

# Radiation Effects Modeling with the CEPTRE Code

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**SAND 2007-???**

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company,  
for the United States Department of Energy's National Nuclear Security Administration under contract  
DE-AC04-94AL85000.



# Outline

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- Sandia Missions
- Overview of CEPTRE
- First- and Second-Order Methods
- Algorithmic Details
- Results
- Summary



# Sandia National Laboratories: a Short Introduction

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- Started in 1949 as a division of Los Alamos
- Two locations: Albuquerque NM and Livermore CA
- Currently about 7,500 employees
- Diverse mission includes:
  - Ensuring the safety of the nuclear weapons stockpile
  - Energy and infrastructure
  - Nonproliferation
  - Homeland security



# Nuclear and Radiation Science Activities at Sandia

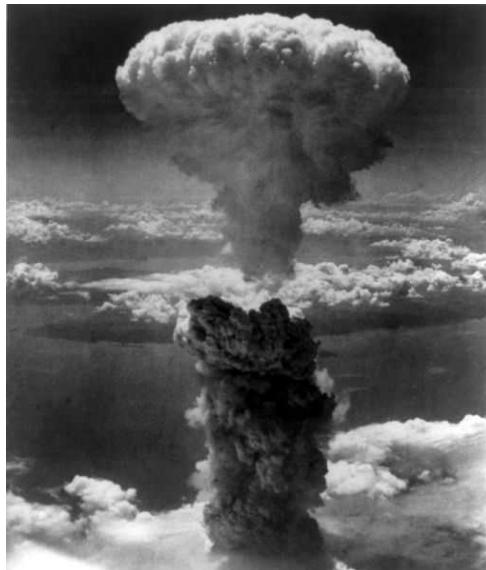
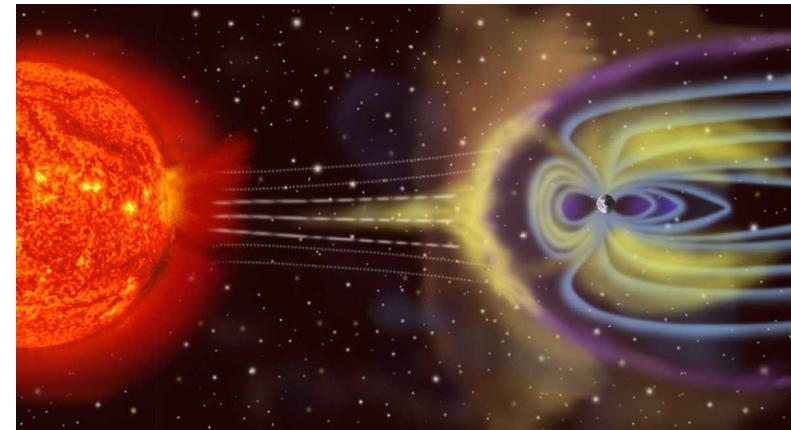
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- Radiation effects
  - Experiments
  - Simulation
- Nuclear power systems
  - Advanced power conversion technologies
  - Hydrogen production
  - Spent fuel management (Yucca Mountain)
- Space nuclear power and propulsion
- Inertial fusion energy (Z-pinch)



# Protecting Components and Systems in Harsh Radiation Environments

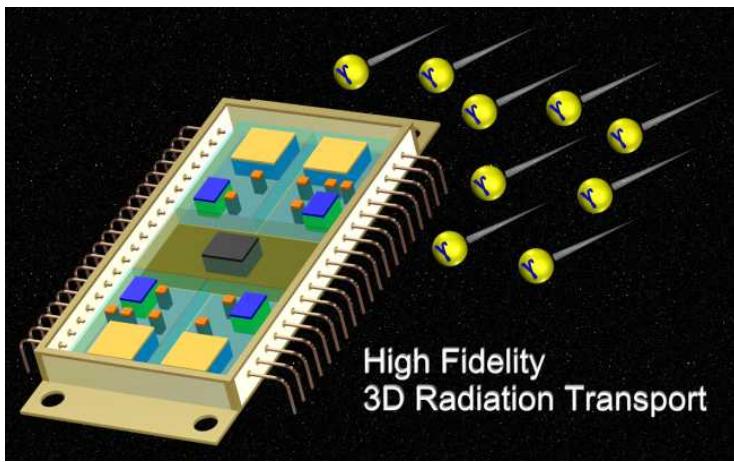
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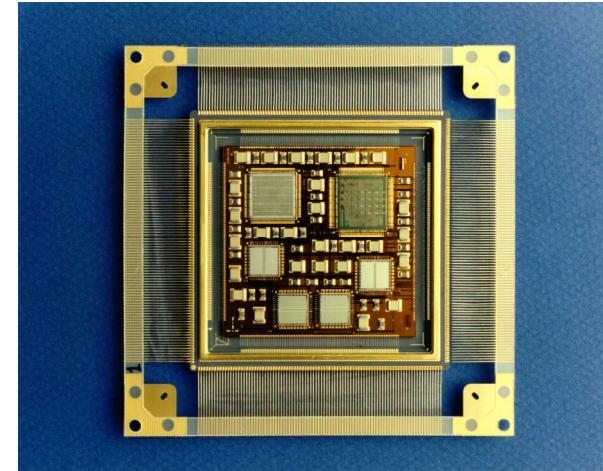


# Radiation transport is fundamental to understanding the effects produced in nuclear & space radiation environments

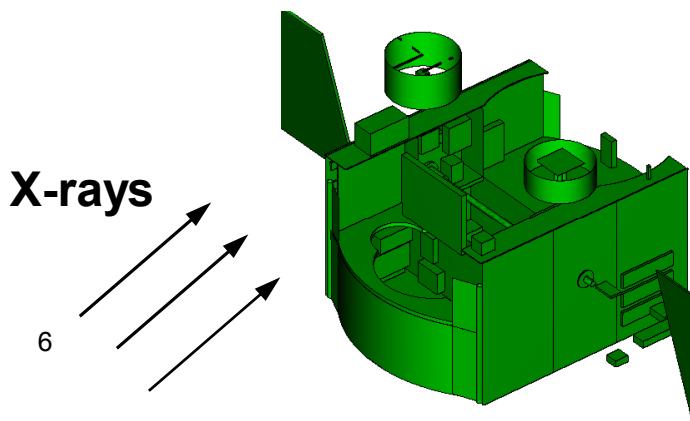
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***The transport of coupled photon, electron, and positron radiation from 1.0 keV to 20.0 MeV***



**Goal:** Predict the effect of radiation on electrical components (e.g. ICs, cables)

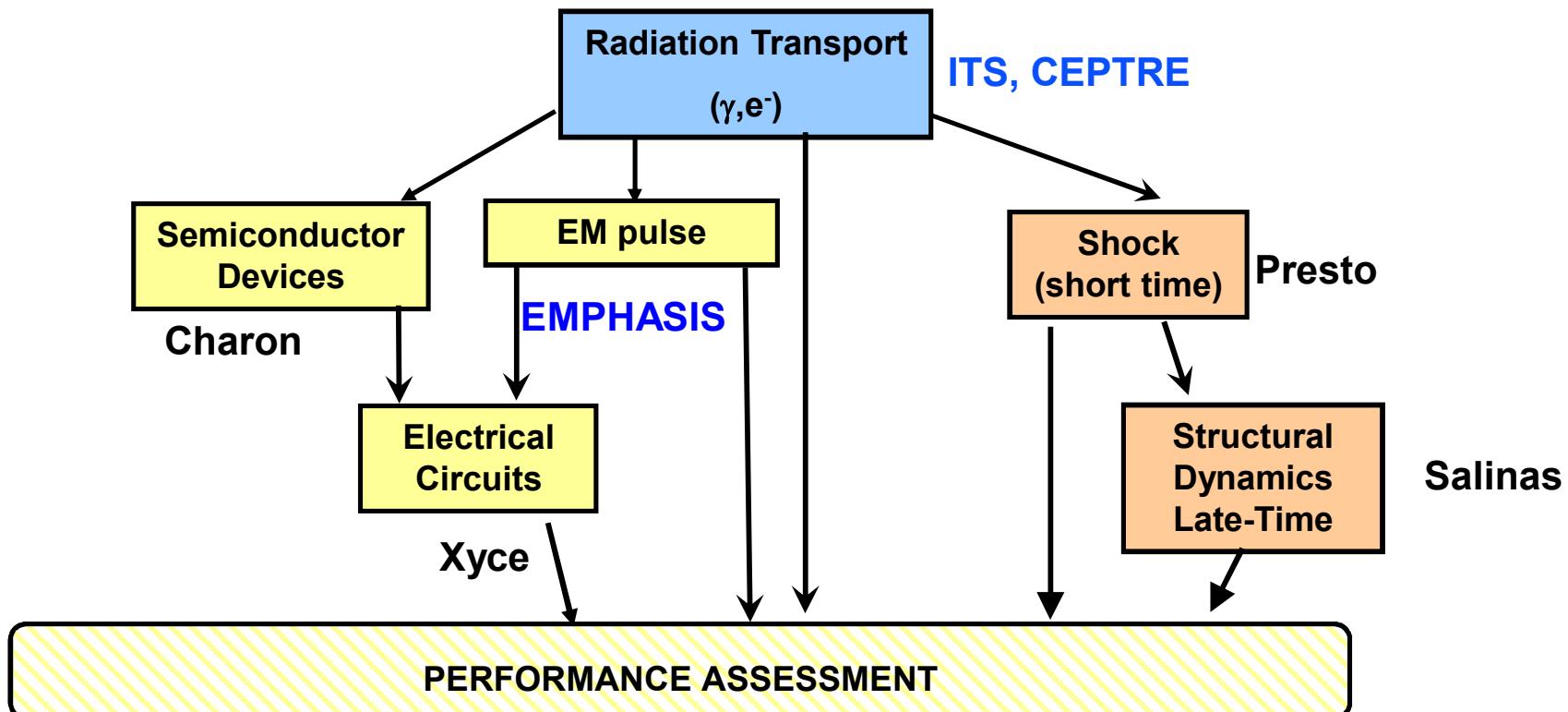


**Goal:** Predict the effect of radiation on materials and structures (mechanical effects)

**Satellite**



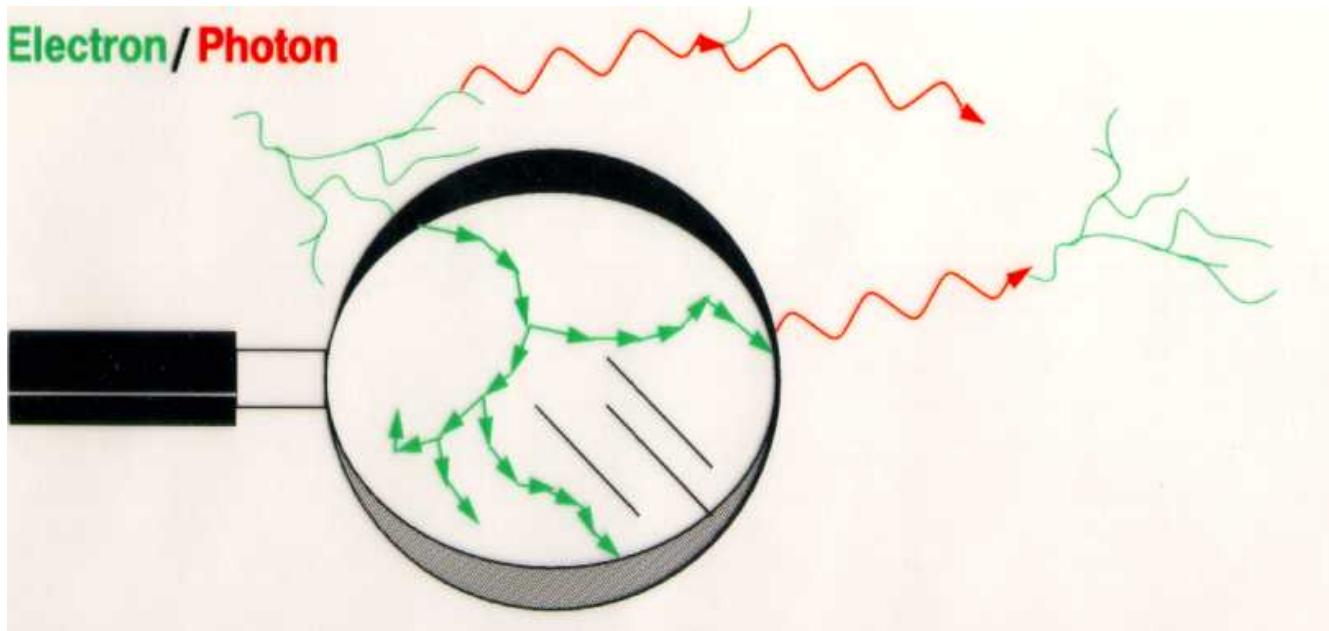
# ASC Program : Codes & Effects





# Coupled photon-electron particle cascade

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*Electrons and photons have  
radically different transport scales!*



# Simulation challenge: radically different cross sections for electrons and photons

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Particle Energy (keV)	Photon MFP (micron)		Electron MFP (micron)	
	silicon	gold	silicon	gold
100	24,000	100	.03	.006
30	4,000	21	.01	.003

For Monte Carlo, analog simulation of electrons would be prohibitively computationally expensive

- *Solution: Condensed-history models*

For deterministic, proper treatment of electron cross sections would require excessive discretization in energy and angle and would be prohibitively computationally expensive.

- *Solution: Boltzmann-Fokker-Planck (BFP) transport equation*



# Two totally different methods are available in computational physics to model radiation transport

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## Monte Carlo Methods (ITS)

*Computer simulation of random walk by statistical sampling*

- Runtime limited
  - Memory not generally a limitation
- Complex 3D modeling capability exists
- Efficient for computing integral quantities
  - Total charge crossing a surface
  - Total dose in a region
- Easily adaptable to parallel computers

## Deterministic Methods (CEPTRE)

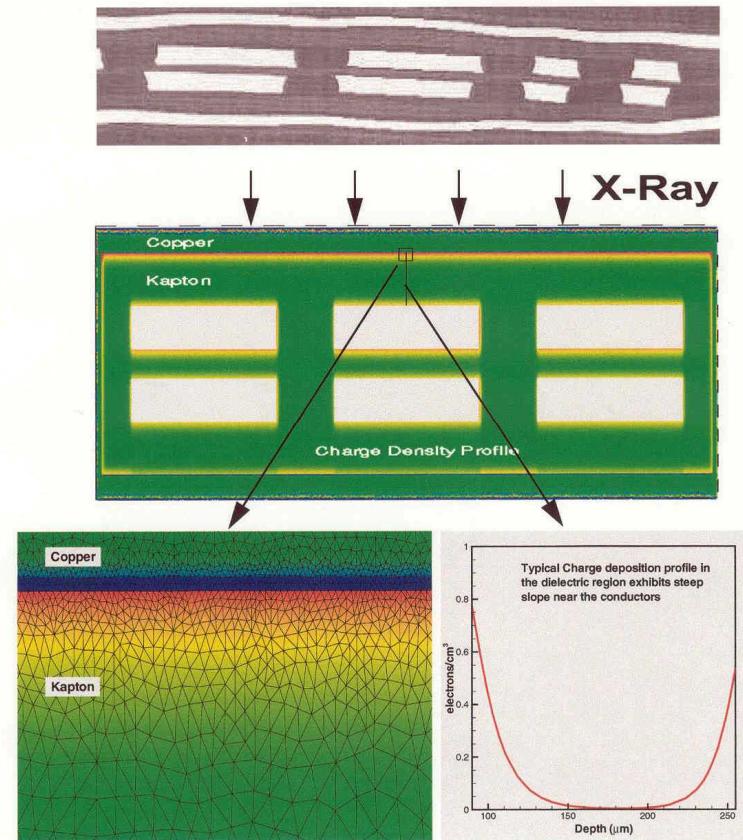
*Numerical solution of the mathematical equation describing the transport*

- Memory and/or runtime limited
- Complex 3D modeling capability is being developed
- Essential for computing differential quantities
  - charge deposition distributions
  - energy deposition distributions
  - space, energy, and angle dependent emission quantities



# Cable SGEMP Simulation Needs

- Requires accurate resolution of dose-enhancement and charge profiles near conductor/dielectric interfaces
- Results in extremely small mesh cells near the material interfaces





# CEPTRE

## Coupled Electron-Photon Transport for Radiation Effects

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- Initial code for SGEMP simulations
- Implemented in Nevada framework
- 2nd-order formulation only
- Even/odd parity, SAAF, LS
- 2D/3D, linear/quadratic elements
- Large matrix solve



# SCEPTRE

Sandia's Computational Engine  
for Particle Transport for Radiation Effects

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- Second generation code
- Standalone code (no framework)
- 1st- and 2nd-order formulations
- $S_n$  and  $P_n$
- Even/odd parity, SAAF, LS
- 2D/3D, linear/quadratic elements
- Large matrix solve or source iteration



# SCEPTRE Status

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- Many components and subsystems complete and functional
- Some notable holes remain
- Lots of unit tests

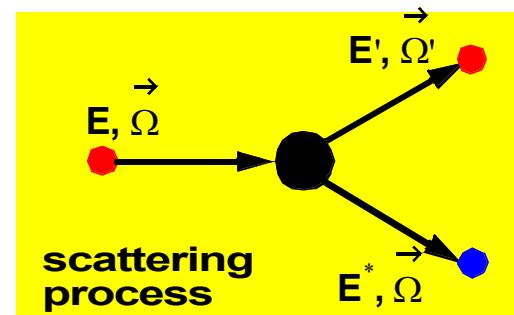




# CEPTRE/SCEPTRE Details

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- Time-independent, deterministic, coupled electron-photon transport code on unstructured meshes
- Numerical solutions to the Boltzmann transport equation which describes the particle distribution in phase space ( $r, E, \Omega$ )
- Physics of particle-media interactions properly characterized by cross sections
- Discretization of Phase Space
  - *Multigroup* approximation in energy along with Legendre expansion of scattering cross sections
  - *Discrete-Ordinates* (also *Spherical Harmonics*) approximation in direction
  - *Finite-Element* approximation in space





# Forms of Boltzmann Transport Equation

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First-order:

$$[\Omega \cdot \nabla + \sigma_t] \psi(r, \Omega) = M \Sigma D \psi(r, \Omega) + Q(r, \Omega)$$

Second-order:

$$[-\Omega \cdot \nabla \mathcal{R}^{-1} \Omega \cdot \nabla + \mathcal{R}] \psi(r, \Omega) = Q(r, \Omega) - \Omega \cdot \nabla [\mathcal{R}^{-1} Q(r, \Omega)]$$

*The continuous forms are equivalent, but their discretized forms have different properties*



# Multigroup energy differencing

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Integrate the continuous-energy Boltzmann equation over an energy band ("group"):

$$\int_{E=E_g}^{E_{g+1}} dE ( )$$



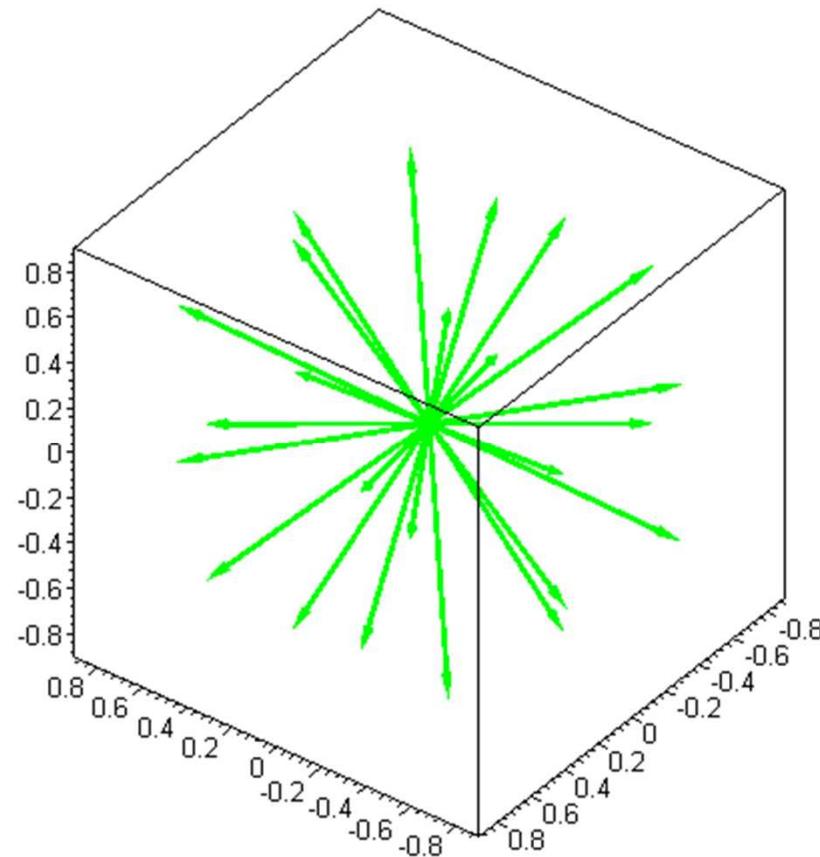
$$[\Omega \cdot \nabla + \sigma_{t,g}] \psi_g(r, \Omega) = M \sum_{g' \neq g} M \Sigma_{g \rightarrow g} D \psi_g(r, \Omega) + \sum_{g' \neq g} M \Sigma_{g' \rightarrow g} D \psi_{g'}(r, \Omega) + Q_g(r, \Omega)$$



# Angular differencing: discrete ordinates

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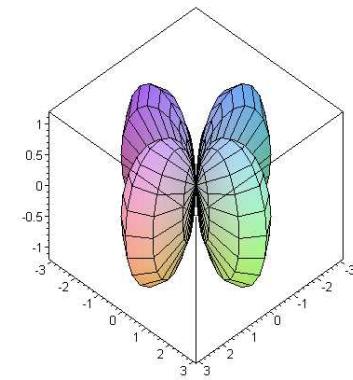
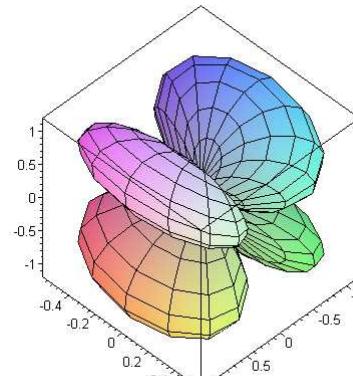
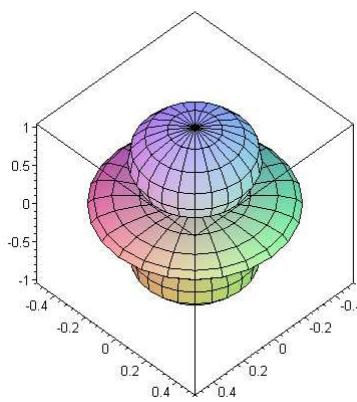
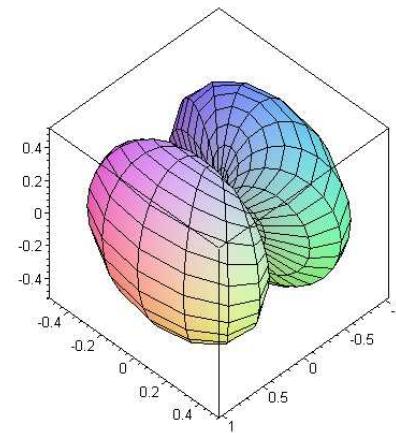
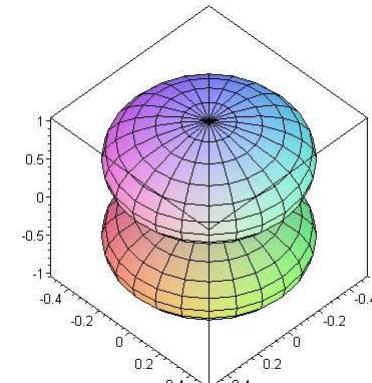
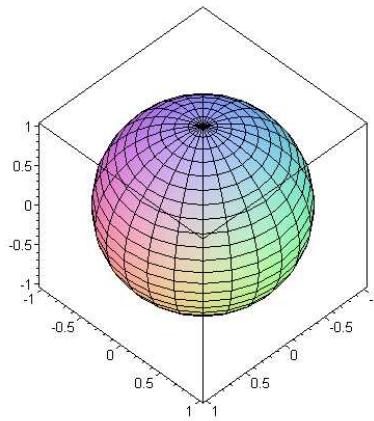
- Collocation in angle
- Compute solution in discrete directions
- Use numerical quadrature to compute angular integrations





# Angular differencing: spherical harmonics

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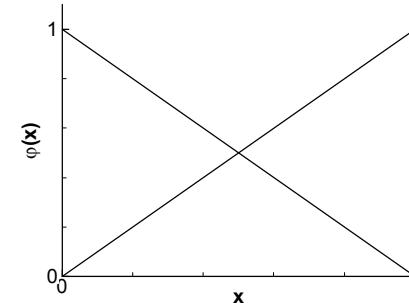
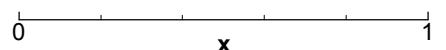




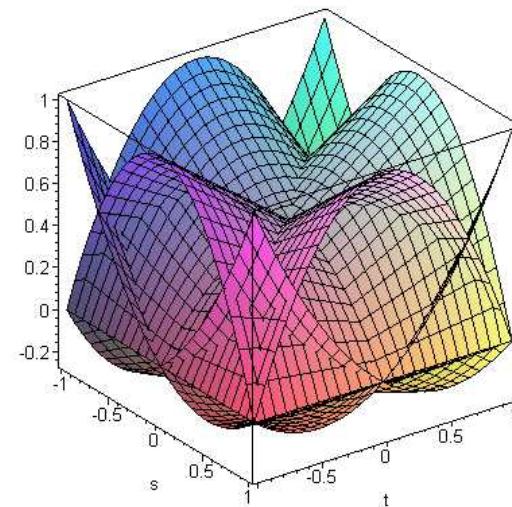
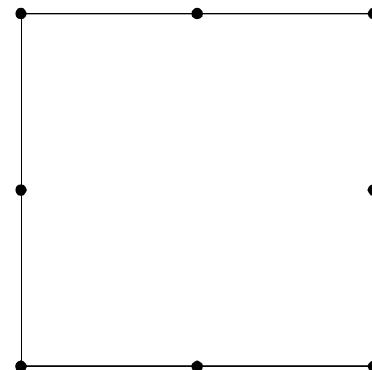
# Spatial differencing: finite element method

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1D  
Linear



Quadratic  
Quadrilateral

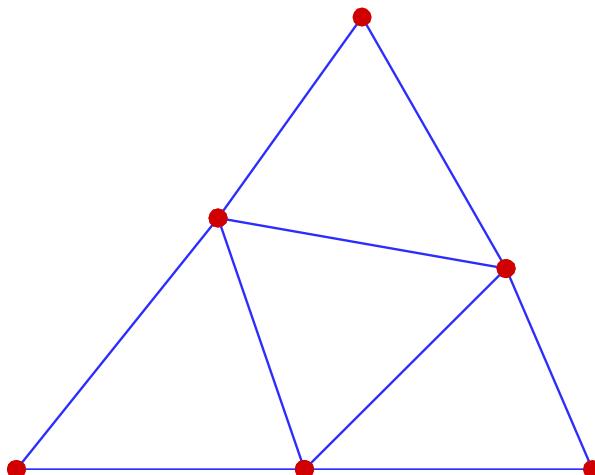




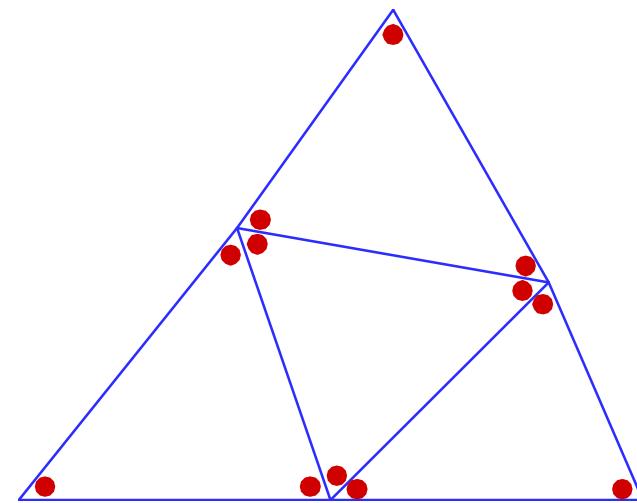
# Continuous vs. Discontinuous FEM Representations

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- Solution may be continuous or have discontinuities (shadow boundaries)
- Continuous mesh smaller problem size
- Discontinuous elements may be more accurate



Linear-continuous elements

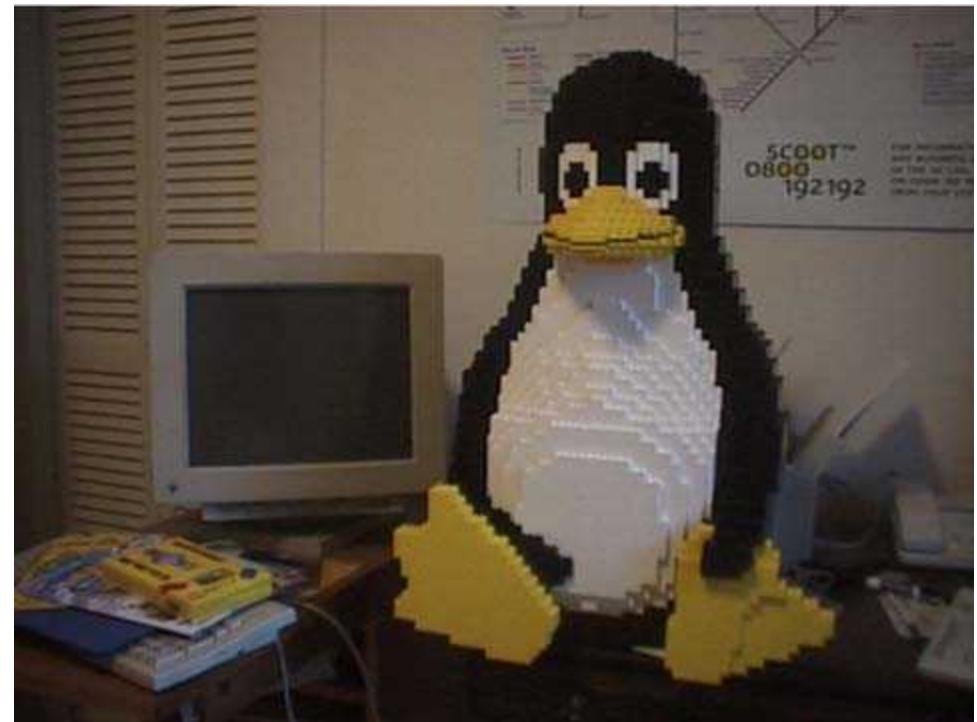
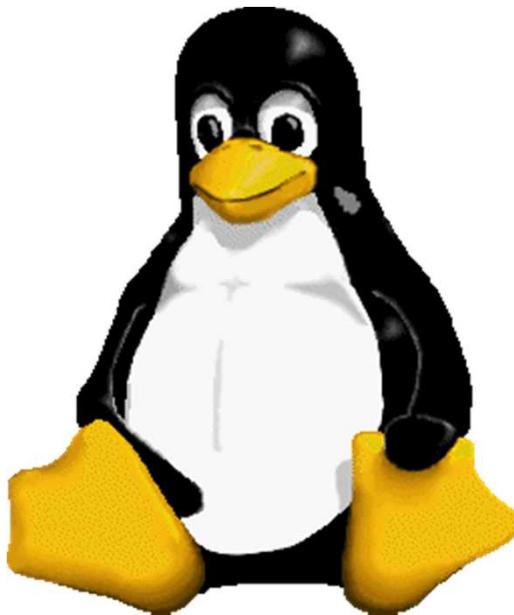


Linear-discontinuous elements



# Cartesian representation has limited fidelity

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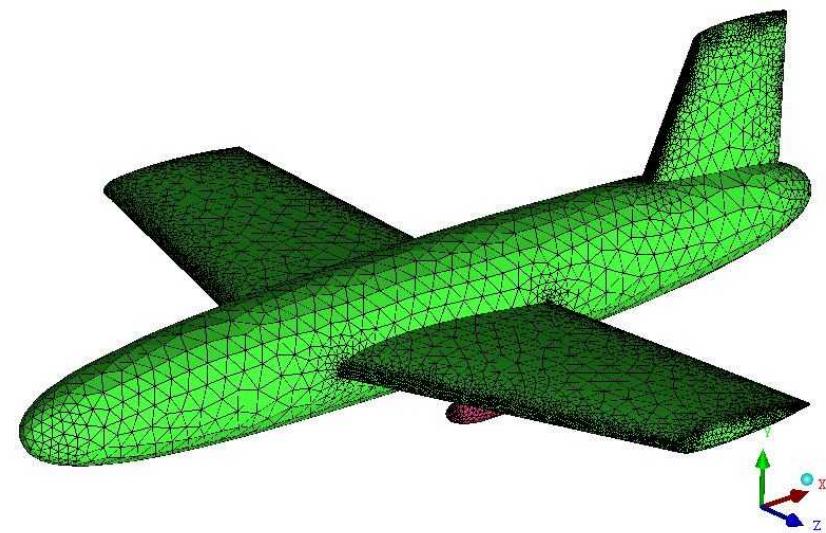
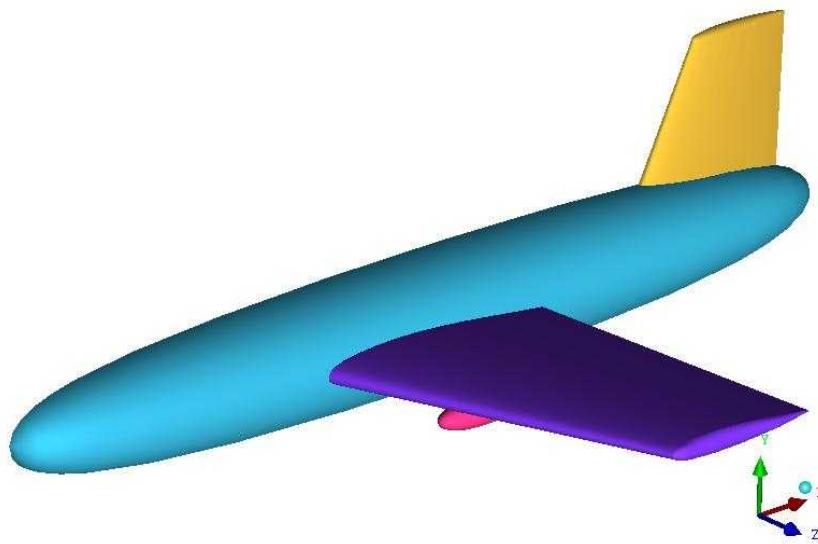
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# Better geometry representation with unstructured finite elements

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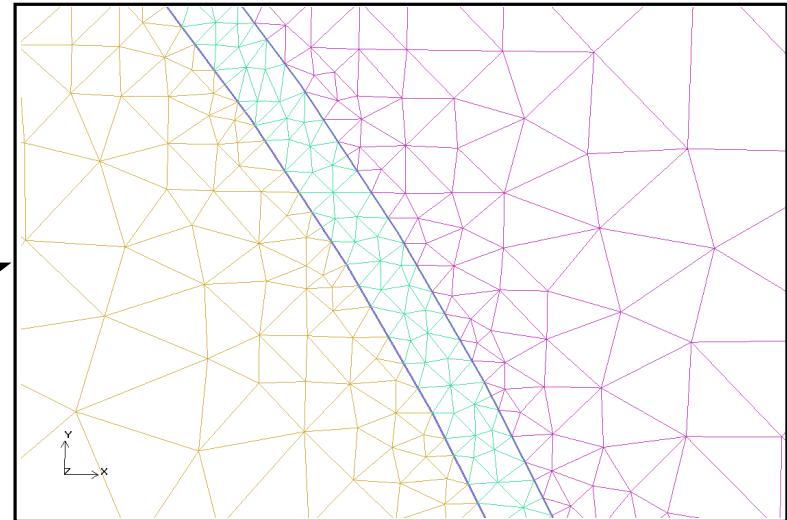
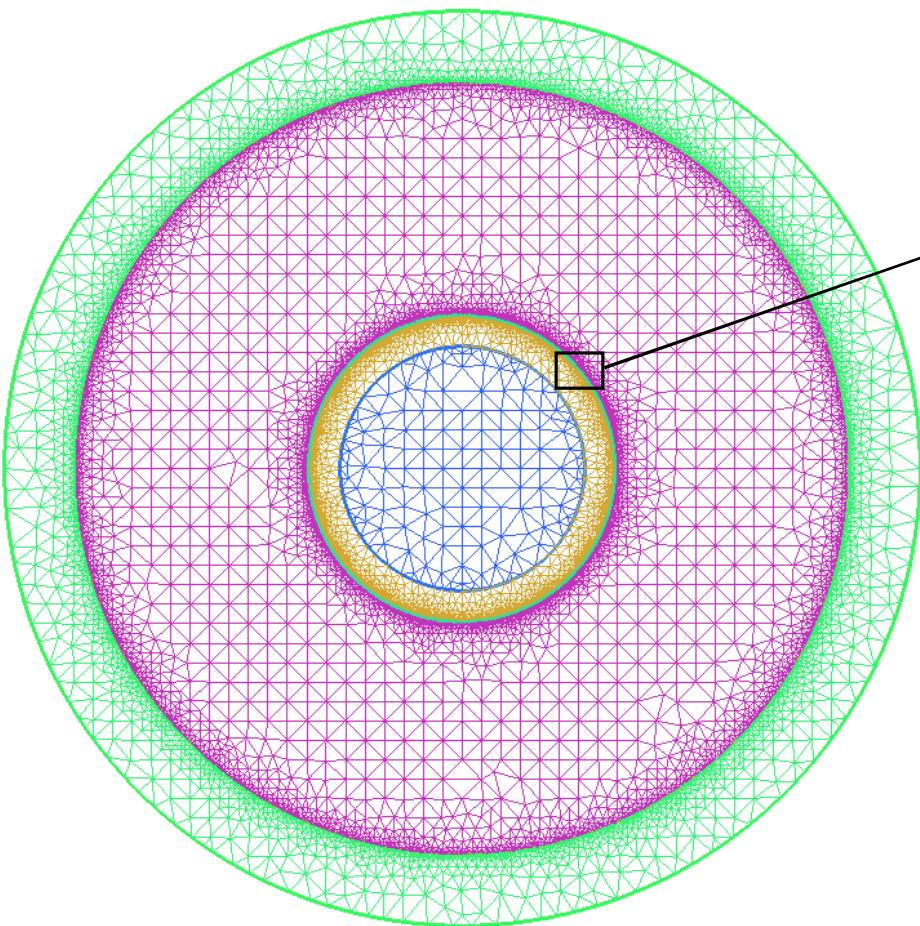




# Extremely fine spatial mesh required near material interfaces for SGEMP analysis

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## Coaxial cable

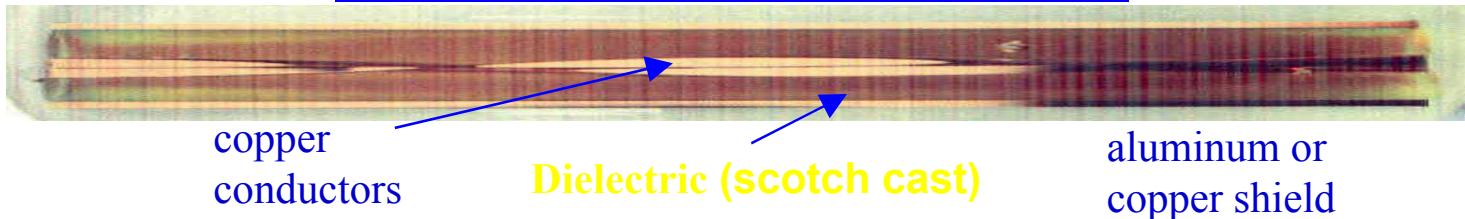


Resolving steep gradients in  
electron distribution near  
conductor/dielectric  
interfaces key to accurate  
solution

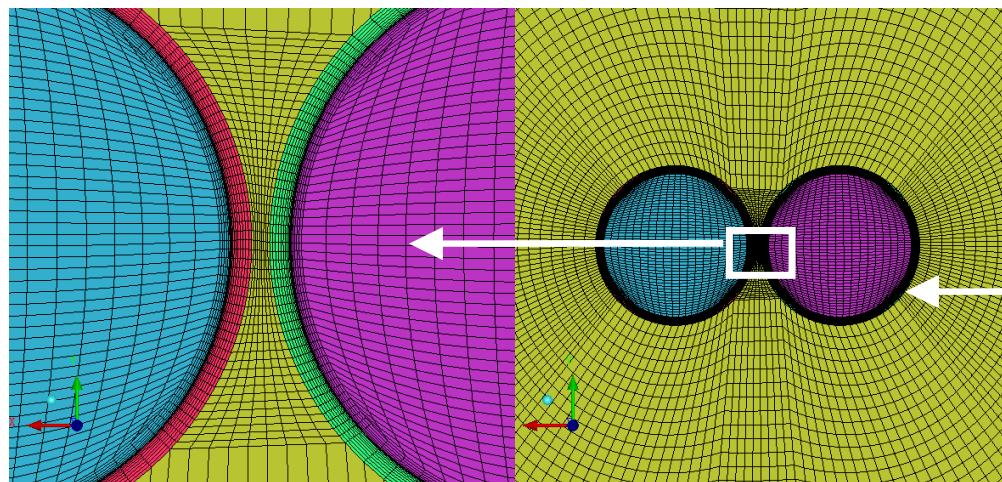


# Cable SGEMP Analysis of a Twisted Shielded Pair (TSP) Coaxial Cable

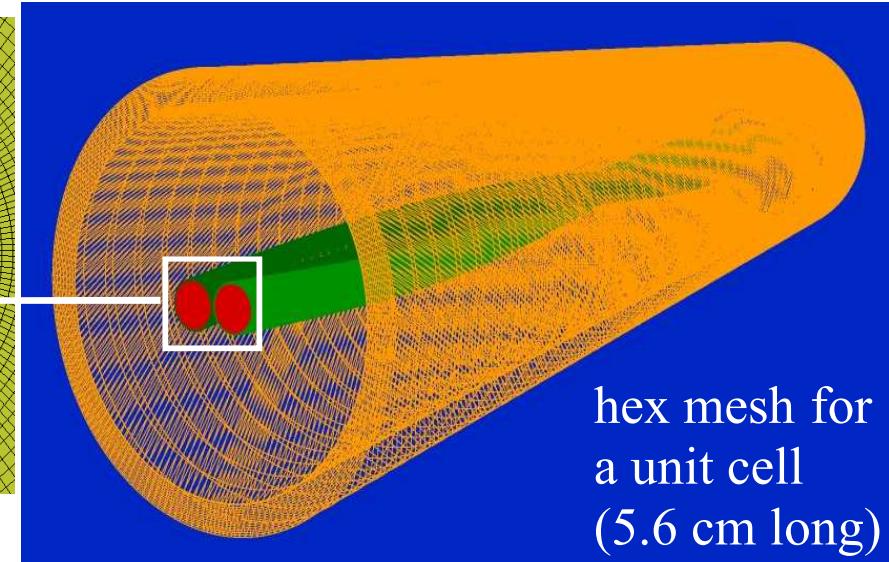
cross sectional view of a TSP cable



hex mesh cross sections



$\mu\text{m}$ -size elements near the center conductors





# Discrete equations may be solved in different ways

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## Conventional Source Iteration



First-order forms:  
scattering sources are  
fixed during an iteration  
(directions decoupled)

## CEPTRE



Second-order forms: all  
terms simultaneously  
solved (huge matrix)

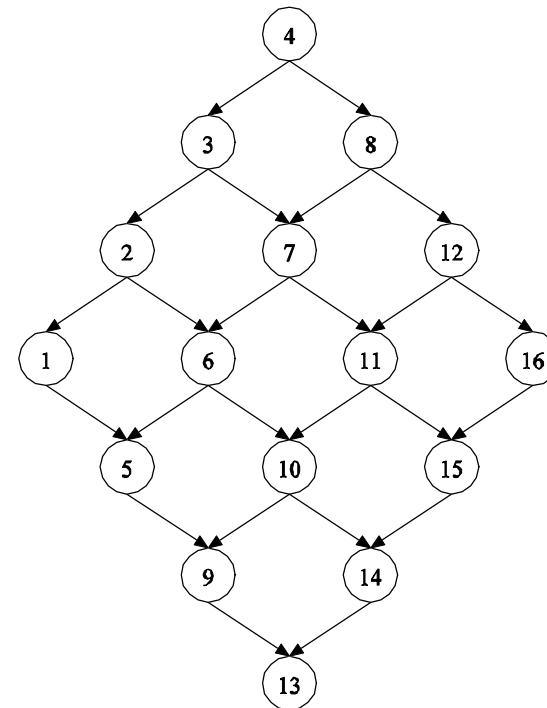


# Sweeps of Structured Meshes

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$$\Omega$$

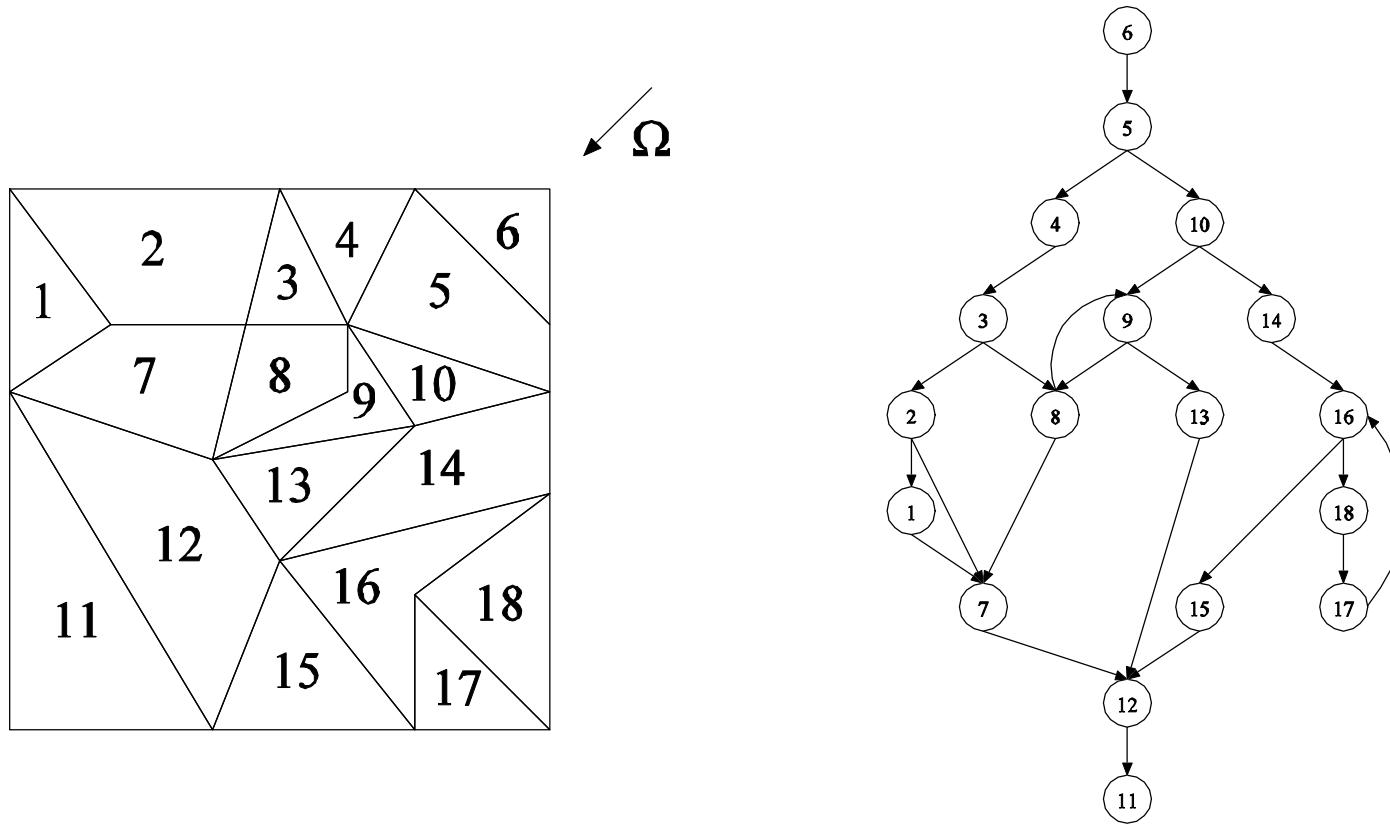
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16





# Sweeps of Unstructured Meshes

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# Parallel Sweeps of Unstructured Meshes

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- Mesh decomposition
- Eliminate cycles in sweep graph
- Sweep ordering
- Communication pattern
- Violations of sweep graph
- Iterative preconditioners

*The main problem for parallel sweeps is  
the struggle for concurrency!*



# Second-order forms

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Even-parity equation:

$$[-\Omega \cdot \nabla \mathcal{R}_O^{-1} \Omega \cdot \nabla + \mathcal{R}_E] \psi^E(r, \Omega) = Q^E(r, \Omega) - \Omega \cdot \nabla [\mathcal{R}_O^{-1} Q^O(r, \Omega)]$$

Odd-parity equation:

$$[-\Omega \cdot \nabla \mathcal{R}_E^{-1} \Omega \cdot \nabla + \mathcal{R}_O] \psi^O(r, \Omega) = Q^O(r, \Omega) - \Omega \cdot \nabla [\mathcal{R}_E^{-1} Q^E(r, \Omega)]$$

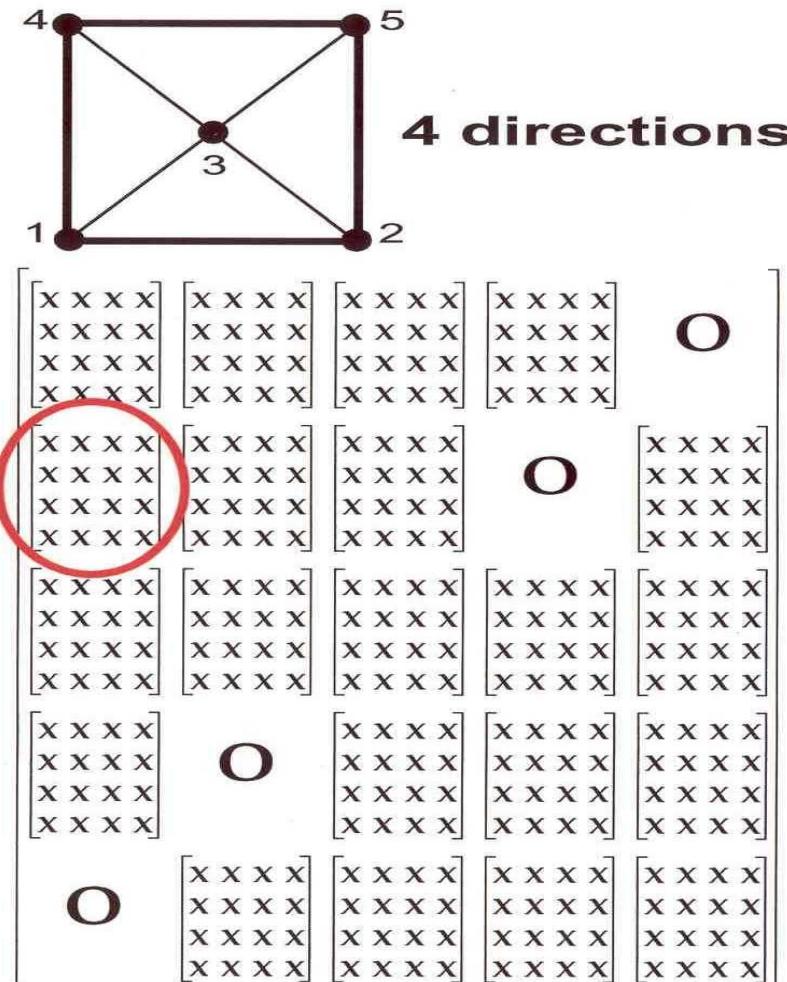
Self-adjoint angular flux (SAAF) equation:

$$[-\Omega \cdot \nabla \mathcal{R}^{-1} \Omega \cdot \nabla + \mathcal{R}] \psi(r, \Omega) = Q(r, \Omega) - \Omega \cdot \nabla [\mathcal{R}^{-1} Q(r, \Omega)]$$



# Second-order matrix

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- Sparse block matrix
- Symmetric Positive Definite System
- Number of block rows:
  - $N_{\text{nodes}}$
- Block size
  - $N_{\text{directions}} \times N_{\text{directions}}$
- Blocks are full due to coupling from scattering
- Tailor-made for VBR data format
- Storage  $\sim (N_{\text{directions}})^2 \times N_{\text{nodes}}$
- Run time  $\sim (N_{\text{directions}})^2 \times (N_{\text{nodes}})^{1.5}$



# First- and second-order forms exhibit different runtime behaviors

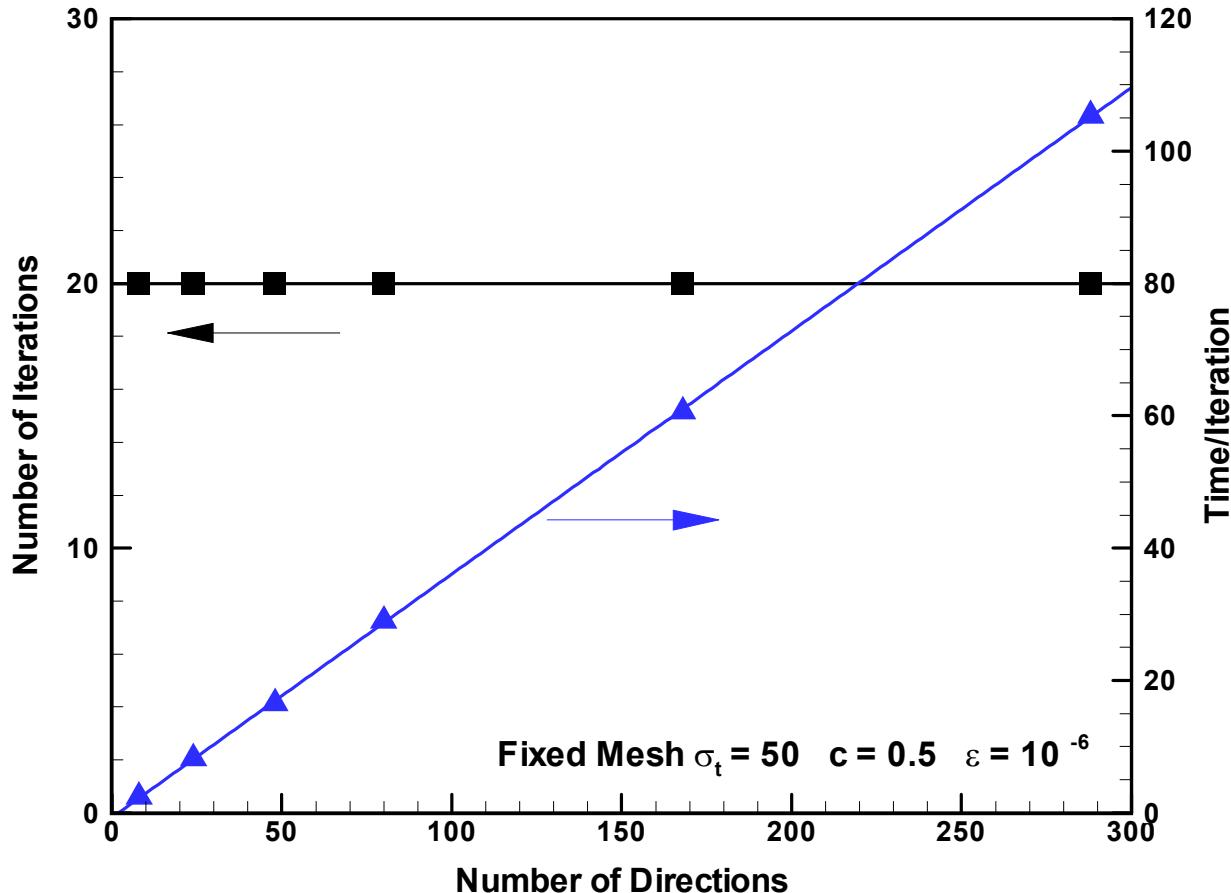
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Computing Conditions	1 <sup>st</sup> order		2 <sup>nd</sup> order	
	Time/Source Iteration	# Iterations	Time/CG Iteration	# Iterations
Baseline	$N_D N_E P^{-(d-1)/d}$	$[\ln(1/c_n)]^{-1}$	$N_D^2 N_E P^{-1}$	$\langle \sigma_a h \rangle^{-1}$
Large $N_D$	$N_D N_E P^{-1}$	$[\ln(1/c_n)]^{-1}$	$N_D^2 N_E P^{-1}$	$\langle \sigma_a h \rangle^{-1}$
KBA Decomposition	$N_D N_E P^{-1}$	$[\ln(1/c_n)]^{-1}$	$N_D^2 N_E P^{-1}$	$\langle \sigma_a h \rangle^{-1}$
Diagonal Preconditioner	N/A	N/A	$(c_1 N_D + c_2 N_D^{-2}) N_E P^{-1}$	$\langle \sigma_a h \rangle^{-1}$
Extended Transport Correction Preconditioner	$N_D^2 N_E P^{-(d-1)/d}$	$[\ln(1/c_{n,eff})]^{-1}$	N/A	N/A



# Scaling with number of angles, first-order

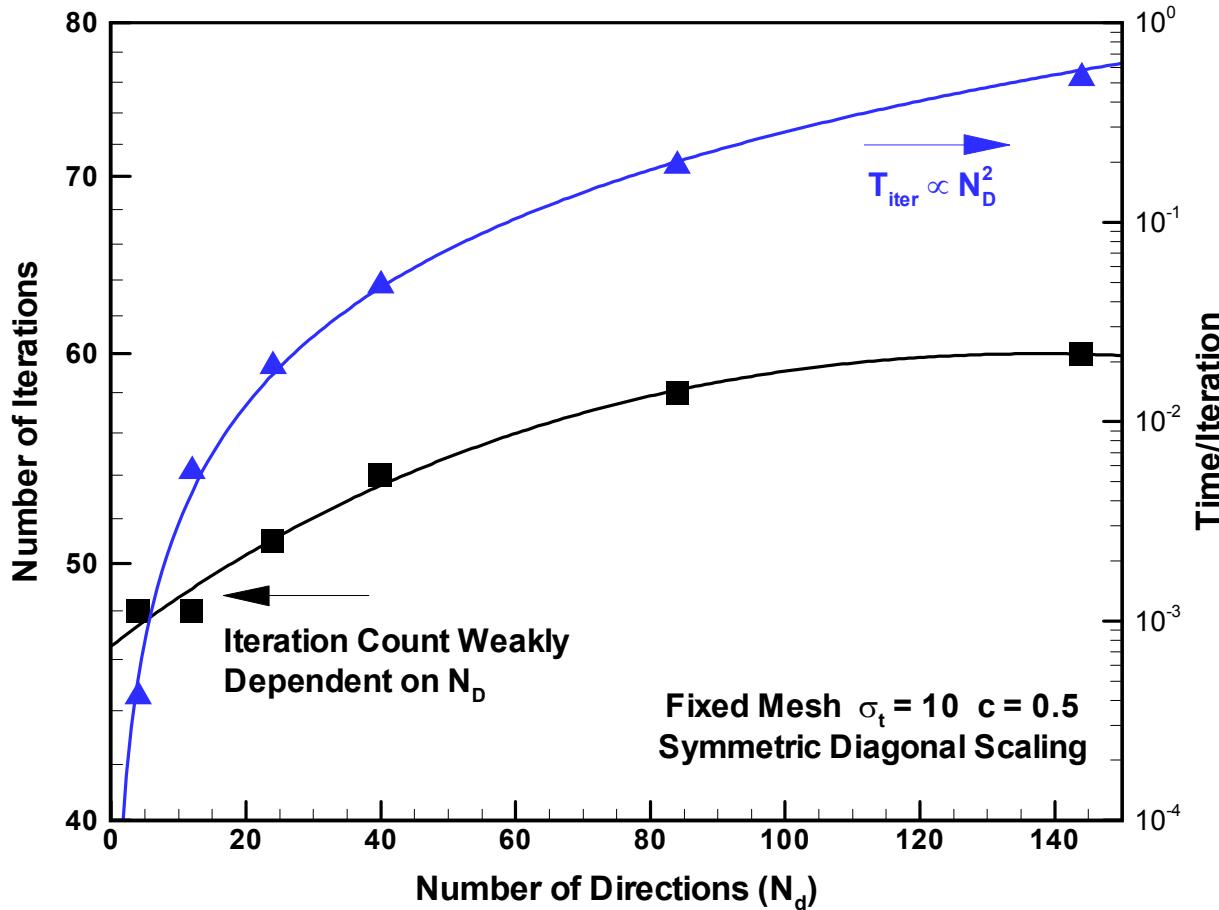
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# Scaling with number of angles, second-order

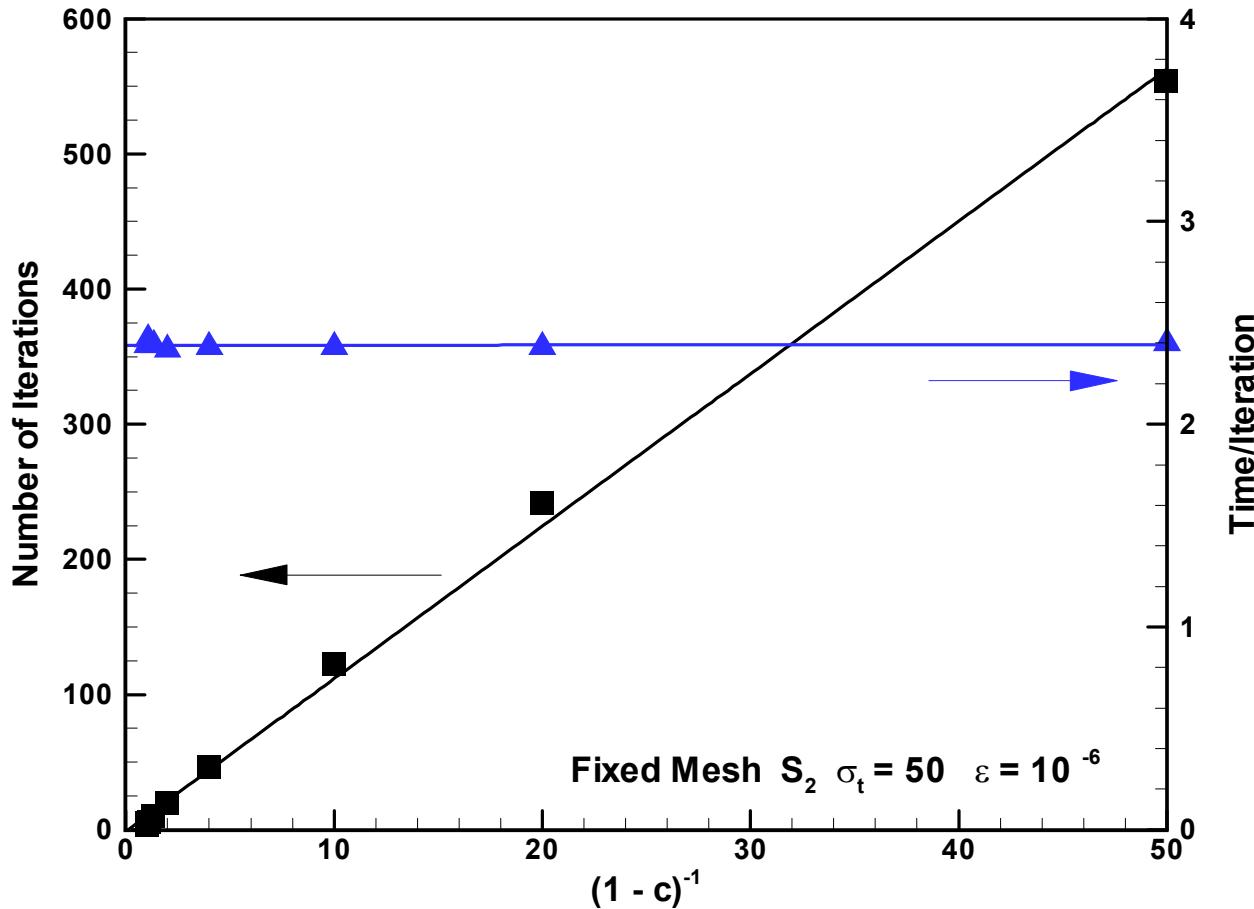
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# Scaling with scattering ratio, first-order

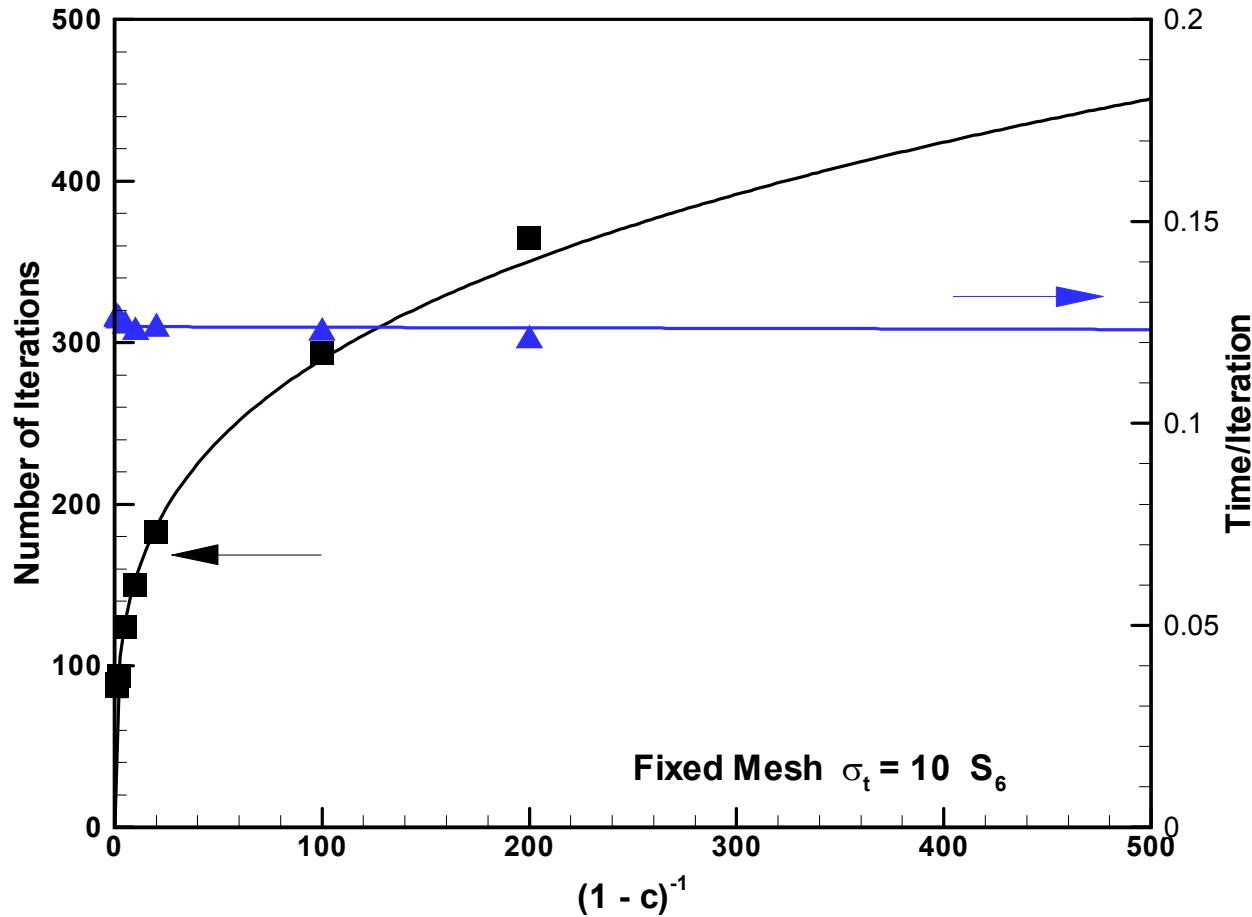
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# Scaling with scattering ratio, second-order

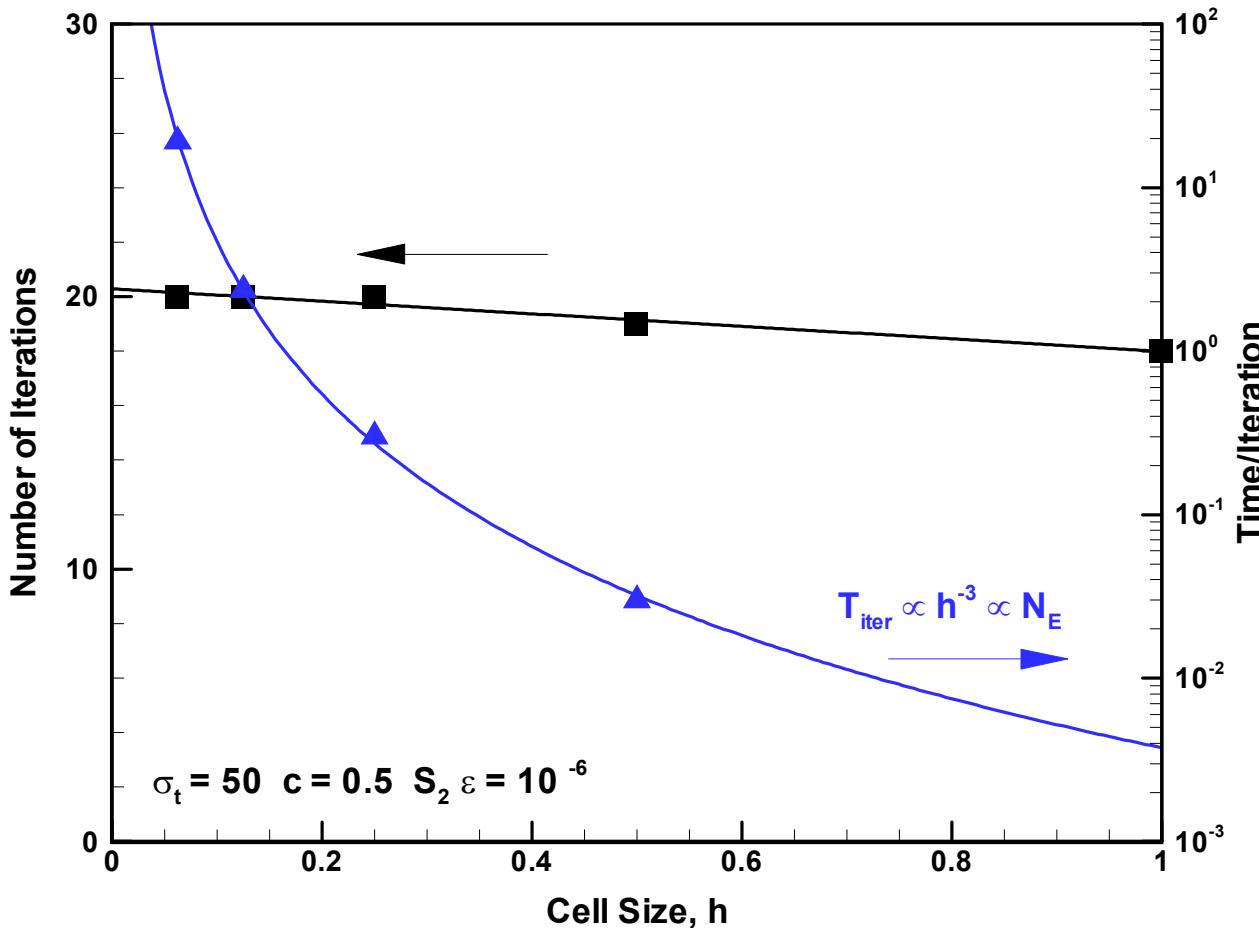
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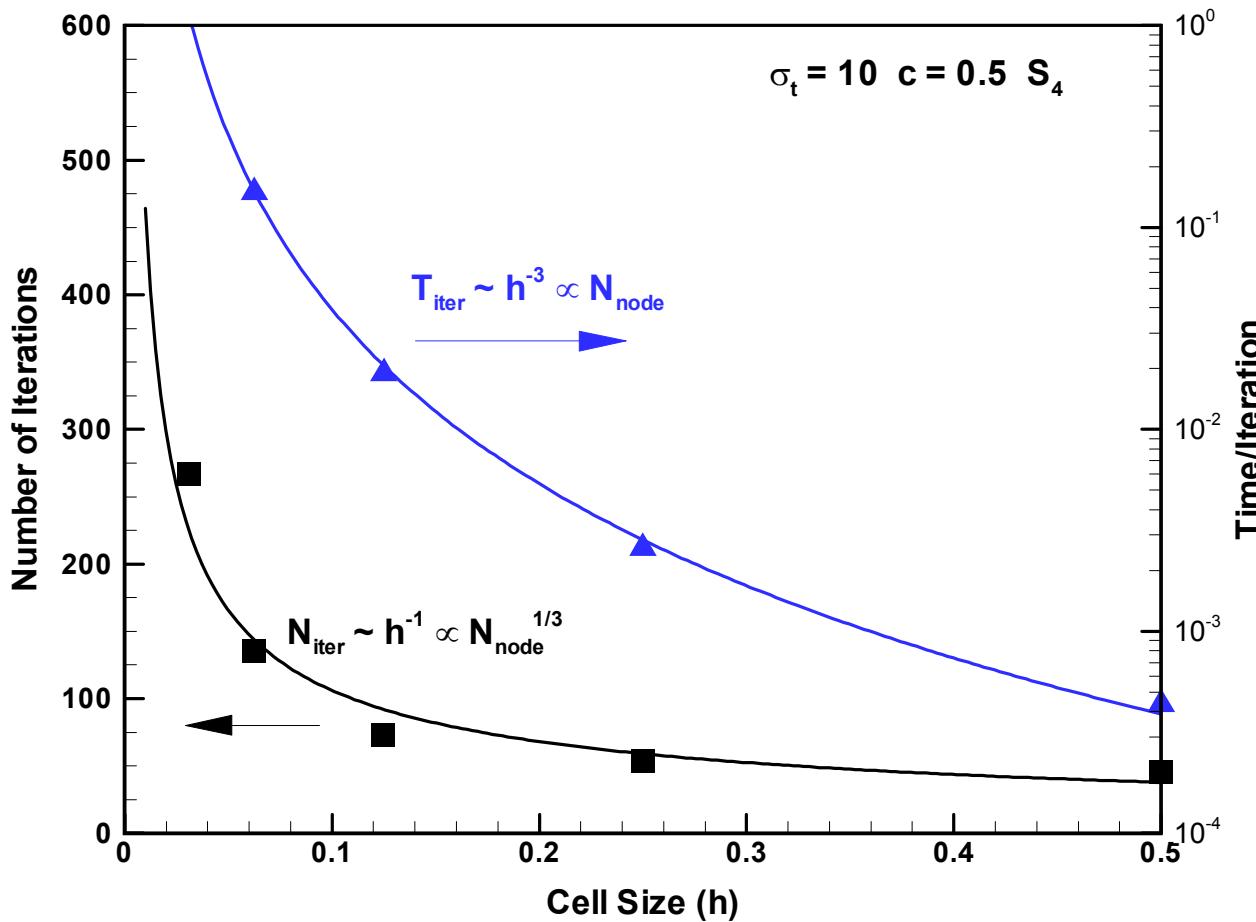
# Scaling with cell thickness, first-order (3D)

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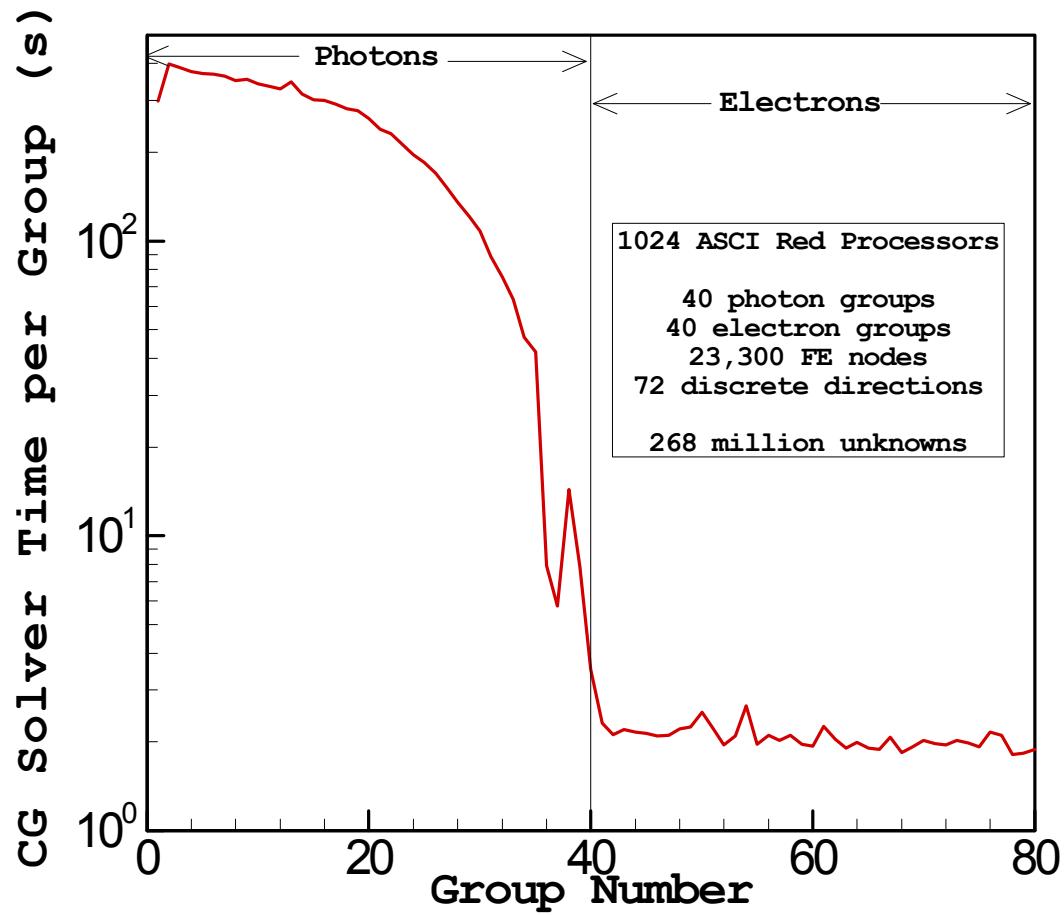
# Scaling with cell thickness, second-order (3D)





# Optical thickness differences for photons/electrons affects second-order performance

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# Currently implementing hybrid solver

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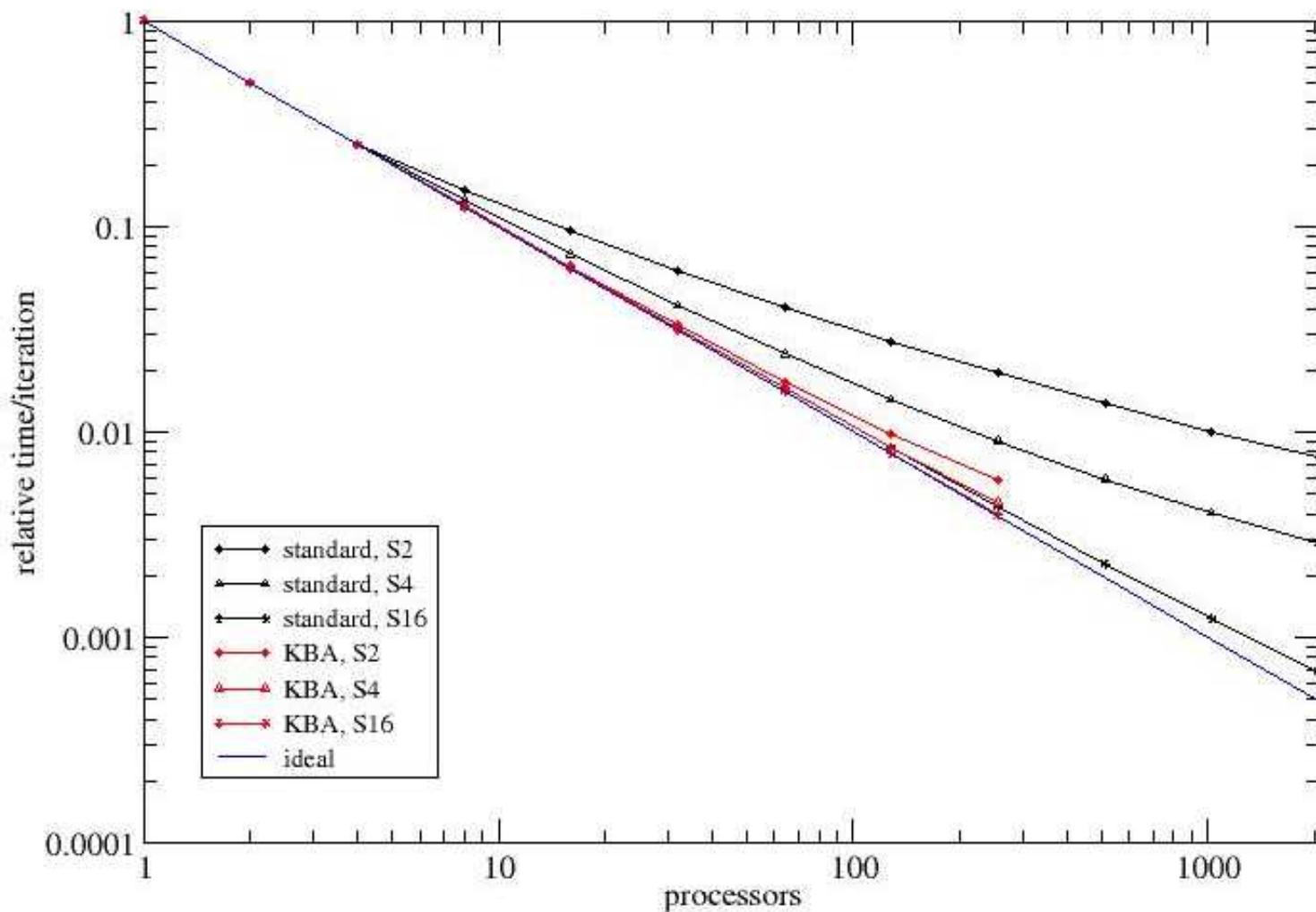
- First-order solver performs well for optically thin problems
- Second-order solver performs well for optically thick problems
- Proposed solution: Use different solver for each energy group / particle type



# First-order theoretical parallel scaling (strong)

128x128 mesh, S16, comm ratio = 0

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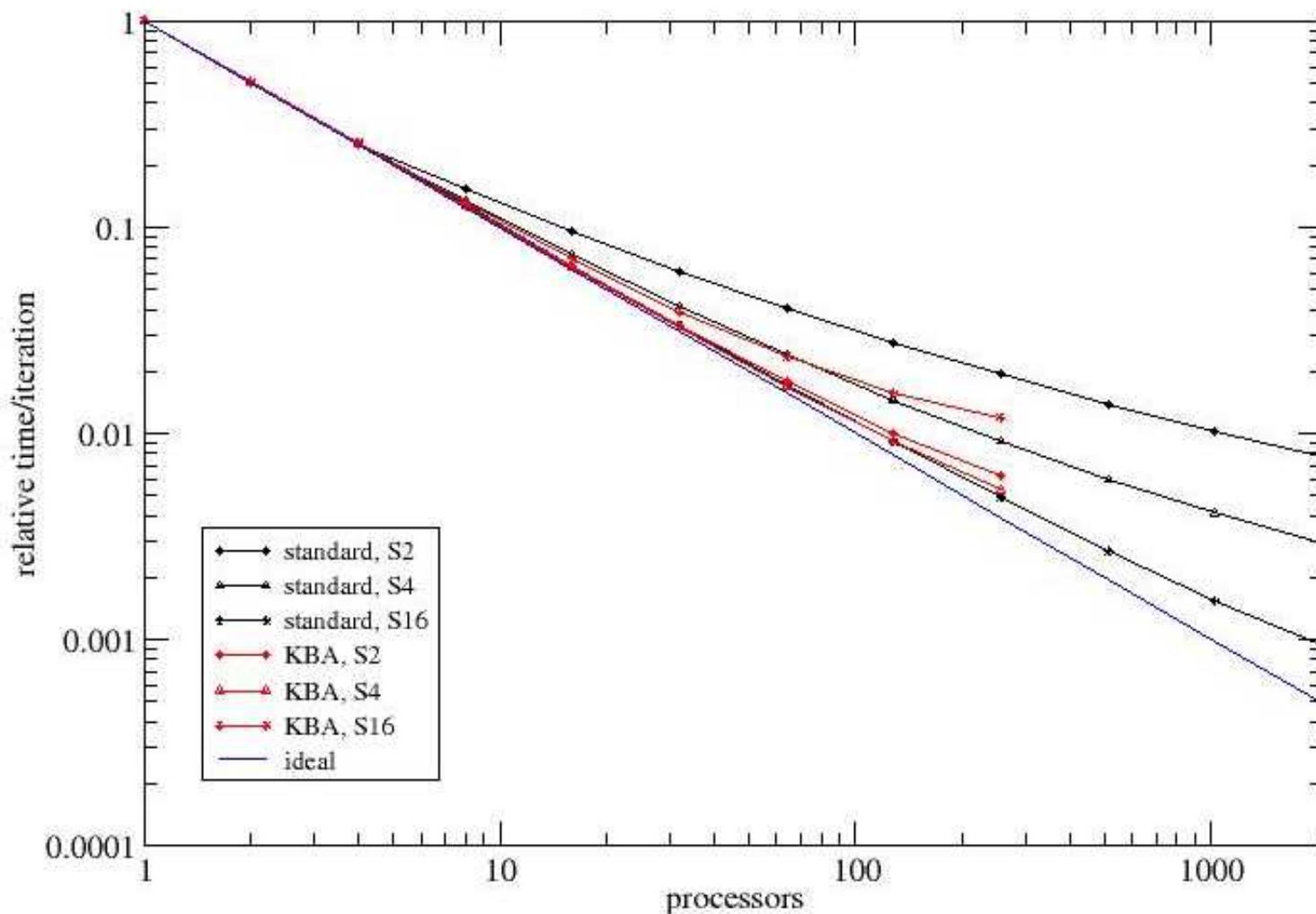




# First-order theoretical parallel scaling (strong)

128x128 mesh, S16, comm ratio = 1

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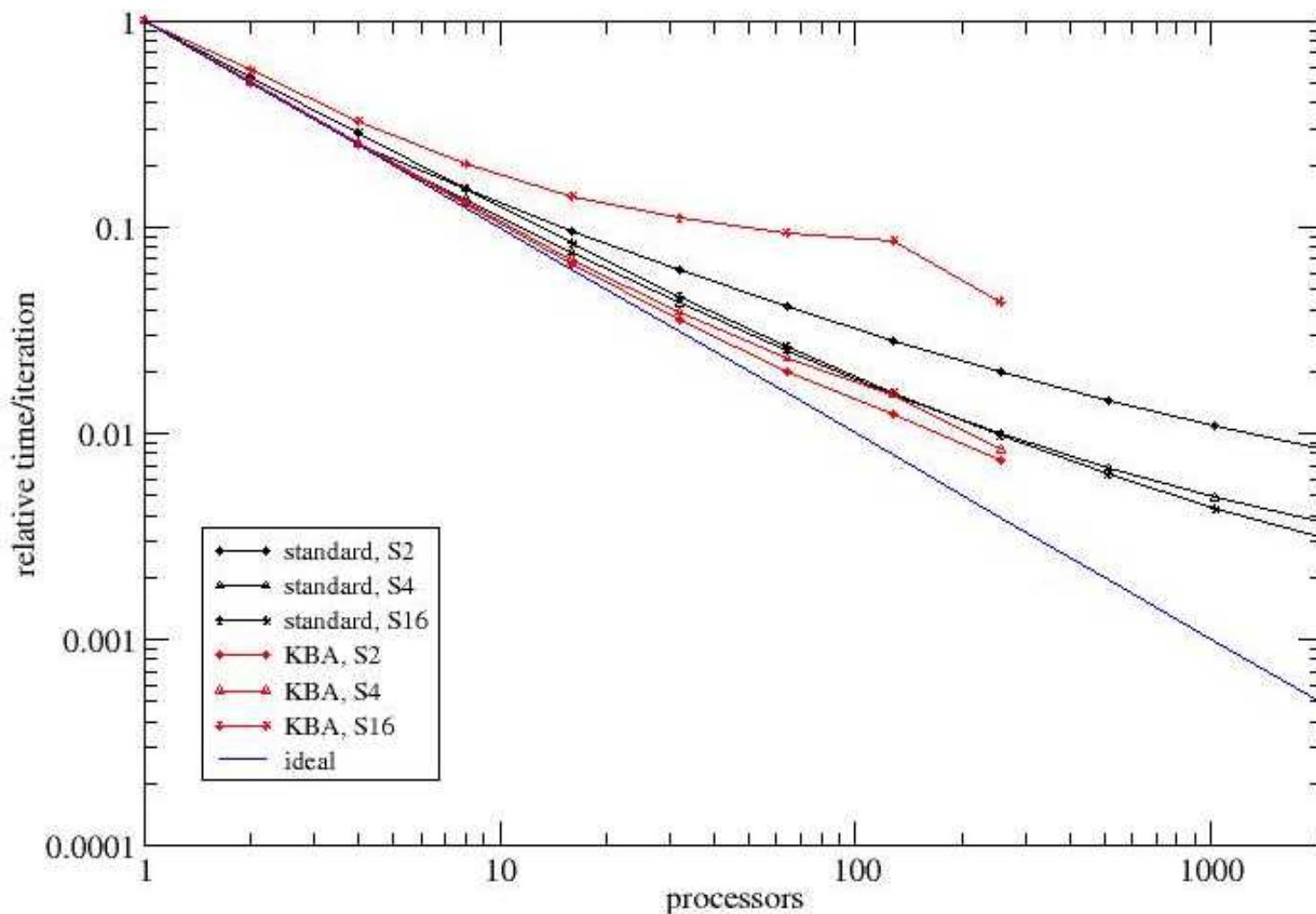




# First-order theoretical parallel scaling (strong)

128x128 mesh, S16, comm ratio = 10

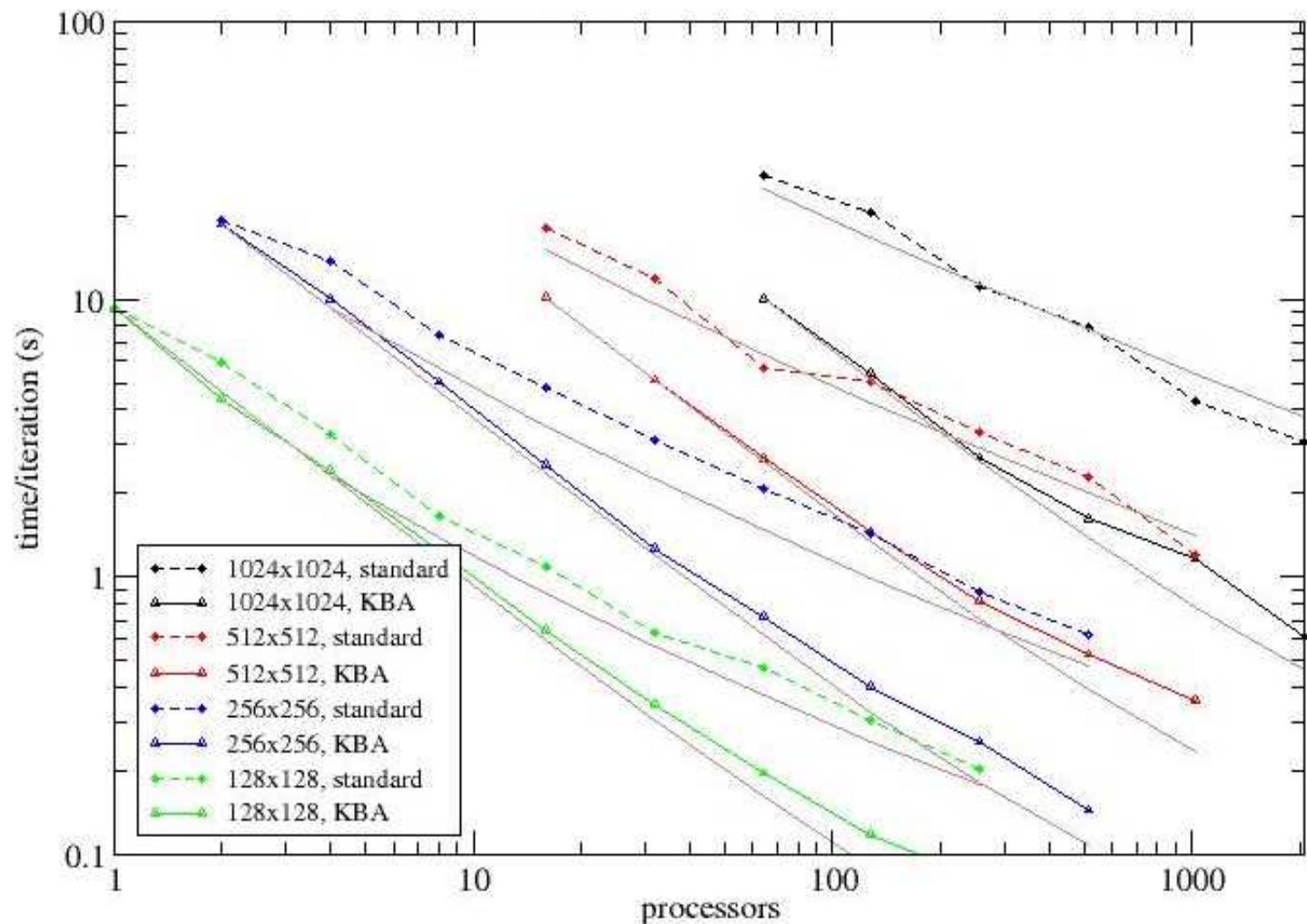
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# Sceptre first-order parallel scaling (strong)

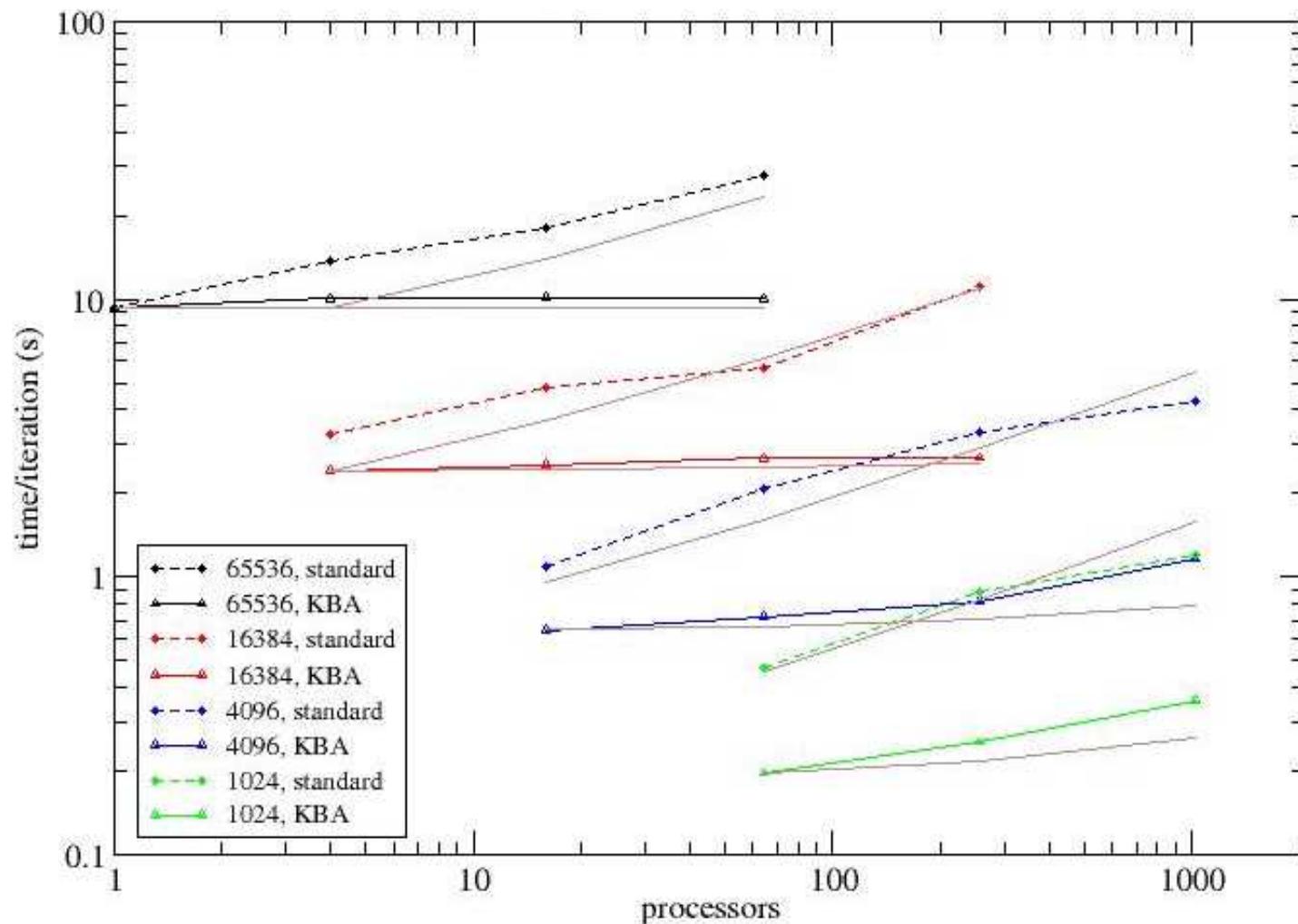
tri3 meshes, S8/S2, Red Storm





# Sceptre first-order parallel scaling (weak)

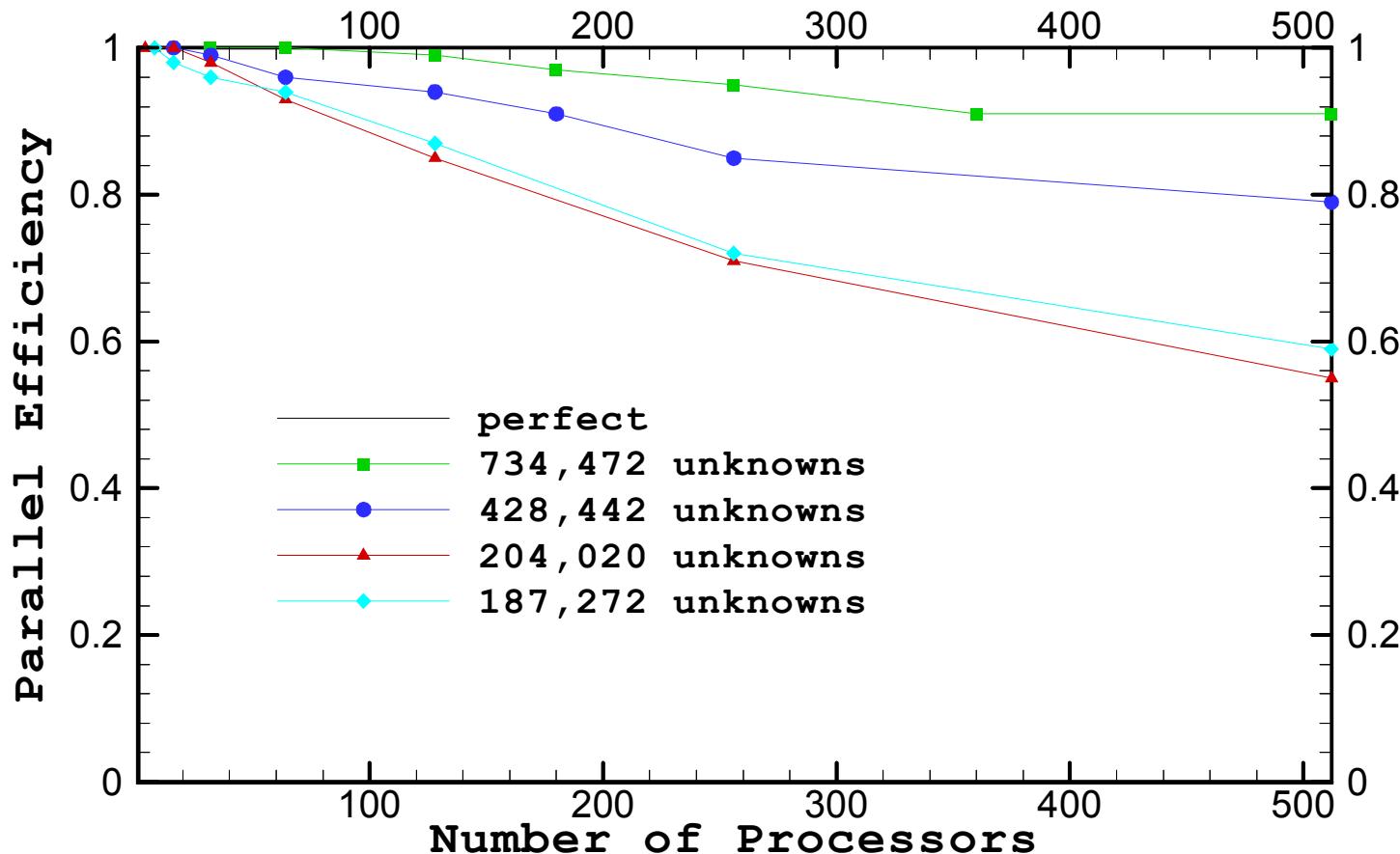
tri3 meshes, S8/S2, Red Storm





# CEPTRE parallel scaling, second-order

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# V & V: Verification and Validation

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- **VERIFICATION**: The process of determining that a computational software implementation correctly represents a model of a physical process (Are we solving the equations right?)
- **VALIDATION**: The process of determining the degree to which a computer simulation is an accurate representation of the real world for a specific application (Are we solving the right equations? Is the right physics modeled?)



# Verification: Method of Manufactured Solutions (MMS)

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- Need: closed-form/exact solutions useful for code verification
- Challenge: limited number of closed-form solutions to the transport equation
- MMS approach: specify ("manufacture") a solution and use transport equation to determine the source term and boundary conditions; use these as code inputs
- Unlimited number of closed-form solutions available with this approach

*Instead of searching for a (possibly non-closed-form) solution to a problem, we find a problem to a solution*

# 3D Manufactured Solution Example

- Unit cube
- Unit total cross section
- Linearly anisotropic scattering,  $c=1/2$
- S8 quadrature
- Tet4 and Tet10 elements

Assumed solution:

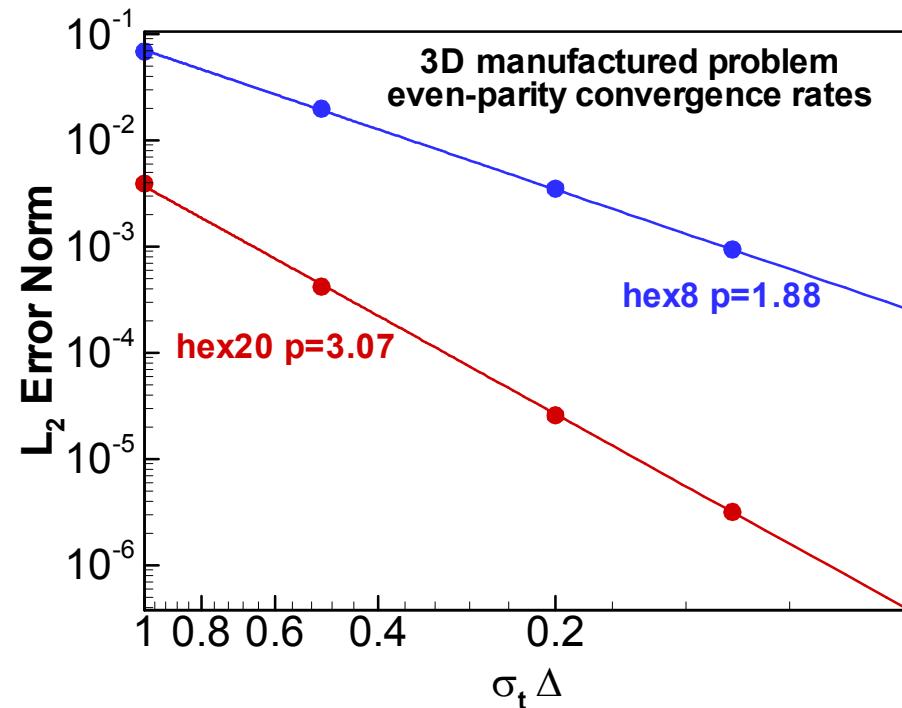
$$\Phi(x, y, z, \mu_z) = \mu_z^2 e^{-x} e^{-y} e^{-z}$$

Distributed source:

$$Q(x, y, z, \mu_x, \mu_y, \mu_z) = \left[ \mu_z^2 (1 - \mu_x - \mu_y - \mu_z) - \frac{1}{6} \right] e^{-x} e^{-y} e^{-z}$$

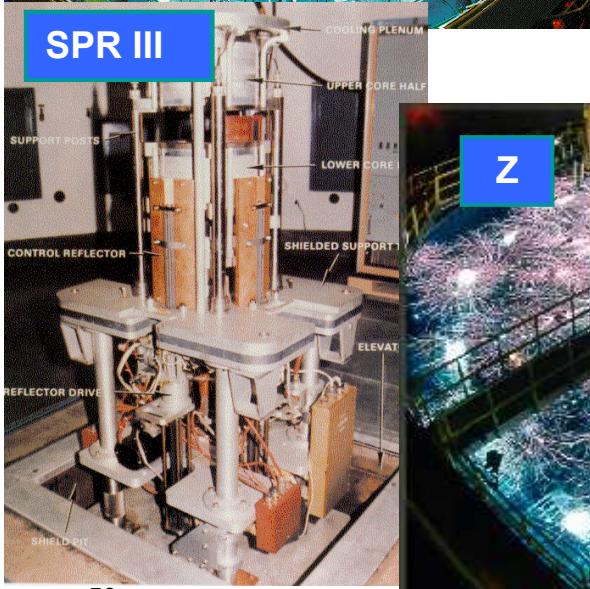
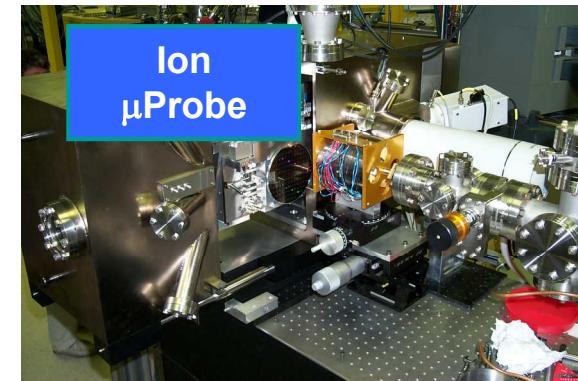
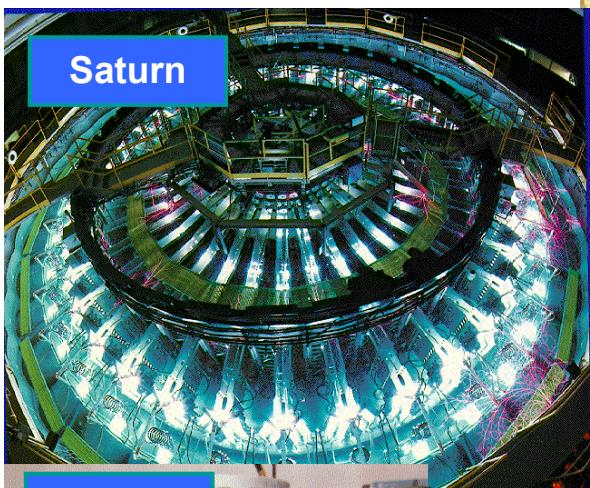
Boundary conditions:

$$\Phi_b(x, y, z, \mu_z) = \mu_z^2 e^{-x} e^{-y} e^{-z}, \text{ for } (x, y, z) \in \partial V$$



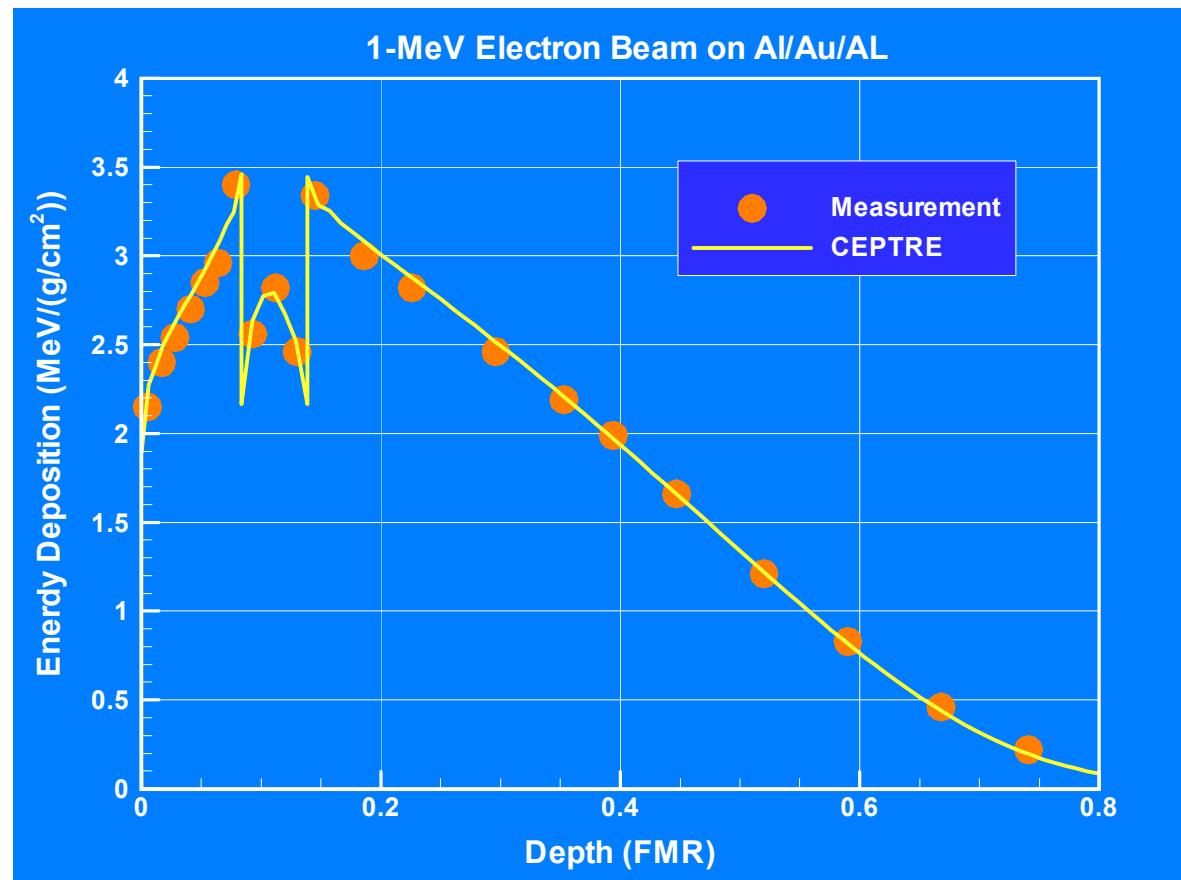
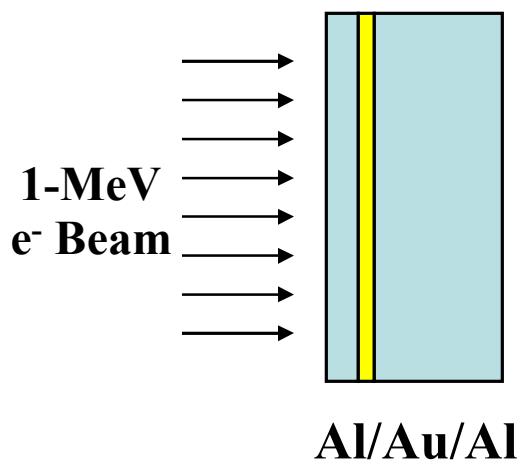


# Sandia radiation facilities used for testing, research, and code validation





# CEPTRE Validation: Energy Deposition 1-MeV Electron Beam on Al/Au/Al



G. J. Lockwood, et al, "Calorimetric Measurement of Electron Energy Deposition  
In Extended Media," SAND79-0414 (1980)



# Future Work

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- Couple first-order and second-order transport solvers
- Develop multi-stage preconditioning strategy using AMG (Algebraic MultiGrid) (tools available in Trilinos)
- Coarse photon mesh/fine electron mesh
- Develop more efficient parallel strategies
- Compare continuous vs. discontinuous finite elements
- Compare higher-order interpolation functions vs. finer mesh for increasing accuracy
- Compare spherical harmonics vs. discrete ordinates



# Summary

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- First- and second-order forms have complementary strengths and weaknesses
- Both forms are (being) implemented in CEPTRE/SCEPTRE
- Either form may be used separately, or together as a hybrid solver
- These forms are not restricted to electron/photon problems