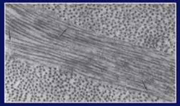
Modeling the Anisotropic Viscoelastic Behavior of the Cornea

SAND2007-2351P

Presented to the Engineering Sciences External Review Panel
Sandia National Laboratories, April 16-17, 2007

Thao D. Nguyen, Reese E. Jones, Mechanics of Materials, Physical and Engineering Sciences, Livermore, CA
Brad L. Boyce, J. Mark Grazier, Microsystem Materials, Materials Performance and Aging, Albuquerque, NM



Objective:

- Develop new Sandia capability for combined in-vitro experimentation and modeling for nonlinear anisotropic viscoelastic behavior of hydrated soft biological materials.
- Develop a physics-based predictive model of the microstructural underpinnings of the mechanical and structural behavior of the cornea.

Overview

Approach:

- Conduct uniaxial tensile strip tests (creep and cyclic loading) to characterize viscoelastic behavior.
- Construct anisotropic viscoelastic model of the cornea based on deformation mechanisms of matrix/fibrils.
- Develop inflation tests of intact cornea with digital image correlation to characterize physiological anisotropic viscoelasticity.
- Develop finite element model of pressurization experiments to post process experimental data and obtain physiological properties.

Applications:

- Biomedical: Predictive models for the mechanical behavior of the cornea for development of improved diagnostics, surgical design tools, and corneal prosthetics.
- Advance Materials: Understanding the structural underpinnings of corneal mechanical behavior for design of bio-mimetic materials with enhanced lexibility, strength/weight ratio, and energy dissipation.

Team Members:
8776: RJones (PI), T. Nguyen; 1824: B. Boyce,J . Grazier
Collaborators: UCSF (Dept. of Ophth.):
S. McLeod (MD); B. Sands (MD)

Constitutive Relations

Homogenizes stress response of stroma: $\mathrm{S}_s = 2\frac{\partial W_s}{\partial \mathbf{C}}$

$$\boldsymbol{\sigma}_s = \frac{\mu}{\sqrt{I_3}}\left(\mathbf{b} - I_3^{-\gamma}\mathbf{1}\right) + \frac{1}{2\pi\sqrt{I_3}}\int_{-\pi}^{-\pi} 2\frac{\partial W_f^{eq}}{\partial \lambda}\lambda^2 \mathbf{m} D(\theta,\mathbf{X})\, d\theta + \sum_k^N \frac{1}{2\pi\sqrt{I_3}}\int_{-\pi}^{-\pi} 2\frac{\partial W_{f_k}^{neq}}{\partial \lambda_k^e}\lambda_k^{e^2}\mathbf{m} D(\theta,\mathbf{X})\, d\theta$$

Viscous flow rule for lamellae

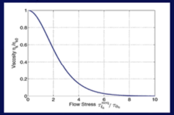
$$\frac{\dot{\lambda}_k^v}{\lambda_k^v} = \frac{1}{\eta_{f_k}}\tau_{f_k}^{neq}$$

Phenomenological lamellar viscosity model

$$\eta_{f_k}\left(\tau_{f_k}^{neq}\right) = \eta_{0_k}\left[\frac{\tau_{0_k}}{\tau_{f_k}^{neq}}\sinh\left(\frac{\tau_{f_k}^{neq}}{\tau_{0_k}}\right)\right]^{-1}$$

reference value
activation stress

Non-equilibrium lamellar stress

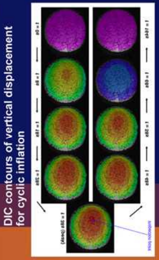


Homogenized flow rule for stroma

$$\boldsymbol{\sigma}_s^{neq} = \sum_k^N \frac{1}{\sqrt{I_3}}\underbrace{\left[\frac{1}{2\pi}\int_{-\pi}^{-\pi}\eta_{f_k}\lambda_k^{e^2}\mathbf{m}\otimes\mathbf{m}D(\theta,\mathbf{X})\,d\theta\right]}_{\eta_s \text{ Homogenized viscosity tensor}} : \mathbf{b}_k^{e^{-1}}\left(-\frac{1}{2}\mathcal{L}_v\mathbf{b}_k^{e^{-1}}\right)\mathbf{b}_k^{e^{-1}}$$

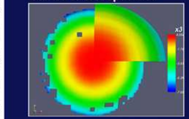
Digital Image Correlation of Bulge Experiments

Inflation/bulge testing of intact cornea

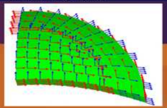


DIC contours of vertical displacement for cyclic inflation

Fit of finite element model of cornea to test specimen



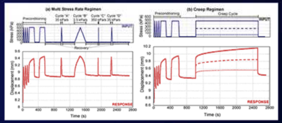
Model of distribution of lamellar orientation



Cornea Tensile Strip Tests

B.L. Boyce, R. E. Jones, T. D. Nguyen, "Stress-controlled viscoelastic tensile response of bovine cornea", accepted to Journal of Biomechanics, 2007.

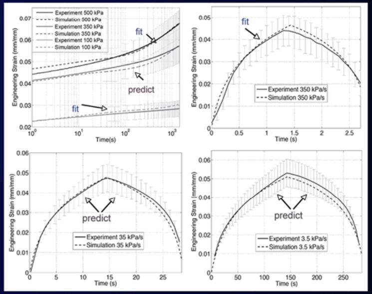
- Bovine corneas obtained from Sierra Medical.
- Corneas harvested within 24 hours, tested in 48 hours of slaughter
- Tensile strips 7.0 mm x 1.25 mm cut from corneas with metal dye
- Strips extracted along Nasal-Temporal (NT) and Inferior-Superior (IS) meridians.
- Tests conducted in Cytosol Opthalmic Balanced Saline Solution at 30C.
- Cornea strip fixtured using pneumatic compression grips.
- Tests conducted using MTS servohydraulic machine under stress control to accommodate material remodeling during preconditioning.



Loading Regimen

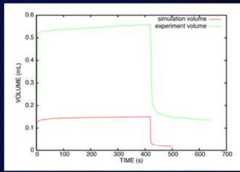
- Developed for repeatability.
- Maximized dwell time for full recovery.
- Same precondition used for all load levels.

Cyclic Uniaxial Tension Simulation

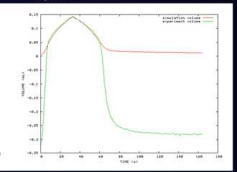


Prelimary Comparing Modeling and Simulations

Creep Experiment 8 kPa (60 mmHg)



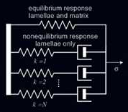
Cyclic Inflation 0.25 kPa/s Comparing Experiments and FEM

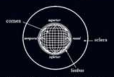


- Currently, the FEM model of the cornea is too stiff, possibly because constitutive model developed from tension test did not account for the crimping of the collagen fibrils of the unloaded cornea.

* Refine the experiments to include a preload at intra-occular pressure.

Rheological Model





Lamellar orientation vector:
$\mathbf{P} = (\cos\theta, \sin\theta)$, $\mathbf{M} = \mathbf{P}\otimes\mathbf{P}$
Lamellar density function: $D(\theta, X)$

Kinematics:
$\mathbf{F} = \mathbf{F}_k^e\mathbf{F}_k^v$

- Affine deformation assumptions

$$\lambda\mathbf{p} = \mathbf{F}\mathbf{P} \longrightarrow \lambda = \sqrt{\mathbf{C}:\mathbf{M}}$$

–Mapping to intermediate configuration

$$\lambda_k^v\tilde{\mathbf{P}}_k = \mathbf{F}_k^v\mathbf{P} \quad \lambda_k^v = \sqrt{\mathbf{C}_k^v:\mathbf{M}} \quad \lambda_k^e = \sqrt{\mathbf{C}_k^e:\tilde{\mathbf{M}}_k}$$

Free energy density of stroma

$$W_s\left(\mathbf{C}, \mathbf{C}_k^e, \tilde{\mathbf{M}}_k\right) = W_m(\mathbf{C}) + \overline{W}_f\left(\mathbf{C}, \mathbf{C}_k^e, \tilde{\mathbf{M}}_k\right)$$

- Isotropic contribution of matrix: $W_m(\mathbf{C}) = \frac{\mu}{2}(I_1 - 3) + \frac{\mu}{2\gamma}\left(I_3^{-\gamma} - 1\right)$
- Anisotropic contribution of lamellae: $\overline{W}_f\left(\mathbf{C}, \mathbf{C}_k^e, \tilde{\mathbf{M}}_k\right) = \frac{1}{2\pi}\int_{-\pi}^{-\pi} W_f\left(\lambda(\theta), \lambda_k^e(\theta)\right) D(\theta, \mathbf{X})\, d\theta$

Free energy density of lamella

$$W_f\left(\lambda, \lambda_k^e\right) = W_f^{eq}(\lambda) + \sum_i^N W_{f_k}^{neq}\left(\lambda_k^e\right)$$

- Phenomenological potential for lamellae:

$$W_f^{eq}(\lambda) = \alpha^{eq}\left(\exp\left[\beta\left(\lambda^2 - 1\right)\right] + \frac{\beta}{\lambda^2}\right)$$

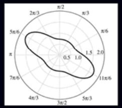
$$W_{f_k}^{neq}\left(\lambda_k^e\right) = \alpha_k^{neq}\left(\exp\left[\beta\left(\lambda_k^{e^2} - 1\right)\right] + \frac{\beta}{\lambda_k^{e^2}}\right)$$

Probing Anisotropic Viscoelastic Response

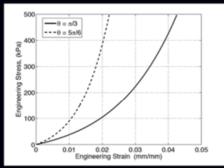
Density distribution function for limbus

$$D(\theta) = \sin^8\left(\theta - \frac{4\pi}{3}\right) + 0.727$$

(Pinsky et. al 2005, Aghamohammadzadeh et. al. 2002)

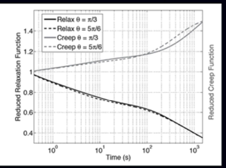


Short-time Stress Response



Preferred orientation (higher lamellar density) significantly stiffer.

Creep/Relaxation Response



Time dependent behavior is orientation dependent!

Conclusion/Future Work

- Developed anisotropic viscoelastic model for the tensile behavior of cornea based on the collagen fibril architecture.
  - Applies multiple nonlinear stress-activated non-equilibrium processes.
  - Anisotropic internal variable formulation specifies flow rule at lamellar level not at tissue level.
  - Properties of stroma (composite level) related to properties of lamellae (fiber level) and lamellar distribution.
  - Consistent homogenization scheme used for effective moduli and viscosities.
  - Model can be applied to other fiber-reinforced soft tissues and composites.
  - Can be modified for viscoplasticity
- Very good agreement with results of uniaxial tensile and cyclic tests.
- Developing digital-image-correlation inflation experiments to evaluate physiological anisotropic viscoelastic behavior of the cornea
- Developing finite element model of the inflation experiments to post-process inflation experiments and obtain physiological properties.

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract No. DE-AC04-94AL85000.