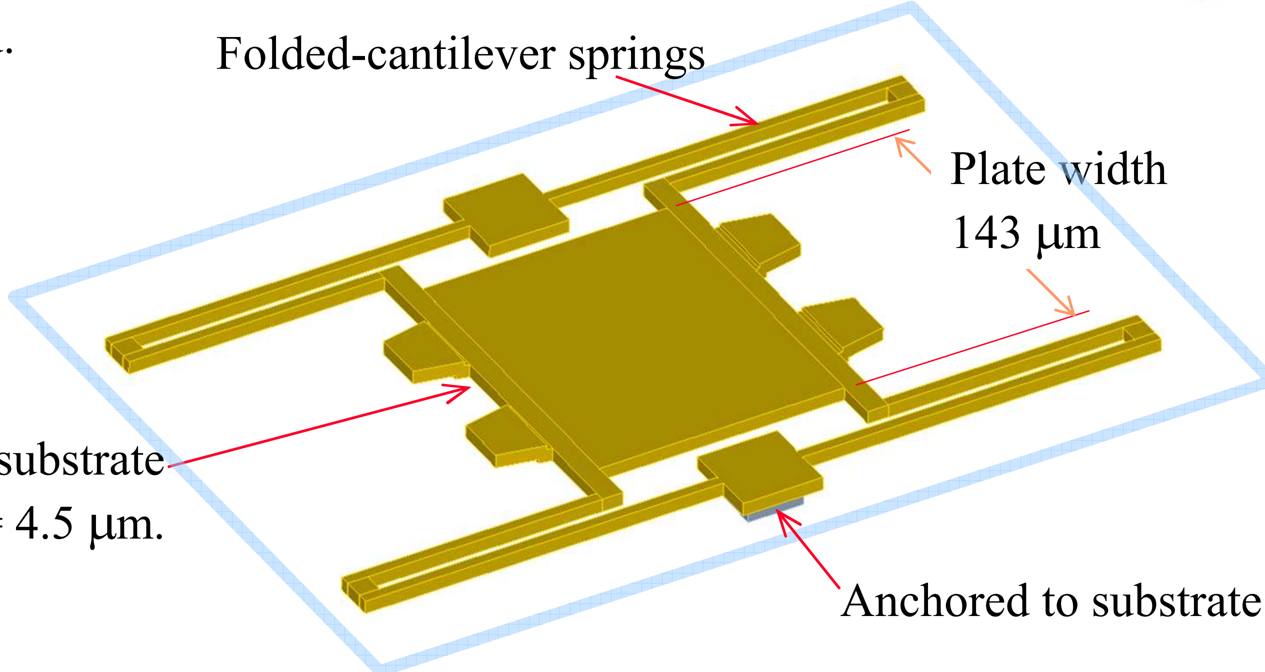
$$\eta = \frac{0.634 + 1.572(\Lambda/G)}{1 + 0.537(\Lambda/G)} \qquad \chi = \frac{1 + 8.834(\Lambda/G)}{1 + 5.118(\Lambda/G)} \qquad \zeta = \frac{0.445 + 11.20(\Lambda/G)}{1 + 5.510(\Lambda/G)}$$

G = gas film (gap) thickness. *Λ/G* is modified Knudsen number $\Lambda = \frac{2-\alpha}{\alpha}\lambda$ $\quad 0 \le \Lambda/G \le 1$

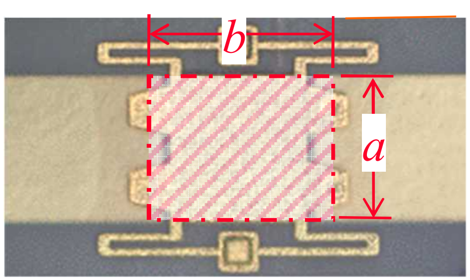
α = accommodation coefficient. (For this test device *a* = 1).

Measurement Is Done on Oscillating Plates

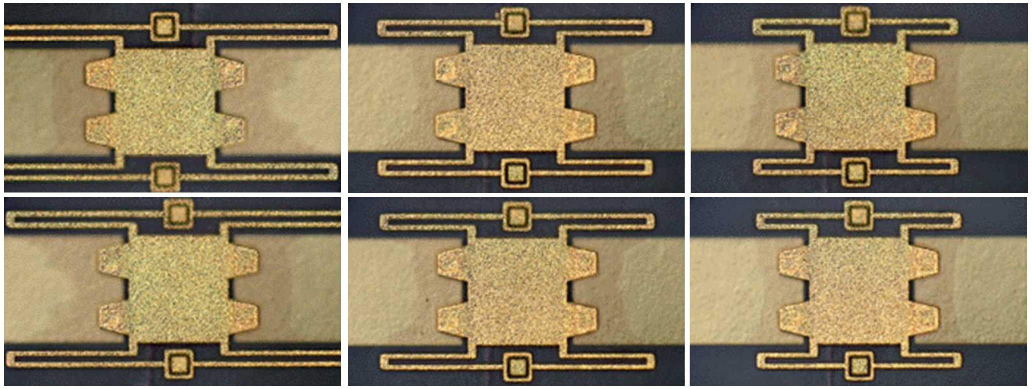
- Structure is electro-plated Au.
- Thickness around 5.7 μm.
- Substrate is alumina.



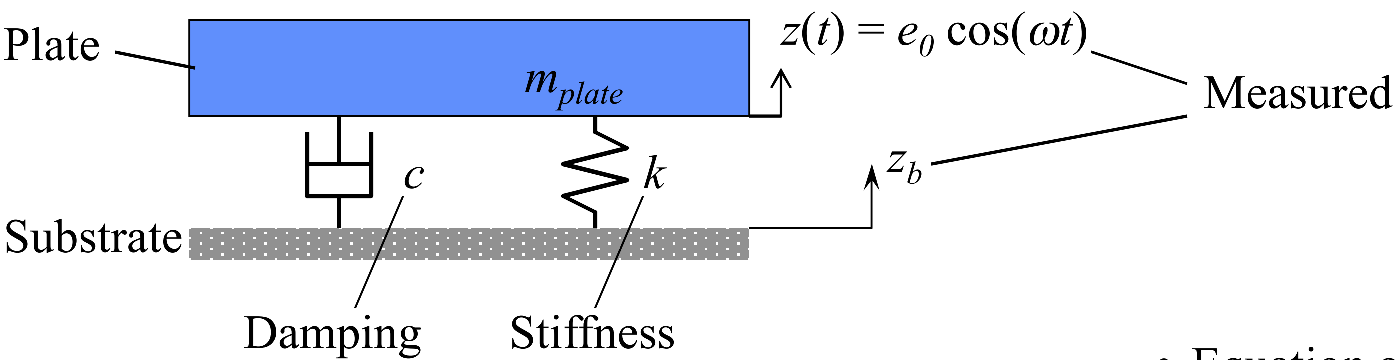
- Assumed width *a* and length *b*, where *ab* = true plate area.



- 2x3 array of plates



The Test Structure Can Be Modeled as SDOF.



- Equation of motion

$$m\ddot{z} = k\left(z_b - z\right) + c\left(\dot{z}_b - \dot{z}\right)$$

- Frequency response function (FRF) from base displacement to plate displacement:

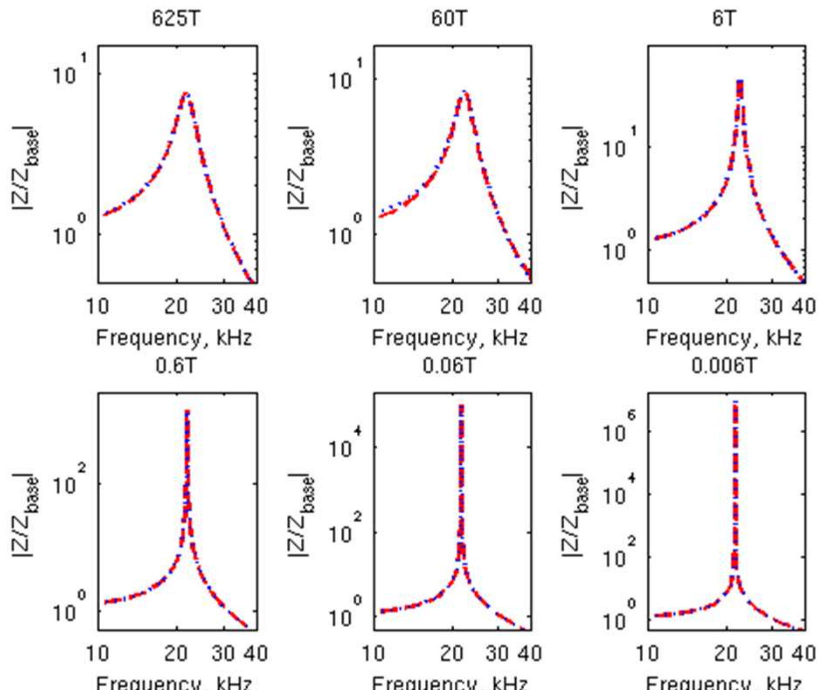
$$\frac{Z(\omega)}{Z_b(\omega)} = \frac{m\omega^2}{-m\omega^2 + j\omega\left(c_s + c_{gas}\right) + \left(k_s + k_{gas}\right)} + 1$$

non-squeeze-film (c_s) · squeeze-film (c_{gas})

damping

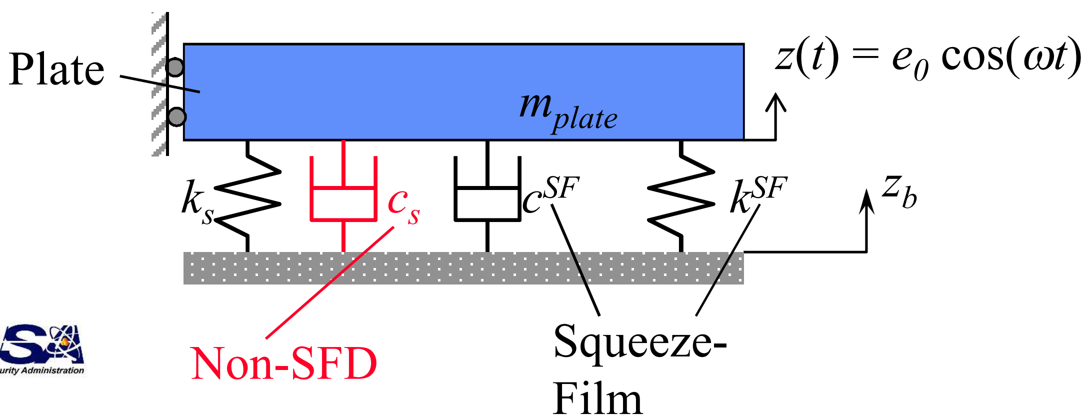
Prediction Shows Squeeze Film Causes High Damping

- Blech's model with assumed structural parameters.
- Assume structural/solid damping = 0.



- In atmospheric air, $Q \sim 10$.
- At 10^{-4} atm., $Q \sim 10^7$.

- Damping *measured* in vacuum is caused by non-squeeze-film damping (Non-SFD).



$$m_{plate}\ddot{z} + c\dot{z} + kz = \tilde{f}$$

$$c = c_s + c^{SF}, \quad k = k_s + k^{SF}$$

Need to Compare Computed Damping Factor c with Measured Damping Ratio ζ.

- Models predict ***damping factor*** c in the equation of motion

$$m_{plate}\ddot{z} + c\dot{z} + kz = f_{ext}(t)$$

- Measurement method gives ***damping ratio*** ζ in the equation of motion

$$\ddot{z} + 2\zeta\omega_n\dot{z} + \omega_n^{\ 2} z = \hat{f}_{ext}(t)$$

- To compare prediction with measurement, use the relationship between c and ζ

  - For Blech's model,

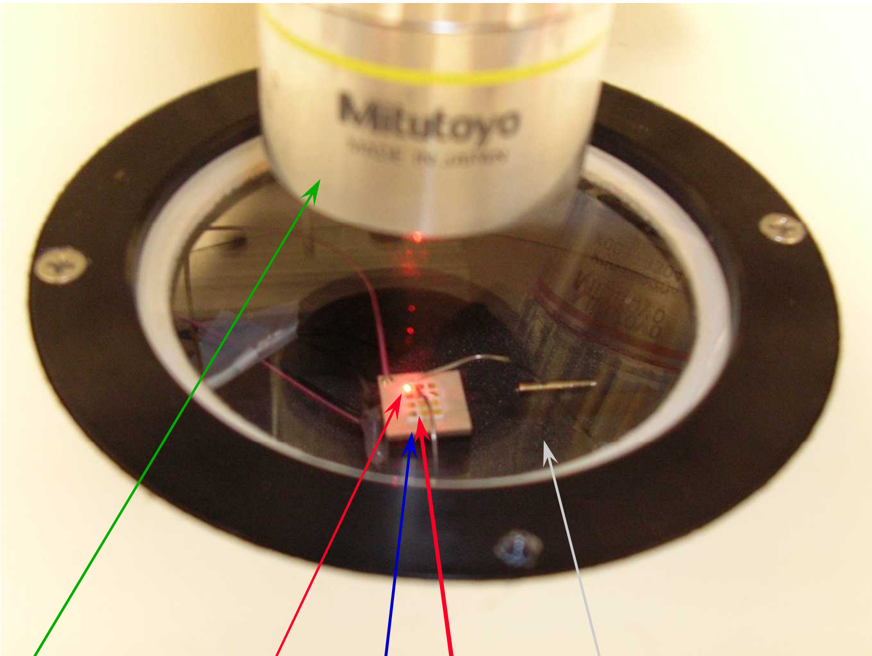
$$\zeta_{Blech} = \frac{c_{Blech}}{2m_{plate}\omega_n}$$

  - For Veijola's model,

$$\zeta_{Veij} = \mathrm{Re}\left(\frac{F_{Veij}}{j\omega_d e_0 2m_{plate}\omega_n}\right)$$

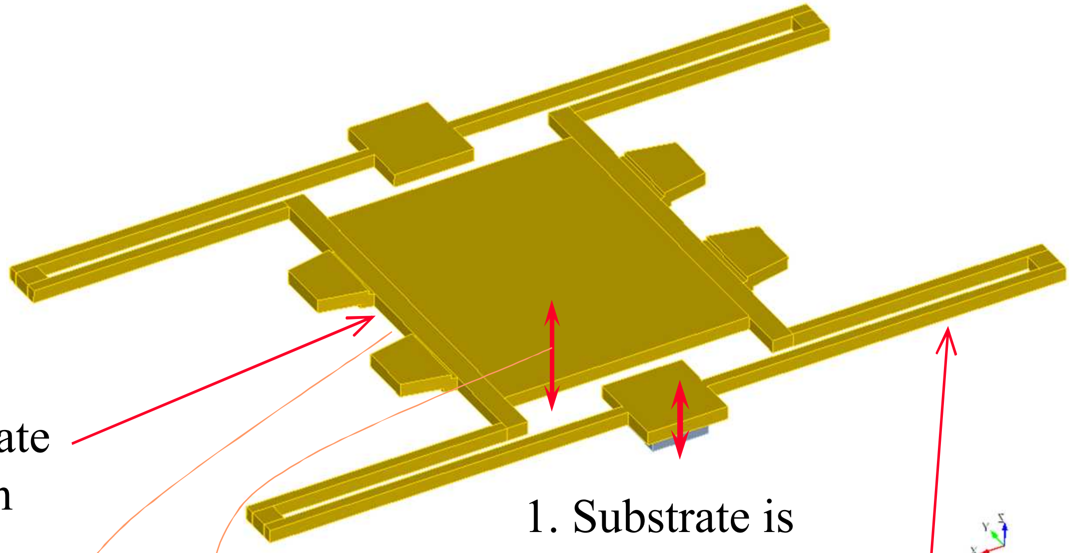
e_0 = amplitude, m
h_{plate} = plate thickness, m
j = √-1
m_{plate} = plate mass, kg
ω_n = natural frequency, rad/s

Measurement Uses Laser and Vacuum.



- Excitation: Base displacement with piezoelectric actuator.
- Sensing: Scanning Laser Doppler Vibrometer.

Oscillating Plate Is Shaken through Its Support.



Air gap between plate and substrate. Mean thickness = 4.5 µm.

1. Substrate is shaken up and down.

2. Plate moves up and down.

3. Springs flex.

4. Air gap is compressed and expanded by plate oscillation.

- Frequency response function (FRF) from base displacement to plate displacement:

$$\frac{Z(\omega)}{Z_b(\omega)} = \frac{m\omega^2}{-m\omega^2 + j\omega(c_s + c_{gas}) + (k_s + k_{gas})} + 1$$

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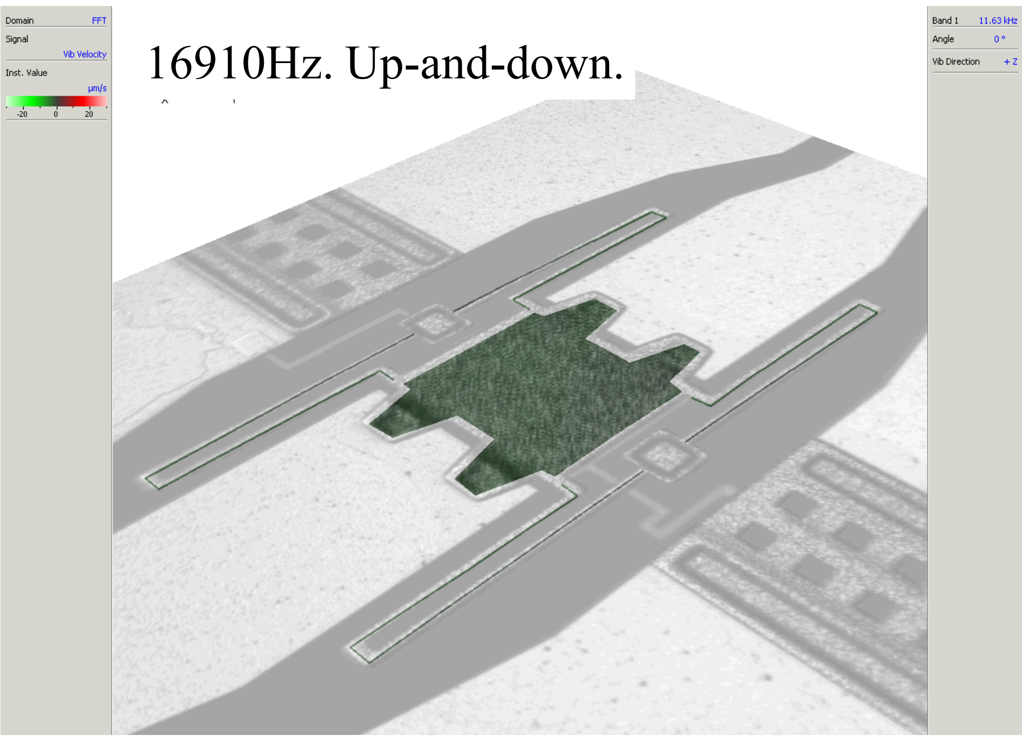
Experimental Modal Analysis Gives Natural Frequency and Damping.

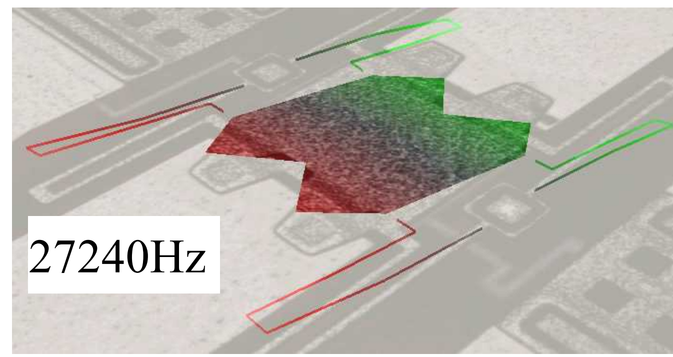
- Modal analysis was done in 2k-200 kHz.
- Tests were repeated at different air pressures from atmospheric (640 Torr) to near-vacuum (<1 milliTorr).

Measured deflection shape, first mode.

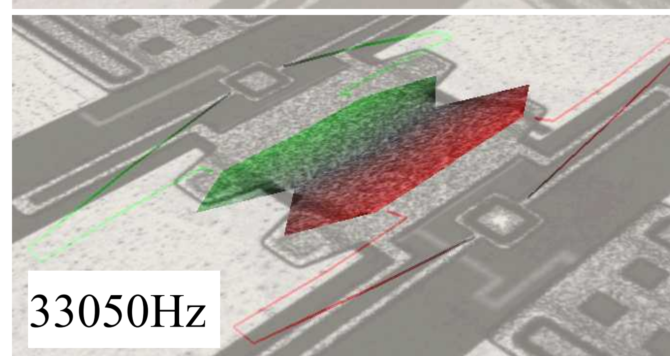


16910Hz. Up-and-down.

Higher modes are not considered.

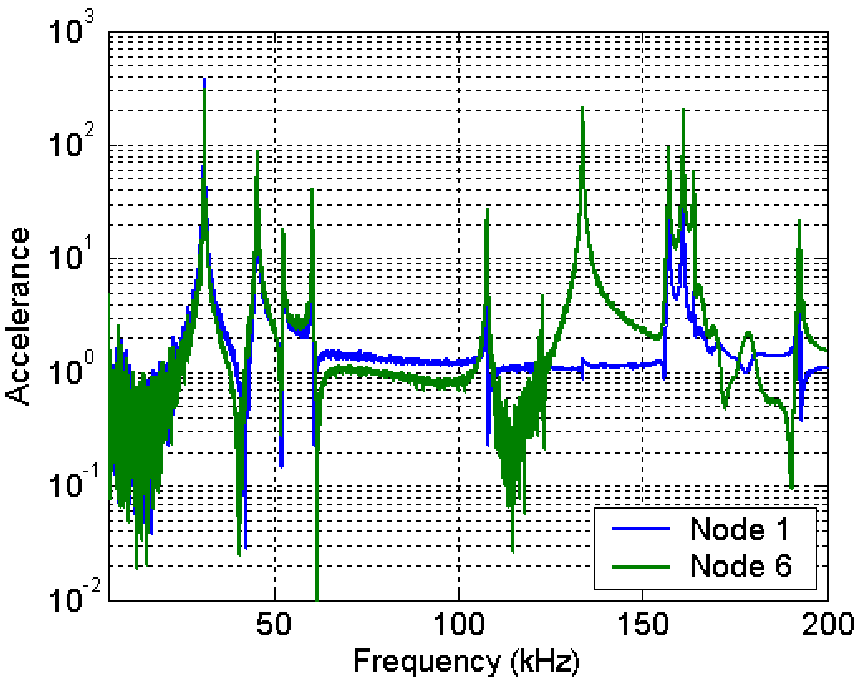


27240Hz

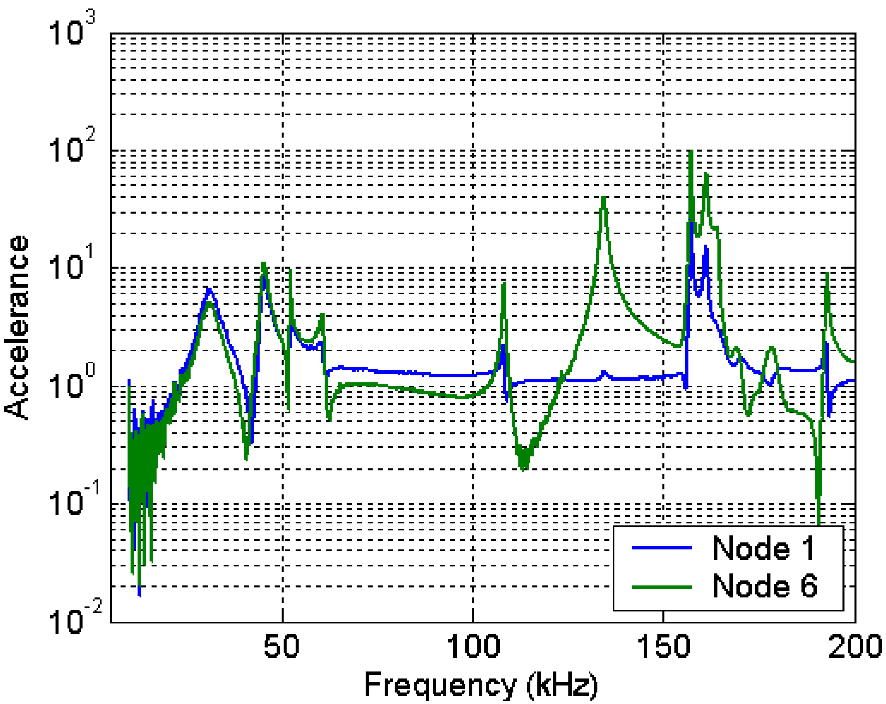


33050Hz

Atmospheric Pressure Suppresses Plate Oscillation.



Near vacuum (0.06 Torr)



Atmospheric (625 Torr)

- From Frequency Response Functions (FRFs), a commercial modal analysis program computed the natural frequencies, mode shapes, and damping ratios.
- Only the lowest-frequency mode will be discussed here.
- Atmospheric air damped the first resonant response by two orders of magnitude.

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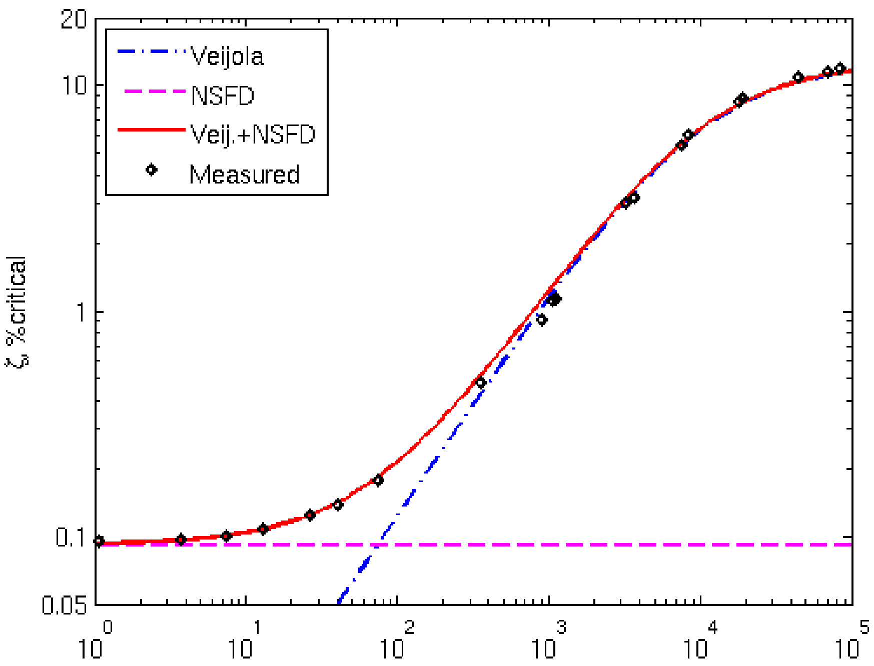
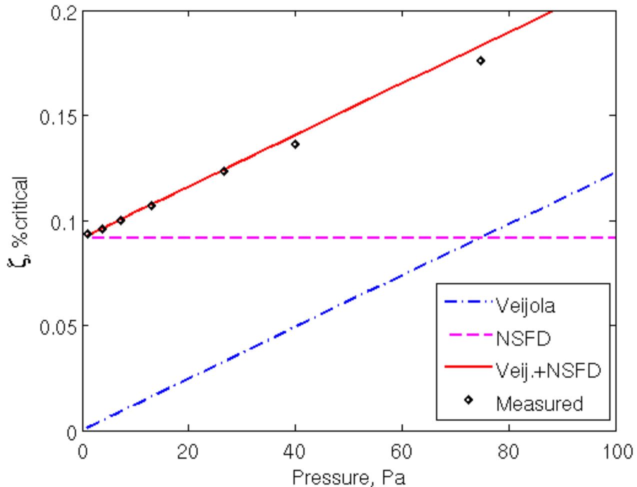
Non-Squeeze-Film Damping is Estimated from Asymptote.

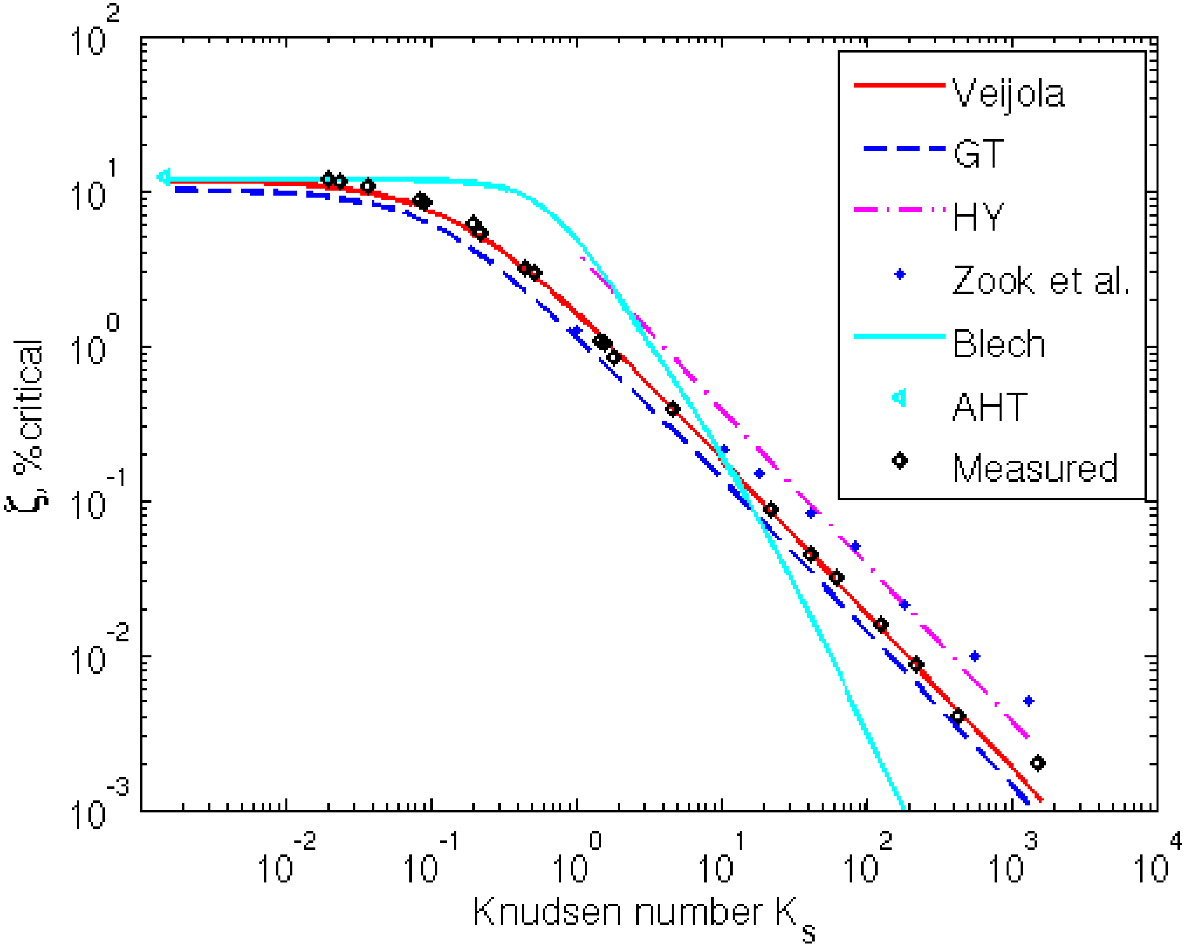




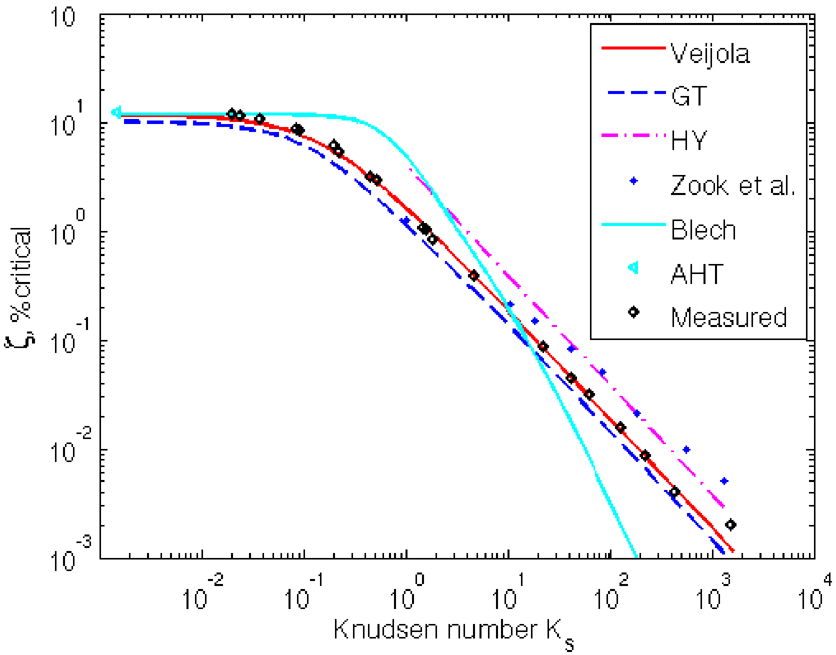
At low pressures, Non-Squeeze-Film Damping (NSFD) is the dominant damping.

To obtain squeeze-film damping from measured total damping, NSFD is subtracted out.

Most Models Predictions Differ from Measured Data



Conclusions:



- On rigid plates with width ~150 µm, oscillating around 4.5 µm above the substrate, squeezed air film can cause large damping.
- Continuum models such as Blech's model, Andrews et al.'s model, and Veijola's model are not less accurate than molecular models.
- For the conditions tested here, in atmospheric air the simplest model mentioned by Andrews et al. is as good as any more sophisticated models.
- In the high squeeze number regime (low pressures or high frequencies), the Veijola model is shown to match experimental data accurately.

Acknowledgment

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- David Epp for some of the modal analysis.
- Chris Dyck and Bill Cowan's team for providing the test structures.
- Carl Diegert for the confocal microscope photographs.
- Jim Redmond and Steve Kempka for technical guidance and programmatic support.

Questions ??

Thank you!

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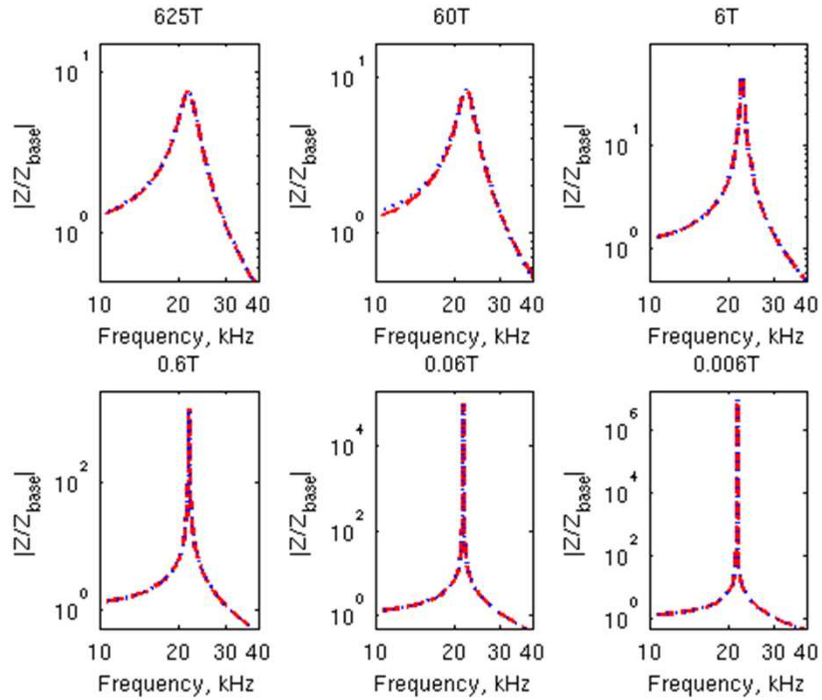
Backup Slides

Which Frequency to use for Predicting Damping Ratio

- Models predict damping factor *c* as function of frequency.
- Which frequency should be used?
- Try 1 and 2 below

1. Curve-fit frequency response function to obtain a damping ratio ζ.



2. Use Resonant frequency in the models.

- 1 and 2 turns out to give practically the same damping factor.
- 2 requires less work.

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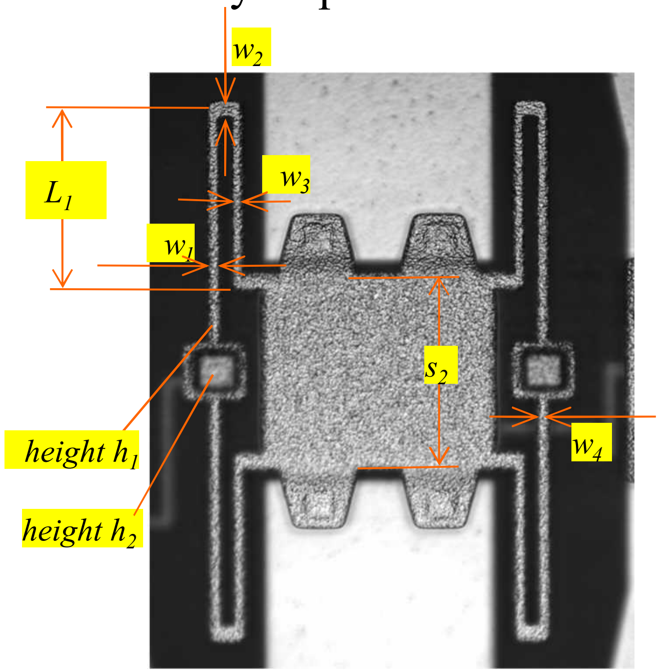
Measurement Was Repeated on Array of Plates.

- Every structure under test was different.
  - by design
  - also due to fabrication variations.

- Dimensions of the 2x3 array of plates were measured with interferometry.



w2	14.6	13.6	13.6
w3	9.7	8.8	8.8
w4	8.8	9.7	9.7
L1	225.1	175.4	126.7
s1	0	0	0
s2	143.2	144.2	144.2
h1	5.7	5.7	5.7
h2	10.4	10.3	10.2

w1	9.7	10.7	10.7
w2	14.6	13.6	13.6
w3	8.8	9.7	8.8
w4	9.7	9.7	9.7
L1	226.1	175.4	126.7
s1	0	0	0
s2	144.2	144.2	144.2
h1	5.7	5.7	5.7
h2	10.4	10.6	10

- Squeezed-film damping ratios were predicted using measured dimensions, measured natural frequency.

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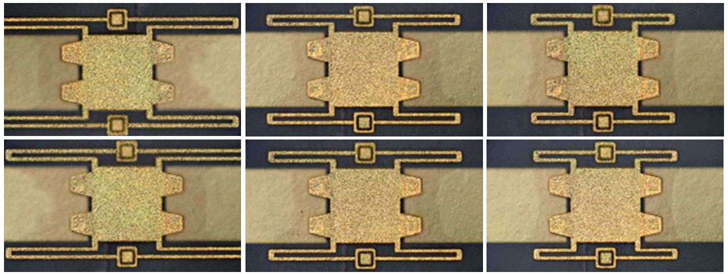
Measurement Gives Natural Frequency and Damping.

Numbers in tables correspond to position in array

Des. and Fab. by Chris Dyck, SNL
Photograph by Carl Diegert, SNL



Natural frequency, Hz

10509	17350	21721
12003	18507	29071

- Shorter springs result in higher natural frequencies, as expected.
- The two rows were significantly different.
  - Fabrication variation
  - Much more common in MEMS than in macro world

Damping ratios ζ, % of critical

Atm P=625Torr		
10.05	7.61	5.35
10.36	7.72	5.93

P = 60 Torr		
5.58	3.57	2.26
4.93	3.56	2.58

P = 6 Torr		
1.03	0.59	0.49
1	0.79	0.41

P = 60 mTorr		
0.49	0.12	0.2
0.3	0.34	0.09

P = 6 mTorr		
0.54	0.07	0.1
0.12	0.28	0.09

- Lower pressure results in lower damping, as expected.
- Curve-fitting was not reliable at 6 mT.
  - $\zeta < 0.1\%$.
  - Window was needed to reduce leakage, but distorted damping.

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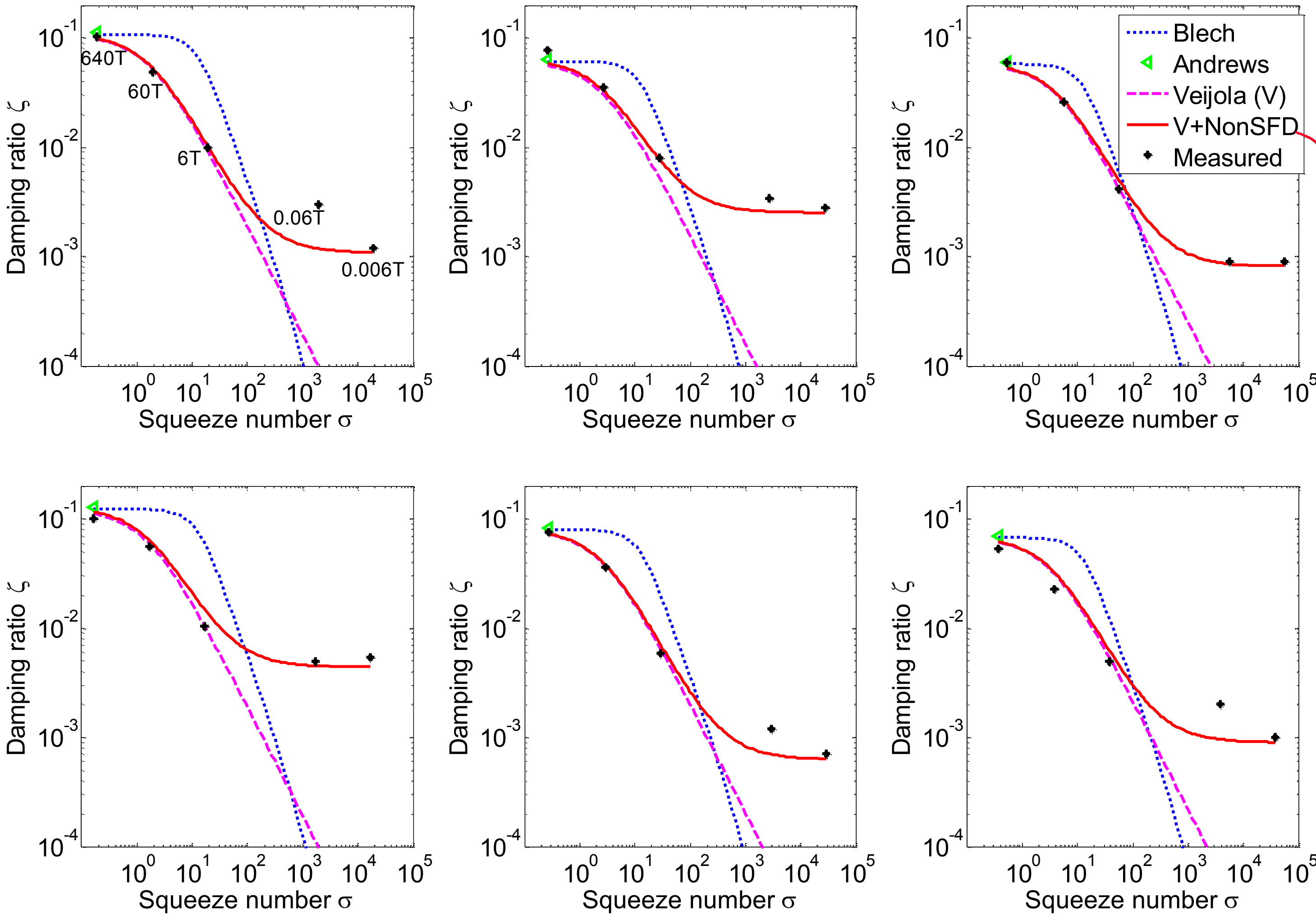
Some Model Predictions Agree with Measurement.



Squeeze number

$$\sigma = 12\mu\left(\frac{a}{h}\right)^2\left(\frac{\omega}{P}\right)$$

- Non-squeeze-film damping accounts for solid structural and other unknown damping.
- At high pressure, nonSFD does not contribute much.
- At near-vacuum:
  - Gas damping is negligible.
  - Assume 90% of damping is nonSFD.