

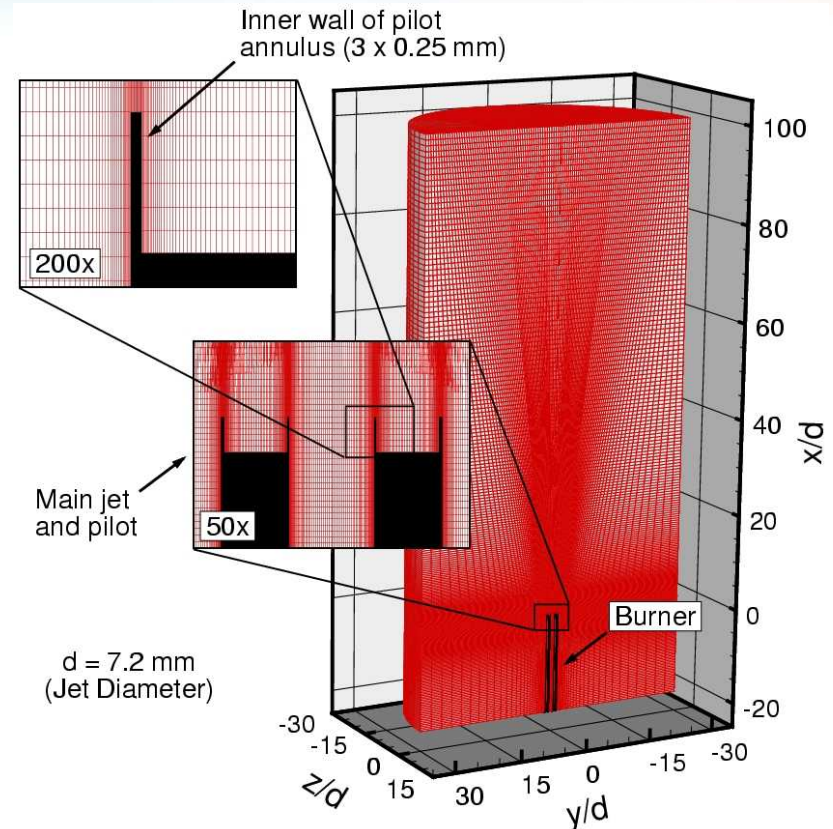
Quality Optimization of Staggered Adaptive Grid Systems in Generalized Coordinates

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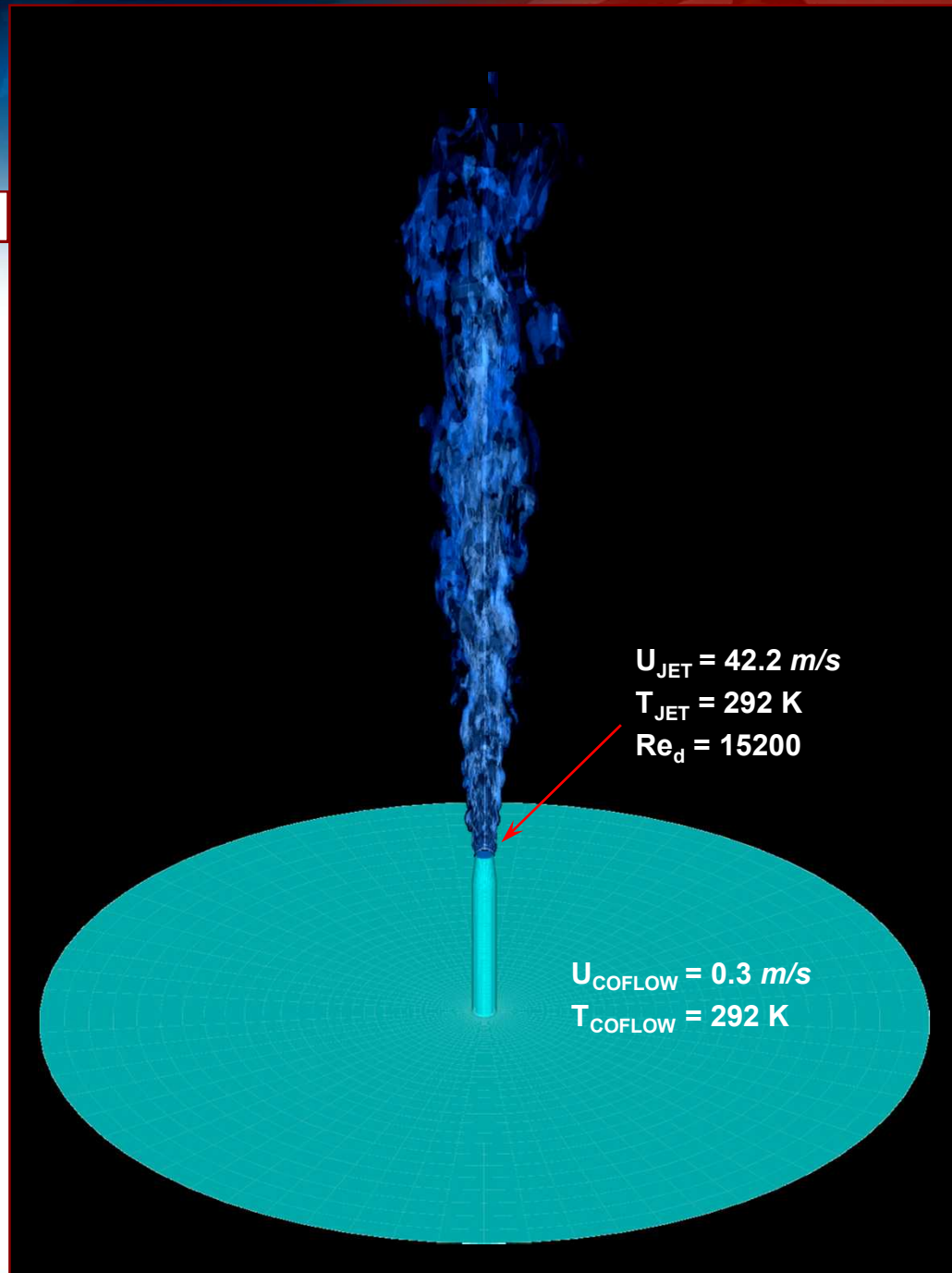
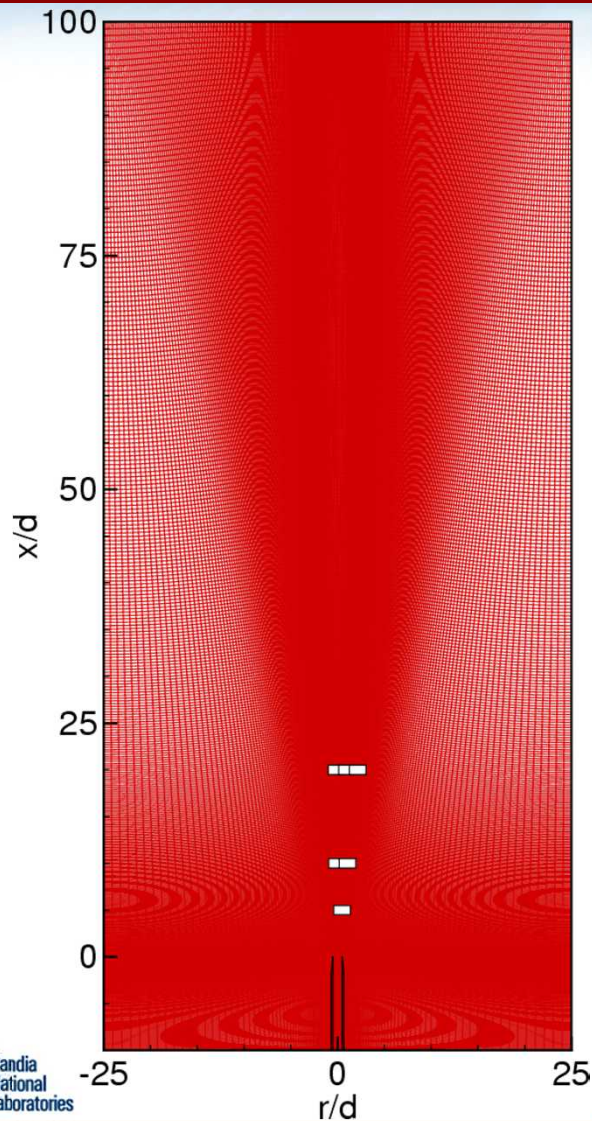
Problem

- **Grid quality affects the accuracy of CFD calculations**
 - Distortion of cells must be minimized
 - Continuity between grid metrics must be maintained
 - Stringent algorithmic properties associated with grid enforced
- **Application of LES (for example) requires use of accurate techniques that represent wide ranges of scales**
 - Generalized (body-fitted) coordinates to handle complex geometry
 - Optimal grid refinement strategies to minimize CPU time
 - Staggered grid system to maintain algorithmic quality
- **When this combination is used, issues related to curvature and alignment of cell centroids must be considered**

***This project focuses on
incorporating curvature effects in generalized coordinates
to optimize the quality of a staggered grid system***

CH₄/H₂/N₂ Flames

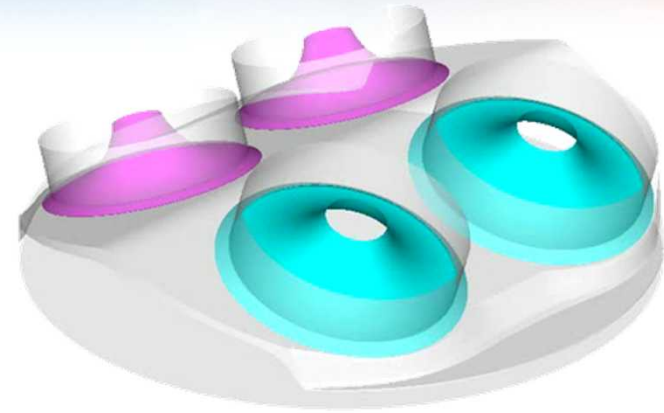
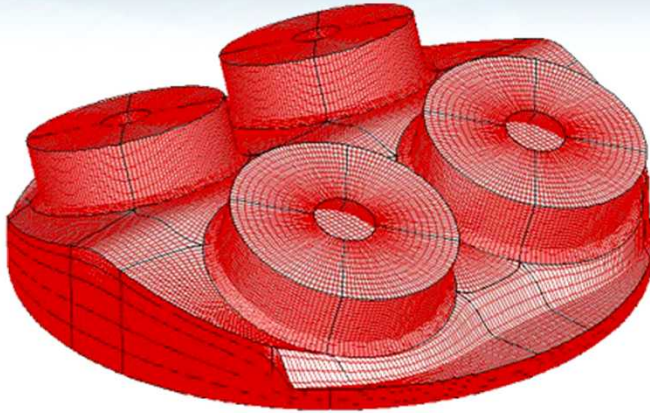
Cross-section of 6-million cell 3D curvilinear grid (80cm x 40cm)



Multiblock Grid Topology for Optical IC-Engine

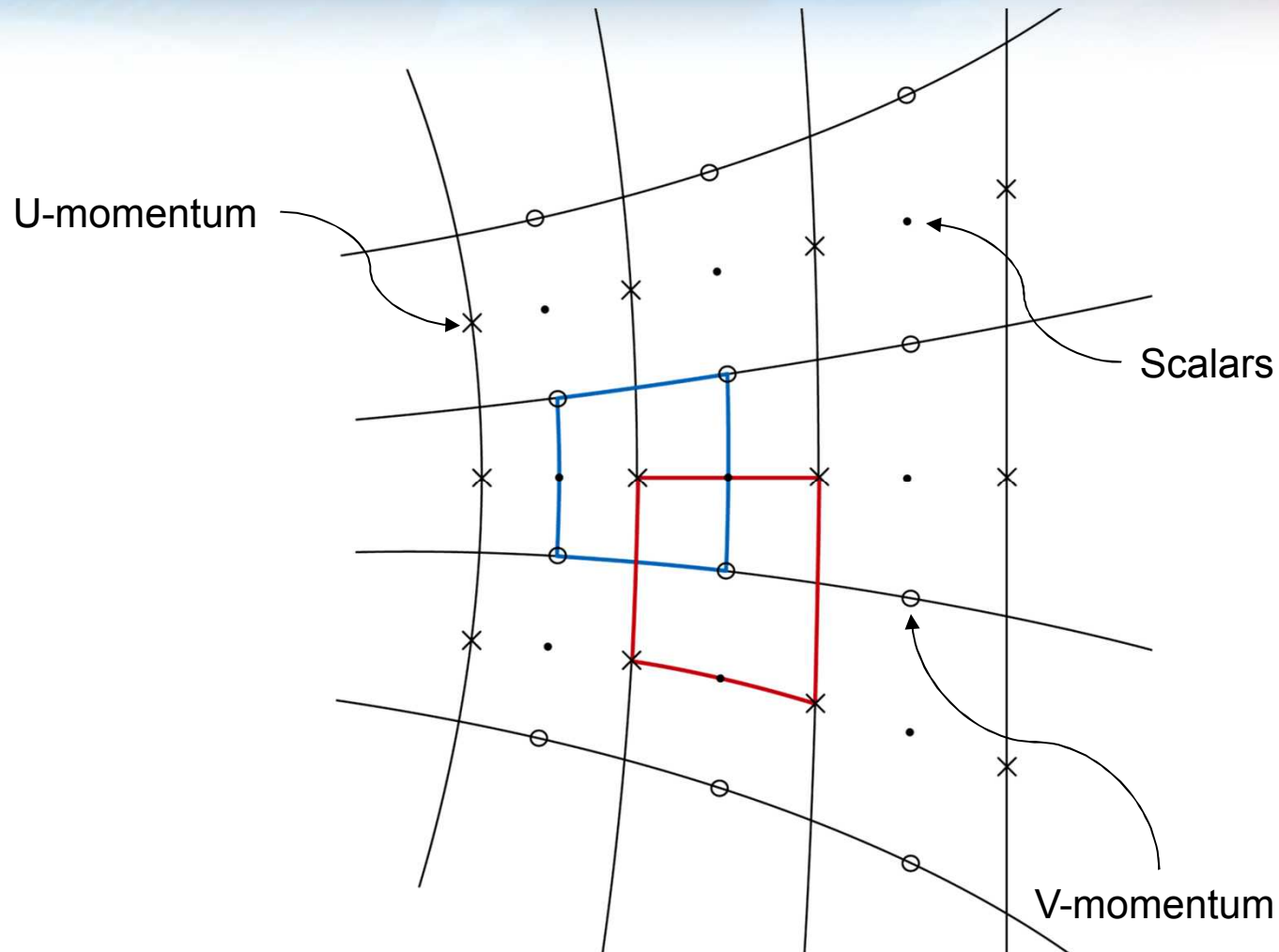
H₂ICE Engine Configuration:

- Baseline: 3-million cells, 137 blocks
- Production: $O(10)$ -million cells
- Code now interfaced with ANSYS ICEM

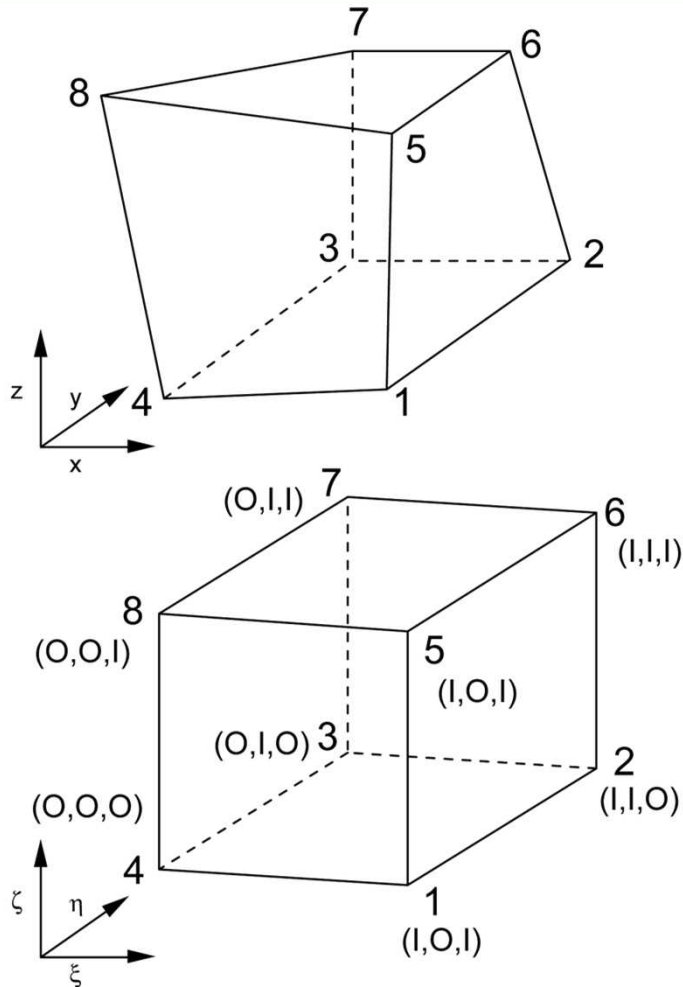


Oefelein, J. C., Sankaran, V. and Drozda, T. G. (2008). Large eddy simulation of in-cylinder combustion processes in IC-engines, *Paper 08PFL-736*, SAE World Congress, April 14-17, Detroit, Michigan, USA.

What is a Staggered Grid



Generalized Curvilinear Mapping

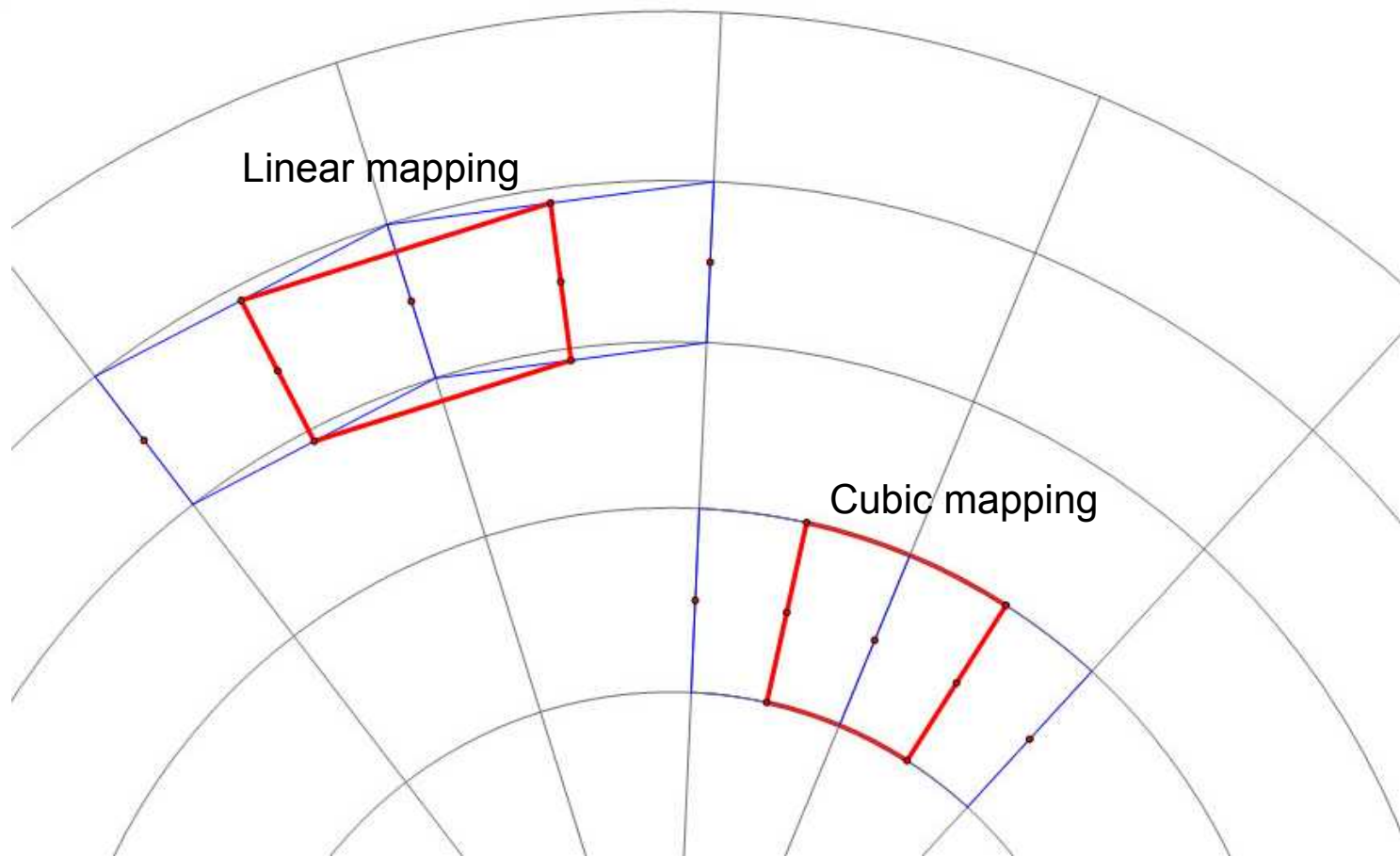


$$\begin{aligned}\bar{x} = & [\bar{x}_1\xi(1-\eta) + \bar{x}_2\xi\eta \\ & + \bar{x}_3(1-\xi)\eta + \bar{x}_4(1-\xi)(1-\eta)](1-\zeta) \\ & + [\bar{x}_5\xi(1-\eta) + \bar{x}_6\xi\eta \\ & + \bar{x}_7(1-\xi)\eta + \bar{x}_8(1-\xi)(1-\eta)]\zeta\end{aligned}$$

$$V = \int_0^1 \int_0^1 \int_0^1 \frac{\partial(x, y, z)}{\partial(\xi, \eta, \zeta)} d\xi d\eta d\zeta$$

$$A = \int_0^1 \int_0^1 \frac{\partial(x, y)}{\partial(\xi, \eta)} d\xi d\eta$$

Linear Mapping Can Lead To Errors Due to Voids and Misalignment of Centroids



Solution ... Derive Tri-Cubic Mapping Function in Generalized Coordinates

$$\bar{x} = \sum_{i=1}^4 \sum_{j=1}^4 \sum_{k=1}^4 c_{ijk} \xi^{i-1} \eta^{j-1} \zeta^{k-1}$$

$$\frac{\partial \bar{x}}{\partial \xi} = \sum_{i=1}^4 \sum_{j=1}^4 \sum_{k=1}^4 (i-1) c_{ijk} \xi^{i-2} \eta^{j-1} \zeta^{k-1}$$

$$\frac{\partial \bar{x}}{\partial \eta} = \sum_{i=1}^4 \sum_{j=1}^4 \sum_{k=1}^4 (j-1) c_{ijk} \xi^{i-1} \eta^{j-2} \zeta^{k-1}$$

⋮

$$\frac{\partial^2 \bar{x}}{\partial \xi \partial \eta} = \sum_{i=1}^4 \sum_{j=1}^4 \sum_{k=1}^4 (i-1)(j-1) c_{ijk} \xi^{i-2} \eta^{j-2} \zeta^{k-1}$$

⋮

$$\frac{\partial^3 \bar{x}}{\partial \xi \partial \eta \partial \zeta} = \sum_{i=1}^4 \sum_{j=1}^4 \sum_{k=1}^4 (i-1)(j-1)(k-1) c_{ijk} \xi^{i-2} \eta^{j-2} \zeta^{k-2}$$

64 Coefficients (c_{ijk}) Obtained Using Function Value and Derivatives at 8 Vertices

	1	2	3	4	5	6	7	8
\bar{x}								
$\partial\bar{x}/\partial\xi$								
$\partial\bar{x}/\partial\eta$								
$\partial\bar{x}/\partial\zeta$								
$\partial^2\bar{x}/\partial\xi\partial\eta$								
$\partial^2\bar{x}/\partial\eta\partial\zeta$								
$\partial^2\bar{x}/\partial\zeta\partial\xi$								
$\partial^3\bar{x}/\partial\xi\partial\eta\partial\zeta$								

Future Work

- **Finish derivation of tri-cubic mapping function in generalized coordinates using spline techniques**
- **Use analytic function to calculate closed form expressions for the cell centroids, volumes, and face area metrics**
- **Reduce analytic solutions to form that can be coded as a subroutine in staggered-mesh LES solver**
- **Begin to investigate issues related to grid continuity and spurious oscillations (i.e., tension splines)**