

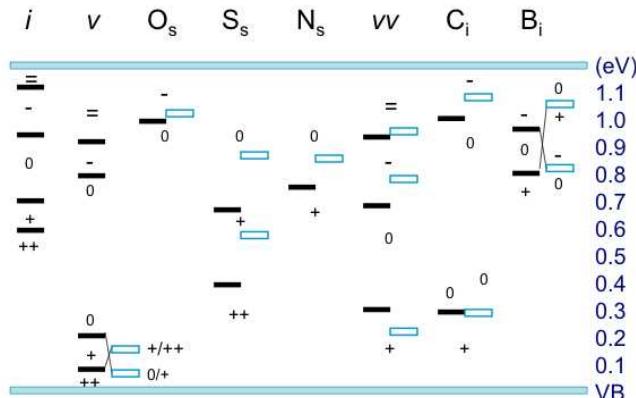
Is the density functional theory “band gap problem” truly a problem? — Defects in silicon

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$$\mathcal{H}\Psi = E \Psi \square \longrightarrow$$



It is the mark of an educated mind to rest satisfied with the degree of precision which the nature of the subject admits and not to seek exactness where only an approximation is possible.

- Aristotle

Motivation: Electrical effects of radiation damage

Issue: radiation effects (n,e, γ ,ions) on electronics

- satellites
- weapon electronics

Historical approach: testing

Radiation testing \longleftrightarrow **Systems response**
n,e, γ ,ions

Renewed interest:

- long-term aging: enhanced low dose rate sensitivities
long-term (decades) radiation damage is different
- fast transients: SPR facility decommission
fast burst neutron test facility going away

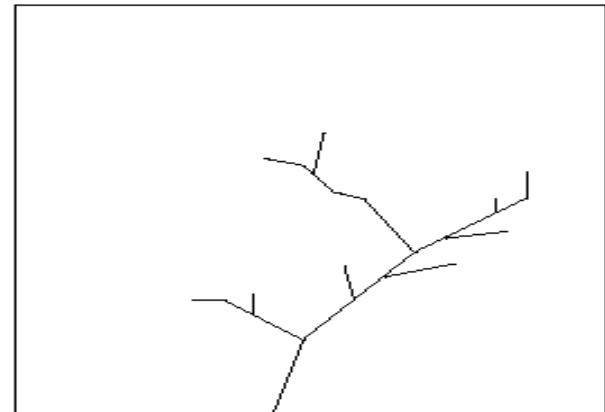
Radiation damage: from atoms to devices

Initial defect distribution

Radiation creates displacement damage:

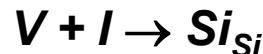


and charge carriers (electrons and holes)

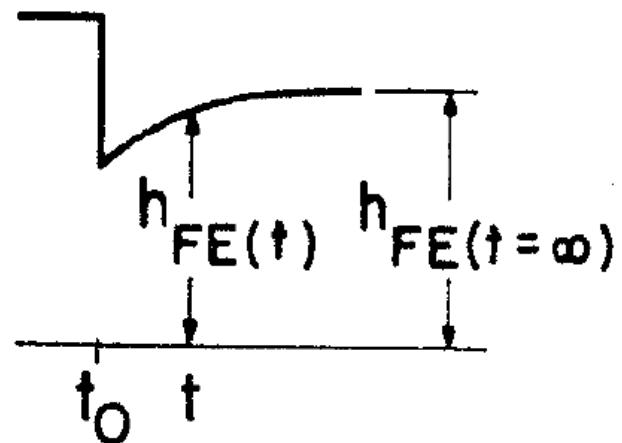
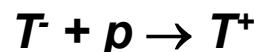


Defect evolution

Defects react with each other, and with other dopants and impurities:



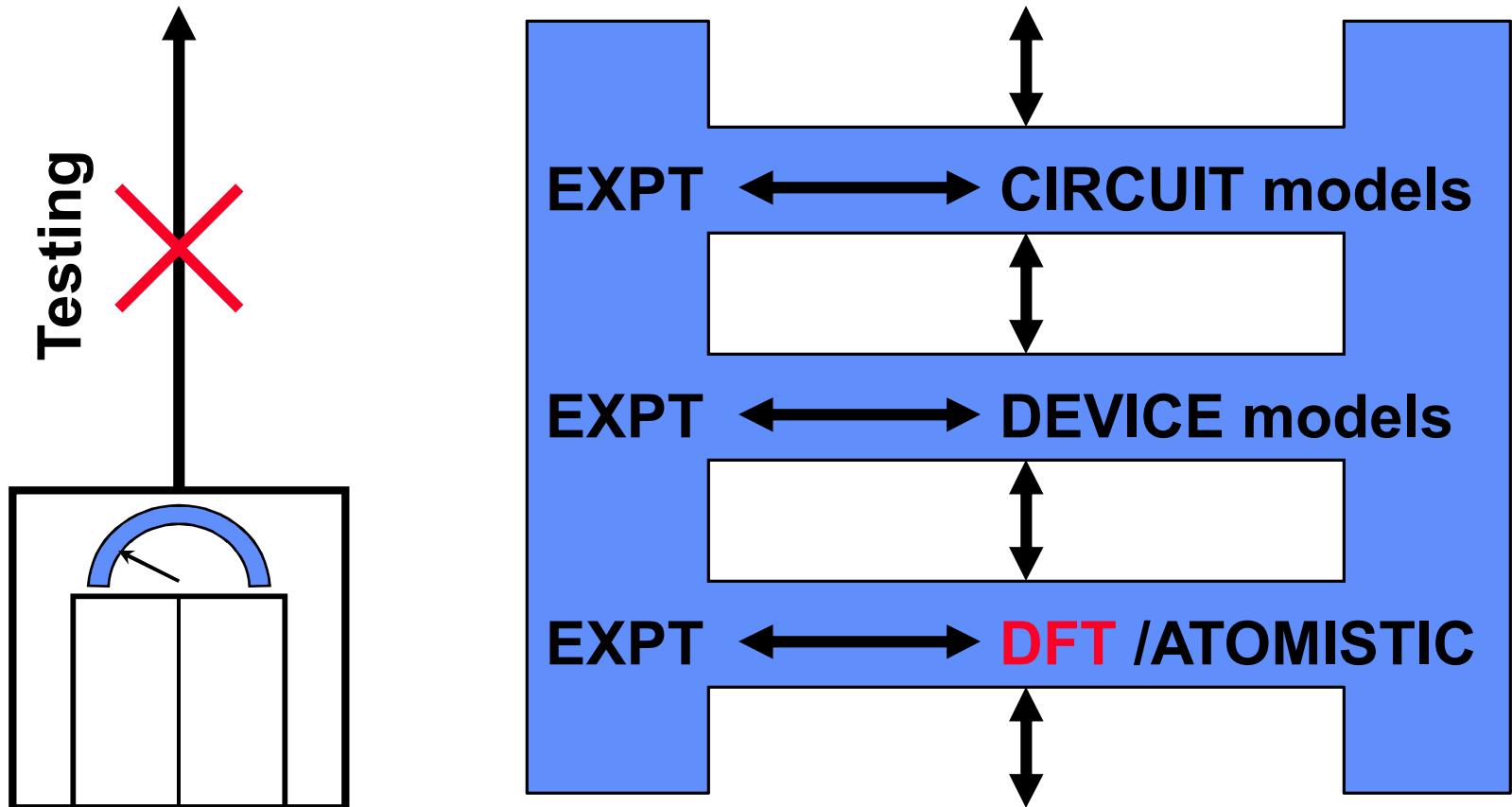
Defects recombine electrons and holes, modifying currents:



Radiation damage creates an evolving chemistry of defects. Those defects modify the performance of electronic devices.

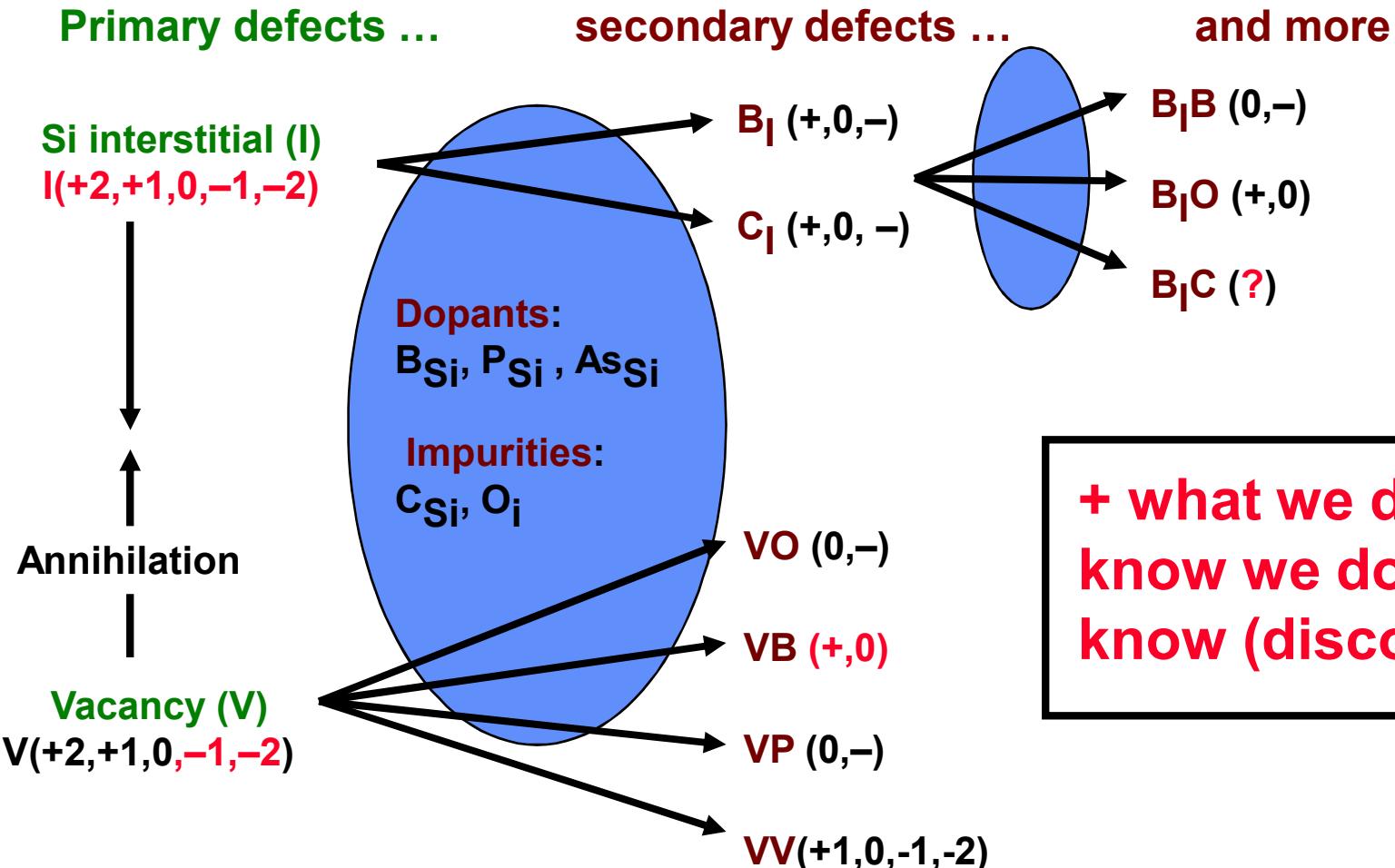
Multiscale ladder for radiation damage

Electrical system response



Radiation damage

The radiation defect universe

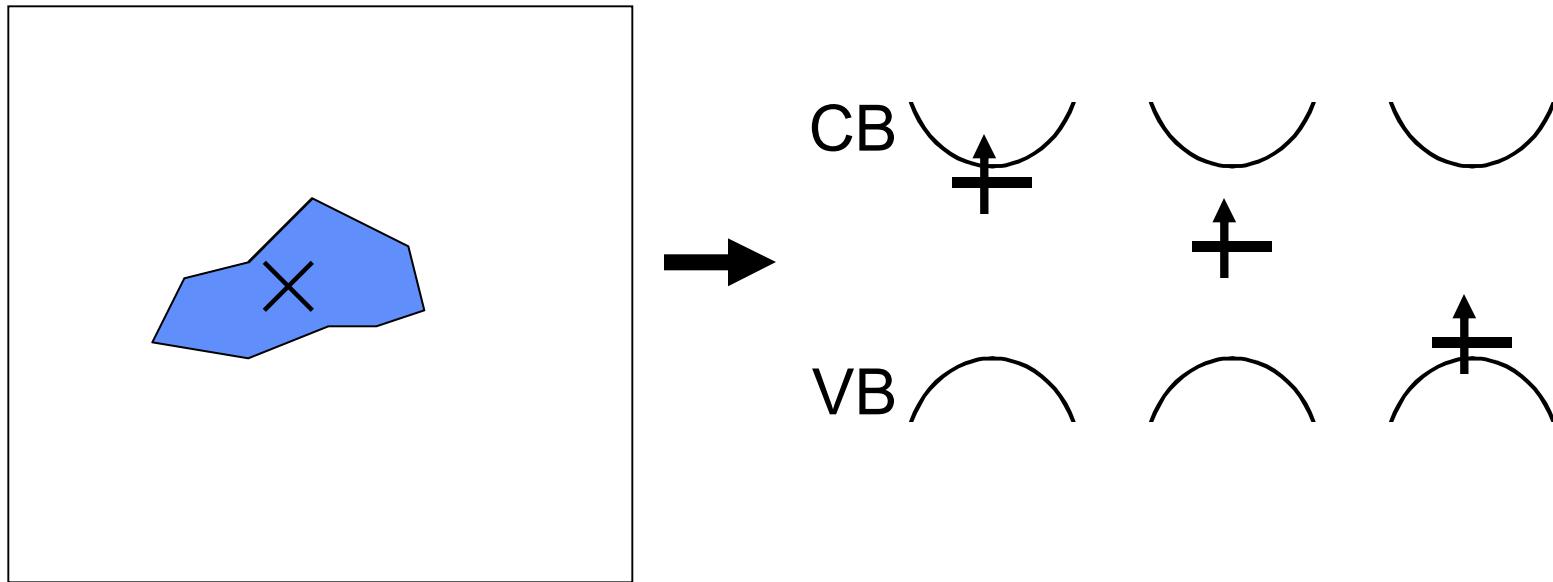


+ what we don't
know we don't
know (discovery)

Need DFT - density functional theory - to fill gaps in defect physics: defect band gap energy levels, diffusion activation

Radiation damage and defects

Radiation damage ...



produces defects ... and introduces electronic transitions

... and we would like to quantify these transitions

What do we know, what do we need?

- **Experimental record incomplete and messy**

- defect level measurements typical Uncertainty(U): $\geq kT \sim 0.03$ eV
 - e.g.: $S(2+/+)$: optical CB -0.61, thermal: -0.55:-0.59
 - $vv(0/+)$: VB+0.20:+0.26, $vv(-/0)$: CB-0.39:-0.44
- often larger: $B_i(-/0)$: $U=0.08$, $N_s(0/+)$: $U=0.12$, B_V : a mess ($U \sim 0.10$)
- $U(\text{expt}) = 0.03$ eV (best), ~ 0.1 eV (otherwise)
- incomplete knowledge (B_V , P_V , B_iX , ...) ... $U=\infty$

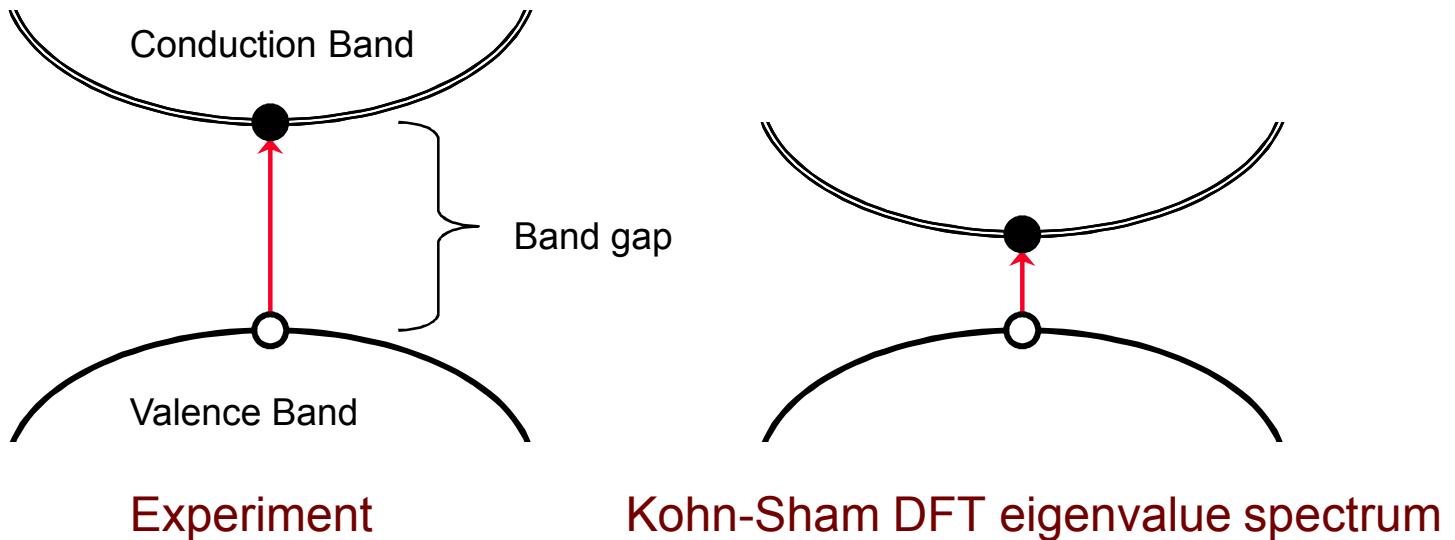
- **Density functional theory: *unproven* for levels**

- DFT structural energetics accurate to no better than ~ 0.1 - 0.2 eV
- **best accuracy to hope for with DFT: ~ 0.1 - 0.2 eV**

- **Accuracy requirement: $kT=0.03$ eV?, 0.1 - 0.2 eV?**

- if 0.03 eV, then even experiment is not good enough - we're doomed
- current device (CHARON, REOS, 1D) simulations using 0.1 eV data
- **apparent target requirement: 0.1 eV (device sims, and expt. record)**

The DFT band gap problem



DFT gap. i.e., in KS eigenvalues, significantly underestimates experiment

[L.J. Sham and M. Schlüter, PRL **51**, 1888 (1983); PRB **32**, 3883 (1985)]

Si: expt: 1.2 eV, DFT/LDA: 0.5 eV

GaAs: expt. 1.5 eV, DFT/LDA: 0.5 eV

The band gap defines the energy scale for defect levels

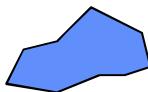
Fundamental impediment to quantitative predictions?

The Supercell Approximation

Fast Fourier Transforms are convenient means to solve 3D Poisson Equation.

DFT codes typically assume periodic boundary conditions.

However, our finite defect is not periodic ...



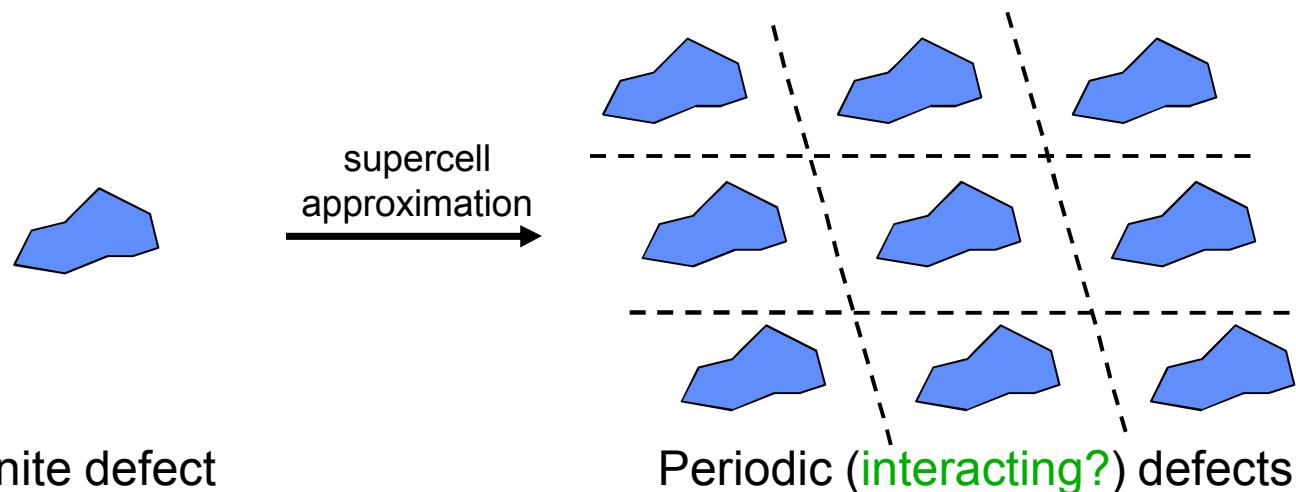
Finite defect

The Supercell Approximation

Fast Fourier Transforms are convenient means to solve 3D Poisson Equation.

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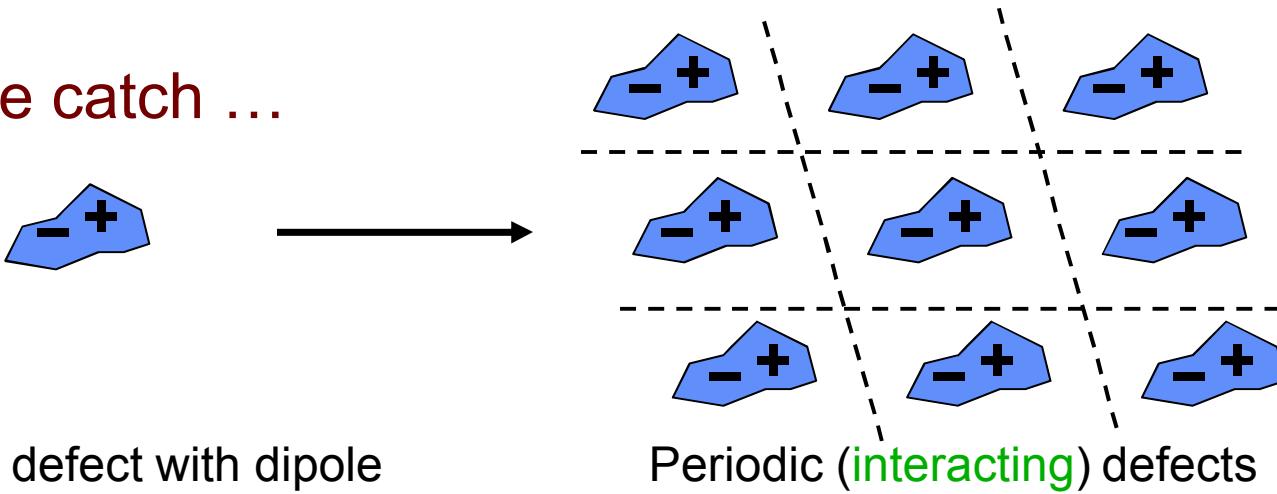


The supercell Idea:

Surround perturbed defect region with enough material to buffer defects.
In the limit of large enough supercells, approach an isolated defect.

The Supercell Approximation

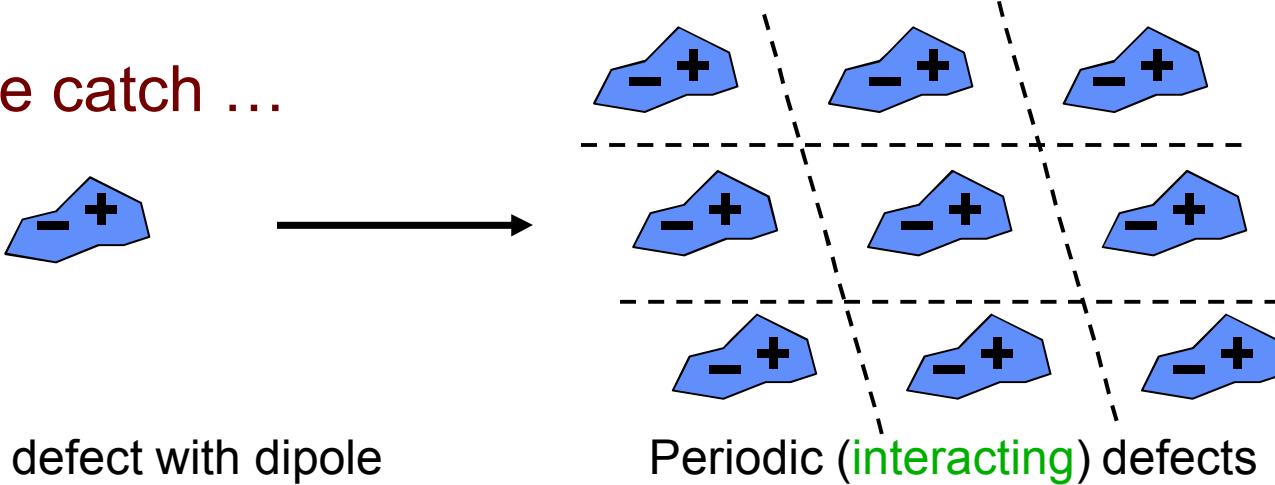
the catch ...



DFT expense limits size of supercell - defects interact

The Supercell Approximation

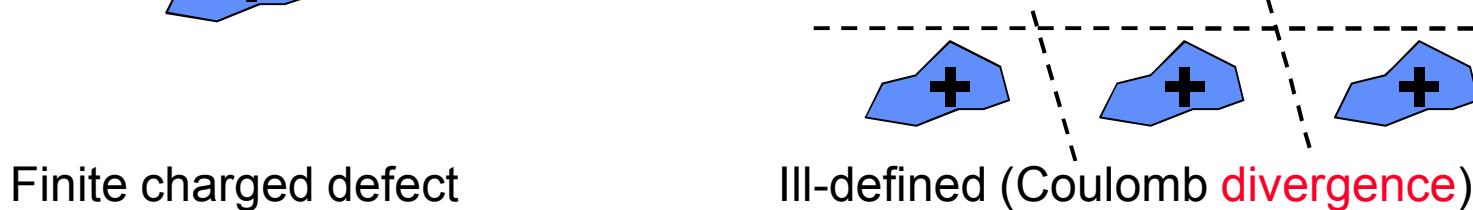
the catch ...



Finite defect with dipole

Periodic (interacting) defects

even worse ...



Finite charged defect

III-defined (Coulomb divergence)

Interactions and divergence are key issues

Supercell issues

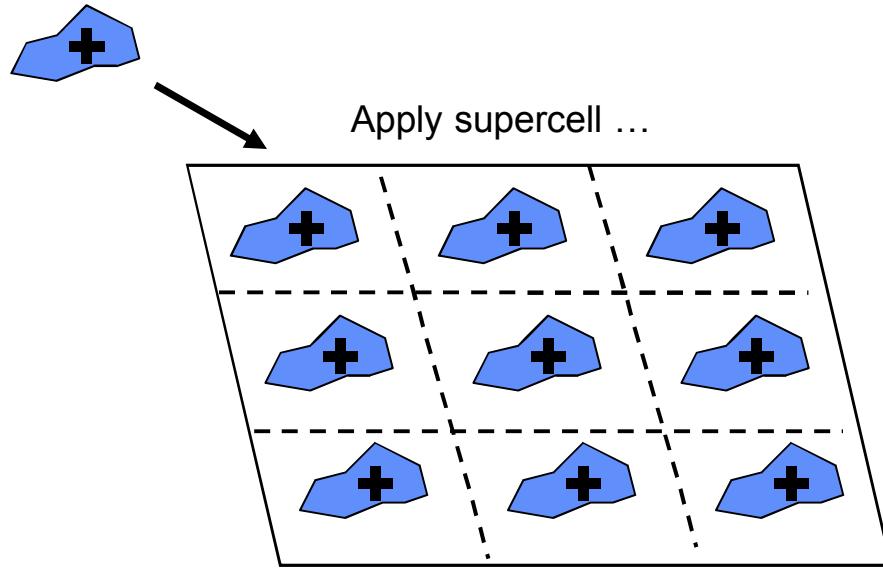
- **Boundary conditions - how to handle net charge**
 - need to eliminate divergence
 - need to install correct long range $q/r \rightarrow 0$ behavior of defect charge
- **Chemical potential for electrons**
 - want transition energies, $(0/-)$, $(0/+)$, need to fix an electron reservoir
- **Finite size effects - Bulk polarization to local charge**
 - supercell has small finite volume, missing bulk dielectric response
- **Finite size effects - Defect level dispersion**
 - defects interact, discrete defect states become bands, overlap CB/VB

And if you get this all right ... is DFT good enough?

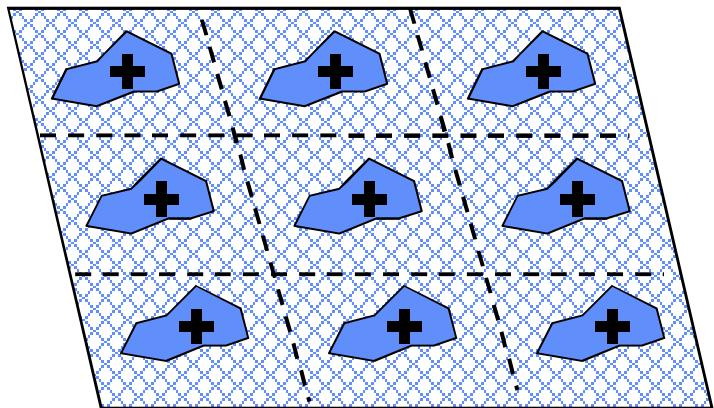
- e.g., is the band gap problem fatal?

Jellium to eliminate divergence?

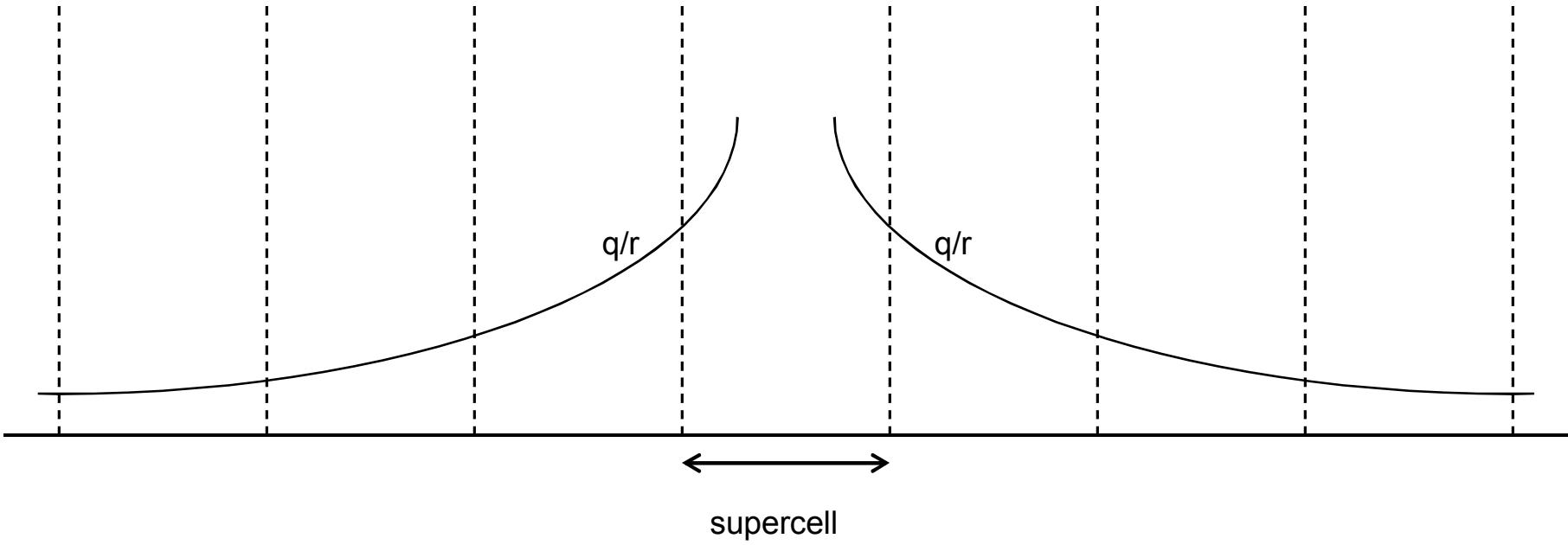
Isolated defect ...



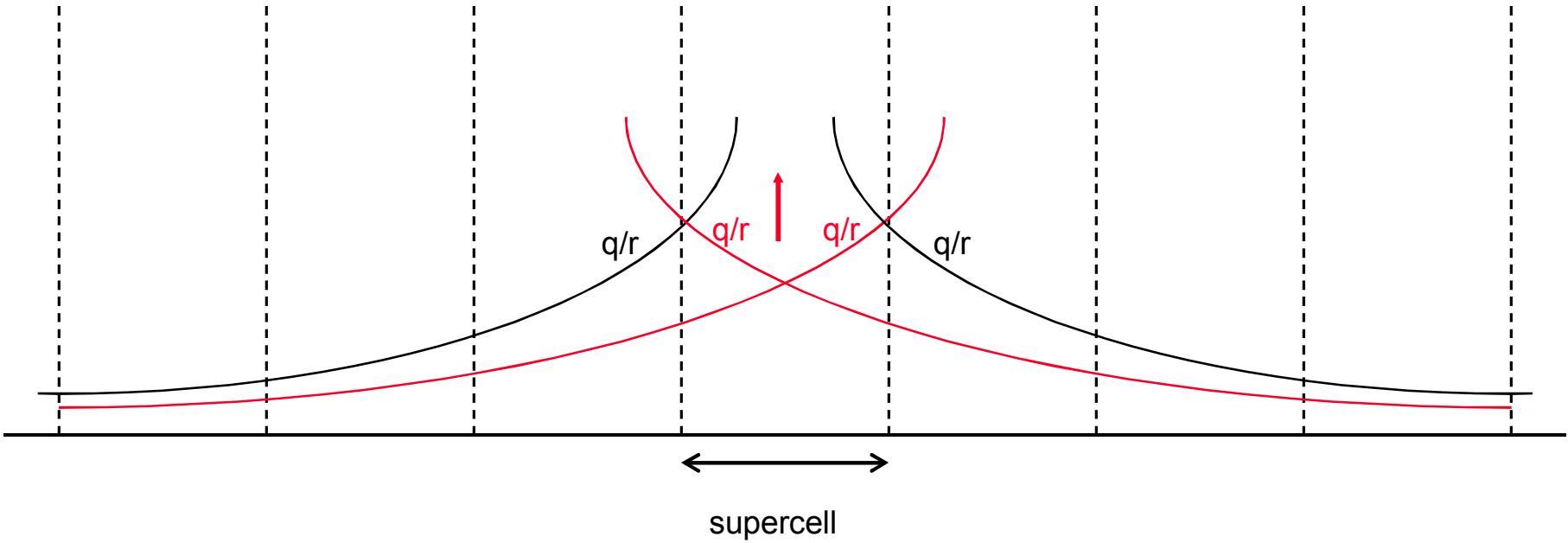
Neutralize with flat background charge:
"jellium"



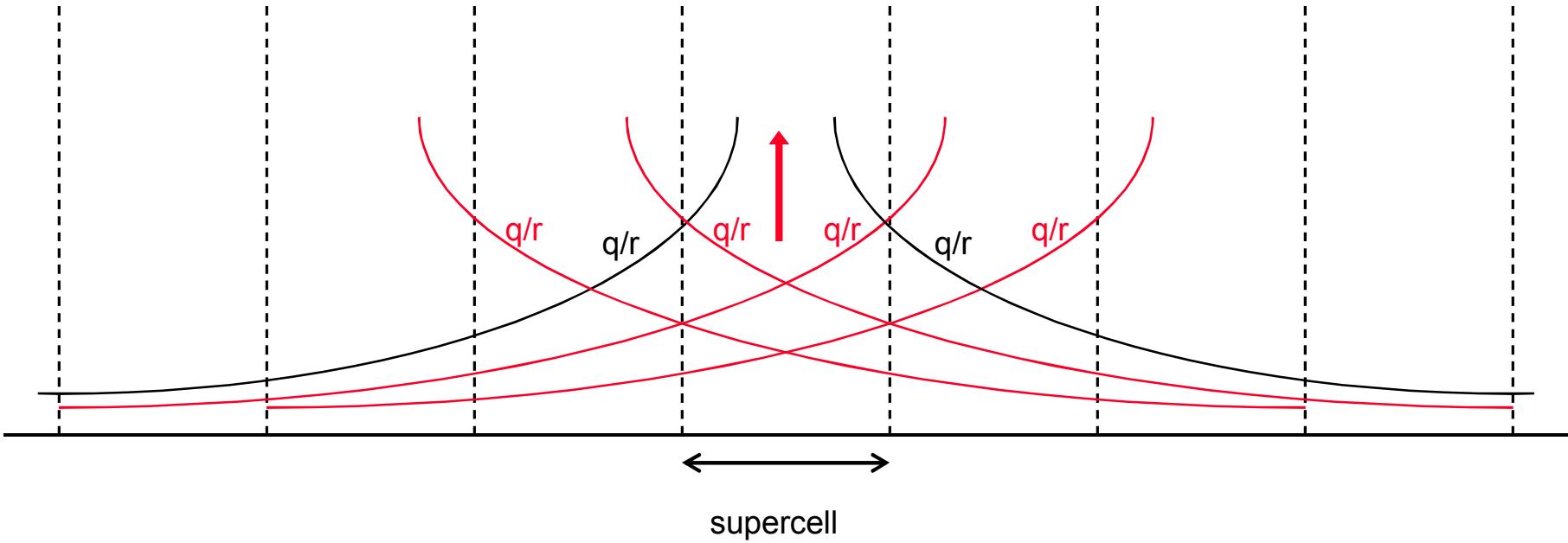
Whence the divergence?



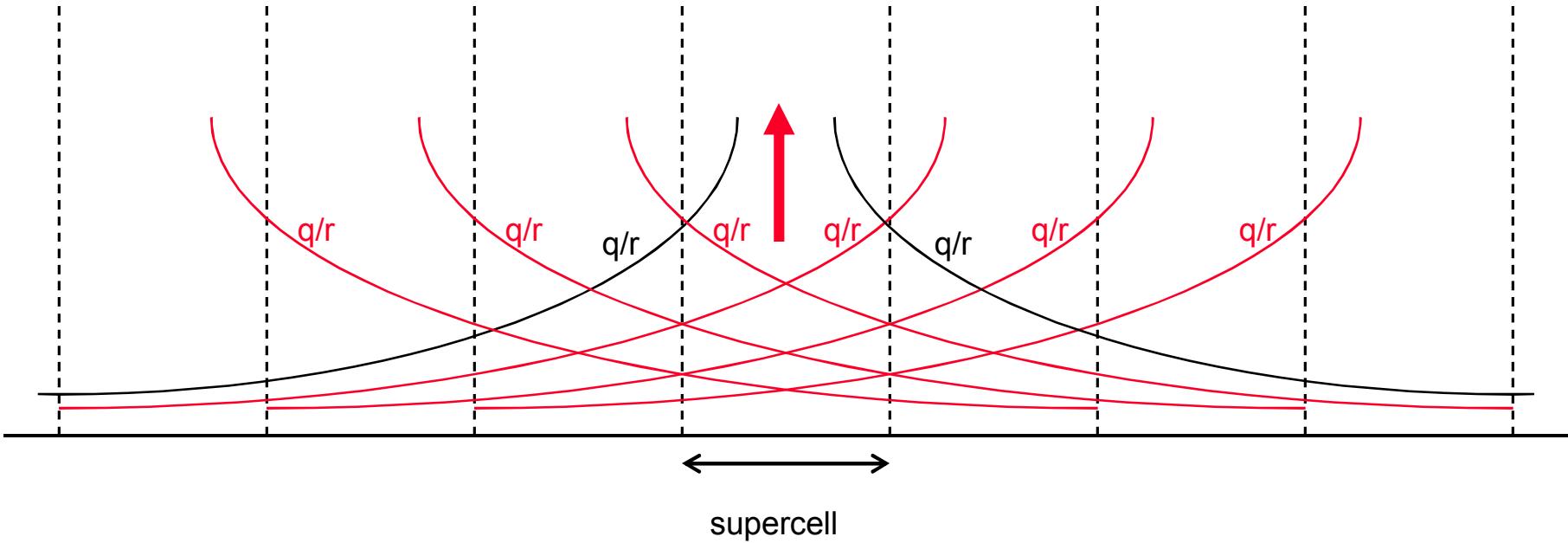
Whence the divergence?



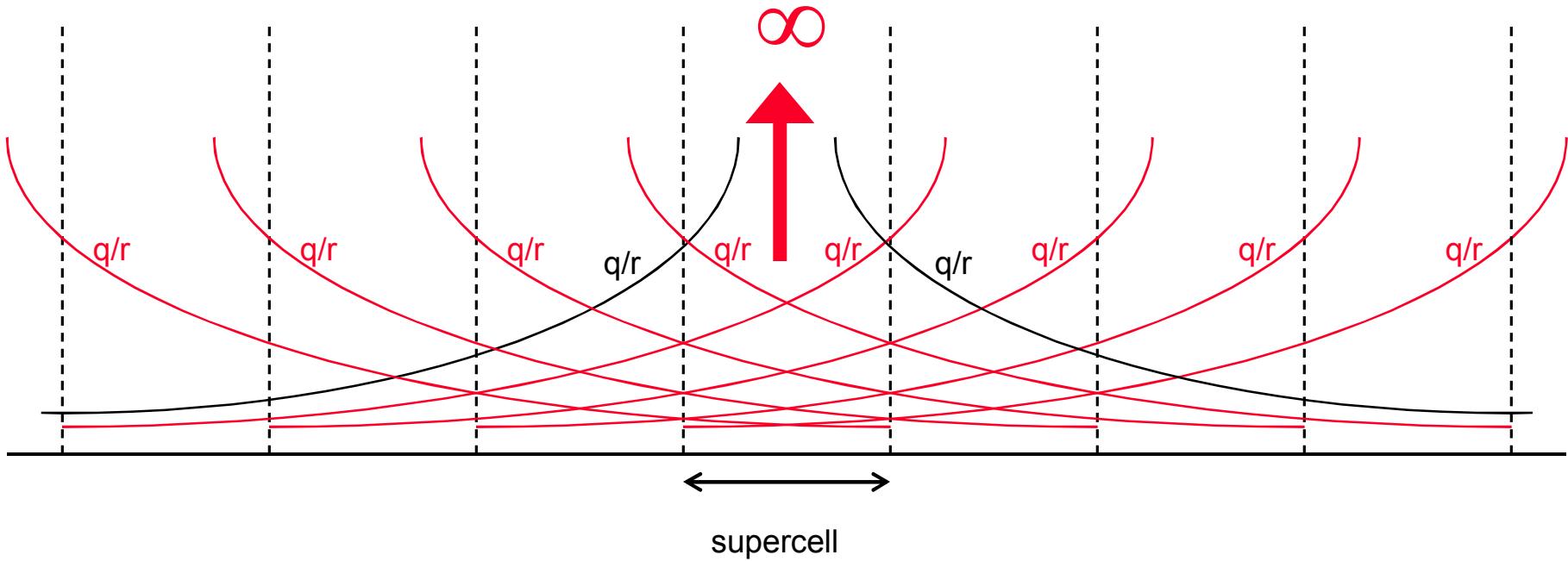
Whence the divergence?



Whence the divergence?



Whence the divergence?

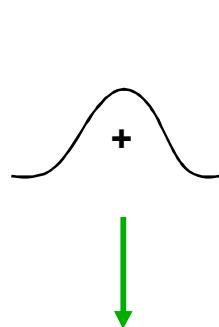


Divergence arises from infinite-ranged q/r potentials from periodic images

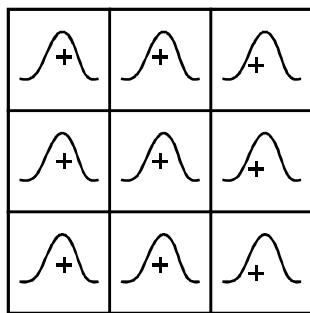
Divergence is not flat

Net charge boundary conditions - jellium

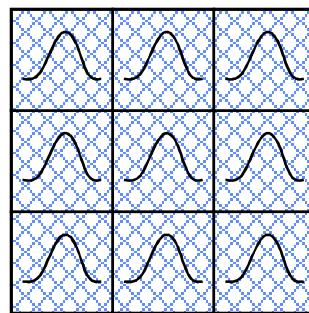
Take isolated charge density...



create cubic supercell ...

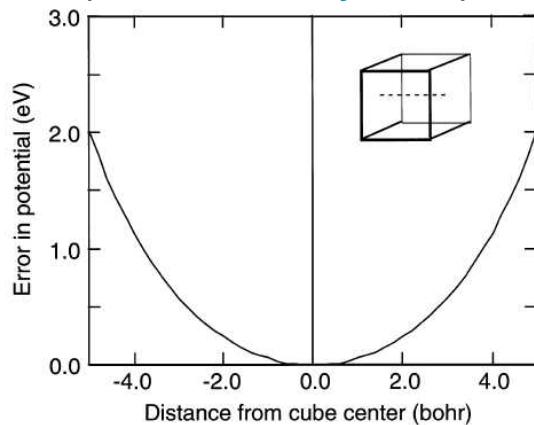


neutralize with "jellium"



Solve Poisson Equation for potential using periodic boundary conditions

Compare exact and jellium potential



Potential error goes as $1/L$ (length)!

L(au)	Cell size (Si)	Median Error(eV)
10.2	8	2.0 eV
20.4	64	1.0 eV
30.6	216	0.67 eV
40.8	512	0.50 eV
51.0	1000	0.40 eV

Error in electrostatic potential over volume of supercell

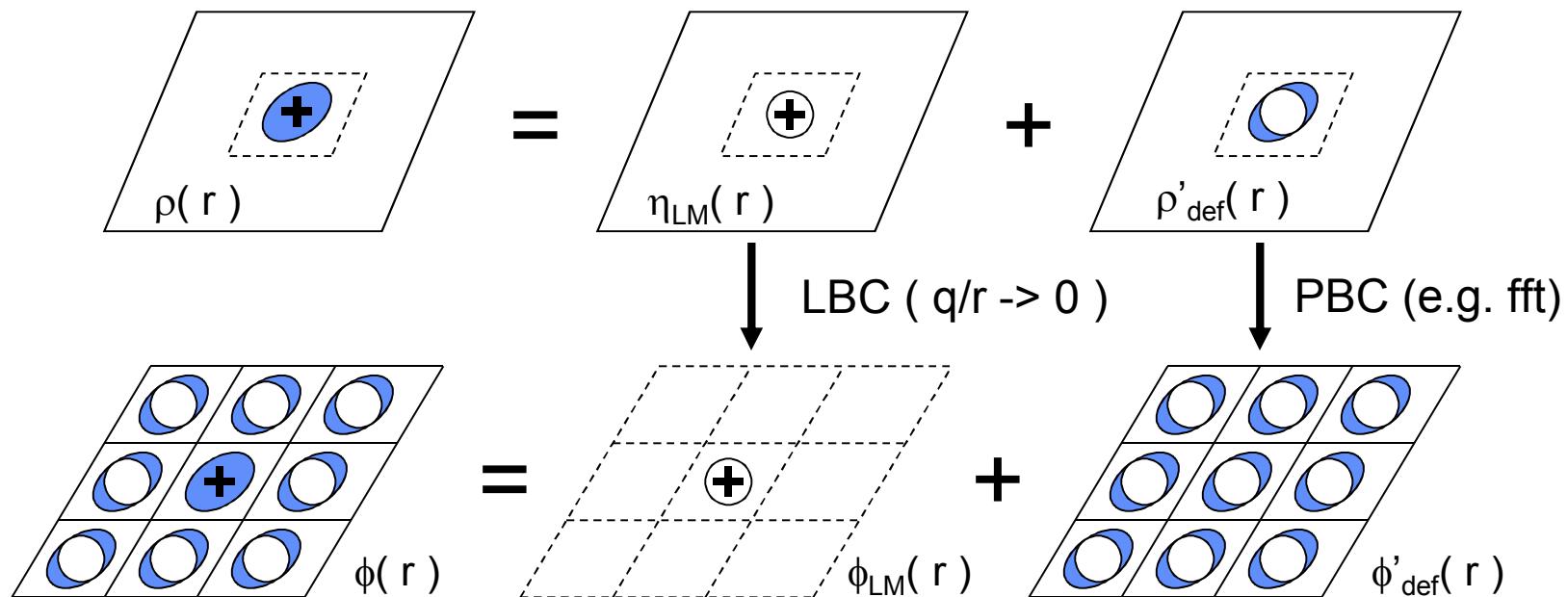
Si band gap: 1.2 eV (expt.), 0.5 eV (DFT)

Standard jellium method has large $O(1/L)$ error in potential. Propagated into density distribution and into energy.

Local Moment CounterCharge (LMCC)

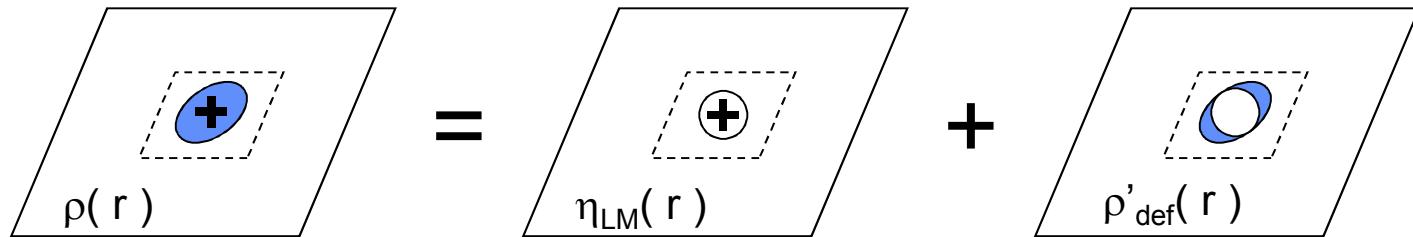
[P.A. Schultz, PRL 84, 1942 (2000)]

- Solution of Poisson Equation is linear in the density
- LMCC: split total density $\rho(r)$ into two pieces ...
 - (1) model local density $\eta_{LM}(r)$ matching multipole (charge) of $\rho(r)$
 - (2) remainder (momentless) density $\rho'_{def}(r) = \rho(r) - \eta_{LM}(r)$



Gives proper $r \rightarrow \infty$ asymptotic boundary condition
 Avoids (not ignores!) Coulomb divergence

A practical method for LMCC



Requirements:

- (a) $\eta_{LM}(r)$ contains the local moments(charge) to be solved
- (b) $\eta_{LM}(r)$ is spatially slowly varying — fft-able
- (c) $\eta_{LM}(r)$ is entirely localized within cell — define vacuum
- (d) potential $\phi(r)$ associated with $\eta_{LM}(r)$ is easily evaluated

Usual suspects:

- point charges violate (b)
- Jellium (flat background) violates (a), (c), and (d)

One (not unique) solution: sum of Gaussians: $\eta_g(r) = \exp(-\alpha r^2)$

$$\eta_{LM}(r) = \sum c_g \eta_g(r - R_g)$$

For charged system: one gaussian

For dipole: pair of gaussians

Quadrupoles and above neglected (good to $O[L^{-5}]$)

Charged cell convergence - Jellium method

PHYSICAL REVIEW B

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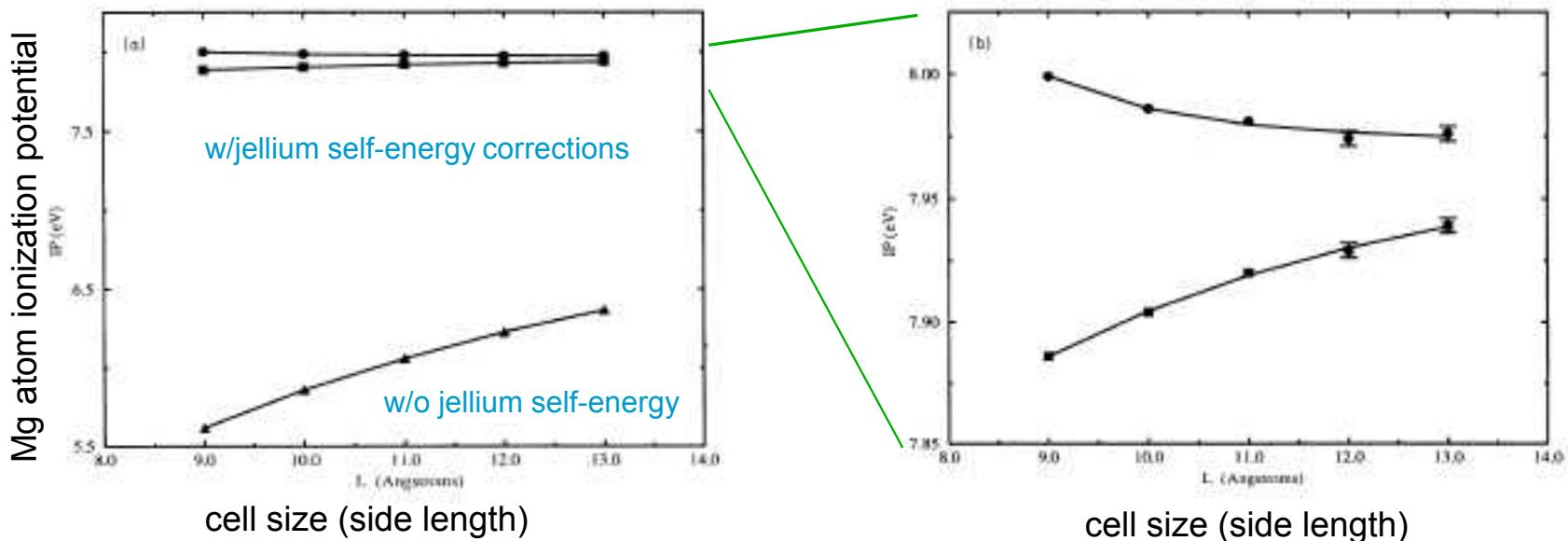
Periodic boundary conditions in *ab initio* calculations

G. Makov and M. C. Payne

Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, United Kingdom

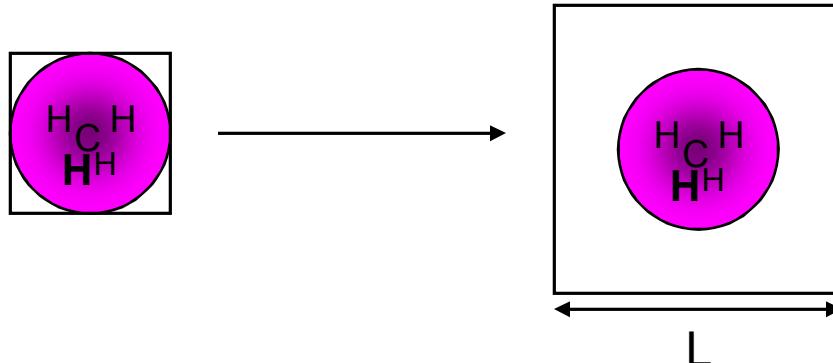
(Received 19 July 1994)

Figure 3



Variation in computed total energy due to incorrect charge potential

Charged cell convergence - LMCC method



Charged, no dipole:



$L = 18.0 - 30.0 \text{ bohr (9.5-15.9 \AA)}$ IP varies $< 10^{-5} \text{ eV}$

Dipole, no charge:



$L = 16.8 - 30.0 \text{ bohr (8.9-15.9 \AA)}$ TE varies $< 10^{-5} \text{ eV}$

Dipole, charge:



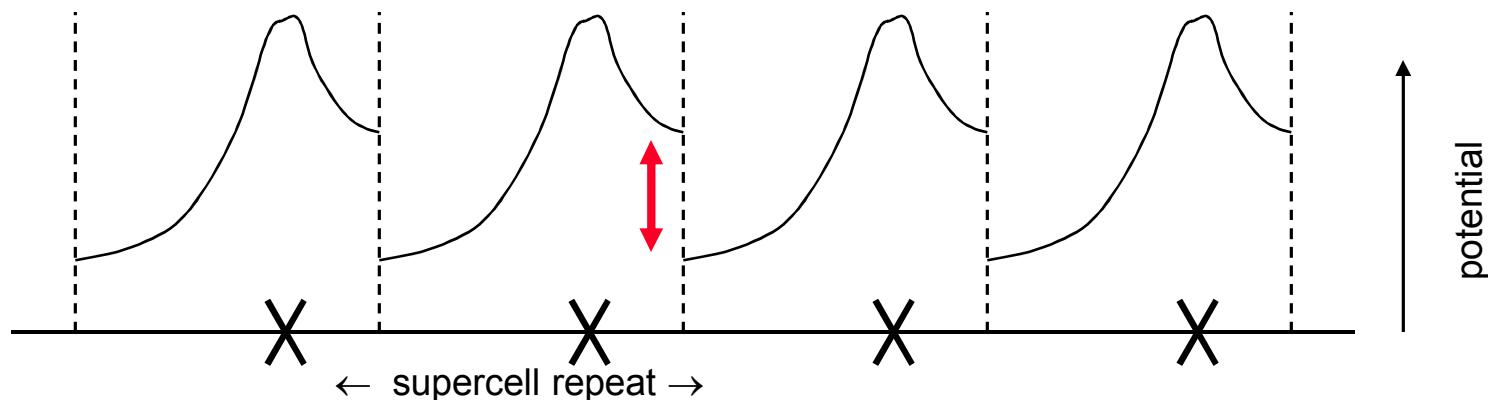
$L = 18.0 - 30.0 \text{ bohr (9.5-15.9 \AA)}$ EA varies $< 10^{-3} \text{ eV}$

Total energy, levels, i.e. full Hamiltonian are all immediately converged.
 -> electrostatic potential correctly represented by LMCC, not just energy

P.A. Schultz, PRB **60**, 1551 (1999)

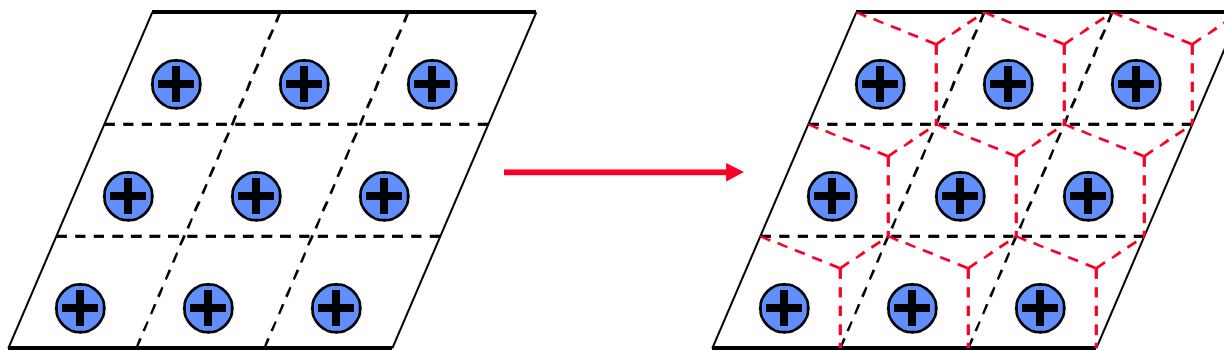
LMCC potential in bulk systems

What is the problem?



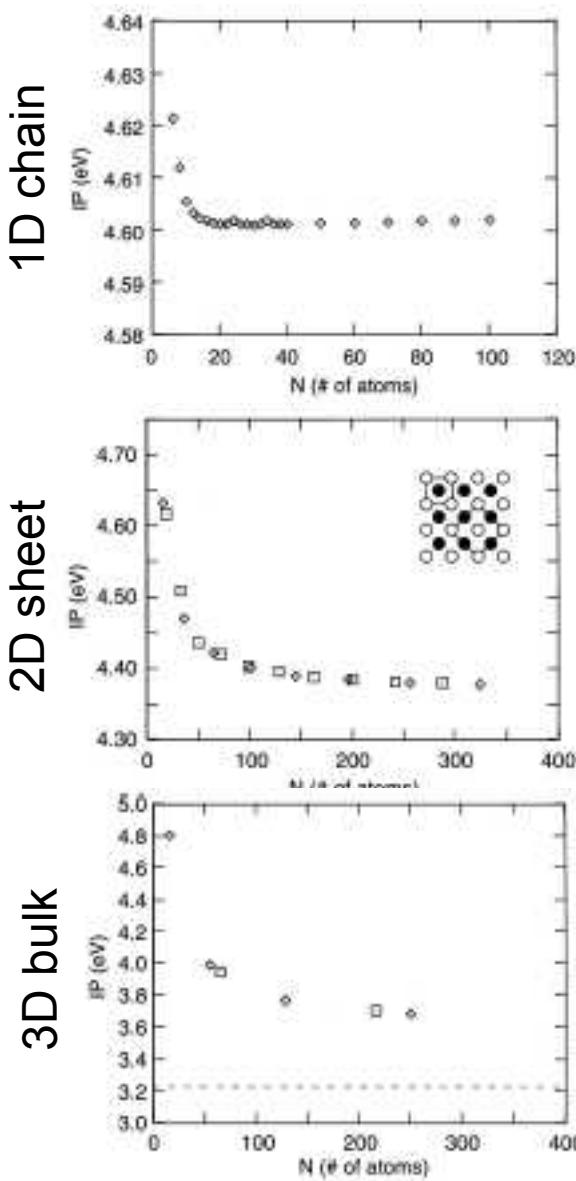
Discontinuity in potential from LMCC at supercell boundary!

The solution: Wigner-Seitz cells around LMCC positions



With WS local volume, LMCC potential is continuous

LMCC: NaCl, Cl vacancy ionization



1D: \leftarrow \rightarrow

Supercell size dependence due to polarization.
 Larger supercell \rightarrow more polarization
 Apparent L^{-3} scaling = 1D classical dielectric screening

2D: single-layer 2D square sheet (polar&non-polar)

Apparent L^{-2} scaling = 2D classical dielectric screening

Insensitive to cell type, polar vs. non-polar

3D: bulk-layer 3D square sheet (fcc&sc cells)

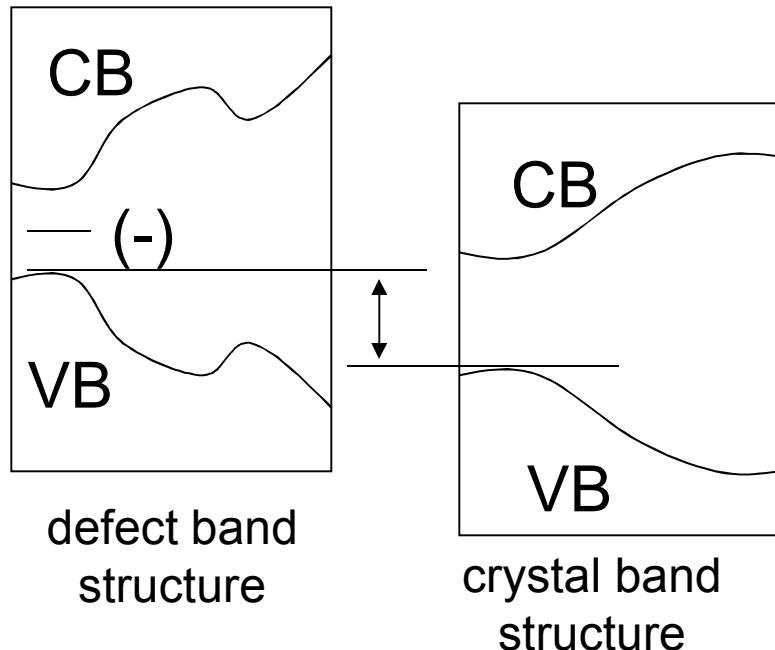
Apparent L^{-1} scaling = 3D classical dielectric screening
 Strictly screening due to large supercell volume
 Insensitive to cell shape

How do you set an energy zero for charge?

Kleinman [PRB 24, 7412 (1981)]: cannot do it in bulk!

Garcia, Northrup, Van de Walle, others: empirical band alignment

- take band feature (e.g., VB, CB,) in defect calculation and “align” with similar feature in bulk calculation



Problems with standard shifts:

1. Which feature? CB,VB top/bottom/c.m.?
2. Defect modifies bands - no clean state
3. Band gap problem: CB/VB dubious, too
4. Band bending by charge
-> unknown uncertainty

Empirically, standard scheme no better than “few tenths of eV”

Garcia, Northrup PRL 74, 1131 (1995)

The electron chemical potential μ_e

- Standard E_{form} of charged defects needs electron reservoir:

$$E_{\text{form}}(q) = E_{\text{defect}}(q) - E_{\text{xtal}}(0) - \sum N_i \mu_i + q \mu_e$$

linked

- Supercells with charge: $\phi_{\text{def}}(r) = \phi_{\text{pbc}}(r) + C_{\text{def}}$

Periodic potential $\phi_{\text{def}}(r)$ only known to within a constant C_{def}

$C_{\text{def}} = \text{fcn}\{\text{defect type, configuration, cell shape, cell size, ...}\}$

$E_{\text{defect}}(q)$ has qC_{def} term in its internal energy

- Standard ad hoc workarounds unsatisfactory - unquantitative

- matching VB,CB edge, band structure features, average potentials ...

- Issue: renormalizing infinities, defect modified bands, band-bending, ...

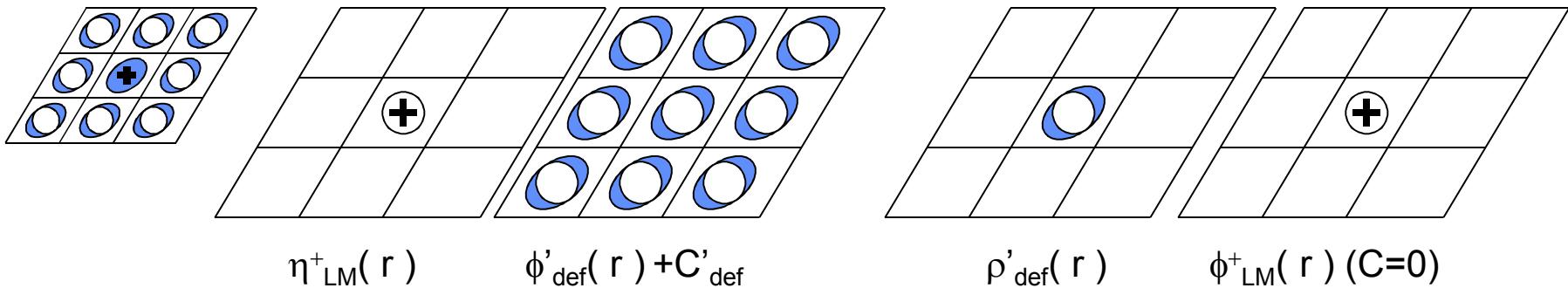
- calibration uncertainty of “few tenths of eV” (Garcia & Northrup) - best case

Needed a more rigorous scheme to fix electron reservoir

Chemical potential shift: Locating a fixed μ_e

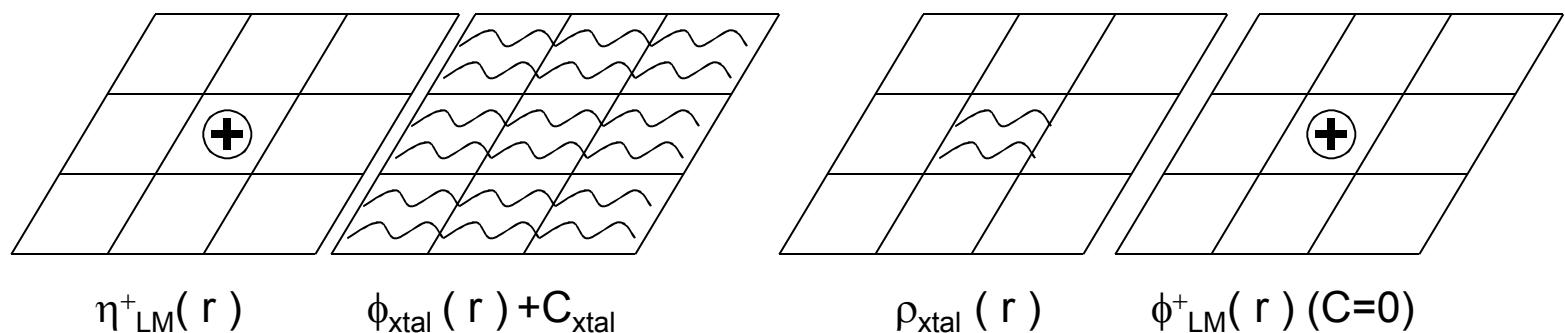
Replace interaction of net charge with periodic defect potential ...

$$E_{\mu 0} = - \int dr \eta^+_{LM} (\phi'_{def} + C'_{def}) + \int_{UC} dr \phi^+_{LM} \rho'_{def}$$



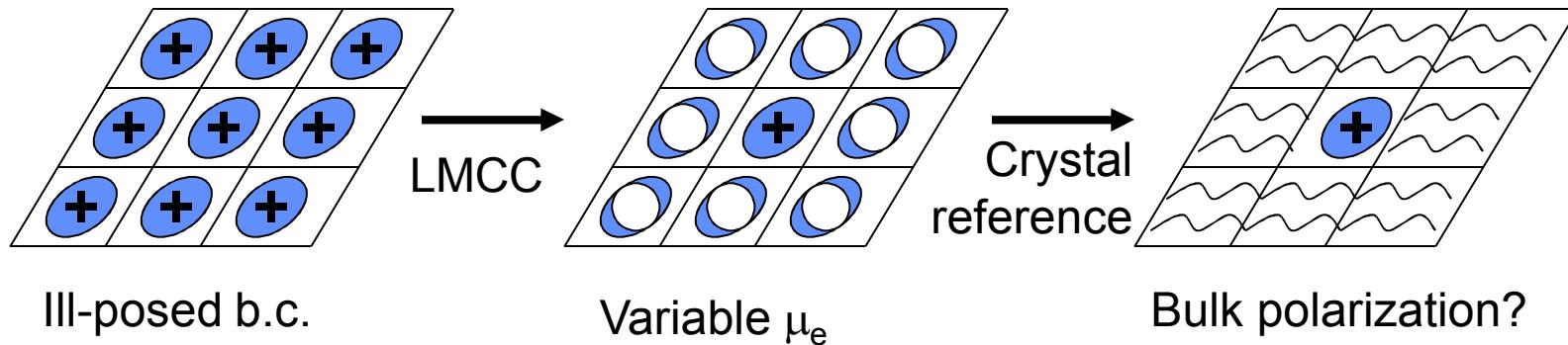
... with crystal:

$$+ \int dr \eta^+_{LM} (\phi_{xtal} + C_{xtal}) - \int_{UC} dr \phi^+_{LM} \rho_{xtal}$$

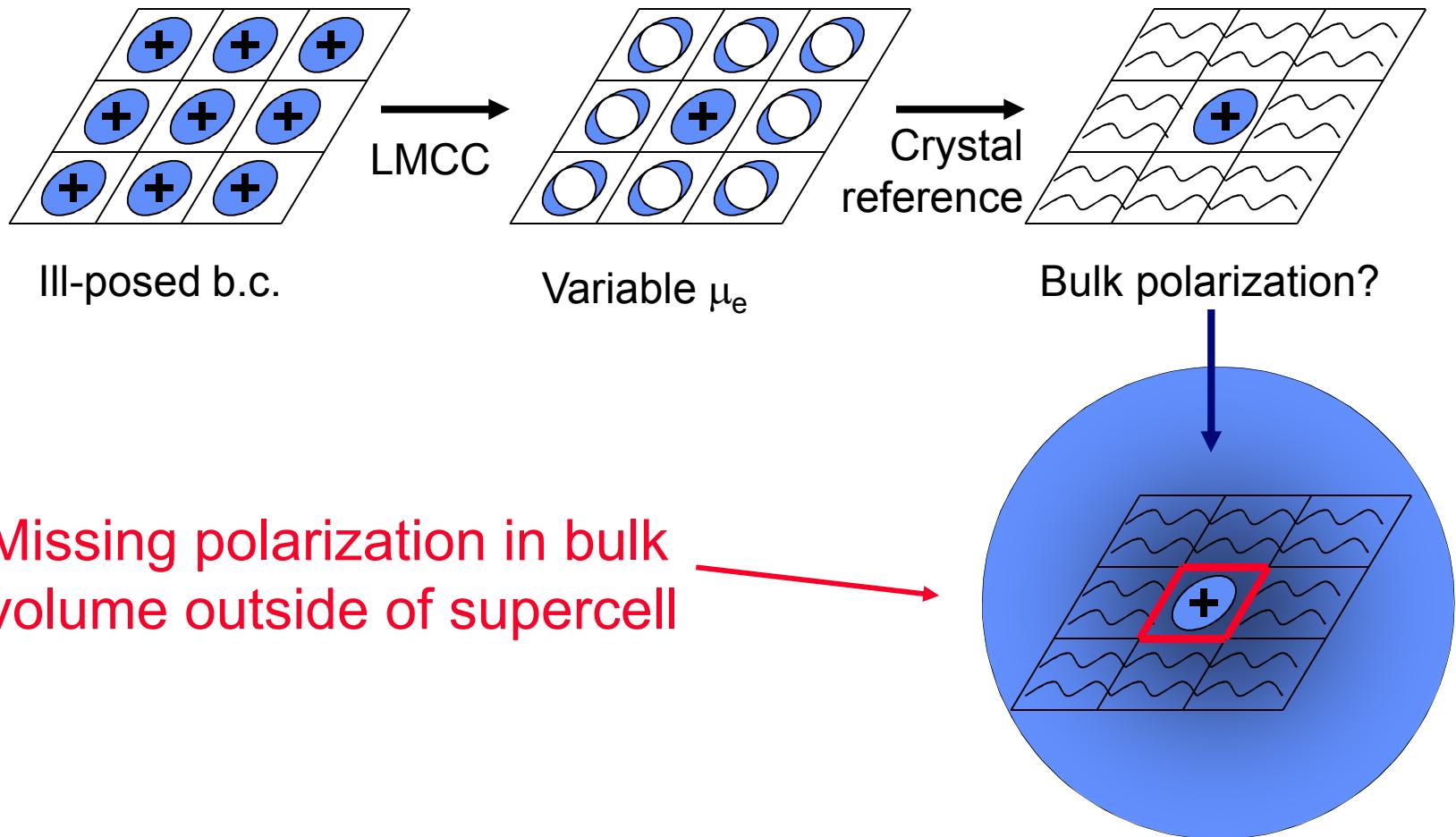


Replace **variable** defect cell C'_{def} , with **fixed** crystal C_{xtal} reference
Not a rigid shift - a valid common electron reservoir for all defects

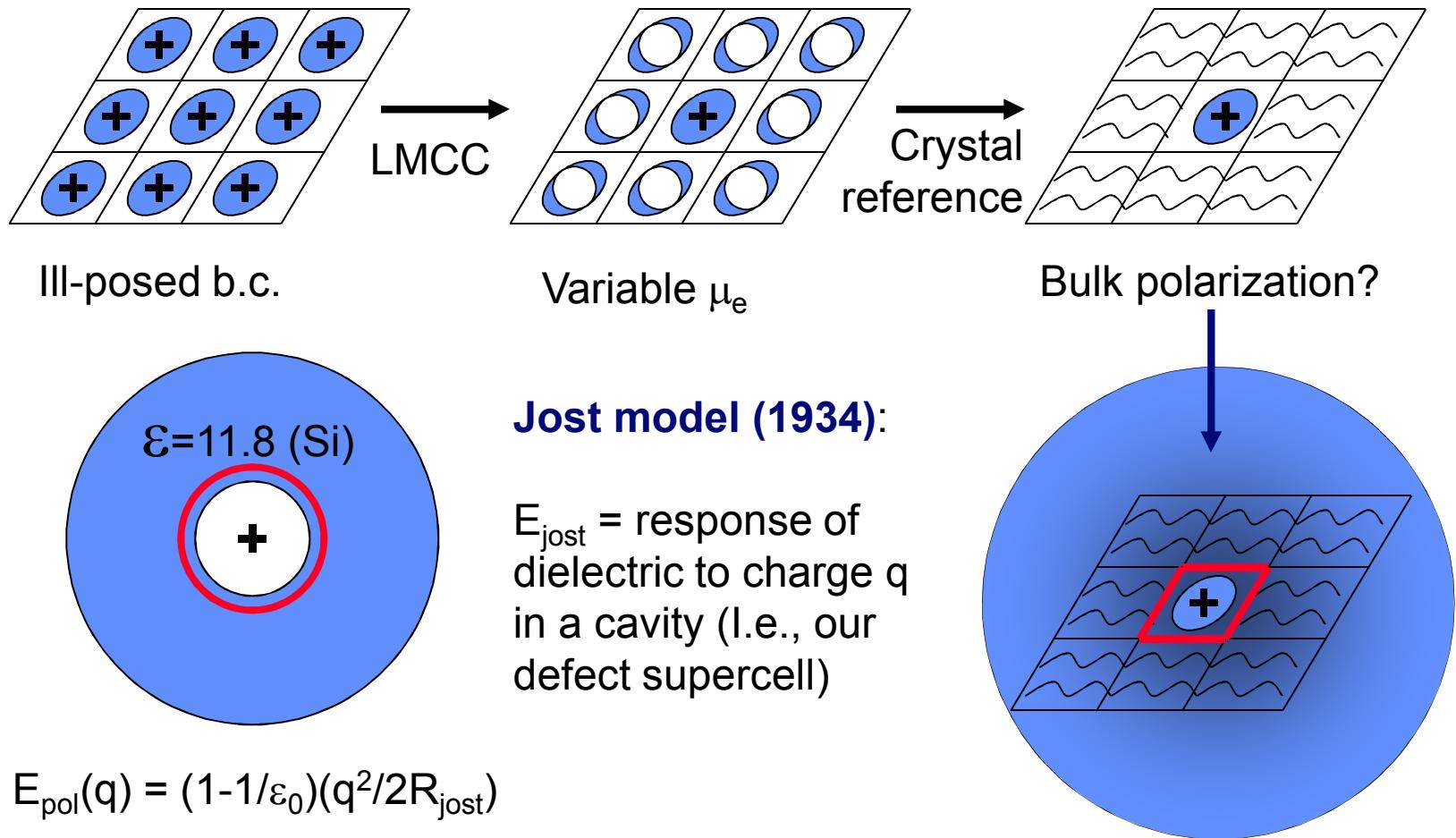
Bulk polarization in a dielectric medium



Bulk polarization in a dielectric medium

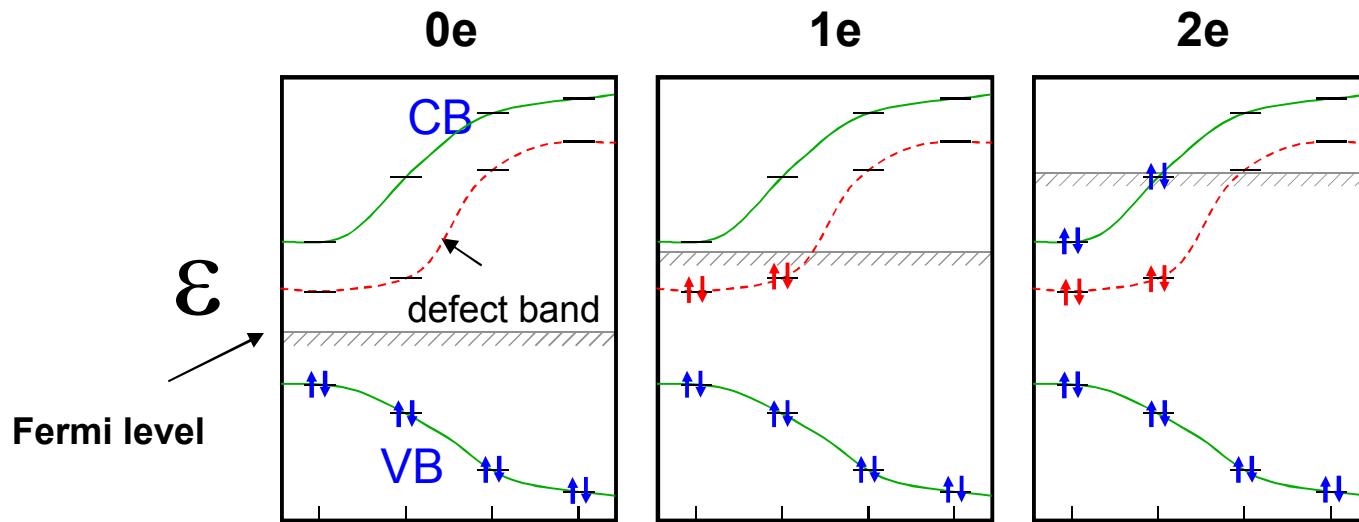


Bulk polarization in a dielectric medium



Bulk polarization included through classical dielectric theory

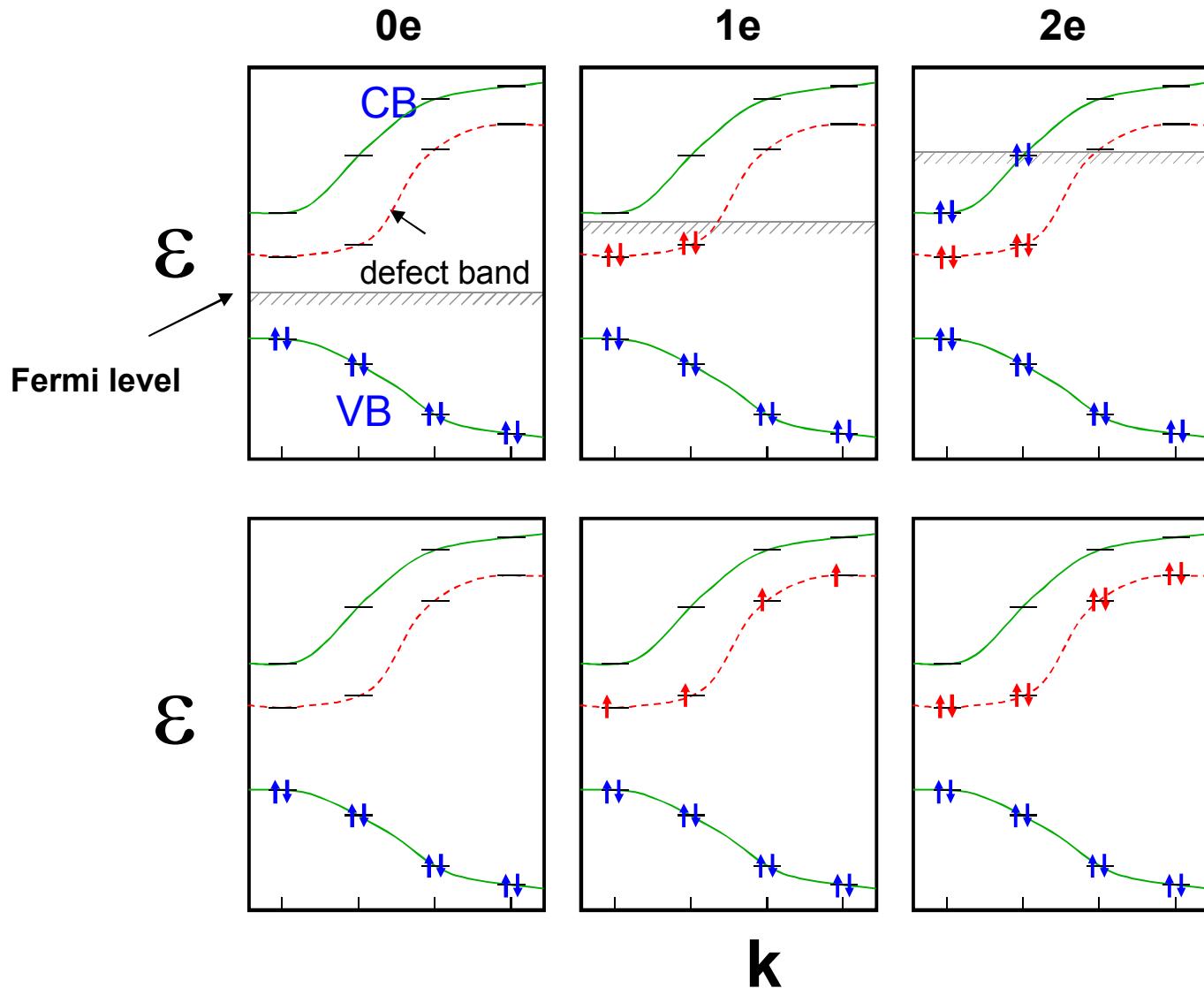
Defect banding



Standard methods:
metallic,
poor model
of defect.

k

Defect banding: Discrete Defect Occupation



DFT Supercell issues

• Boundary conditions - how to handle net charge

- need to eliminate divergence, install correct q/r behavior of potential
- errors in jellium local electrostatic potential: 1.0 eV/64-site Si, fall off as $1/L$
- LMCC method: Peter A. Schultz, PRL 84, 1942 (2000)

• Chemical potential for electrons

- want transition energies, (-/0), (0/+), need rigorous chemical potential
- errors in standard valence band shift schemes: “few tenths” of eV
- developed a scheme to *fix* electron reservoir for defect supercells

• Defect level dispersion

- defects interact, discrete defect states become bands, overlap CB/VB
- errors made by interacting defects - inappropriate model for defect
- Discrete Defect Occupation scheme to populate states

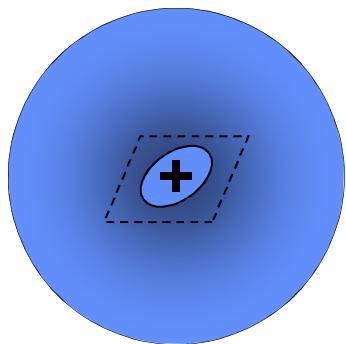
• Bulk response to local charge - finite size effects

- supercell has small finite volume, missing bulk polarization response
- bulk polarization is biggest number in problem - need to get it right
- modified simple Jost model (from 1934!) to model bulk polarization

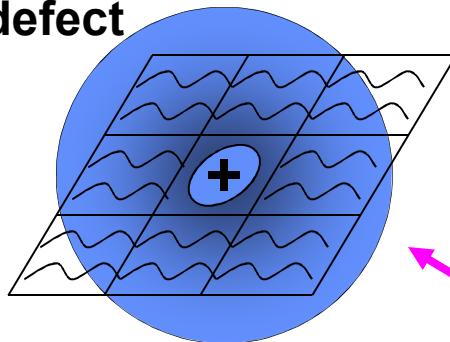
And then ... what about DFT's band gap problem?

A supercell theory for defect energies

Target system:
isolated defect



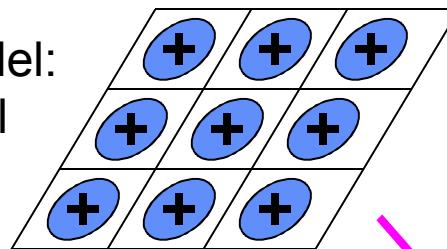
Computational
model for
isolated defect



+ Discrete Defect
Occupation
(defect banding)

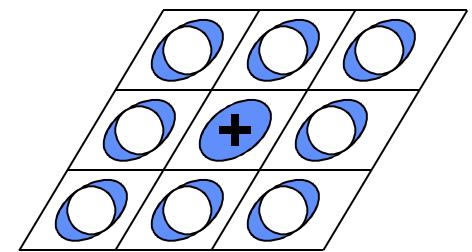
Jost Bulk
polarization

Standard
DFT model:
Supercell

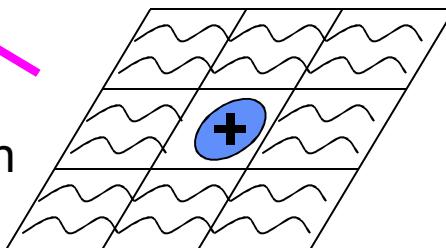


LMCC to fix
boundary
conditions

Finite Defect Supercell Model



Crystal embedding
to fix μ_e



FDSM gives robust computational model for charged defect

Charged Defect Formation Energy

Finite Defect Supercell Model Formation Energy

$$E_{\text{form}}(q) = E_{\text{defect}}(q) - E_{\text{xtal}}(0) - \sum N_i \mu_i + E_{\mu_0}(q) + E_{\text{pol}}(q)$$

$E_{\text{defect}}(q)$: DFT energy with LMCC potential

$- E_{\text{xtal}}(0) - \sum N_i \mu_i$: match number of each type of atom

$E_{\mu_0}(q)$: fix chemical potential μ_e to common electron reservoir

$E_{\text{pol}}(q)$: bulk polarization response

Defect level calculation

$$\Delta E(q/q-1) = E_{\text{form}}(q) - E_{\text{form}}(q-1)$$

Need to set spectrum vs. VB/CB by single marker.

All defect levels for all defects then fixed by continuity.

The computational method

General purpose DFT code: SeqQuest: <http://dft.sandia.gov/Quest>

Molecules, 2D slab, 3D Bulk

Metals and insulators (complex k-points)

Multiple density functionals: LDA and GGA/PBE

Norm-conserving pseudopotentials (“semi-local”)

Well-converged local orbital (Gaussian-based) basis set

Forces, with complete Pulay corrections

Automatic geometry minimization

Tuned, compact implementation

Fast, small, accurate, powerful --- 100's atoms on a desktop

Anatomy of a DFT code

- (1) Create a guess density: $\rho(r) = \sum \rho_{\text{atom}}(r)$
- (2) Construct a Hamiltonian, H , over a basis ϕ_i : $H_{ij} = \langle \phi_i | H[\rho] | \phi_j \rangle$

$$H = T_{\text{kinetic energy}} + V_{\text{nuclear attraction}} + V_{\text{coulomb}}[\rho] + V_{\text{xc}}[\rho]$$

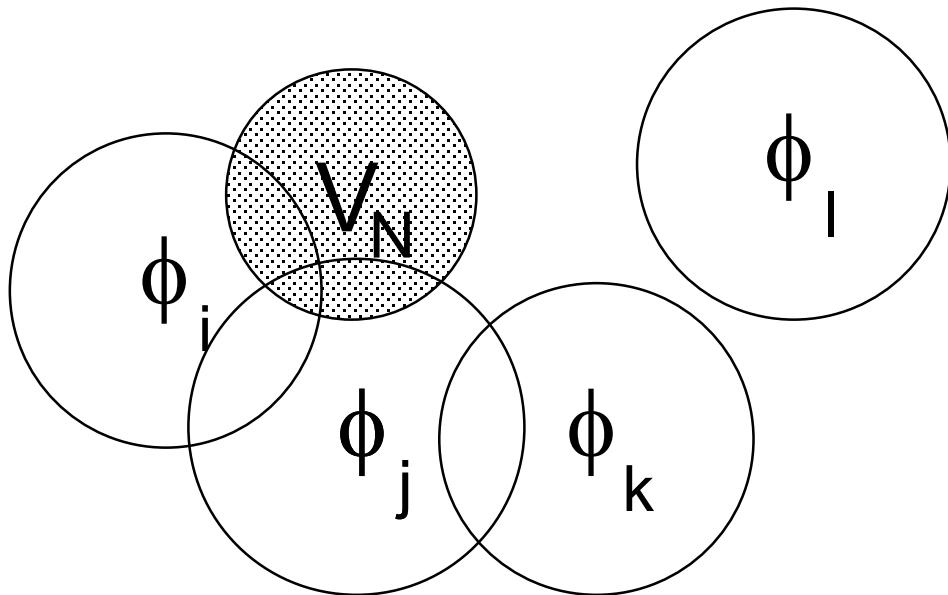
$$V_{\text{nuc}} = \sum V_{\text{atom}}[R] \rightarrow N^3 \text{ Hamiltonian}$$
- (3) Solve for wavefunctions ψ_i :
 $H \psi_i = \varepsilon_i \psi_i \rightarrow N^3 \text{ eigensolve}$
- (4) Compute new density: $\rho(r) = \sum f_i |\psi_i|^2$
- (5) Repeat (2)-(4) until self-consistent

SeqQuest:

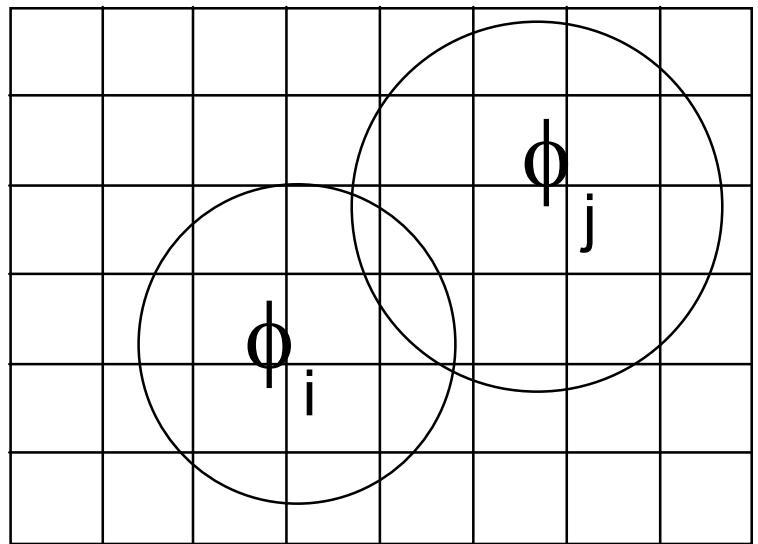
- (1) Map onto a local (Gaussian) orbital basis ... small N
- (2) Reformulate electrostatics ... $O(N)$ Hamiltonian

Quest: two kinds of matrix elements

Analytic local 2 or 3-center:
(iteration-independent setup)

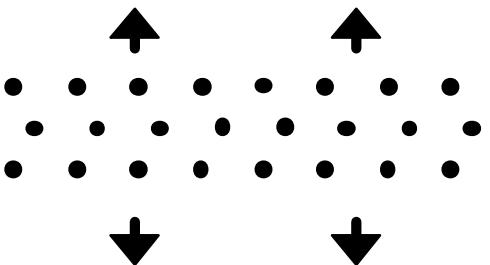


Mesh-based 2-center:
(iteration-dependent scf elements)

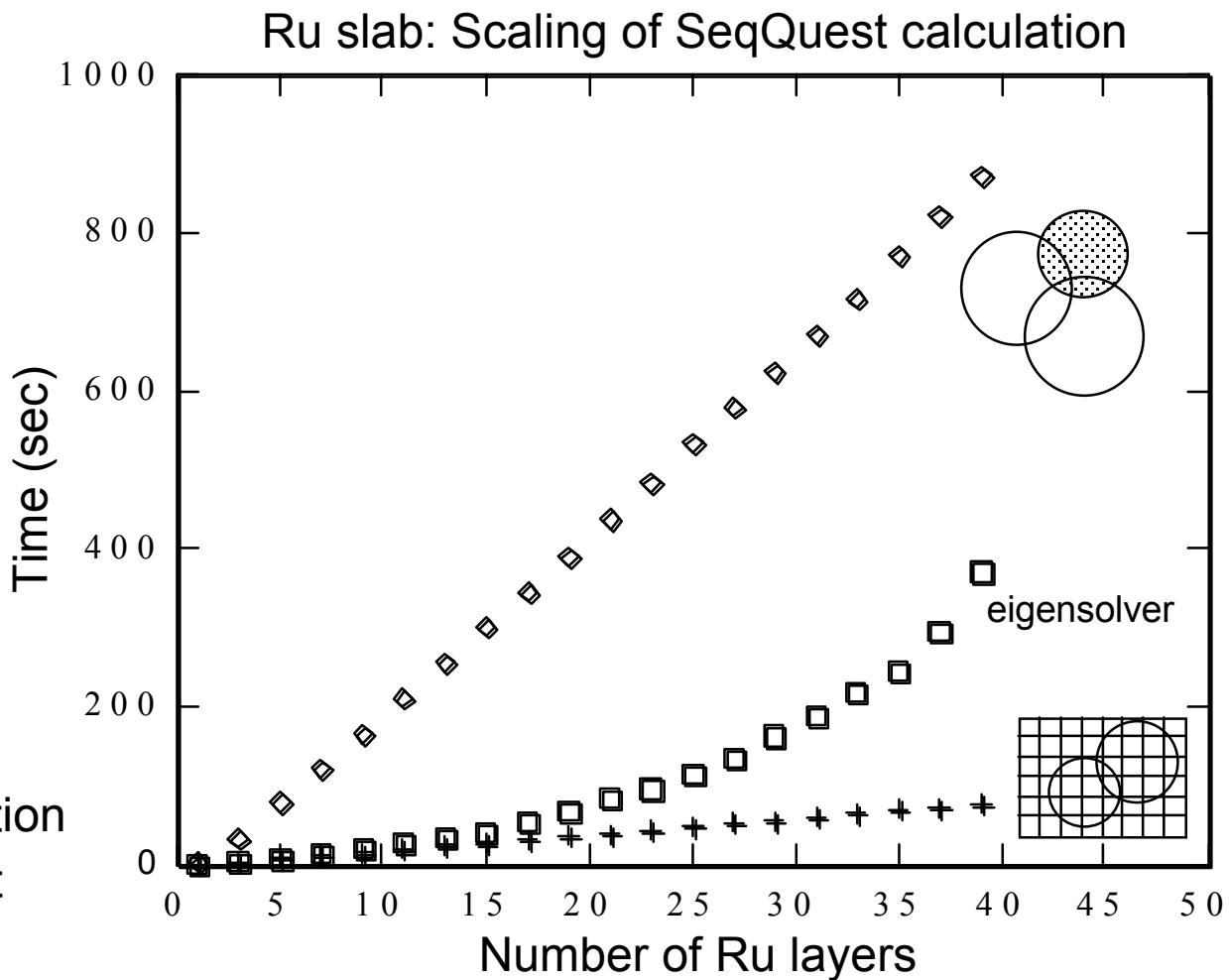


Both are $O(N)$!

Scaling Ru slab



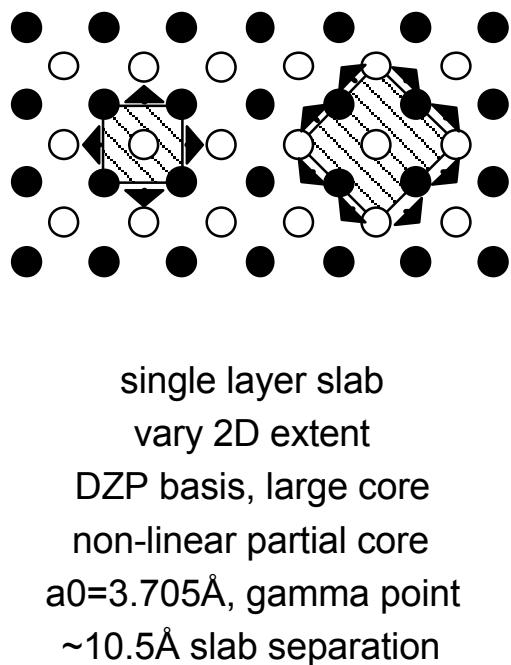
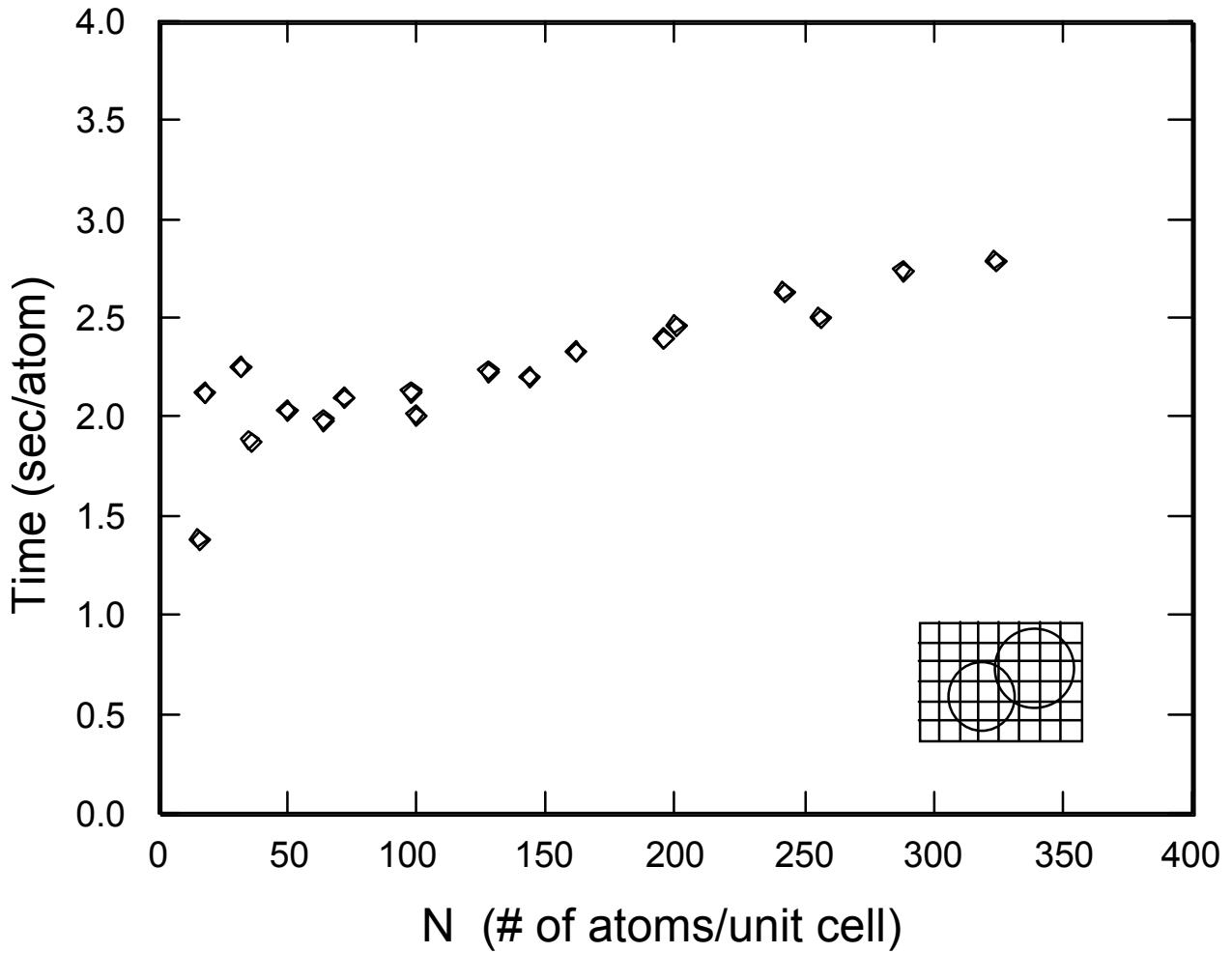
(0001) 1x1 surface
vary layer thickness
 $a_0=2.673\text{\AA}$, $c/a=1.575$
10k/IBZ, $\sim 12\text{\AA}$ slab separation
full DZP basis SeqQuest



Actually achieves $O(N)$, and reaches it quickly

Scaling: NaCl 2D slab

Average (per atom) cost of SCF Hamiltonian



Computational details

Defect supercell calculations

SeqQuest code - periodic, gaussian-basis, pseudopotential code

<http://dft.sandia.gov/Quest>

LDA and GGA-PBE functionals

Full FDSM (LMCC, chemical potential, DDO, bulk screening)

Calculations ranging from 64-site to 512-site supercells

Converged k-point sampling

Final series: 250-site (5x5x5 fcc cell) with 2^3 k -points

Fully relaxed atomic positions

Lattice parameter fixed at theoretical value

LDA: 10.20 bohr (5.40 Å)

PBE: 10.34 bohr (5.47 Å)

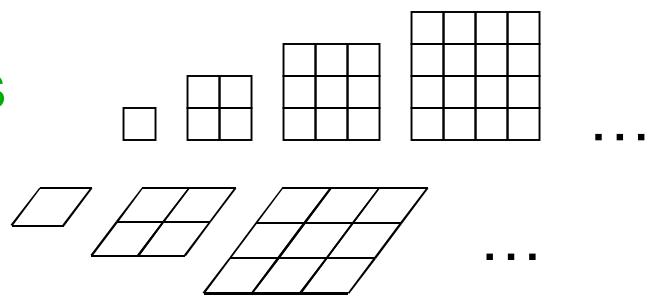
Si supercells and Brillouin Zone sampling

Three regular supercell sequences

cubic: $8N^3$ -sites = 8,64,216,512, ...

fcc: $2N^3$ -sites = 2,16,54,128,250,432, ...

bcc: $32N^3$ -sites = 32,256,1728, ...



Electron states summed over regular discrete grid in reciprocal space, k-points in the “Brillouin Zone”

Density of reciprocal space quadrature inverse to cell size

The larger the cell, the smaller the k-space sample needed

If N_k is dimension of k-space grid, different cells of dimension N and M have formally identical sampling if $NN_k = MM_k$

1x1x1 (8-site) sc cell with 6x6x6 k-sample is formally equivalent to

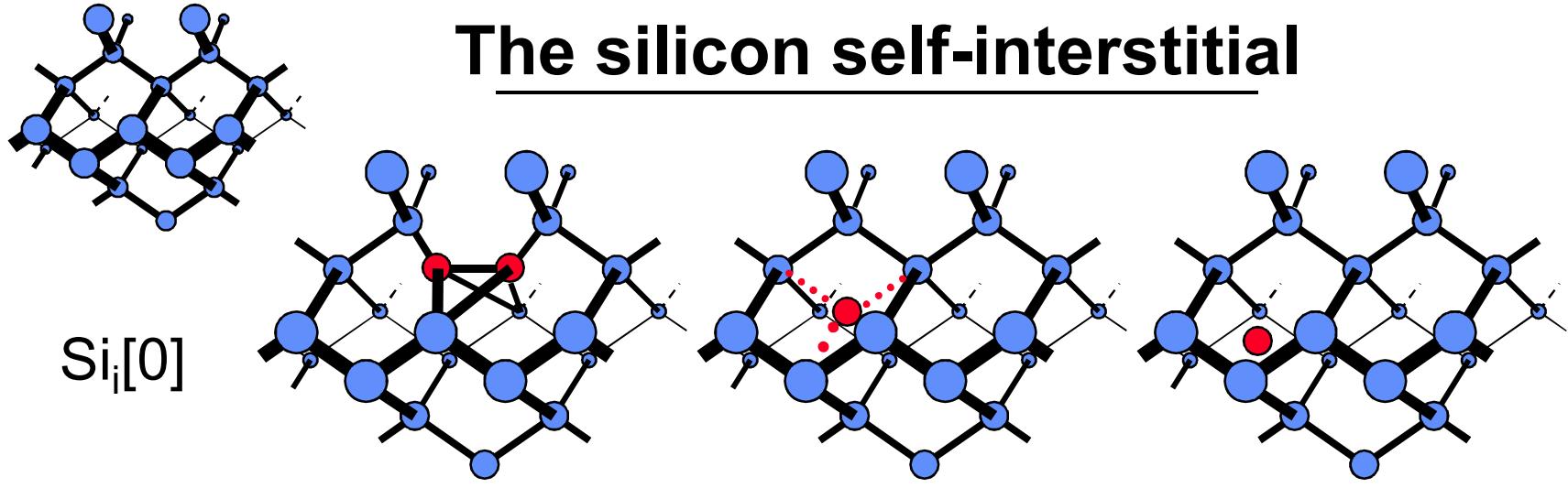
2x2x2 (64-site) sc cell with 3x3x3 k-sample is formally equivalent to

3x3x3 (216-site) sc cell with 3x3x3 k-sample, etc

Computational cost scales as $(\text{site})^3$ with cell size, linearly in k-points.

Desire convergence with smaller cells, k-samples

The silicon self-interstitial



Functional	$E(eV)$	X (110-split)	T (tetrahedral)	H (hexagonal)
LDA		3.37	3.56	3.42
PBE		3.55	3.91	3.62
PW91		3.70	4.09	3.77
AM05		3.16	3.40	3.25

- Ann E. Mattsson, unpublished.

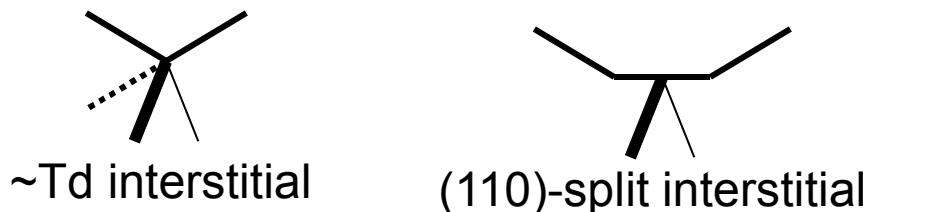
Results for interstitial are sensitive to functional

Note: PBE not the same as PW91 [see: A.E. Mattsson, et al., PRB **73**, 195123 (2006)]

Surface effects! [see: A.E. Mattsson, et al., to be published]

Cell size convergence: silicon self-interstitial

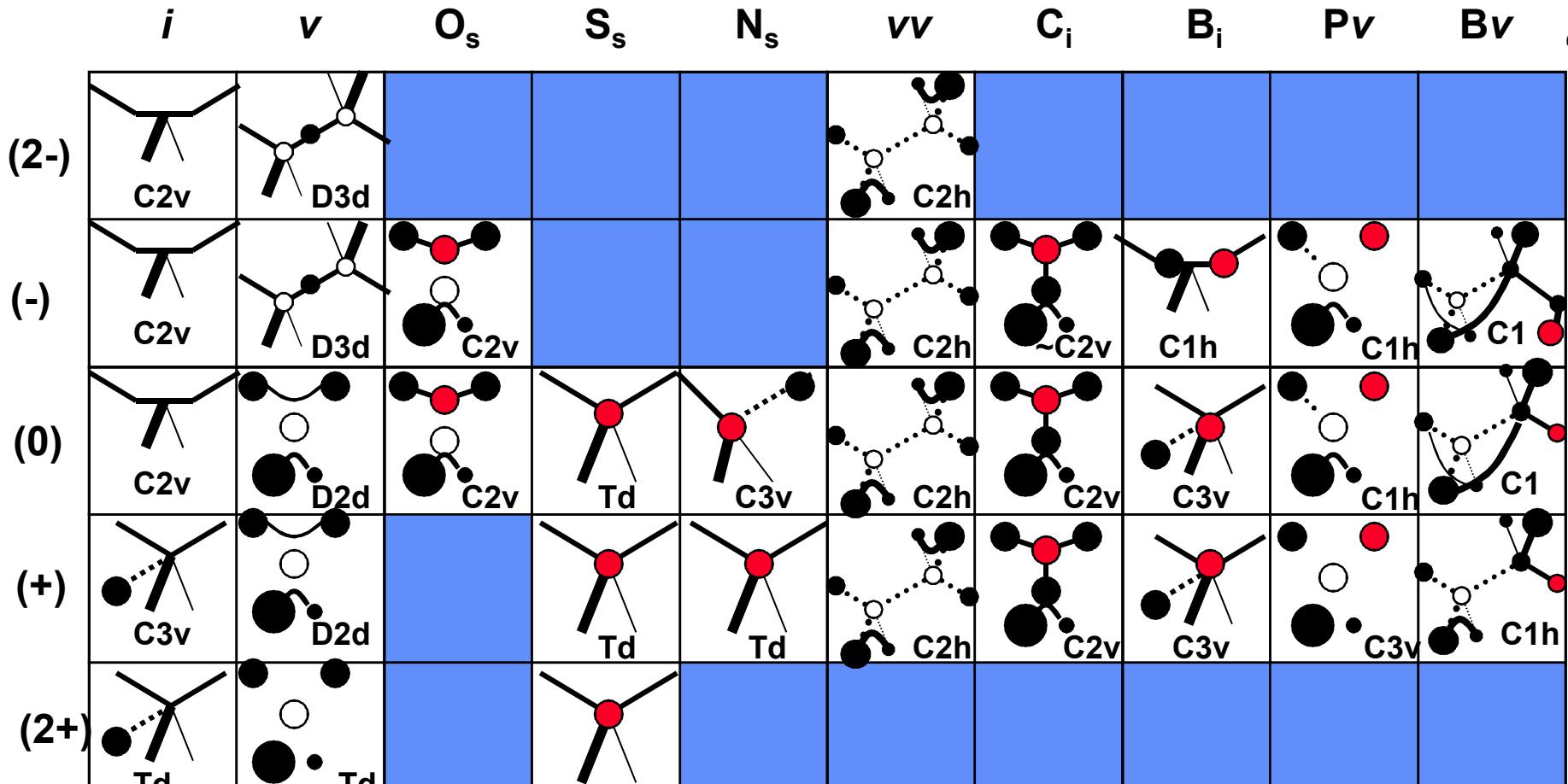
Formation energy (eV)
Si self-interstitial



	Cell	$i(+2)$	$i(+)$	$i(0)$	$i(-)$	$i(-2)$
8×2^3	64	13.49	8.56	3.72	-0.70	-4.88
2×4^3	128	13.35	8.52	3.71	-0.73	-4.94
8×3^3	216	13.48	8.59	3.66	-0.86	-5.16
2×5^3	250	13.34	8.51	3.67	-0.83	-5.14
2×6^3	432	13.34	8.51	x	x	x

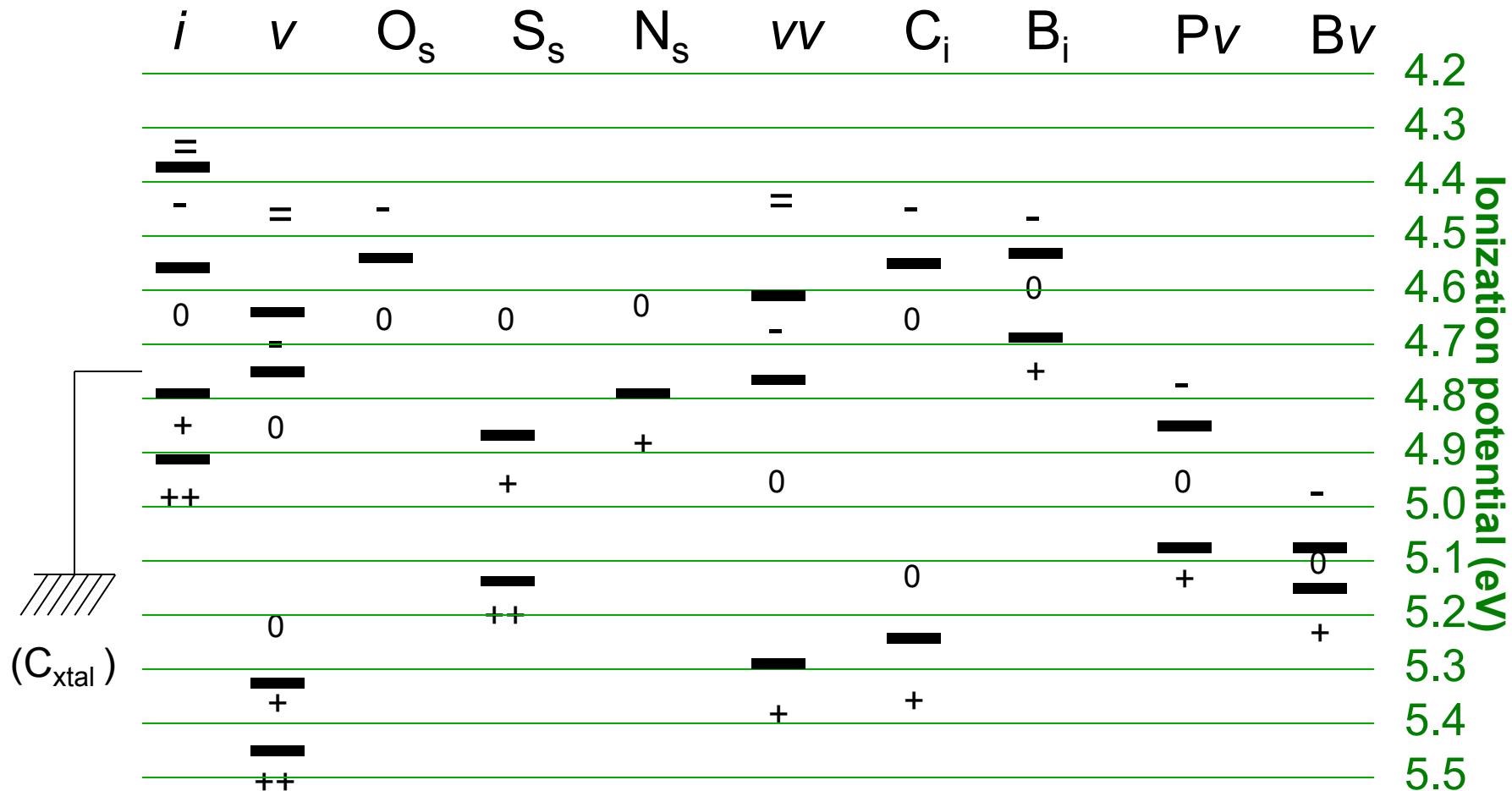
Total electrostatics using proper boundary conditions works!
Convergence for small (250-atom) supercells (except $i(2-)$?).

Si defect structures - DFT/LDA



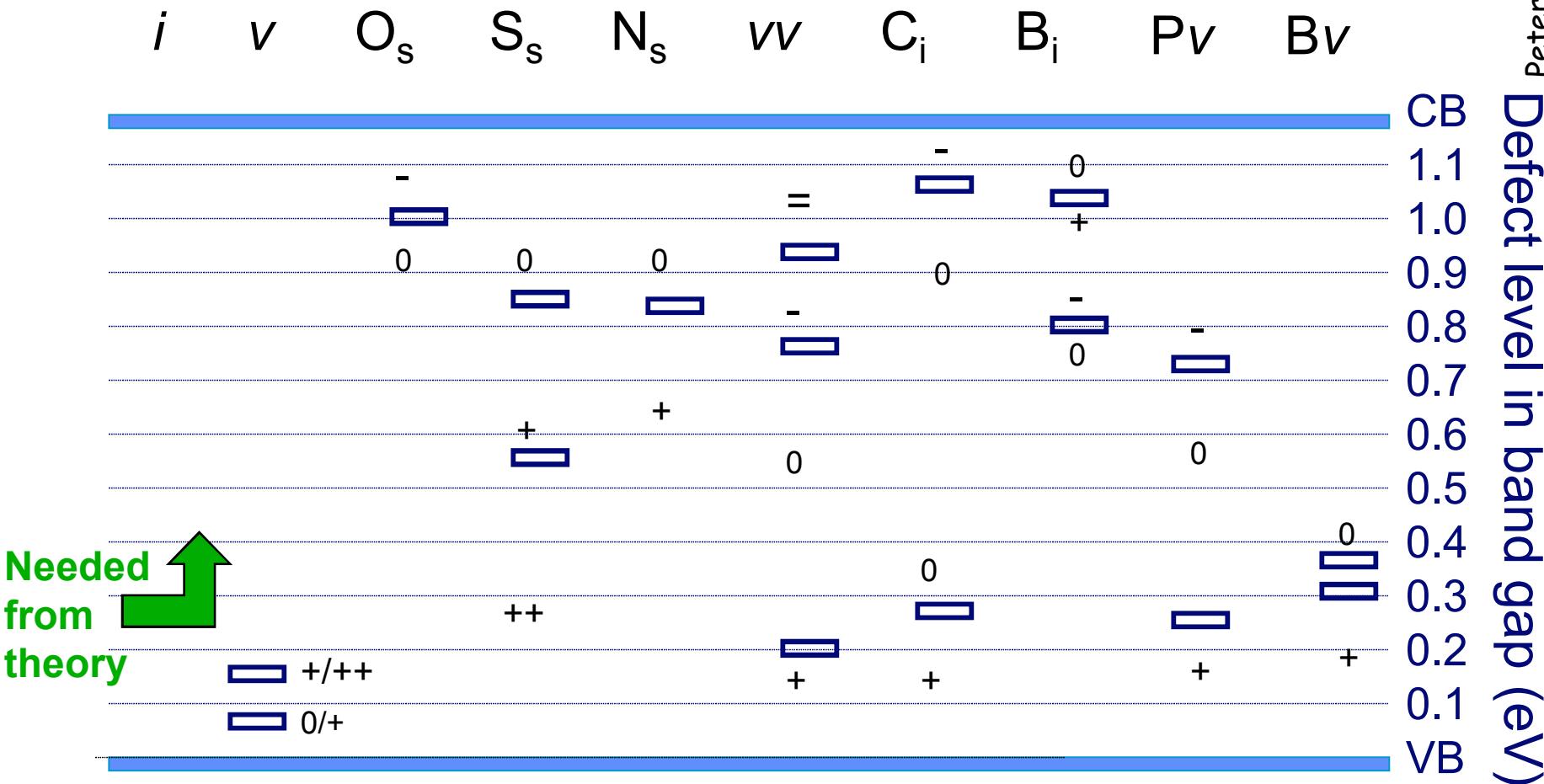
GGA: $C2v < D3d$ for $v(-)$

Si: DFT/LDA defect charge transitions



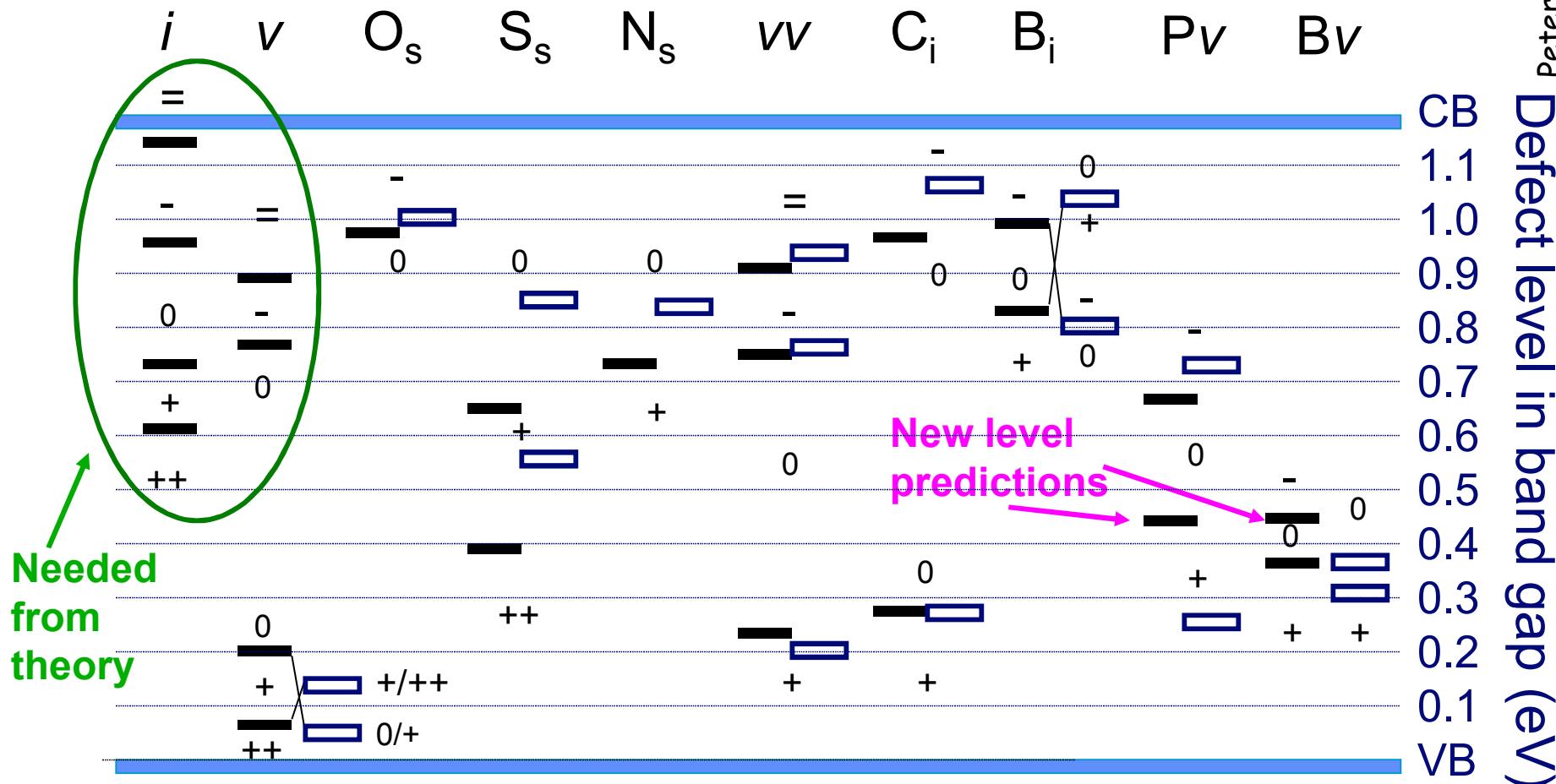
All formation energies grounded to common electron reservoir
 Full width of experimental band gap seen in DFT defect levels

Si: Experimental Levels



Experimental record silent on most important defects!

Si: DFT/LDA vs. Experimental Levels

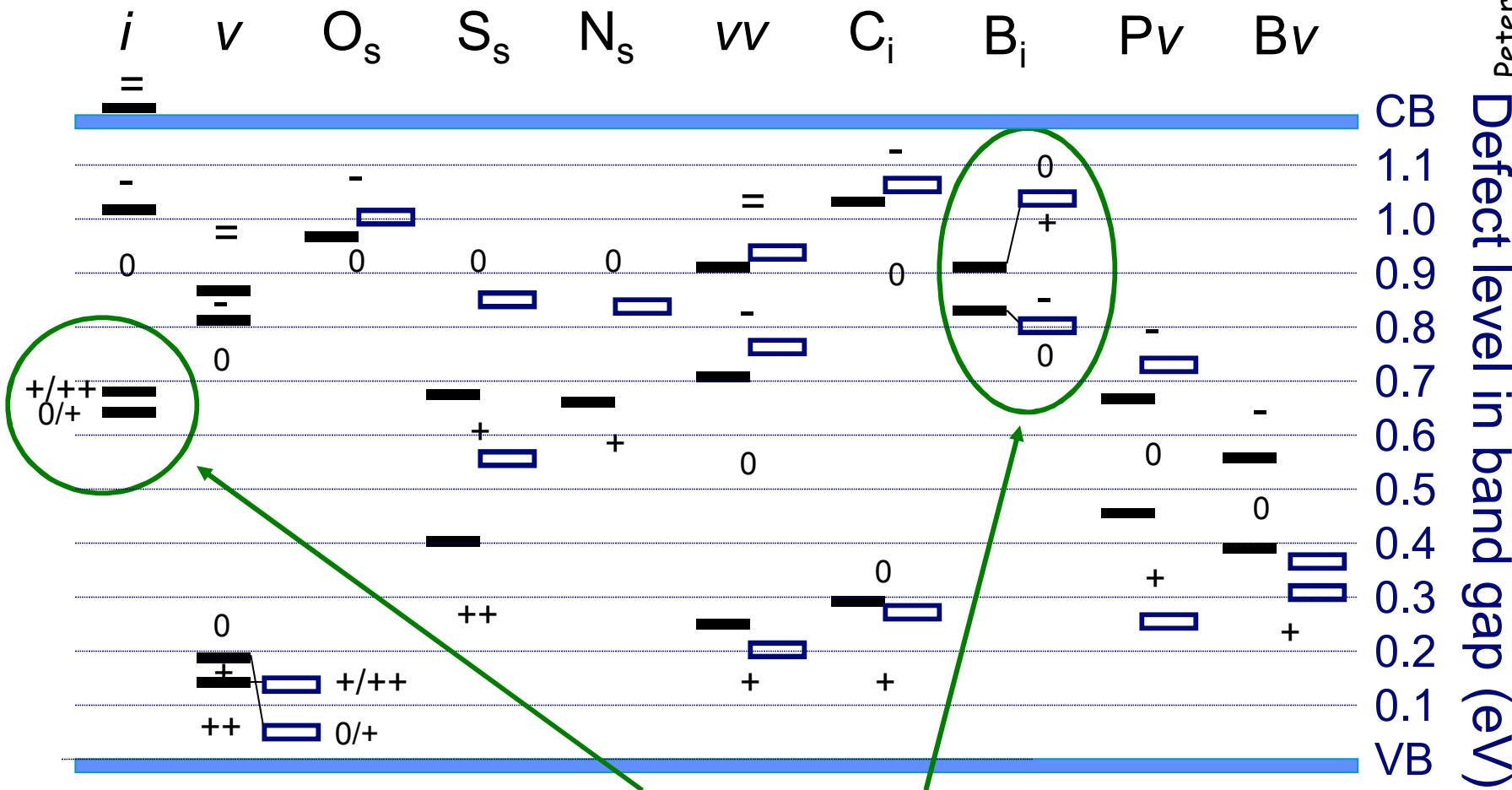


LDA: max error=0.25 eV, mean |error|= 0.08 eV (v-like: 0.04 eV)

Surprises: new charge states for P-v and B-v pair defects

Problems: Boron interstitial (negative-U), vacancy (0+/2+), S(0+/2+)

Si: GGA/PBE vs. Experimental Levels

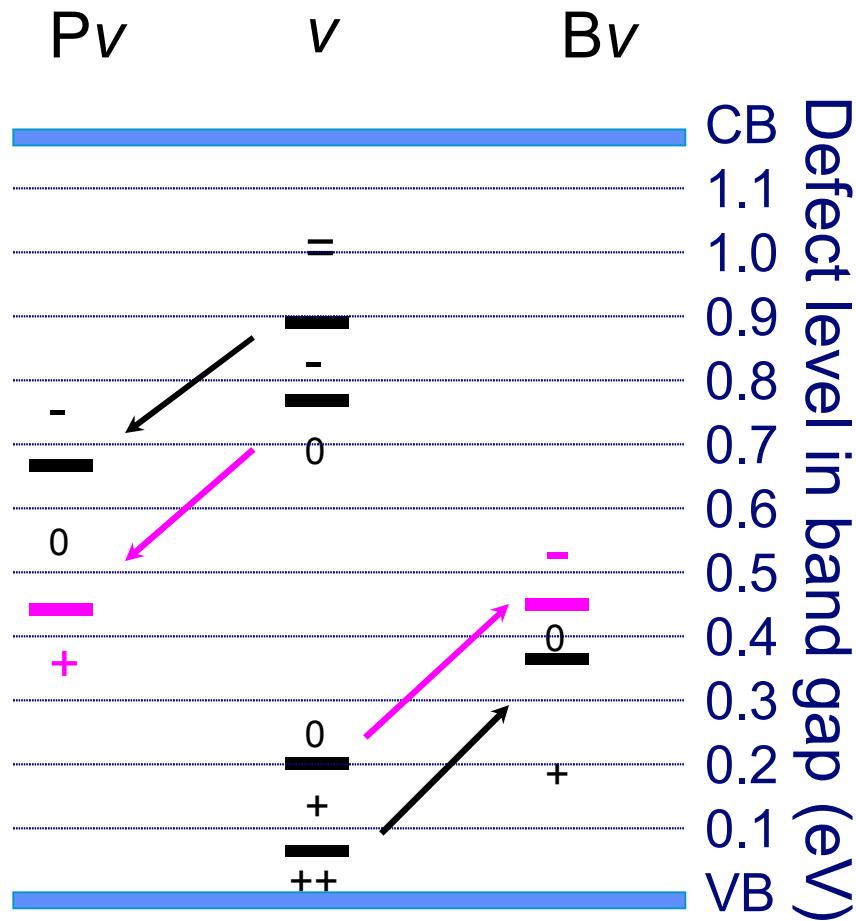
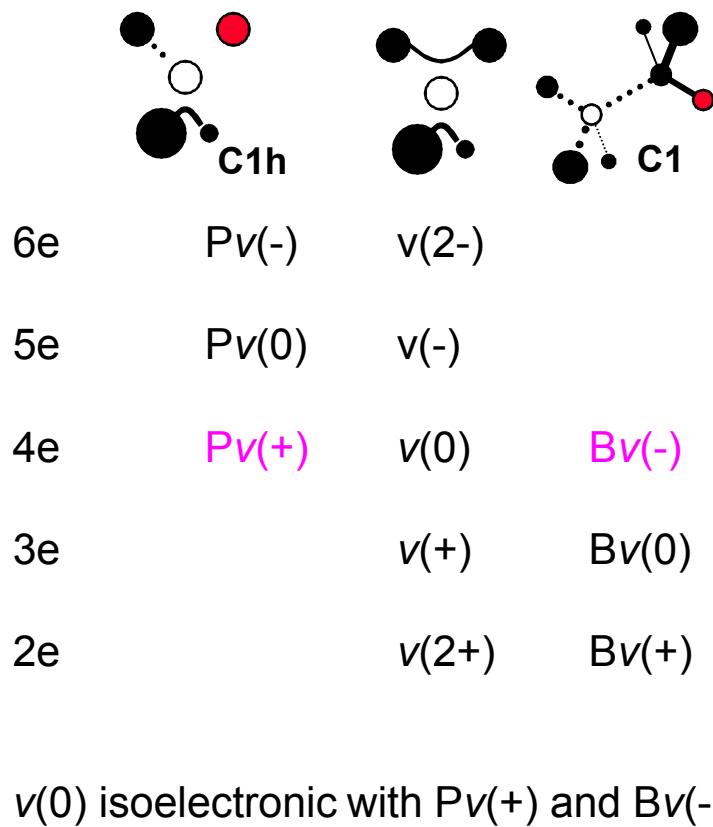


GGA/PBE improves defect levels with bonding changes

PBE max error=0.17 eV, mean $|error|=0.08$ eV, (v -related: 0.05 eV)

New P-v and B-v levels still present

P- v and B- v charge states



By analogy to vacancy, expect new charge states isoelectronic to $v(0)$
DFT finds them, they are real, i.e., >0.25 eV (max DFT error) from edge
New states at midgap \rightarrow effective recombination centers

Summary

- Finite Defect Supercell Model - **robust computational model** of defect
 - fix boundary conditions (LMCC Poisson solver)
 - rigorous chemical potential (common electron reservoir)
 - bulk polarization (bulk screening through modified Jost model)
 - defect banding (discrete defect occupation scheme)
 - still needs refinement (what about elastic effects?)
- Accurate DFT (LDA or PBE) Si defect levels, mean error < 0.1 eV
 - wide variety of defects - intrinsic, primary, secondary, 1st-row, 2nd-row
 - top and bottom of band gap
 - **predictive** despite band gap problem: **new levels in P-v and B-v**
 - evaluated as differences of valid ground state energies, not KS eigenvalues
- Band gap problem?
 - not in computation of localized defect states from total energy calculations!
 - However, still have issue of connecting defect level spectrum to band edge
 - Can we “fix” band gap problem (e.g., w/EXX) without screwing up energies?

Thanks to: Kevin Leung, Ann Mattsson, Art Edwards, Harry Hjalmarson, Renee Van Ginhoven

Contact information: paschul@sandia.gov, <http://www.cs.sandia.gov/~paschul>

Summary - II

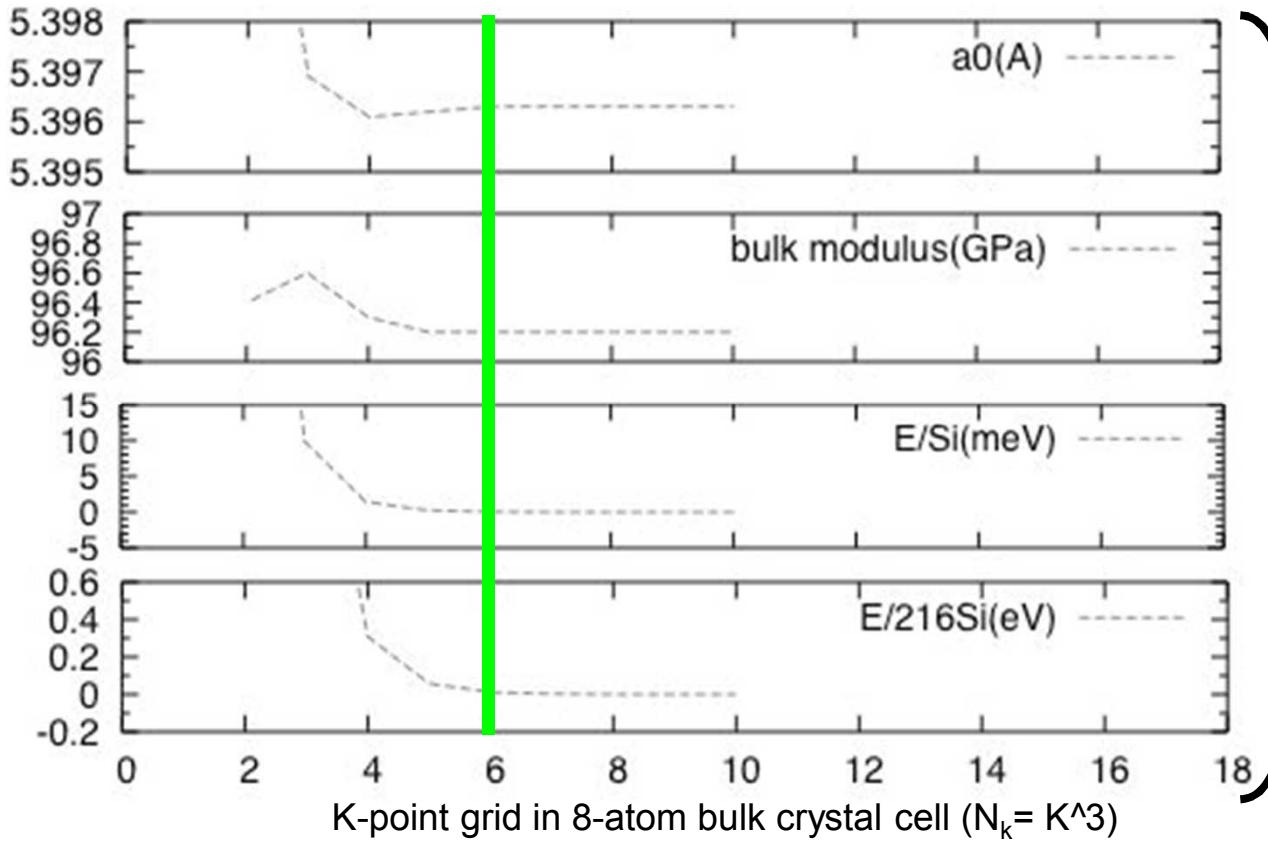
- Beware Black Boxes!
 - know your assumptions, control your approximations
 - what *exactly* is your code doing?
- There is a gap between our tools (DFT codes) and reality (defects)
 - construction of valid (i.e., quantitative) computational model
- Improvements?
 - path forward to better (i.e., more accurate) functionals not clear
 - existing LDA and GGA do very well already
- Questions?

Thanks to: Kevin Leung, Ann Mattsson, Art Edwards, Harry Hjalmarson, Renee Van Ginhoven

Contact information: paschul@sandia.gov, <http://www.cs.sandia.gov/~paschul>

- extra slides -

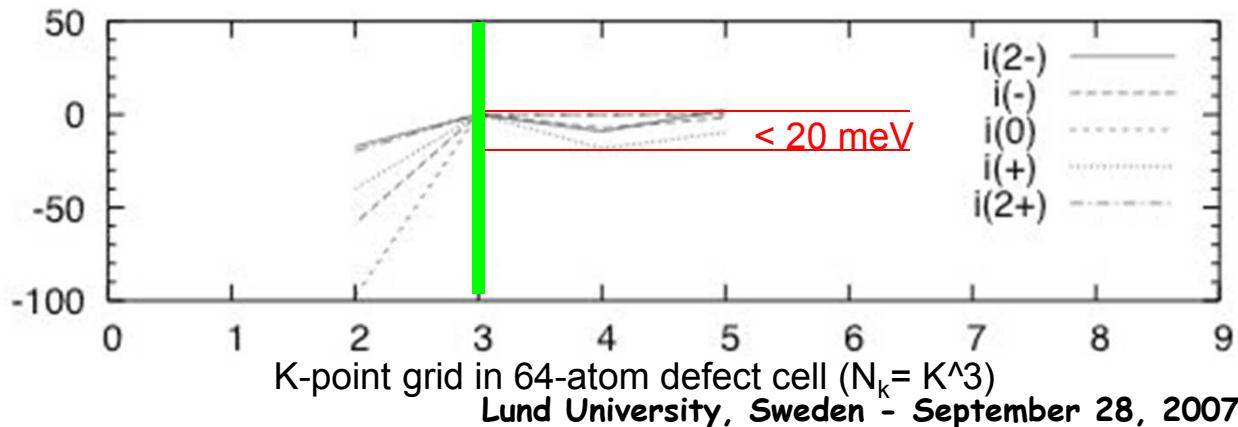
Brillouin Zone sampling: Quest/FDSM



Bulk properties ($a_0, B, E/Si, E_{tot}$) converge quickly:

at 6^3 k w/8-cell
 $= 3^2$ k w/64-cell
 $= 2^3$ k w/216-cell

Defect energies should not vary faster than bulk, IF computational model is valid.



Interstitial formation energies in 64-site cell vary < 20 meV
 $\{10$ meV w/o $I(+)\}$
beyond equivalent of 6^3 k-grid in 8-site.

Radiation Effects in Oxides and Semiconductors (REOS) Simulations

- Harry Hjalmarson

Transient REOS Simulations

2D drift-diffusion equations

p-type silicon; 10^{17} cm⁻³ B doping

Interstitials, vacancies and divacancies included

Evolution of carrier recombination computed

Interstitial energies obtained from DFT/SeqQuest calculations
hand-off between DFT->continuum

Data

Solar cell diodes; *p*-doped region dominant

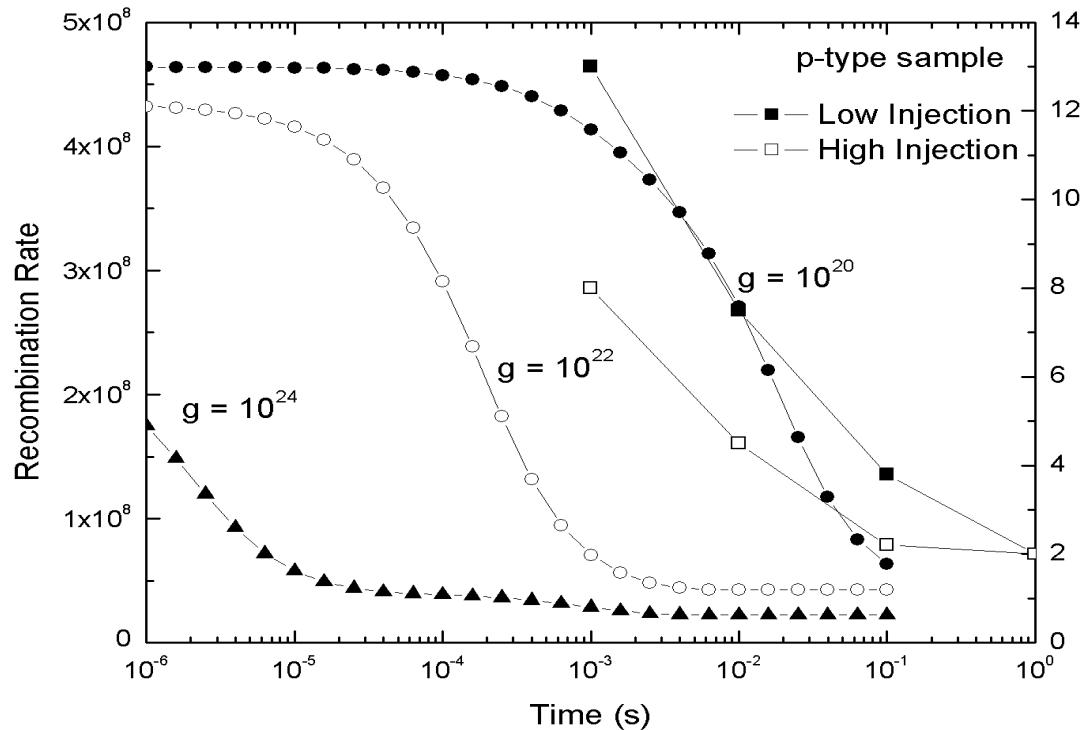
Evolution of carrier recombination measured

Comparison

Reasonable agreement with data (DFT values OK)

Minimal sensitivity to interstitial defect levels

REOS Diode Annealing Factor Simulations with DFT Data



Time scale of simulations and data are comparable
DFT (self-interstitial) data give reasonable results