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Finite-Difference Numerical Simulation of 3D Seismic Wave Propagation

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Why Compute Synthetic Seismic and Acoustic Data?

1) **Fundamental research:** study scientific issues associated with wave propagation in earth and atmosphere environments (radiation, reflection, refraction, scattering, attenuation, dispersion, etc.).

2) **Applied research:** understand practical issues related to remote sensing and imaging with seismic and/or acoustic waves (detection, resolution, sensitivity, parameter estimation accuracy, etc.).

3) Engage in **prediction**, hypothesis testing, or simulation (ground motion, CO2 sequestration monitoring, fluid inclusion effects, etc.).

4) Enhance **interpretation** of field-recorded seismic/acoustic data.

5) **Validate** data processing, analysis, interpretation, imaging, or inversion **algorithms** with *realistic* synthetic data generated from known earth and atmosphere models (Marmousi Model, SEG/EAEG Salt Model, SEAM project).

6) **Design** field or laboratory data acquisition **experiments** or **equipment** (survey planning, illumination studies, borehole tools, core sample apparatus).

7) Develop and **enhance numerical computation** capabilities (algorithm parallelization, memory reduction, execution speedup, FD operators, absorbing boundary conditions).

8) Improve **seismological education** via modern visualization capabilities.

SNL Geophysics Department: Seismic and Acoustic Wave Propagation R&D

R&D Thrust: Development and application of advanced numerical algorithms for simulating 3D seismic and acoustic wavefields propagating within realistic geologic and atmospheric environments:

- isotropic elastic and anelastic (i.e., attenuative/dispersive) solid media.
- fixed and moving fluid (acoustic) media.
- poroelastic (fluid-saturated solid) media.
- *anisotropic (directional) media (both elastic and anelastic) under development.*

Numerical Solution Methodology: Explicit, time-domain, finite-differencing of coupled systems of first-order partial differential equations, representing “full physics” mathematical characterization of continuum-mechanical wave propagation problems.

- TD FD method is simple and flexible, and historically popular in petroleum industry.
- known numerical stability and dispersion properties.
- accommodates point-by-point heterogeneity in medium properties.
- Coupled 1st-order systems have superior geophysical and numerical properties, compared to higher-order PDEs.
- allows straightforward calculation of novel quantities (kinetic and strain energy, particle rotation, Poynting vector) useful for wavefield decomposition.
- readily parallelizable via spatial domain decomposition strategy.

But:

- large-scale or broadband simulations can be very expensive.
- full-physics solution may be difficult to interpret.

Elastodynamic Velocity-Stress System

$$\frac{\partial v_i}{\partial t} - b\frac{\partial \sigma_{ij}}{\partial x_j} = b\left[f_i + \frac{\partial m_{ij}^a}{\partial x_j} \right]$$ (3 equations)

$$\frac{\partial \sigma_{ij}}{\partial t} - \lambda \frac{\partial v_k}{\partial x_k}\delta_{ij} - \mu\left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right] = \frac{\partial m_{ij}^s}{\partial t}$$ (6 equations)

Nine, coupled, first-order, linear, non-homogeneous partial differential equations.

Wavefield variables:
- $v_i(\mathbf{x},t)$ - velocity vector
- $\sigma_{ij}(\mathbf{x},t)$ - stress tensor

Earth model parameters:
- $b(\mathbf{x})$ - mass buoyancy
- $\lambda(\mathbf{x})$, $\mu(\mathbf{x})$ - elastic moduli

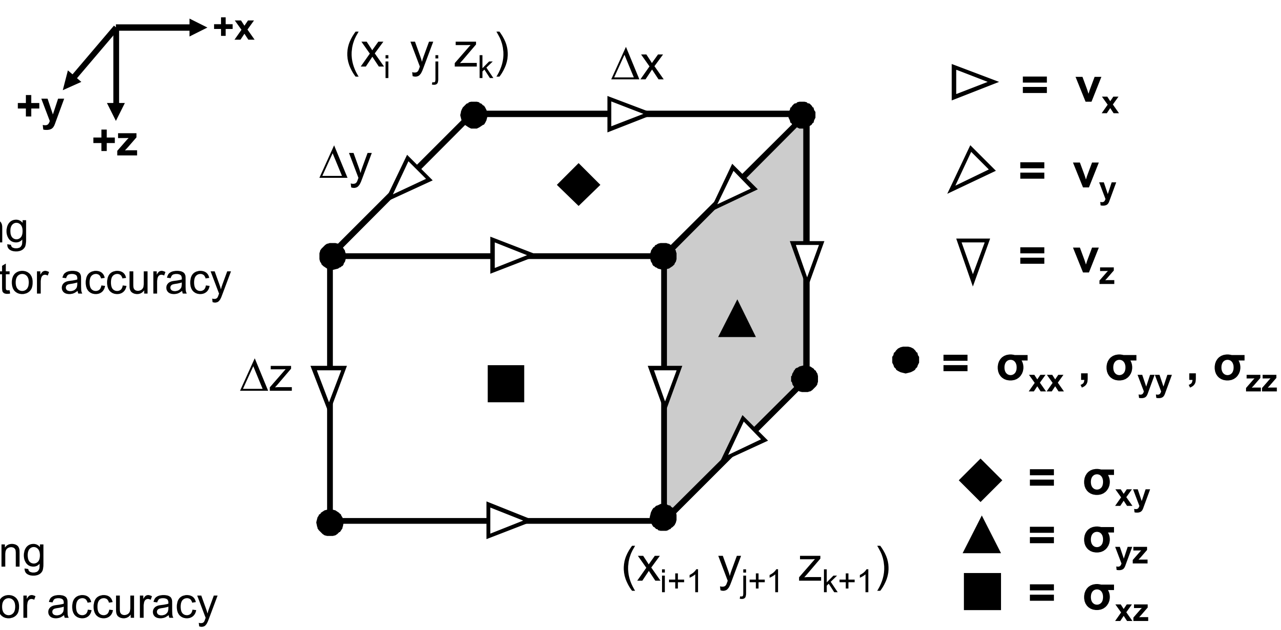
Body sources:
- $f_i(\mathbf{x},t)$ - force vector
- $m_{ij}(\mathbf{x},t)$ - moment tensor

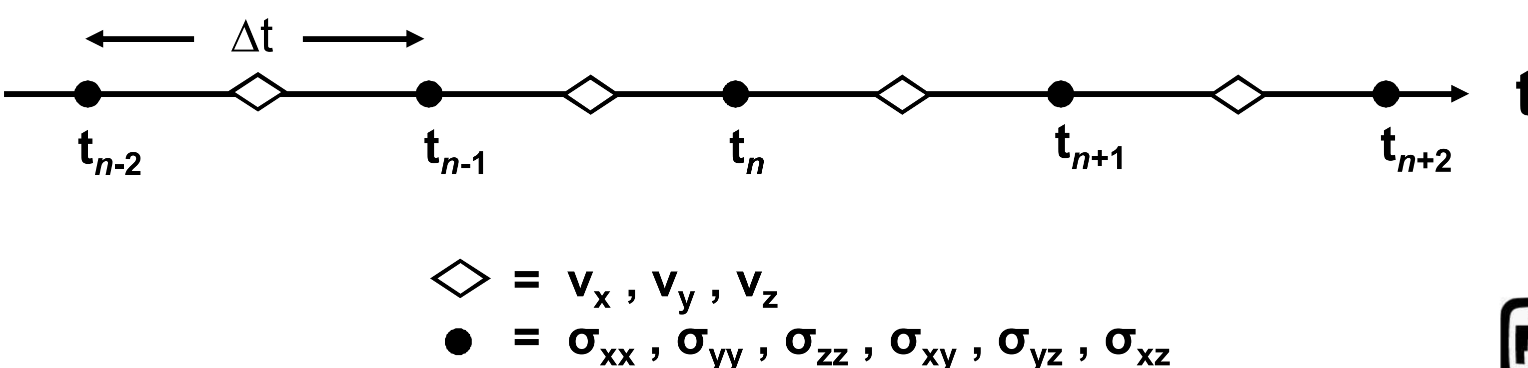
Derived from fundamental principles of continuum mechanics (conservation of mass, balance of linear and angular momentum), an isotropic elastic stress-strain constitutive relation, and linearization to the infinitesimal deformation regime.

Staggered Spatial and Temporal Storage Schemes

3D spatial staggering
⇒ high centered FD operator accuracy

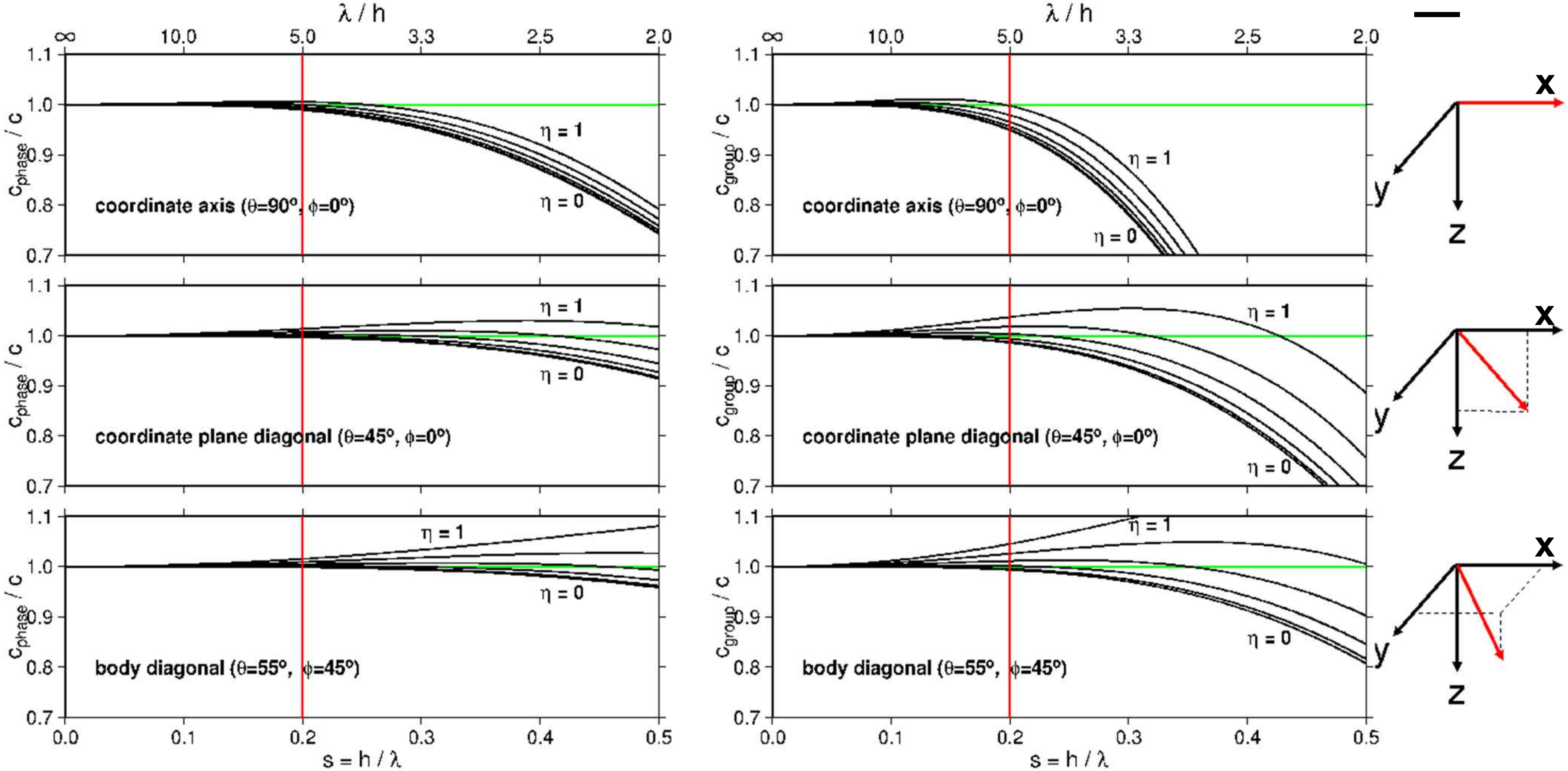
1D temporal staggering
⇒ high centered FD operator accuracy





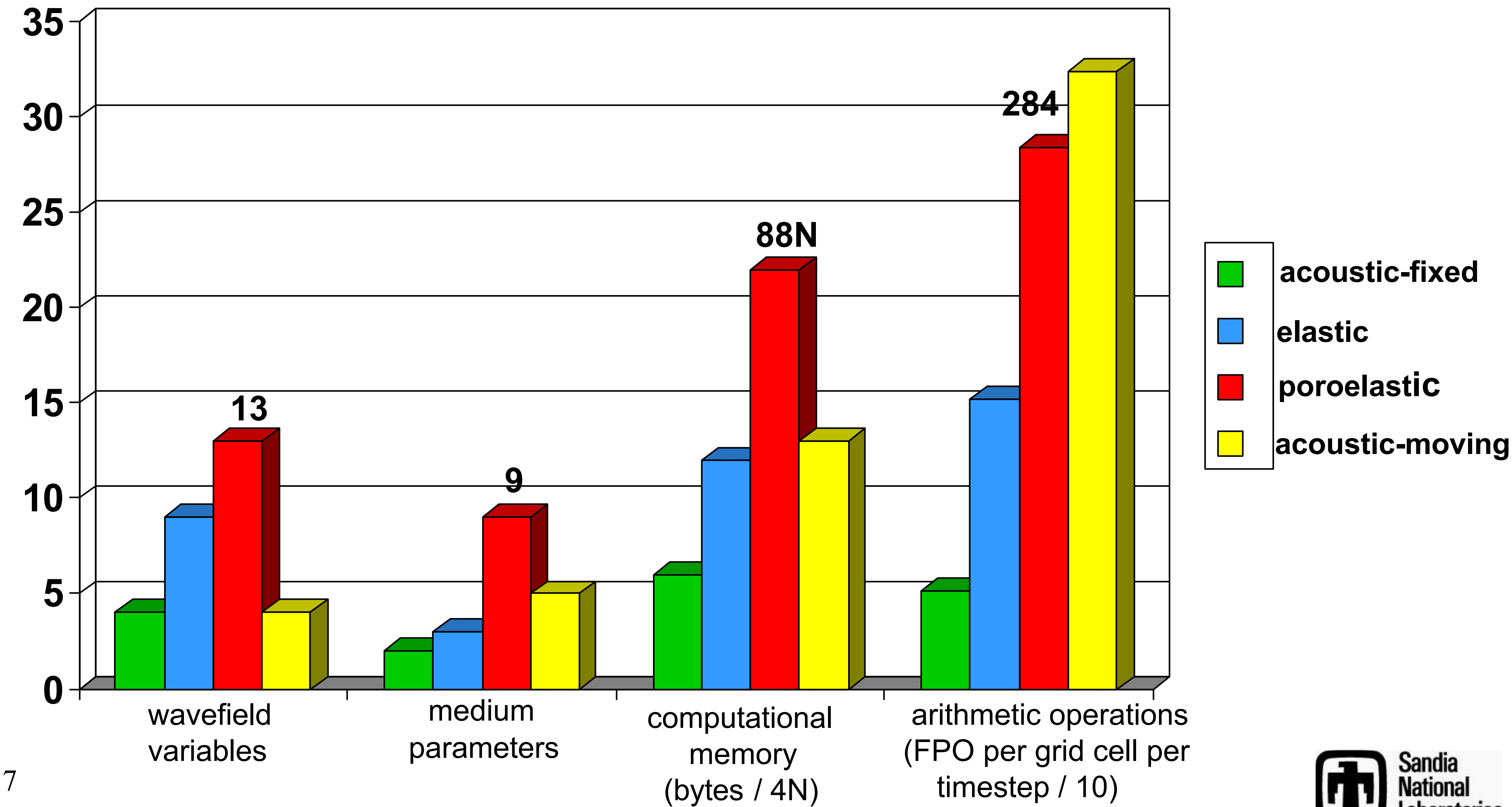
Numerical Dispersion: Phase and Group Speed Curves

Phase Speed Group Speed



Curves appropriate for 3D FD solution of 1st order velocity-stress elastic *and* velocity-pressure acoustic systems on staggered temporal/spatial grids. Stability parameter η ranges from 0 to 1. Vertical red line: the conventional “5 grid intervals per wave-length” rule of thumb for minimal numerical dispersion.

Time-Domain Finite-Difference Algorithm Comparisons: 3D O(2,4) temporal / spatial staggered solution of 1st-order coupled PDE systems for heterogeneous media



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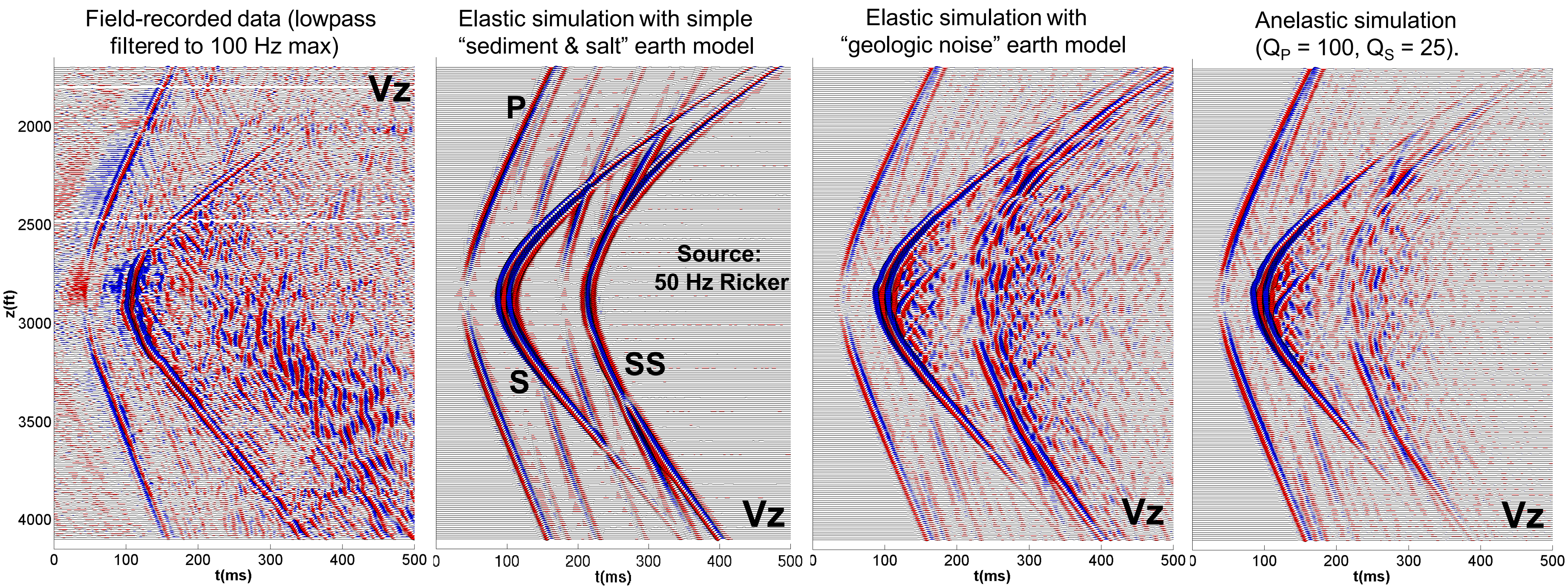
Bayou Choctaw Salt Dome Dual-Well Seismic Data



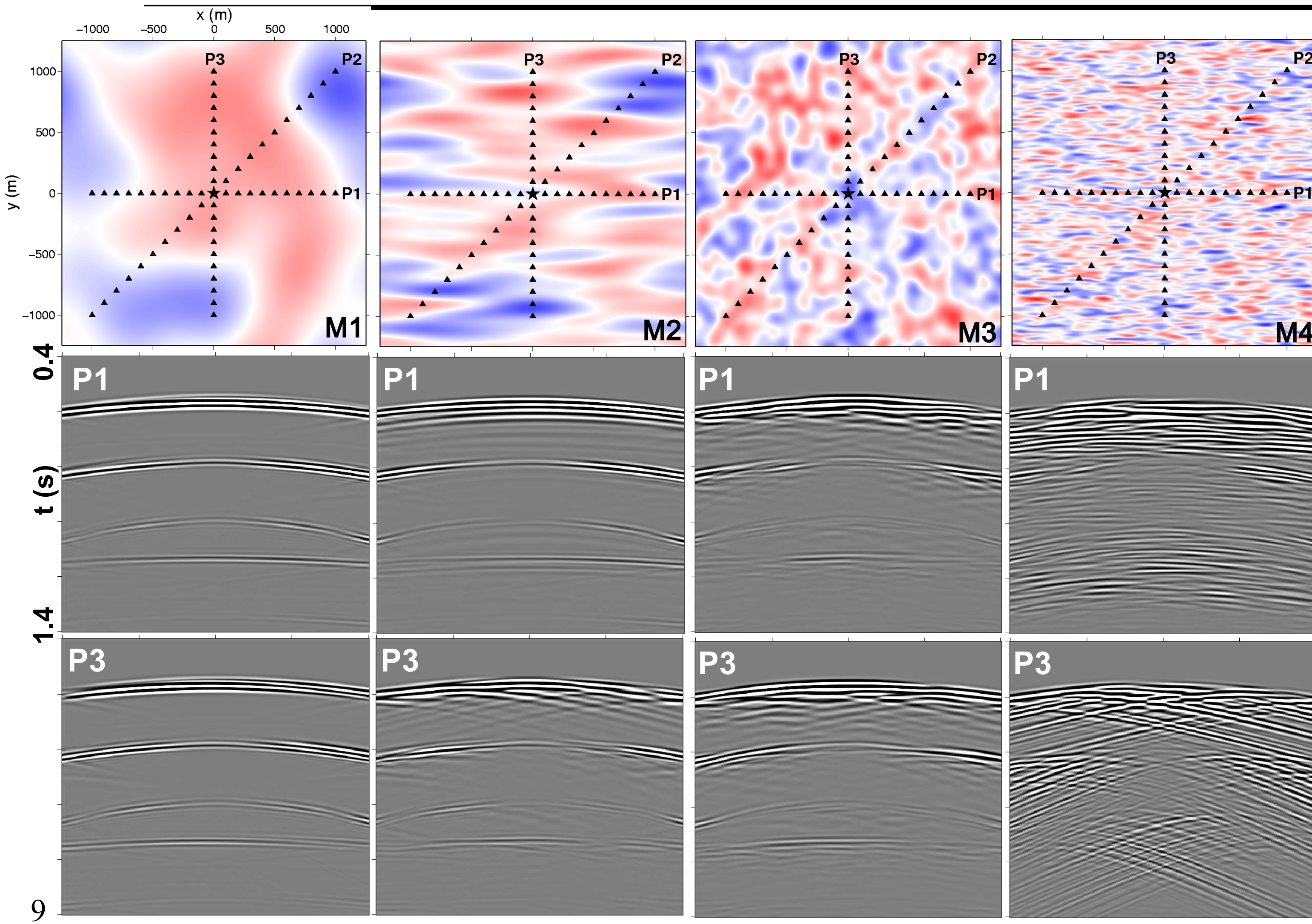
1) Field data: borehole hydraulic vibrator and 3C velocity receivers.

2) Numerous seismic events observed (well-to-well P and S, salt flank reflections, coda).

3) 3D elastic and anelastic modeling used to replicate and interpret field data:

- timing and amplitude of direct P and S; salt flank reflections; rugose salt flank creates coda; attenuation reduces amplitude of strong reflected SS phase.

Permian Basin Seismic Scattering



Horizontal cross-sections: Central Basin Platform deep carbonates.

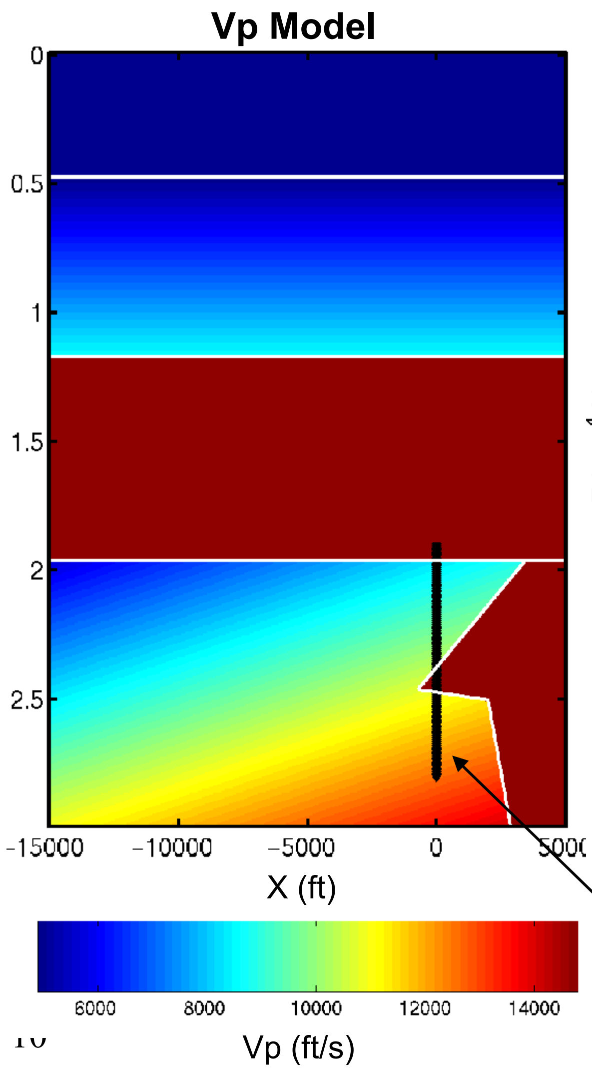
Large/small and isotropic/anisotropic perturbations in Vp.

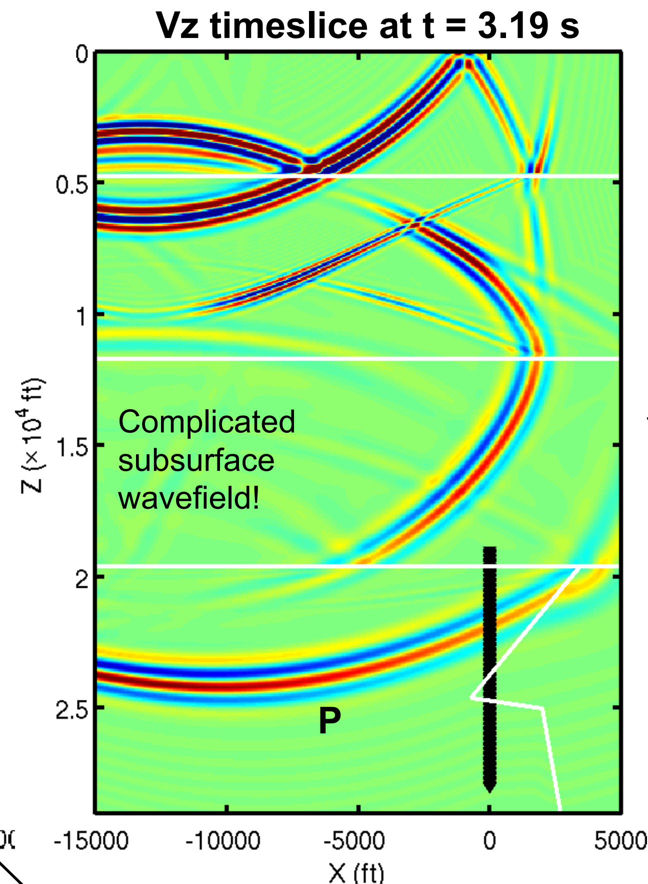
Profile #1

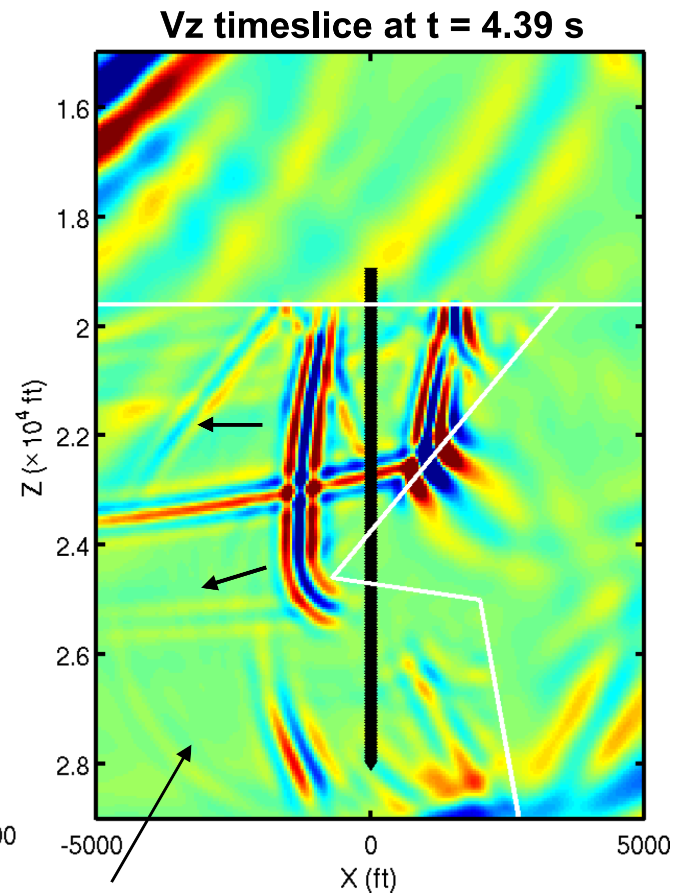
Scattered seismic energy (ΔVz traces) calculated with 3D elastic FD algorithm.

Profile #3

Gulf of Mexico Marine VSP Simulation: Salt Flank Overhang Model





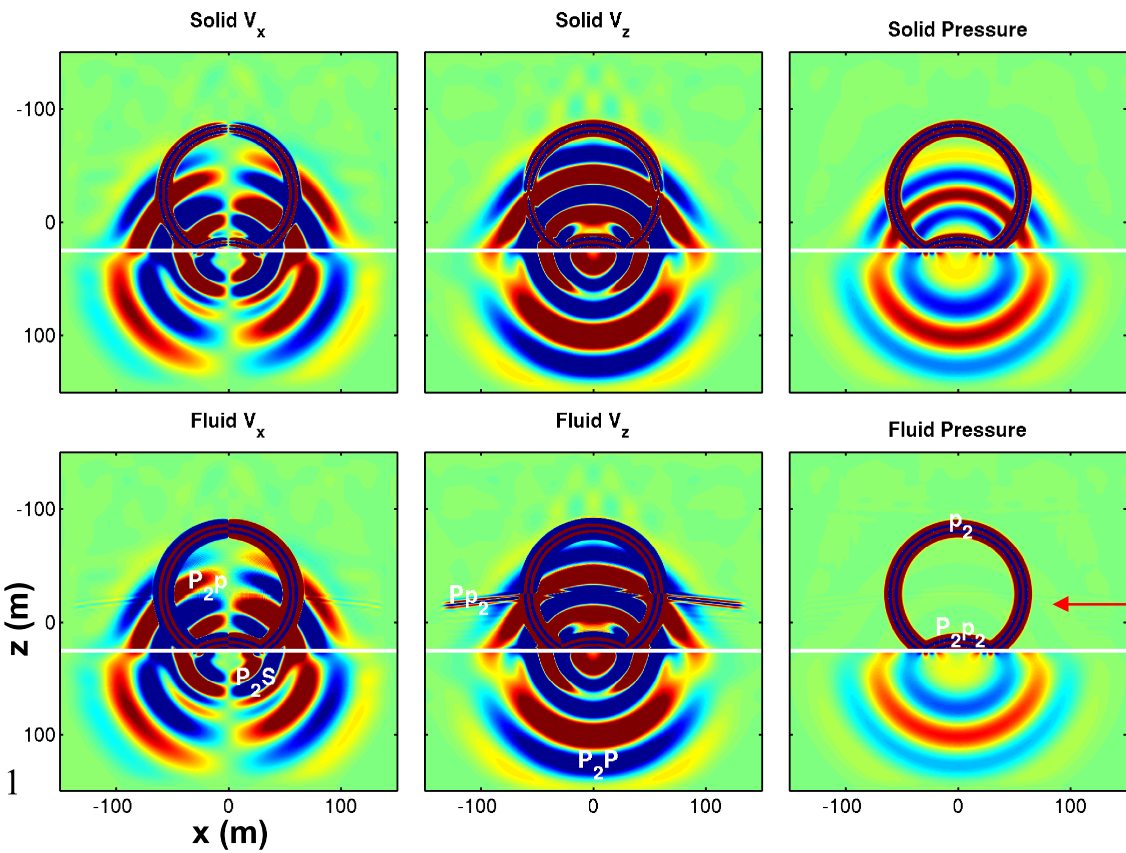


VSP receiver array

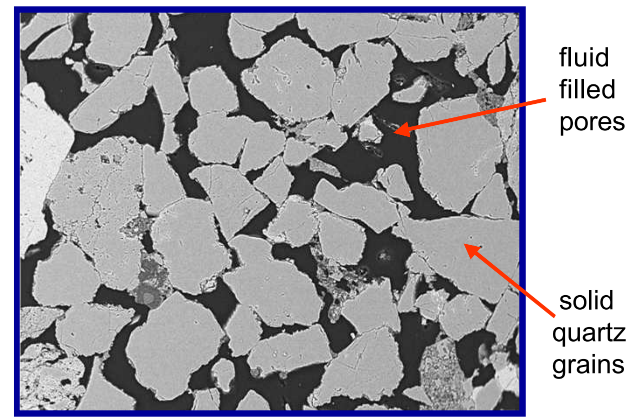
Near-vertical wavefronts propagating out of sediment wedge and across VSP array: *uninterpretable* prior to modeling!

Poroelastic Wave Propagation Modeling

Velocity-stress-pressure finite-difference algorithm, based on Biot theory, simulates 3D wave propagation within a heterogeneous fluid-saturated solid.



Castlegate sandstone



porosity: ~ 28%

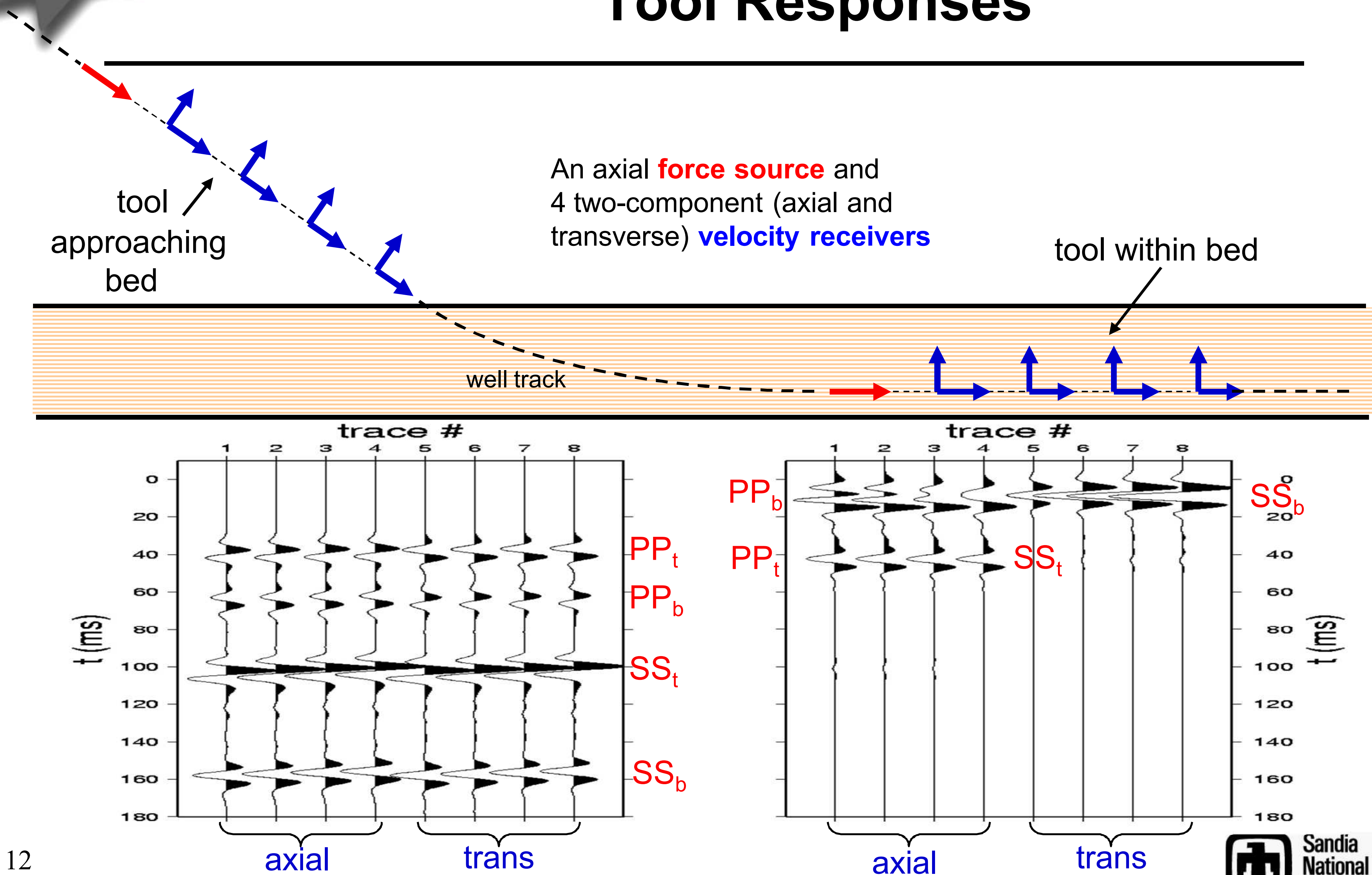
permeability: ~ 0.3 Darcy

Reflection, transmission, and mode conversion of poroelastic waves at gas-brine contact within saturated sandstone.

Note **slow P wave**, predicted by Biot theory, but rarely observed in field data.

Single-Well Seismic Acquisition Tool Responses

An axial **force source** and 4 two-component (axial and transverse) **velocity receivers**



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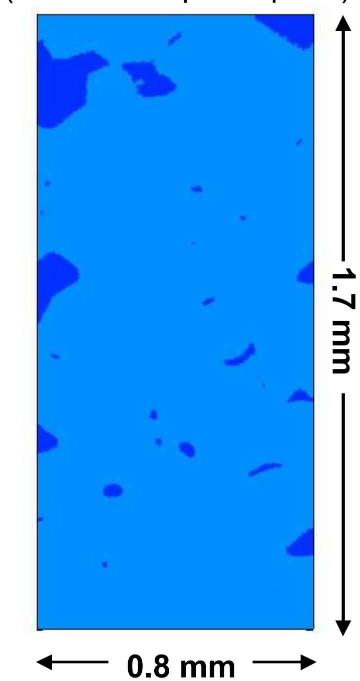
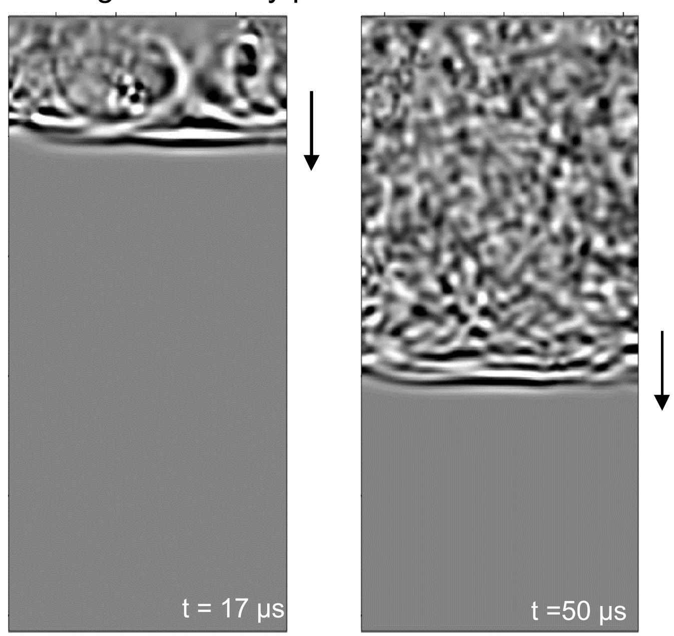
3D Elastic Wave Propagation Within Microscale Models of Porous Media

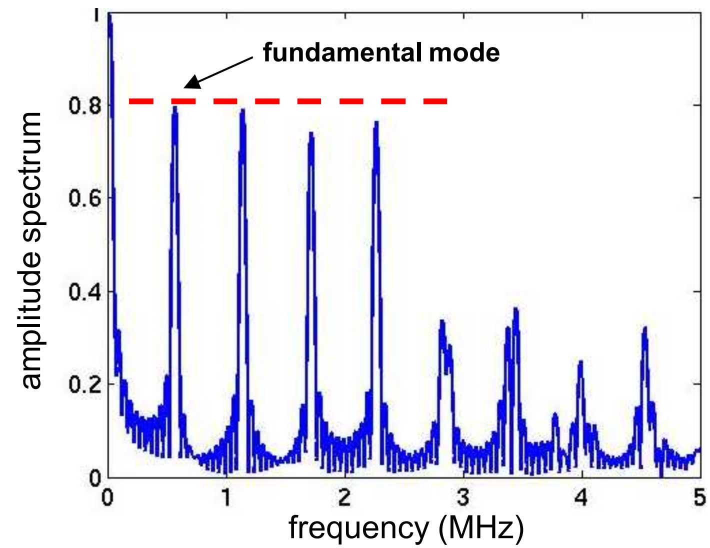
Numerical **resonance spectroscopy** of porous (fluid-saturated solid) media enables inference of microstructural geometric and material properties.

Porous medium section
(dark blue = pore space)



Timeslices of elastic pressure
generated by plane wave source





3D wave modeling within a water-saturated solid elastic framework (ϕ = 0.05). Note:

- strong scattering behind initial wavefront.
- periodic boundary conditions at flanks.
- low frequency spectral peaks related to bulk effective medium properties
- high frequency peaks associated with characteristic dimensions of pore structure.

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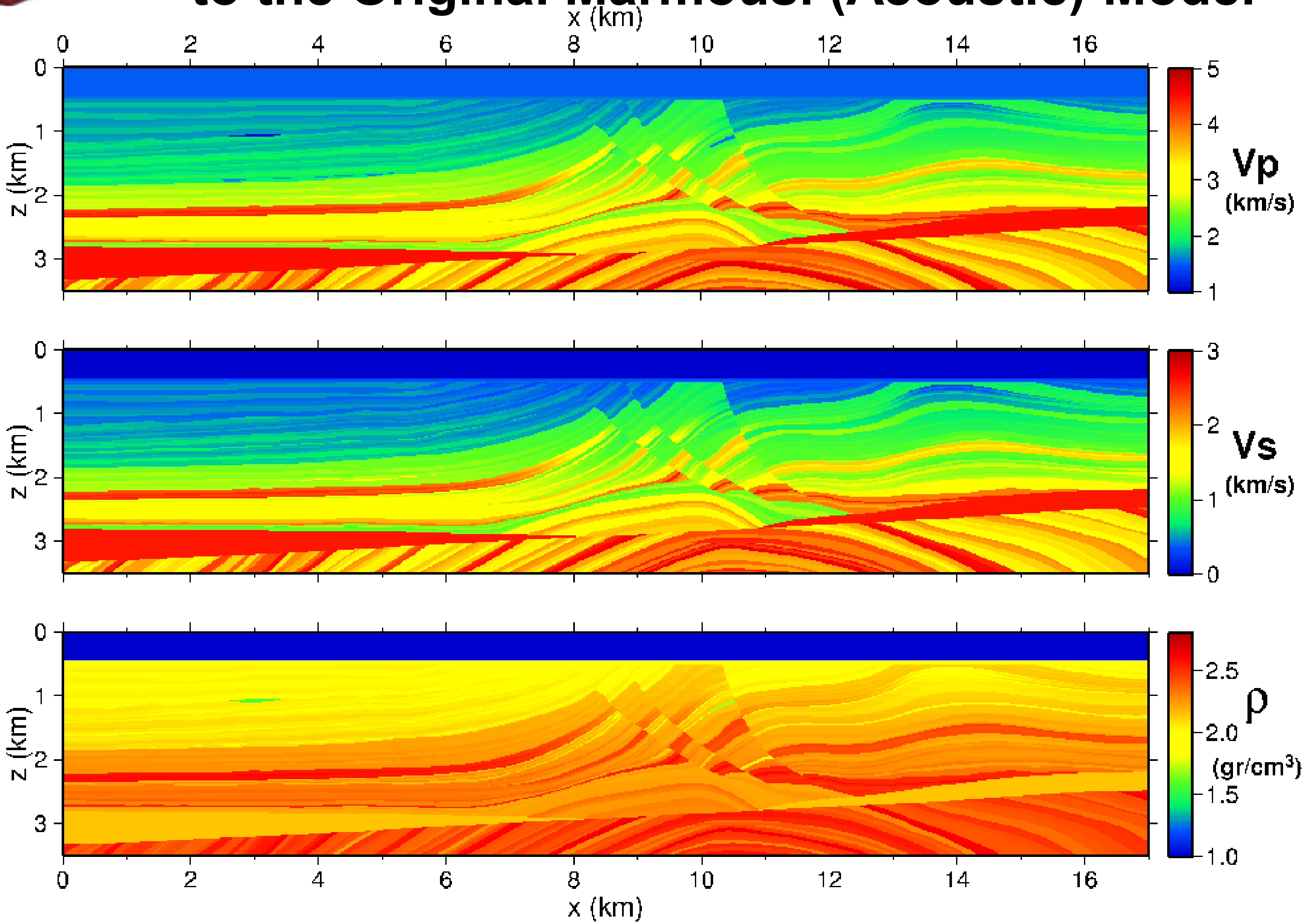
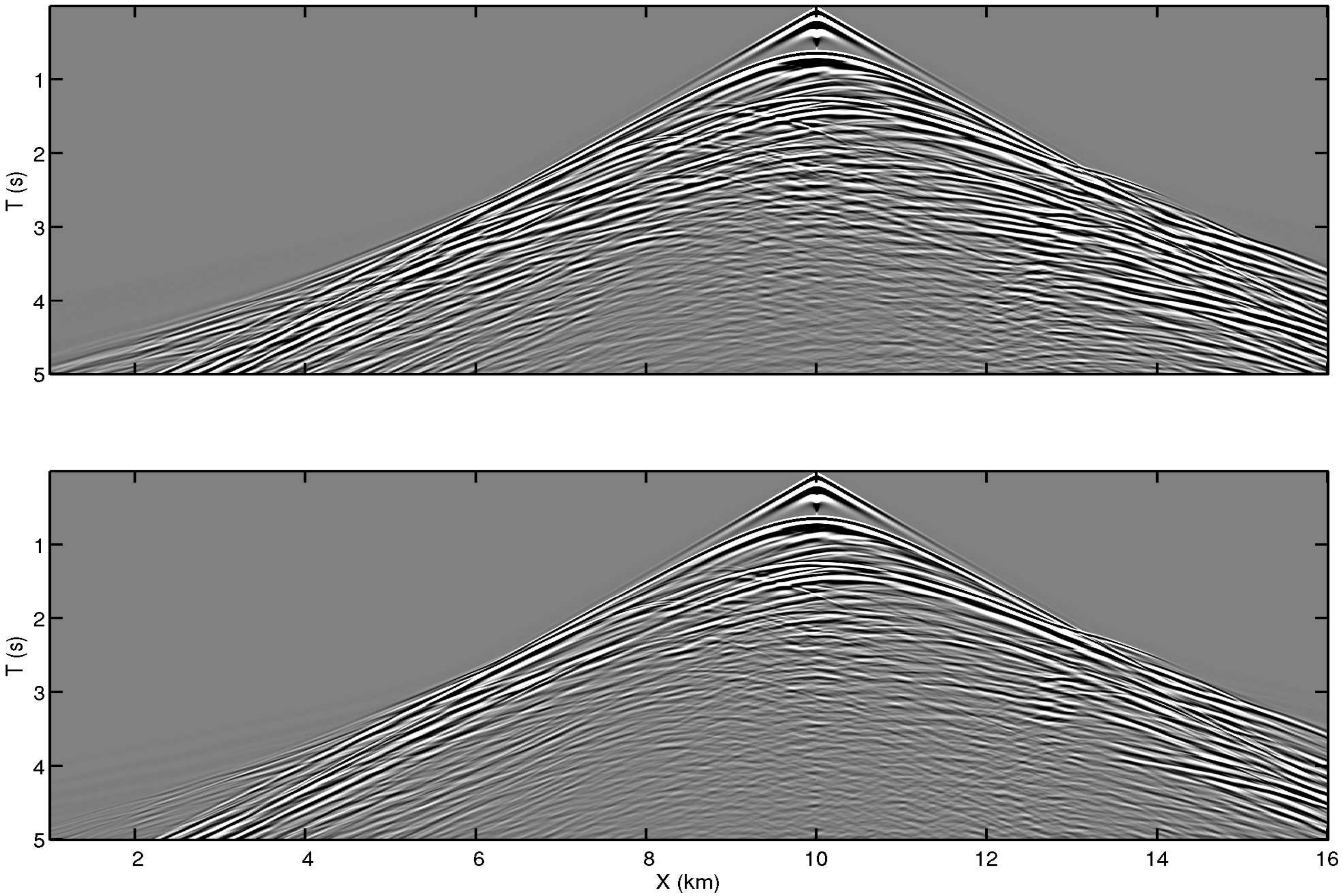
Marmousi2: An Isotropic Elastic Upgrade to the Original Marmousi (Acoustic) Model



After Martin et al.,The Leading Edge, Feb. 2006. SNL 3D elastic modeling conducted with 5 m spatial grid interval ⇒ ~1.2 billion gridpoints!

Pressure Trace Comparison



Velocity-pressure (acoustic) algorithm

Velocity-stress (elastic) algorithm

1501 hydrophones, 5 m below sea-surface, arrayed from x = 1 km to x = 16 km. Note strong similarity of calculated responses.

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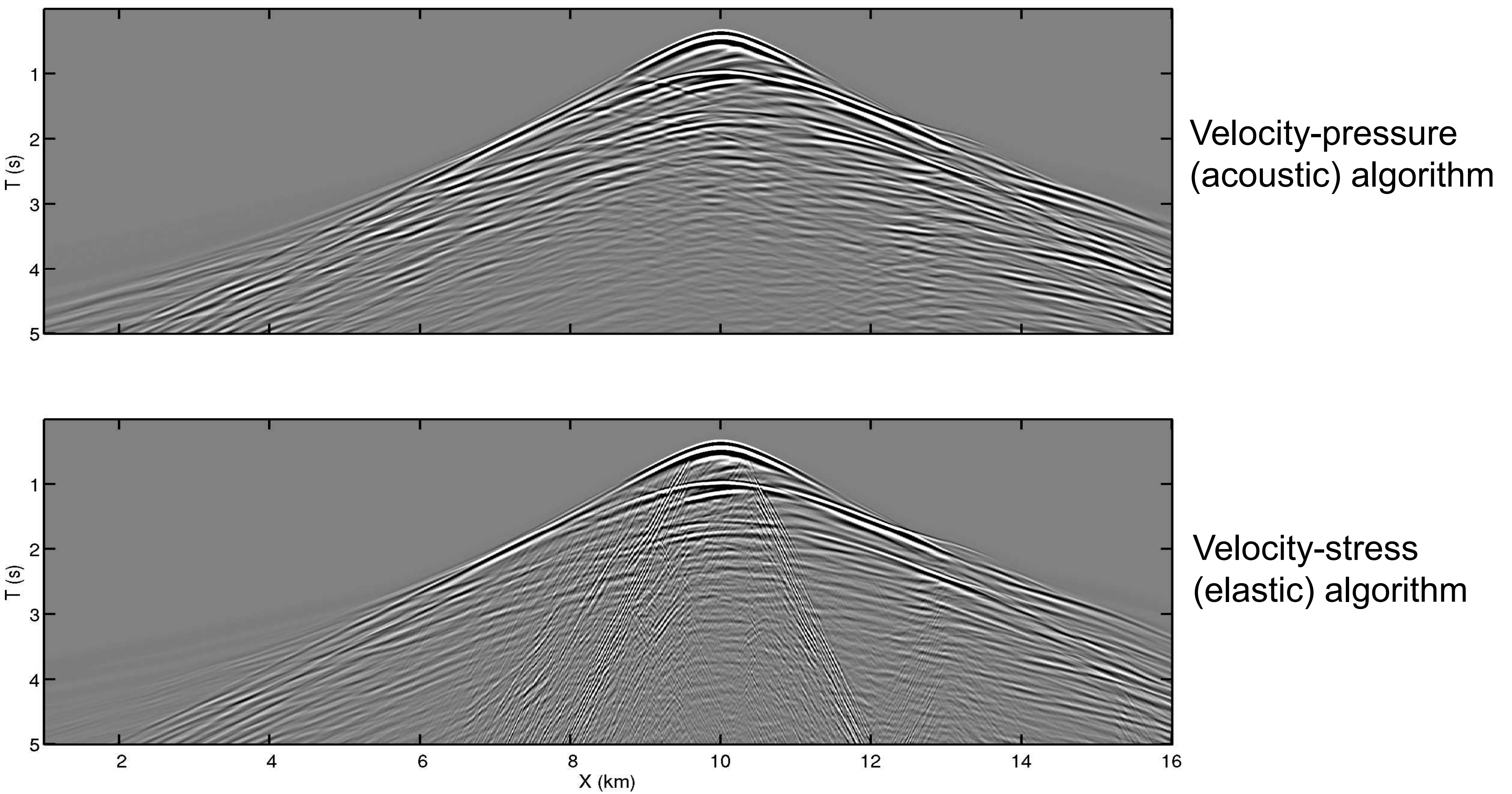
Ocean Bottom Seismometer Trace Comparison



1501 vertical component (Vz) ocean bottom seismometers, located 450 m below sea-surface, arrayed from x = 1 km to x = 16 km. Note strong differences in calculated responses.

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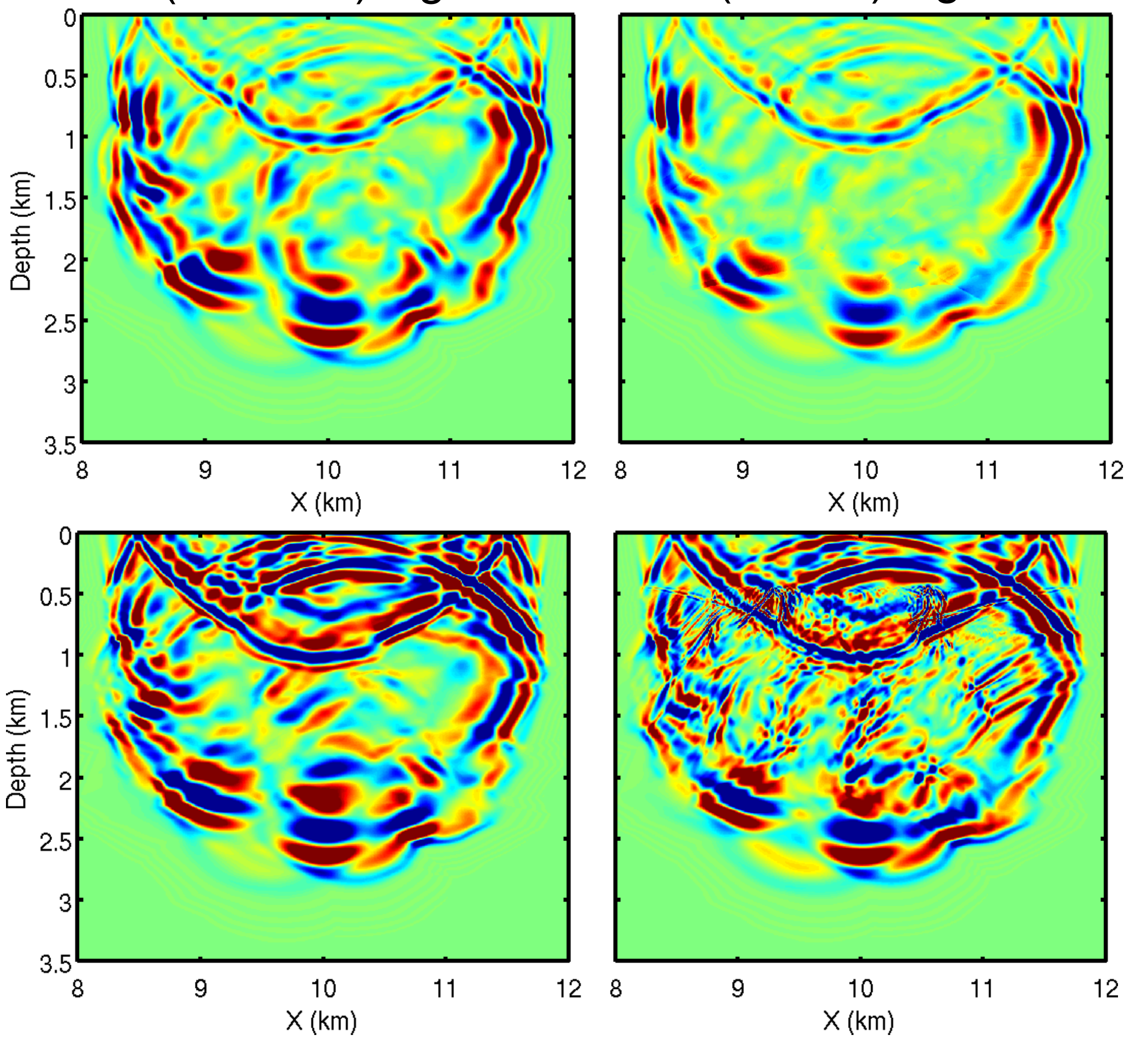
Timeslice Comparisons: Pressure and Vz Particle Velocity

VP (acoustic) algorithm | VS (elastic) algorithm



Pressure timeslices;
t = 1.37 s.

(note similarity)

Vz Velocity Timeslices;
t = 1.37 s.

(note difference)

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Run Time Estimation Equation: The Famous "Fourth-Power Law"

$$T_{\mathrm{run}} = \left[(x_{\max} - x_{\min})(y_{\max} - y_{\min})(z_{\max} - z_{\min})(t_{\max} - t_{\min})V_{\max}\right]$$

$$\times \left[\frac{2\tau}{\eta_1 N_{\mathrm{proc}}}\right]\left[\frac{2f_{\max}}{\eta_2 V_{\min}}\right]^4$$

V_{min} , V_{max} = min, max velocities
f_{max} = max frequency
τ = seconds / gridpoint / timestep
N_{proc} = number of processors

FD numerical factors:

$$\Delta t = \eta_1 \frac{\Delta h}{2V_{\min}} \qquad \Delta h = \eta_2 \frac{V_{\min}}{2 f_{\max}}$$

where Δt = **timestep**, Δh = **grid interval**

Assumptions: Uniform (and identical) grid interval in all 3 coordinate directions; identical parallel processors; perfect parallel scalability; neglects ancillary FD operations (ABCs, free-surface, source insertion, receiver interpolation, model input, data output). Ideally, $\eta_1 = \eta_2 = 1$, i.e., algorithm is run at temporal CFL and spatial Nyquist limits.

Example Cost Calculation: Gulf of Mexico Acquisition Scenario

Parameters for Algorithm Execution Time Estimation:

1) V_{min} = 500 m/s (sub-seabed shear), V_{max} = 5000 m/s (salt).
2) f_{max} = 50 Hz.
3) X = Y = Z = 10 km; T = 10 s.
4) η_1 = 1 (ideal); η_2 = 0.4 (5 Δh per λ_{min}).
5) $\tau = N_{FPO} / R$ with $N_{FPO} \approx 150$ (3D elastic VS with O(2,4) FD), and R =2 GHz (too low?).
6) N_{proc} = 1000 (too high?).

$\Rightarrow$ $T_{run} \approx 130$ hours!

Cost = T_{run} x N_{proc} x P = $13,000 (with P = dollars/ processor hour ~ 0.1)

10,000 source seismic survey implies $130 million total cost!

(for different parameters, just scale result using the fourth-power law!)

Algorithm Research and Development Issues: Faster Speed, Reduced Memory, Higher Accuracy, and Superior Seismics!

Different Media Types:

- anisotropic elastic and anelastic (attenuative/dispersive) media.
- improved treatment of poroelasticity, or “beyond Biot”.

Algorithmic Issues:

- higher order temporal and spatial FD operators.
- optimized FD operator coefficients.
- better ABCs (PML?), allowing effective treatment of the ‘thin model”.
- efficient treatment of piecewise homogeneous or “factorized” media.

Hybrid Algorithms:

- mixed physics/math approach for multiple-media-type models.
- TD finite-integro-difference method for solid absorptive media.
- spatial FD operator order switching for models with large velocity range.

Sources and Receivers:

- multiple simultaneous sourcing for order-of-magnitude speedup.
- compressional/shear wavefield separation via pressure/rotation receivers.
- wavefield directional filtering via Poynting vector implementation.