

Uncertainty for Qualitative Variables

SAND2007- XXXX

Presented to Consequence Management Team
November 12, 2007

John Darby
6462

Sandia National Laboratories



Acknowledgments

- **Approximate Reasoning based on Logic Evolved Decision Techniques developed at LANL by Terry Bott and Steve Eisenhower**
- **Assistance with Mathematics from Jon Helton, Arizona State/SNL**
- **Belief/Plausibility for Fuzzy Sets for Numeric Variables: Ron Yager paper**
- **Adversary Defender Model Concepts**
 - Peter Merkle, SNL
 - Brian Bush, LANL
- **Graphical Ranking Technique**
 - John Cummings, SNL



References

- “Evaluation of Risk from Acts of Terrorism: The Adversary/Defender Model using Belief and Fuzzy Sets”, SAND2006-5777, September, 2006
- “Evaluation of Risk for Acts of Terrorism using Belief and Fuzzy Sets”, Journal of Nuclear Materials Management, Winter, 2007, Volume XXXV, Number 2
- “LinguisticBelief: A Java Application for Linguistic Evaluation using Belief, Fuzzy Sets, and Approximate Reasoning”, SAND2007-1299, March, 2007
- “Linguistic Evaluation of Terrorist Scenarios: Example Application”, SAND2007-1301, March, 2007



Presentation Topics

- **Methodology**
- **Software Tools**



Methodology

- **Uncertainty**
 - Ambiguity
 - Belief/Plausibility Measure
 - Superset of Probability Measure
 - Vagueness
 - Fuzzy Sets
 - Typically used for Numeric Variables
- **Qualitative Variables**
 - Fuzzy Sets
 - Purely Linguistic Fuzzy Sets
- **Combination of Qualitative Variables**
 - Convolution of Evidence over Linguistic Fuzzy Sets
 - Approximate Reasoning
 - Belief/Plausibility Measure for Uncertainty



Uncertainty

- **Ambiguity**
 - Uncertainty as to what will occur **in the future**
 - Dow Jones Industrial Average Close on Dec. 31, 2007
 - Will be one value
 - Ambiguity as to what that value will be
- **Vagueness**
 - Uncertainty as how to categorize a **known** outcome
 - Dow Jones close is 13,876 on Dec. 31, 2007
 - Is this “High” ?
 - What do you mean by “High”?
 - Vagueness can be expressed Linguistically (Words)



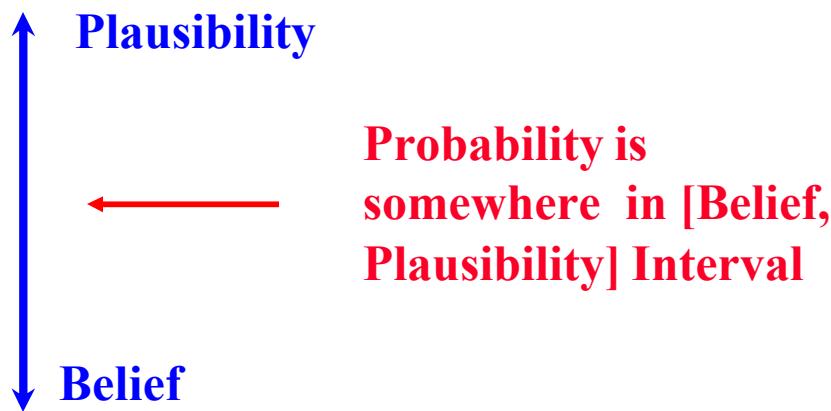
Ambiguity: Aleatory and Epistemic

- For a Fair Coin
 - Uncertainty is **Aleatory** (random)
 - Probability Heads is $\frac{1}{2}$
 - Probability Tails is $\frac{1}{2}$
- But if we cannot toss coin, we do not know coin is fair, we do not even know if coin has Heads and Tails
 - May not be Fair Coin (may be Weighted for Tails)
 - May be Two-Headed or Two-Tailed Coin
 - **Epistemic** (state of knowledge) uncertainty
 - Insufficient information to assign Probability to Heads and Tails
 - For Total Ignorance
 - Belief/Plausibility for Heads is 0/1
 - Belief/Plausibility for Tails is 0/1
- With more information (actually tossing the coin) we can reduce Epistemic Uncertainty
 - If at least one Heads and one Tails occur in a series of tosses, we know coin has Heads and Tails
 - Many tosses needed to assess if coin is fair
- For Fair Coin we cannot reduce aleatory uncertainty



Belief and Plausibility

- Belief / Plausibility form a Lower / Upper Bound for Probability
- Belief is what probability **will** be
- Plausibility is what probability **could** be

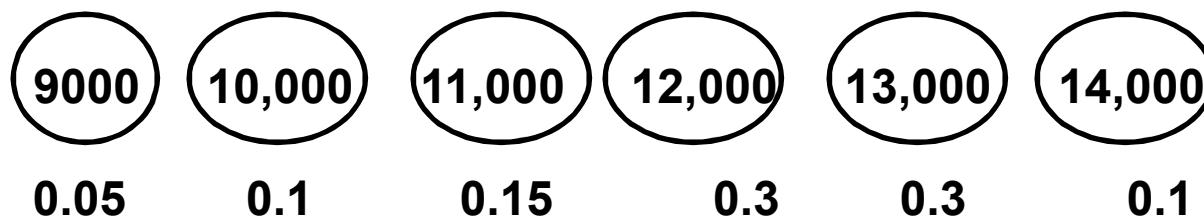


- Similar to a Confidence Interval for a Parameter of a probability distribution; a confidence measure that parameter is in interval, but exactly where in interval is not known
- Belief/Plausibility both reduce to Probability if Evidence is Specific



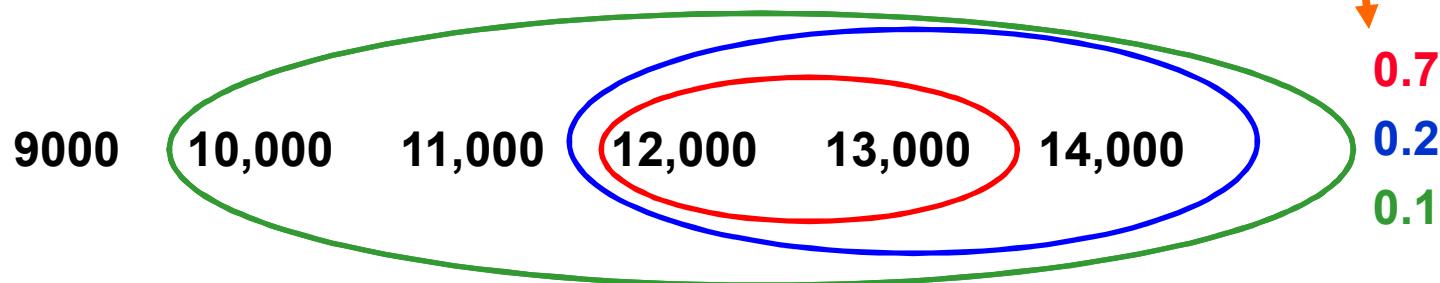
Example of Evidence: Predict Stock Market Close Dec. 31, 2007

- Probability



← Evidence

- Belief/Plausibility



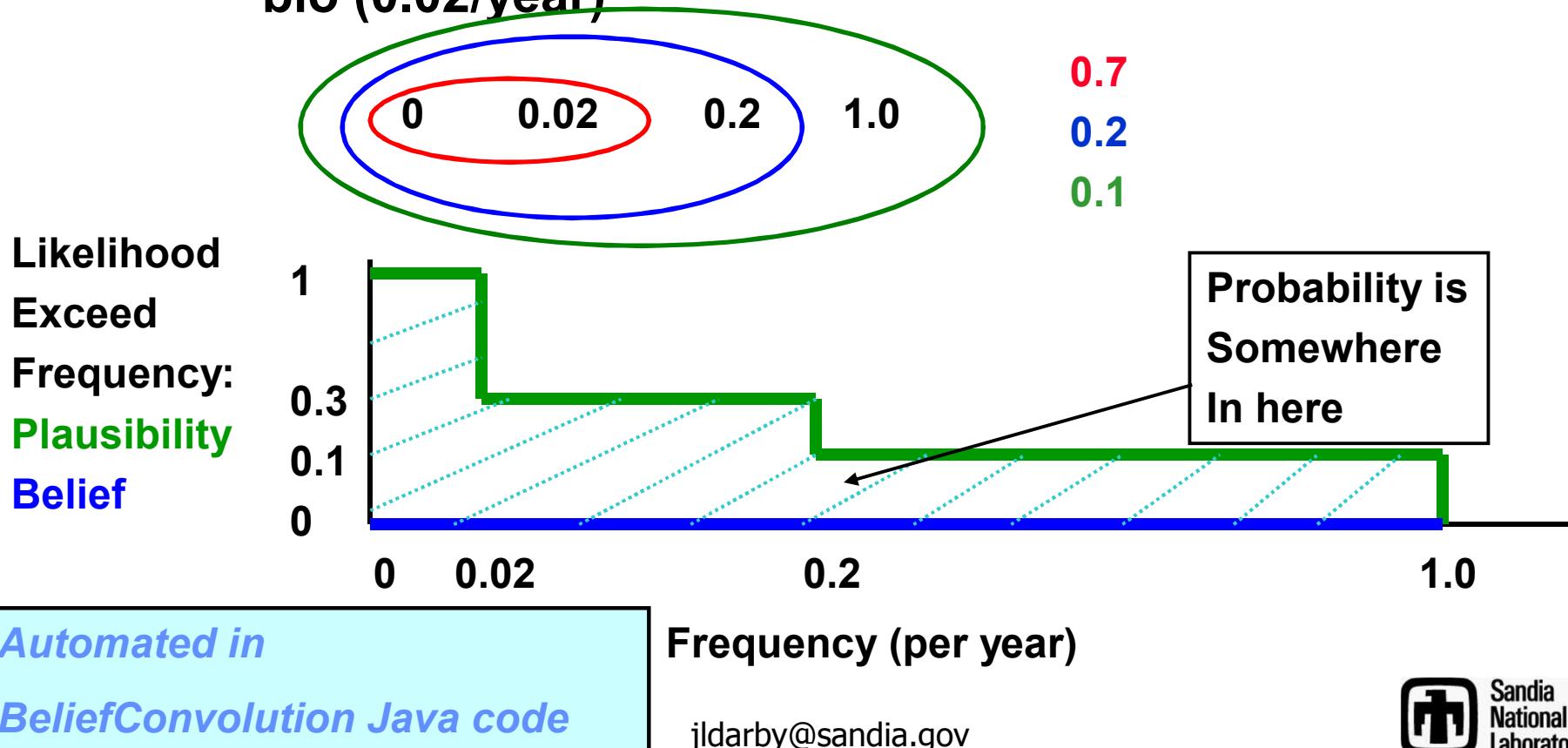


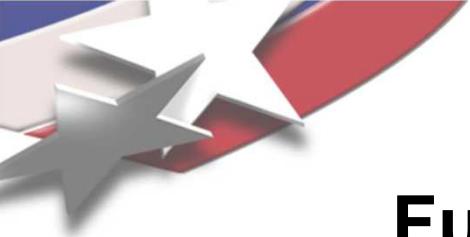
Why is this useful?

- What is Likelihood of Bio-Terror attack against a Major US City?
 - Frequency of Attack (per year) somewhere in $[0, 1]$
 - Point Estimate? **Useless**, too much uncertainty
 - Probability Distribution?
 - Must assign probability to each value in $[0, 1]$
 - Probability for 0 attacks per year is high
 - Probability assigned to 0 affects probabilities for all other values as **must sum to 1.0**
 - Is $P(0) = 0.01, 0.001, 0.00001, \dots$? **Don't Know**

Why is this useful?

- Evidence is about 1 major Attack every 5 years (0.2/year)
 - Assume Expert Opinion is: 10% Chance Attack is bio (0.02/year)

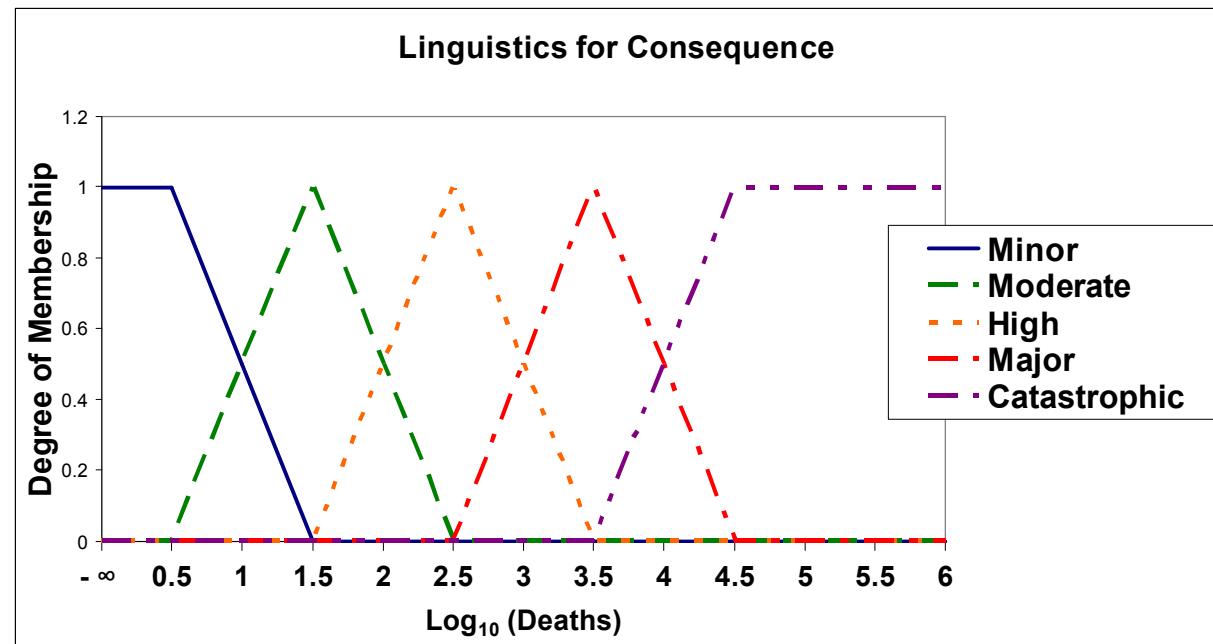




Vagueness

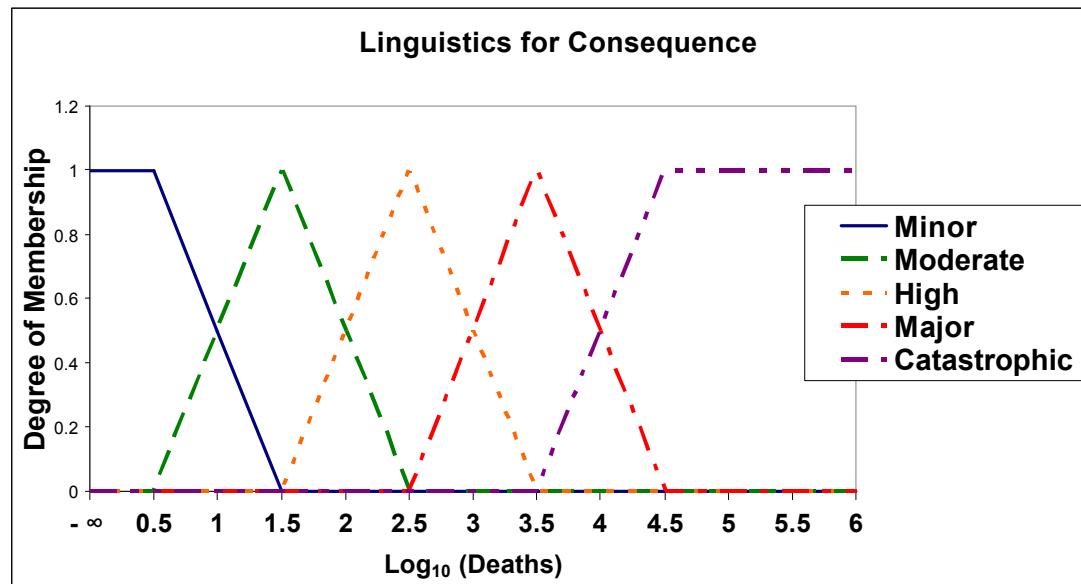
Fuzzy Sets for Numeric Variable

- Represent Variable with **Sets** to **reason at Fidelity Desired**.
Above 30,000 deaths is
“Catastrophic”.
- Use **Fuzzy Sets to Avoid Sharp Distinction**. “Major” Deaths is Between *About* 1000 and *About* 10,000. 999 and 1001 deaths are each part “High” and part “Major”.

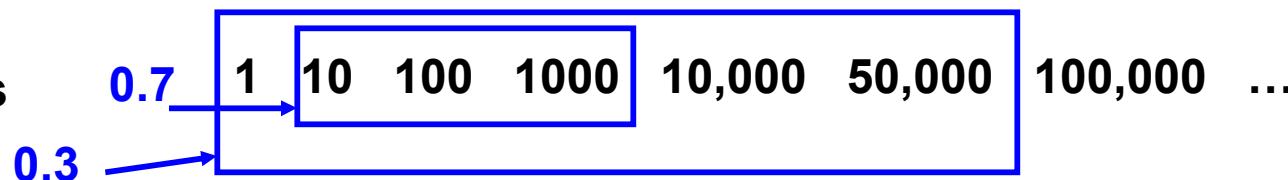


Uncertainty for Fuzzy Sets: Numeric Variable

Fuzzy Sets for Deaths



Evidence For Deaths



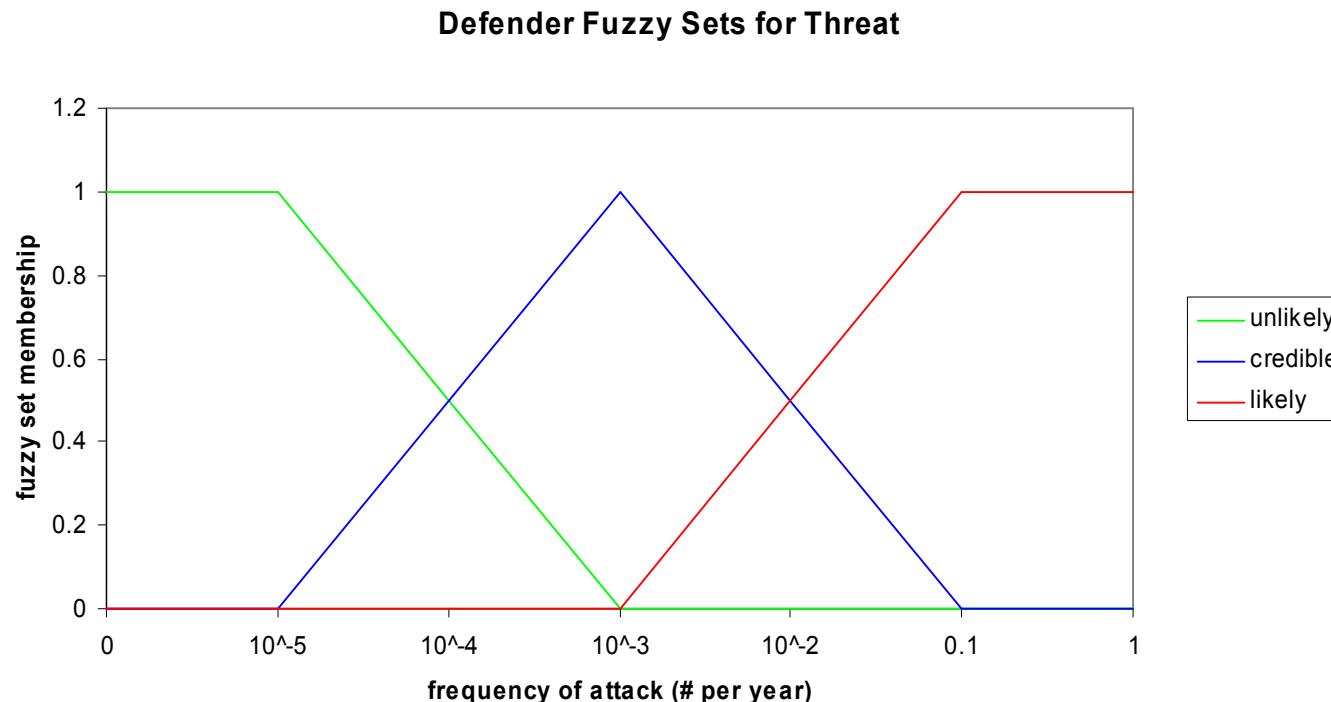
Uncertainty Distribution for Deaths

for Deaths:	Minor	Moderate	High	Major	Catastrophic
Belief / Plausibility	0 / 0.65	0 / 1	0 / 1	0 / 0.65	0 / 0.3



Why is this Important?

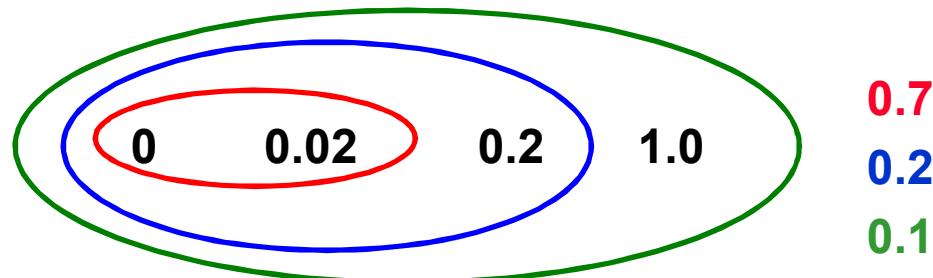
- What is Likelihood of Bio-Terror attack against a Major US City?
 - Evidence is about 1 major Attack every 5 years (0.2/year)
 - Assume Expert Opinion is: 10% Chance Attack is bio (0.02/year)
- Assume Following Fuzzy Sets for Evaluating Frequency of Attack



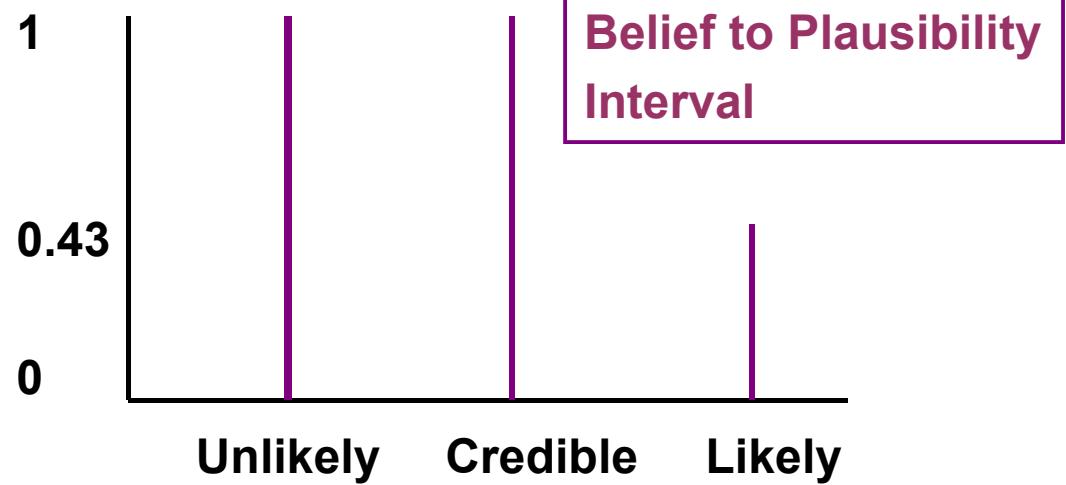


Why is this Important?

Evidence



**Likelihood
For Frequency**



*(Calculated with BeliefConvolution
code using Yager Method)*

jldarby@sandia.gov



Qualitative Variables

- Variable Segregated into **Purely Linguistic Fuzzy Sets**
 - Variable: “Health”
 - Fuzzy Sets: “Bad”, “Moderate”, “Excellent”
- Why Pure Linguistics?
 - **Numeric Scale is Unknown**
 - Is “Health” [0, 10], [0, 10^6], [-700, square root of 42]?
 - **Scaling is Un-Manageable when Combine Variables**
 - Combine “Health” with “Wealth” to Evaluate “Quality of Life”
 - “Wealth” can be Numeric: [\$0, \$50B]
 - What is Numeric Scale for “Health”?
 - What is Numeric Scale for “Quality of Life”?



Fuzzy Sets for Non-Numeric Variable

Adversary Level of Technical Training:

High School

Bachelors

Advanced

Do **NOT** Force Numeric Measure: Requires Arbitrary Scale

Adversary Level of Technical Training:

High School = **1?**

Bachelors = **2?**

Advanced = **3?**

Adversary Level of Technical Training:

High School = **10?**

Bachelors = **100?**

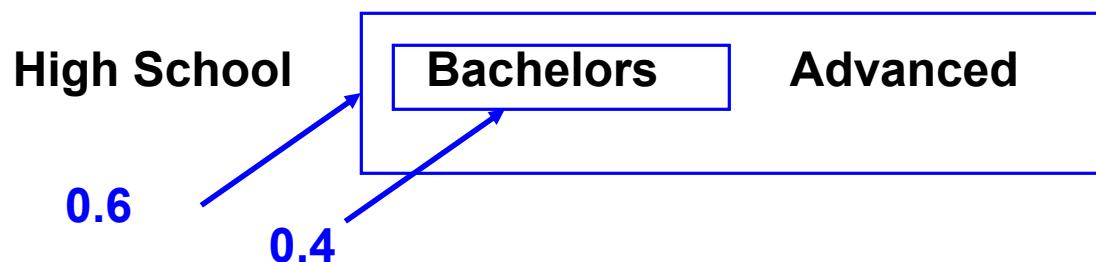
Advanced = **1000?**



Uncertainty for Fuzzy Sets: Non-Numeric Variable

- Fuzzy Sets for Adversary Level of Technical Training
 - High School Bachelors Advanced

- Evidence



- Uncertainty Distribution: Belief / Plausibility

High School	Bachelors	Advanced
0 / 0	0.4 / 1	0 / 0.6



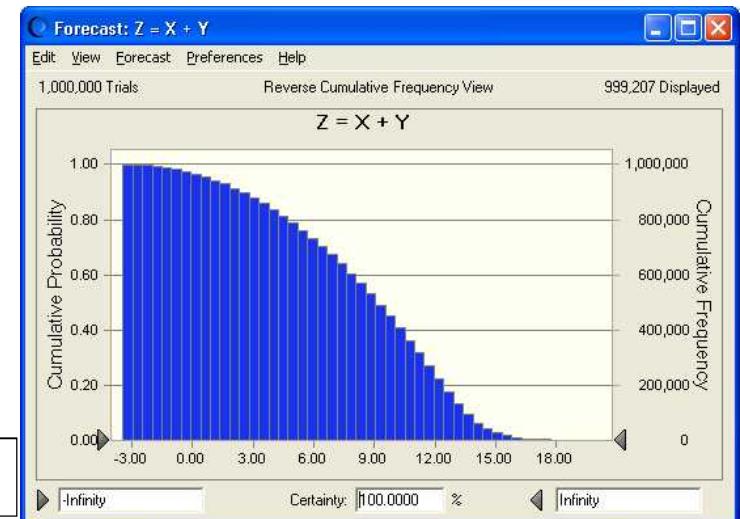
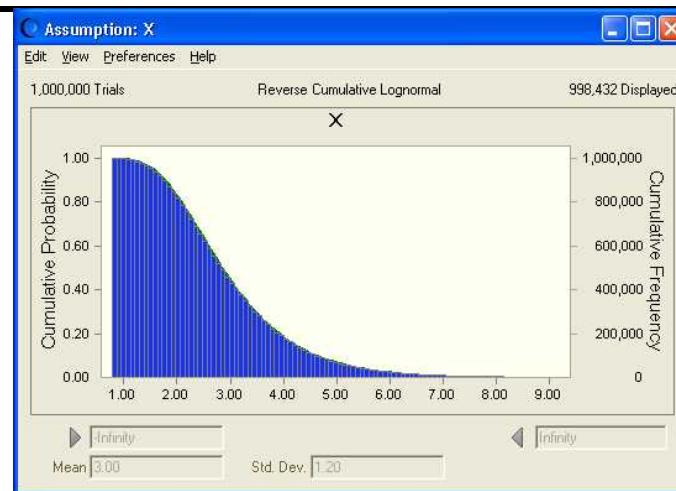
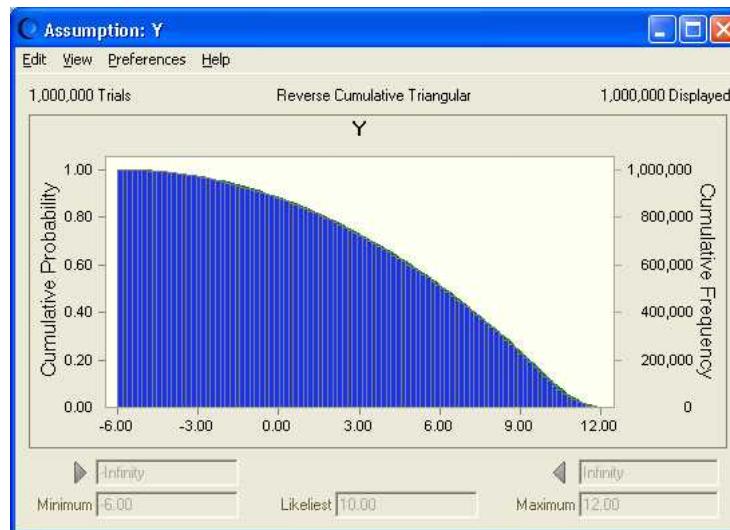
Combining Variables: Convolution of Uncertainty Distributions

- **Belief/Plausibility Distributions**
 - Evidence Over Fuzzy Sets for Each Variable
- **Convolute Distributions** per the Rule Base
 - Mathematics of Belief/Plausibility
- **Same Concept as Convolution of Probability Distributions**
 - Mathematics of Probability

Convolute Probability Distributions: Crystal Ball Software

$$Z = X + Y$$

X and Y Independent



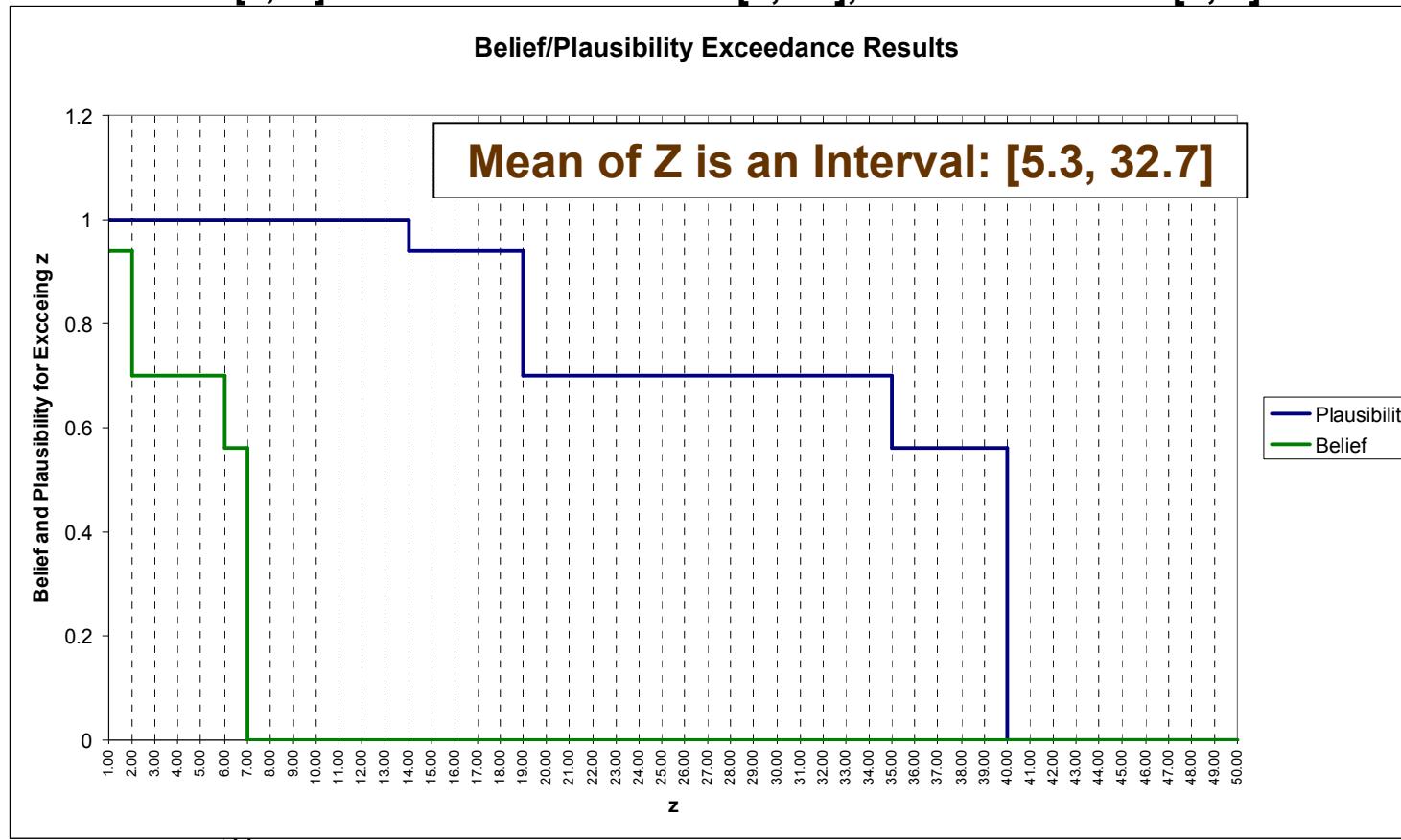
Mean of Z is a Point value: 8.34

Convolute Belief/Plausibility Distributions for Numeric Variables: BeliefConvolution Software

$Z = X + Y$, X and Y Non-Interactive

X over [1, 20] with Evidence: 0.8 for [2, 15], 0.2 for [1,10]

Y over [0,30] with Evidence: 0.7 for [5, 25], Evidence 0.3 for [0, 4]





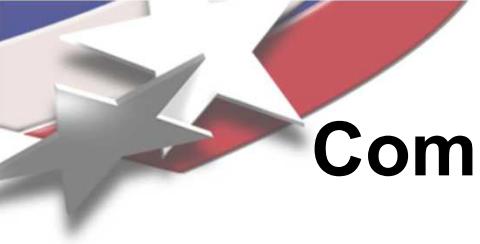
Convolute Belief/Plausibility Distributions for Linguistic Variables: LinguisticBelief Software

- Example Follows



Combining Qualitative Variables: Approximate Reasoning

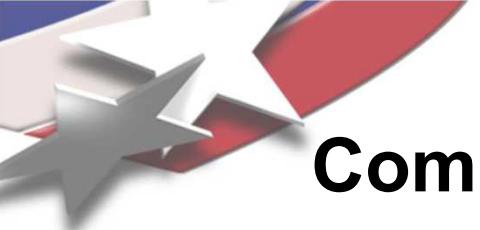
- Mathematics for **Combining Words**
- If we use Words instead of numbers we need a way of combining the Words for Different Variables
- Implemented as A Rule Base for Combining Fuzzy Sets from Different Variables



Combination of Linguistic Variables: Example

Develop the Model: Happiness for Any Individual
Define the Variables and their Fuzzy Sets

- **Basic Variables**
 - **Health**
 - Bad, Moderate, Excellent
 - **Wealth**
 - Poor, Middle Class, Rich
 - **Outlook on Life**
 - Pessimist, Optimist
- **Rule Based Variables**
 - **Quality of Life = Heath x Wealth (x per rule base)**
 - Not so Good, Good
 - **Happiness = Outlook on Life x Quality of Life**
 - Depressed, Accepting, Very Happy



Combination of Linguistic Variables: Example

Develop the Approximate Reasoning Rule Base for Rule Based Variables

Quality of Life

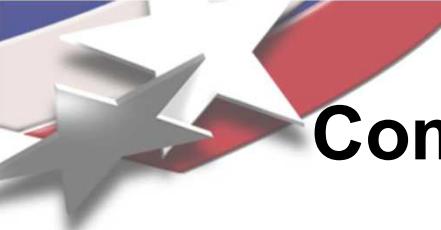
Rules for selected RuleLinguistic

Rules for RuleLinguistic: Quality of Life

Fuzzy Set for Input Linguistic: Health	Fuzzy Set for Input Linguistic: Wealth	Output Fuzzy Set for Rule (blank if rule not set)
Bad	Poor	Not so Good
Bad	Middle Class	Not so Good
Bad	Rich	Not so Good
Moderate	Poor	Not so Good
Moderate	Middle Class	Not so Good
Moderate	Rich	Good
Excellent	Poor	Good
Excellent	Middle Class	Good
Excellent	Rich	Good

Specify Output Fuzzy Set for Selected Rule Choices Are: ▾

Accept Rules as Shown Cancel



Combination of Linguistic Variables: Example

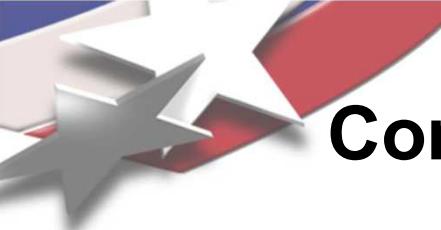
Happiness



The screenshot shows a software dialog box titled "Rules for selected RuleLinguistic" with a sub-section titled "Rules for RuleLinguistic: Happiness". The table displays the following rules:

Fuzzy Set for Input Linguistic: Outlook on Life	Fuzzy Set for Input Linguistic: Quality of Life	Output Fuzzy Set for Rule (blank if rule not set)
Pessimist	Good	Accepting
Pessimist	Not so Good	Depressed
Optimist	Good	Very Happy
Optimist	Not so Good	Accepting

Below the table, a message says "Specify Output Fuzzy Set for Selected Rule" followed by a "Choices Are:" dropdown menu. At the bottom are "Accept Rules as Shown" and "Cancel" buttons.

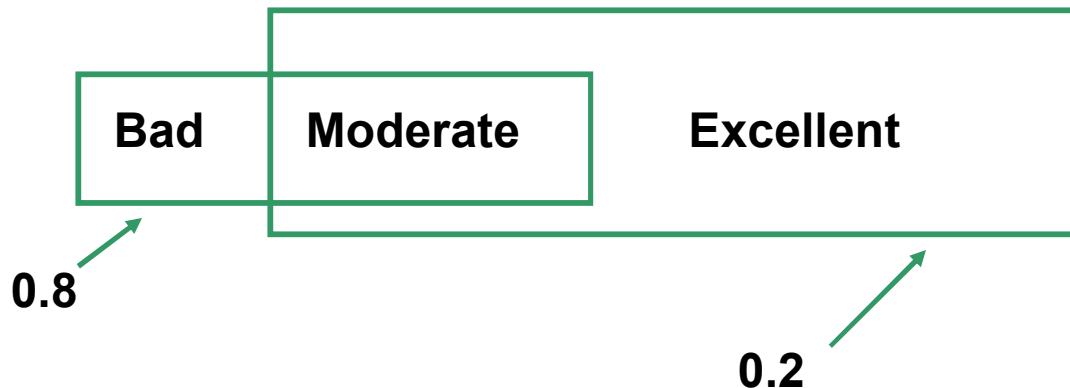


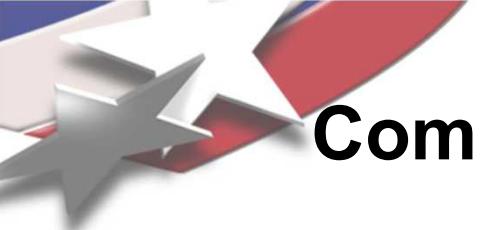
Combination of Linguistic Variables: Example

**Evaluate the Model for Specific Individual:
Happiness for “John”**

Assign Evidence to Fuzzy Sets for Basic Variables

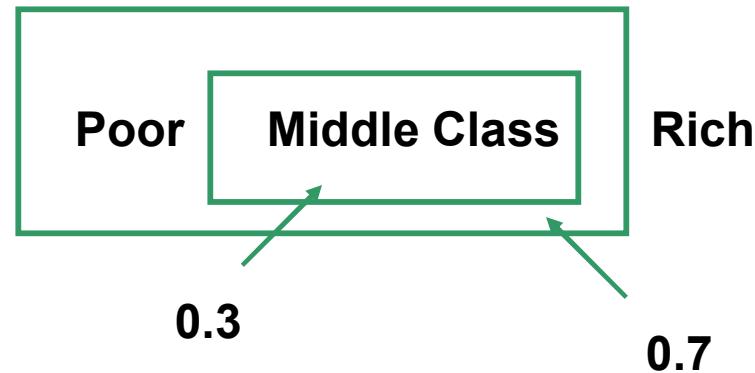
Health

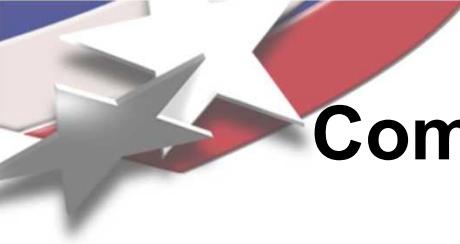




Combination of Linguistic Variables: Example

Wealth

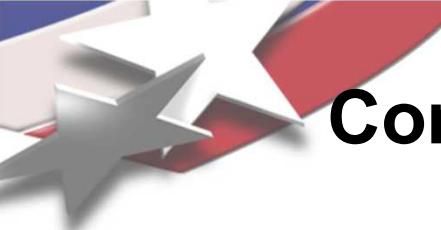




Combination of Linguistic Variables: Example

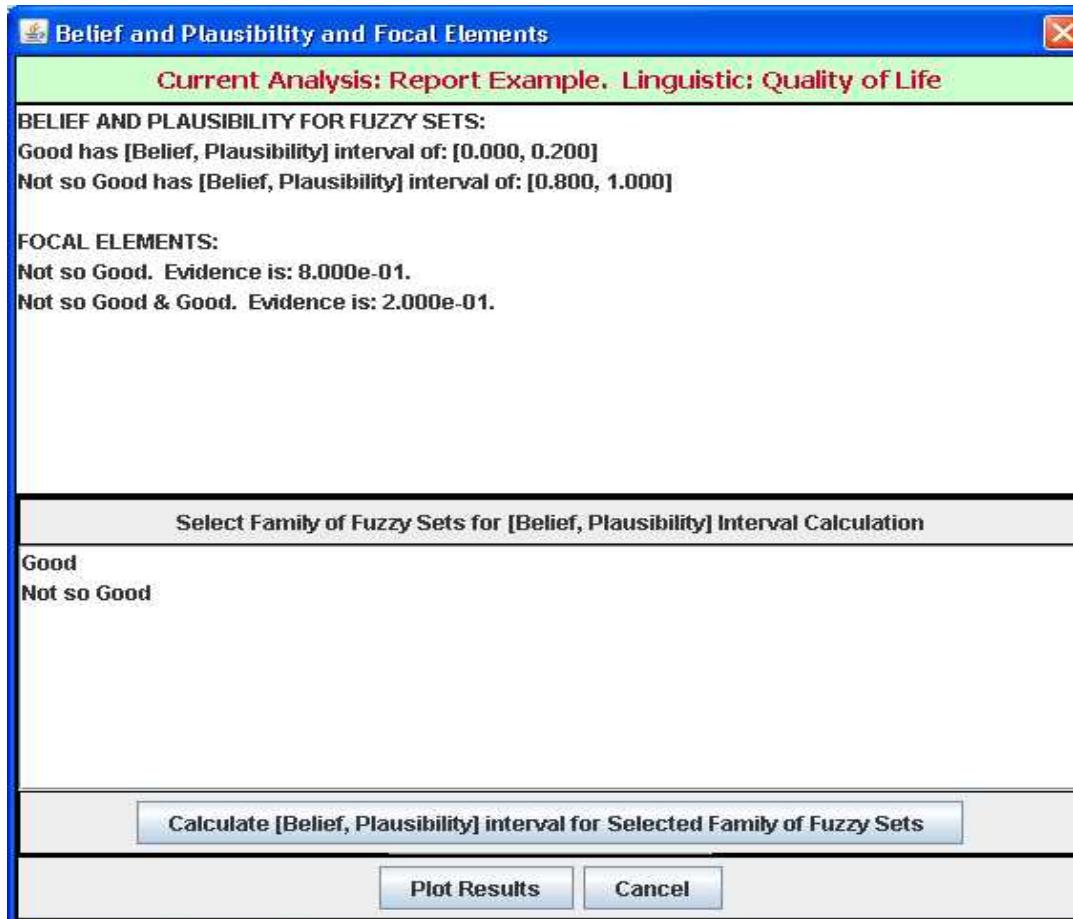
Outlook on Life

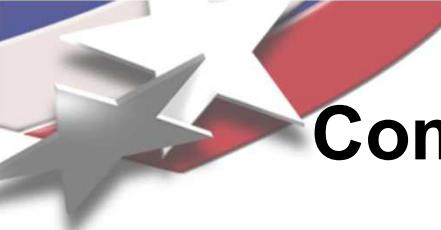




Combination of Linguistic Variables: Example

Evaluate Variable: Quality of Life for John





Combination of Linguistic Variables: Example

Evaluate Variable Happiness for John

 Belief and Plausibility and Focal Elements X

Current Analysis: Report Example. Linguistic: Happiness

BELIEF AND PLAUSIBILITY FOR FUZZY SETS:

Depressed has [Belief, Plausibility] interval of: [0.016, 1.000]
Accepting has [Belief, Plausibility] interval of: [0.000, 0.984]
Very Happy has [Belief, Plausibility] interval of: [0.000, 0.196]

FOCAL ELEMENTS:

Depressed. Evidence is: 1.600e-02.
Depressed & Accepting. Evidence is: 7.880e-01.
Depressed & Accepting & Very Happy. Evidence is: 1.960e-01.

Select Family of Fuzzy Sets for [Belief, Plausibility] Interval Calculation

Depressed
Accepting
Very Happy

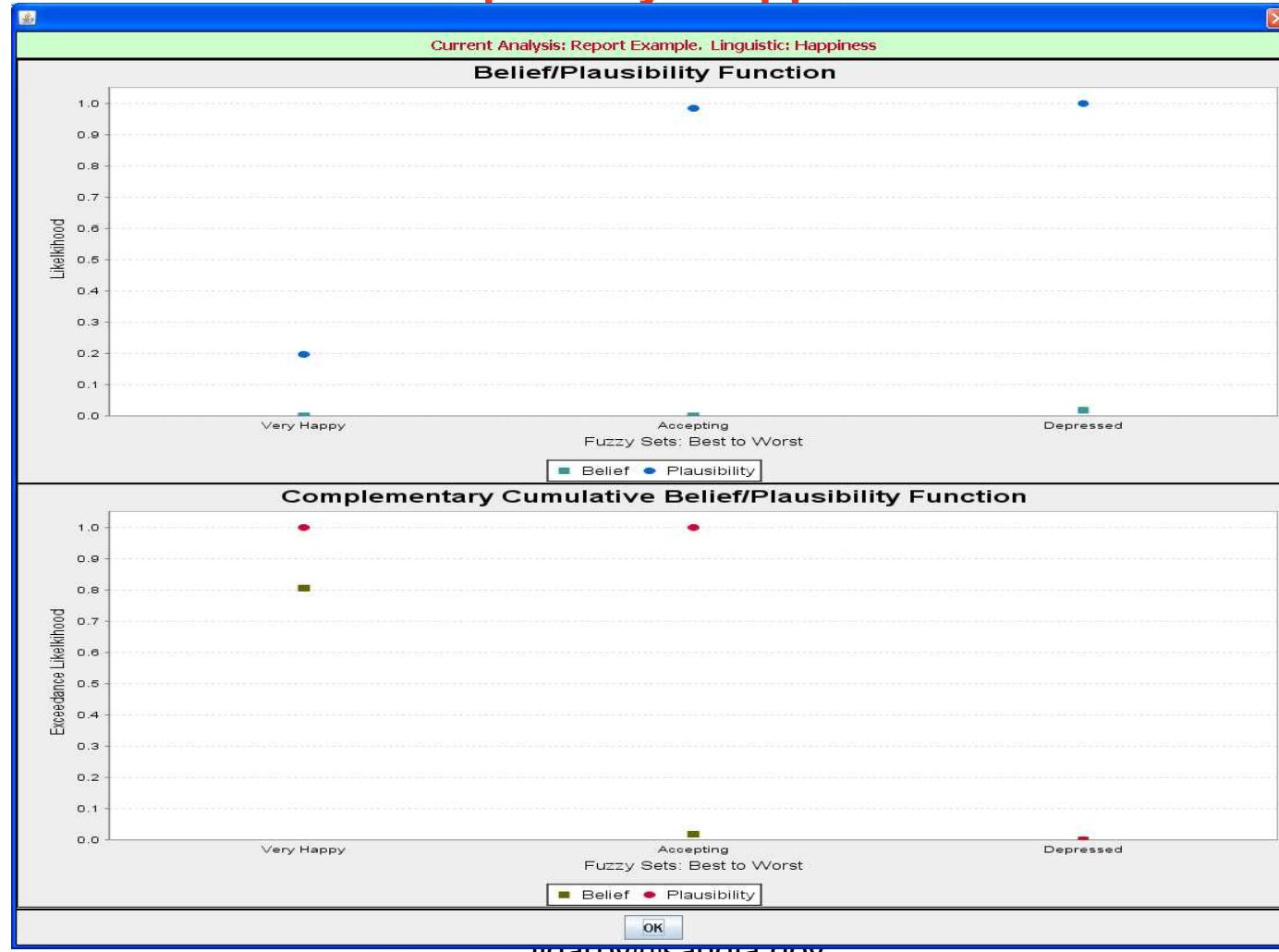
Calculate [Belief, Plausibility] interval for Selected Family of Fuzzy Sets [0.000e+00, 9.840e-01]

Plot Results Cancel

jldarby@sandia.gov

Combination of Linguistic Variables: Example

Summarize Results Graphically: Happiness for “John”





Results Expressed as Complementary Cumulative Belief/Plausibility Functions (CCBPFs) for Linguistic Variable

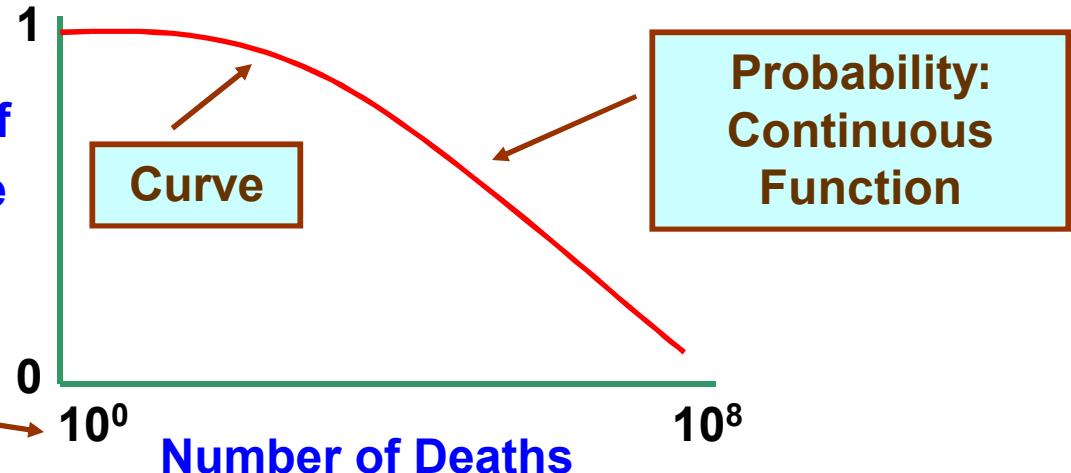
- Linguistic Fuzzy Sets **Ordered** from “Best” to “Worst”
 - CCBPFs are **Non-Increasing**
- “Likelihood” of **Exceeding** Fuzzy Set
- “Likelihood” is **Belief/Plausibility Interval**
- Analogous to Complementary Cumulative Probability Function (CCPF) for Random Variable
 - CCPF Random Variable is a real number
 - discrete or continuous
 - CCBPFs Variable has linguistic fuzzy sets
 - Discrete
 - CCPF is a One Function: a Curve
 - CCBPFs are Two Functions: an Interval



CCPF and CCBPFs

- CCPF

Likelihood of Exceedance

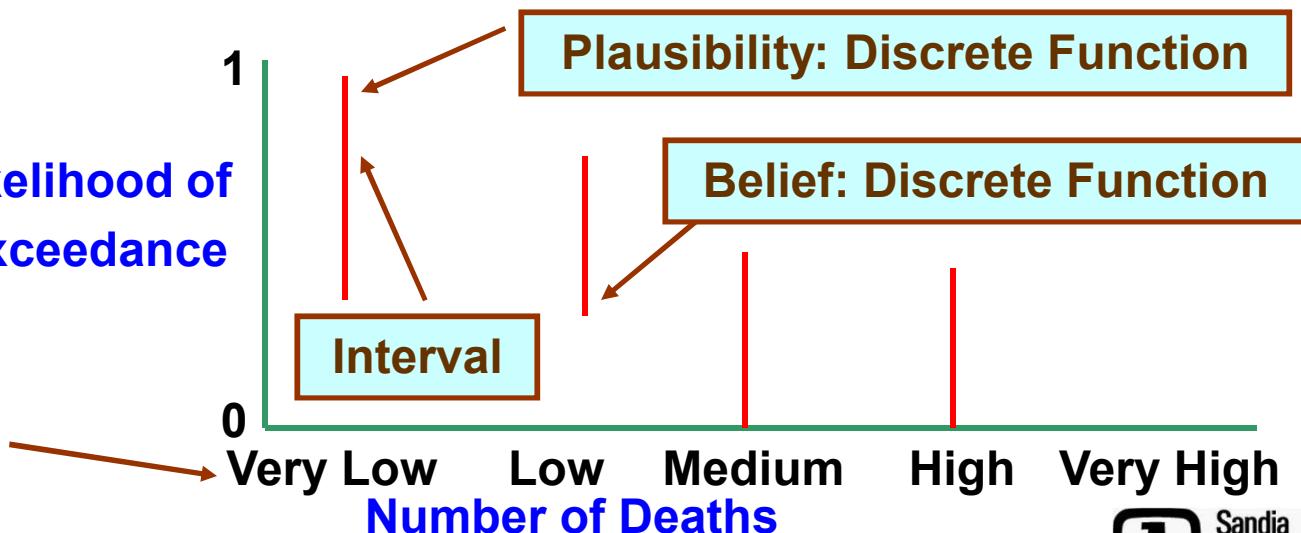


Continuous Variable:
Real Number

- CCBPFs

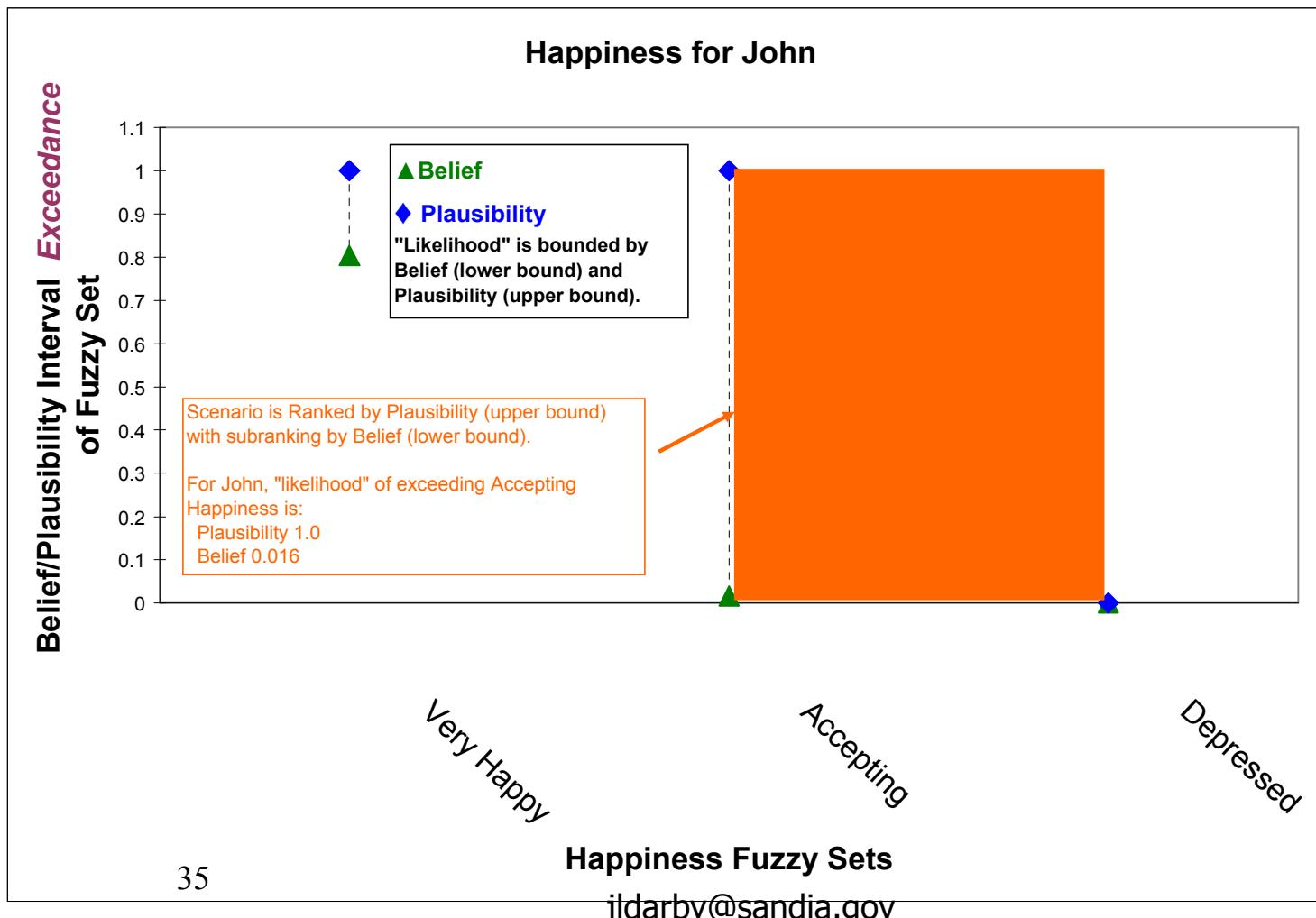
Likelihood of Exceedance

Discrete Variable:
Linguistic Fuzzy
Sets



Combination of Linguistic Variables: Example

Graphical Summary for Ranking: “John”





Custom Software Tools: Java

- **BeliefConvolution**
 - Convolution of **Numeric Variables** with Belief/Plausibility
- **LinguisticBelief**
 - **Evaluation of Linguistic Variables**
 - Linguistic Fuzzy Sets
 - Approximate Reasoning
 - Belief/Plausibility
- **PoolEvidence**
 - **Multiple Experts provide Evidence for Variables**
 - Linguistic Fuzzy Sets
 - **Combine Evidence**
 - Pooled Evidence for Variables
 - **Input for LinguisticBelief**

LinguisticBelief: Example Application

LinguisticBelief Application

File Utilities Help

New Open Save EXIT! Exit

Current Analysis: National Planning Scenario

- Rule Linguistics
 - Long Term Consequence
 - Casualties (Equivalent Dead)
 - Adversary Estimate Consequence
 - Adversary Estimate Information Required
 - Detect Adversary Gathering Information
 - Adversary Estimate Scientific/Engineering Attributes Required
 - Detect Adversary Gathering Scientific/Engineering Attributes
 - Adversary Estimate Attributes Required
 - Adversary Estimate Resources Required
 - Detect Adversary Gathering Attributes
 - Immediate Consequence
 - Detect Adversary Gathering Resources
 - Vulnerability
 - Consequence
 - Adversary Estimate Vulnerability
 - Threat
 - Risk
 - Basic Linguistics
 - Dead
 - Injured/III
 - Infrastructure Damage
 - Evacuation
 - Contamination
 - National Economic Impact

Information for Selected Tree Node

fuzzy sets: credible, Very Low, medium, -- Rule output fuzzy set: Negligible*, * Rule input fuzzy sets: credible, Very Low, high, -- Rule output fuzzy set: Negligible*, * Rule input fuzzy sets: credible, Very Low, very high, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: credible, Low, very low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: credible, Low, low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: credible, Low, medium, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: credible, Low, high, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: credible, Low, very high, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: credible, Medium, very low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: credible, Medium, low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: credible, Medium, medium, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: credible, Medium, high, -- Rule output fuzzy set: Medium*, * Rule input fuzzy sets: credible, Medium, very high, -- Rule output fuzzy set: High*, * Rule input fuzzy sets: credible, High, very low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: credible, High, low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: credible, High, medium, -- Rule output fuzzy set: Medium*, * Rule input fuzzy sets: credible, High, high, -- Rule output fuzzy set: High*, * Rule input fuzzy sets: credible, High, very high, -- Rule output fuzzy set: Extreme*, * Rule input fuzzy sets: likely, Very Low, very low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, Very Low, low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, Very Low, medium, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, Very Low, high, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, Very Low, very high, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, Low, very low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, Low, low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, Low, medium, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, Low, high, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, Low, very high, -- Rule output fuzzy set: Medium*, * Rule input fuzzy sets: likely, Medium, very low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, Medium, low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, Medium, medium, -- Rule output fuzzy set: Medium*, * Rule input fuzzy sets: likely, Medium, high, -- Rule output fuzzy set: High*, * Rule input fuzzy sets: likely, Medium, very high, -- Rule output fuzzy set: High*, * Rule input fuzzy sets: likely, High, very low, -- Rule output fuzzy set: Low*, * Rule input fuzzy sets: likely, High, low, -- Rule output fuzzy set: Medium*, * Rule input fuzzy sets: likely, High, medium, -- Rule output fuzzy set: High*, * Rule input fuzzy sets: likely, High, high, -- Rule output fuzzy set: Extreme*, * Rule input fuzzy sets: likely, High, very high, -- Rule output fuzzy set: Extreme*, . [Belief, Plausibility] Intervals for fuzzy sets are as follows : Fuzzy Set: Negligible [0.000, 0.384], Fuzzy Set: Low [0.000, 0.349], Fuzzy Set: Medium [0.155, 1.000], Fuzzy Set: High [0.000, 0.720], Fuzzy Set: Extreme [0.000, 0.000], .

India
National
laboratories

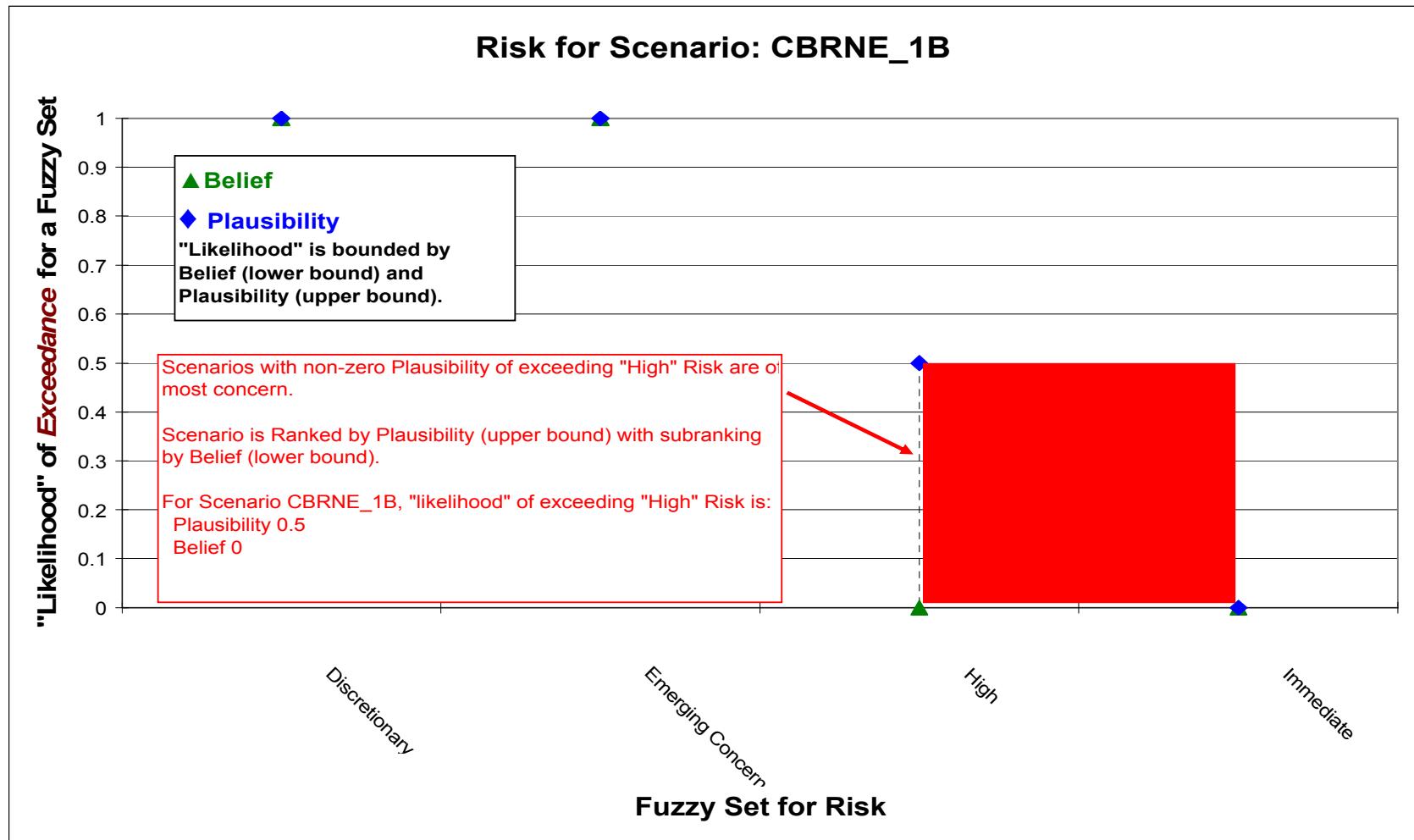
jdarby@sandia.gov



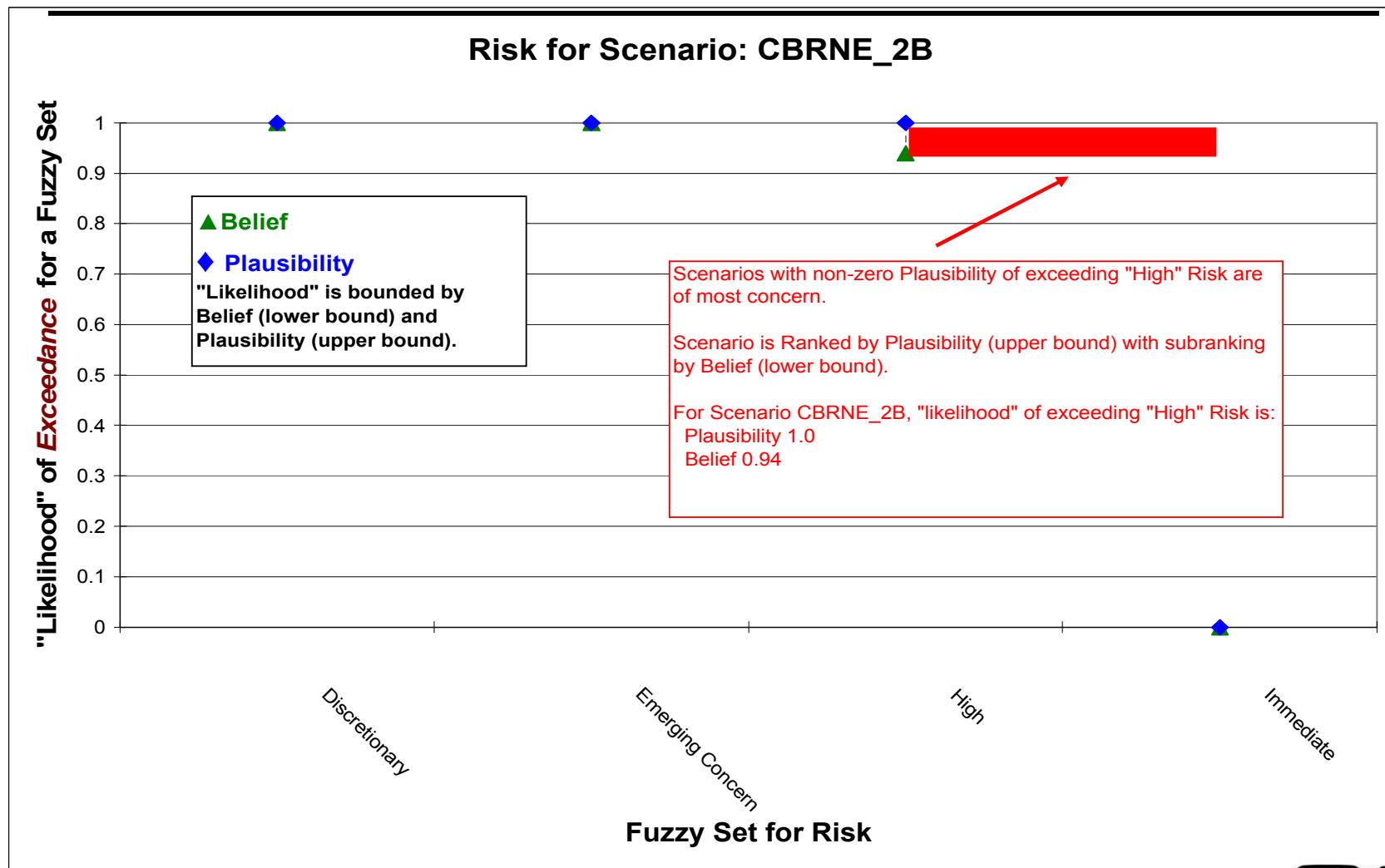
Rank Order Scenarios by Risk

- Rank a scenario by the Highest Non-Zero Plausibility of Exceeding the “Worst” Fuzzy Set
 - For Scenarios with Equal Plausibility, Subrank by Highest Belief
 - Extension of “Probability of Exceedance” approach
 - Uses Fuzzy Sets instead of Numbers
 - Uses Belief/Plausibility Interval instead of Probability
 - Can be “Color Coded”
 - Shown for 3 of 5 scenarios in Following from SAND2007-1301

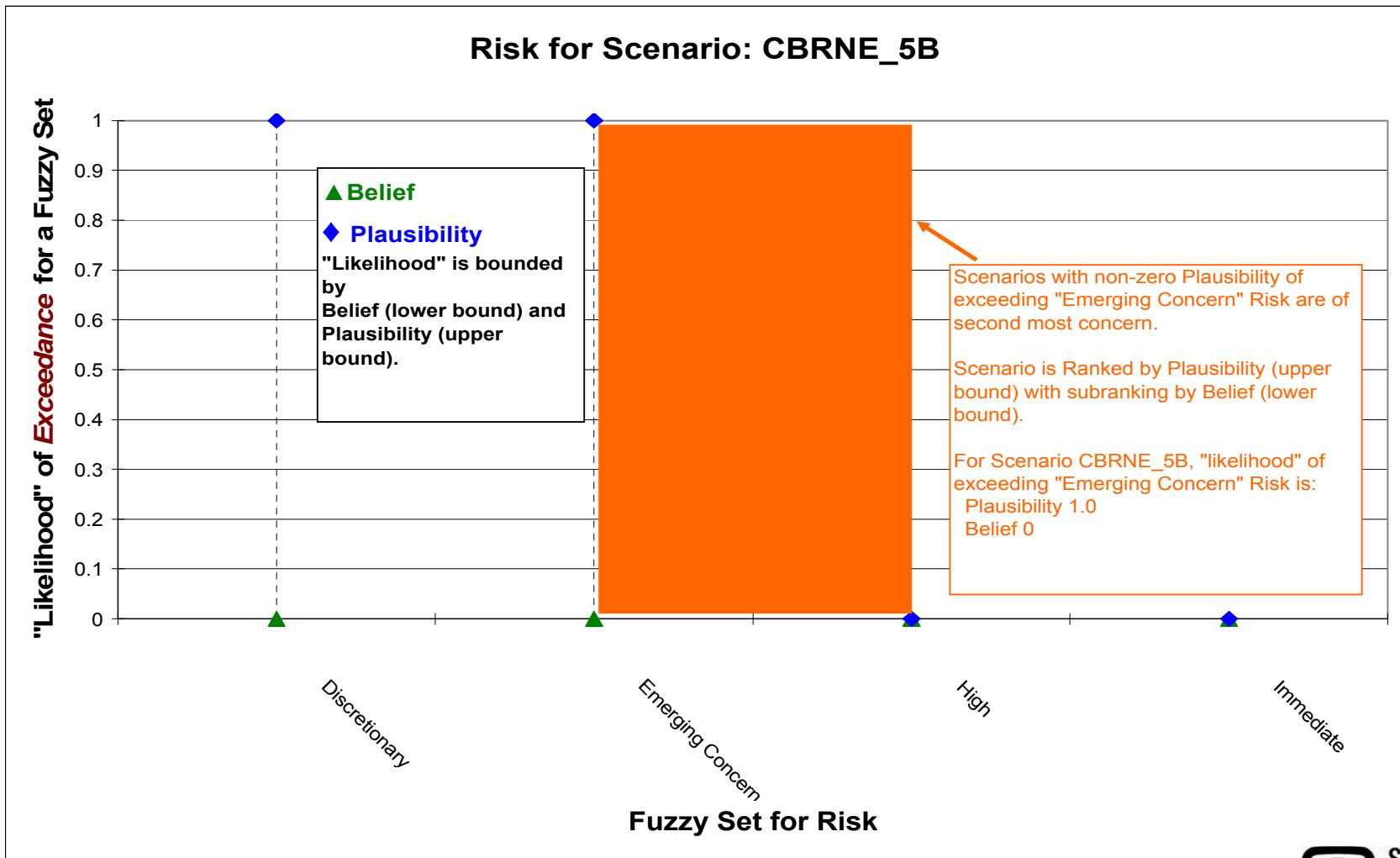
Rank Order Results



Rank Order Results



Rank Order Results





Results of Ranking of All Five Scenarios

RANKING FOR SCENARIOS CBRNE_1B through CBRNE_5B

For *Exceeding* Fuzzy Set “**High**” the Scenarios rank ordered (decreasing) are:

CBRNE_2B has plausibility of exceedance of 1.0 and belief of exceedance of 0.94
CBRNE_3B has plausibility of exceedance of 1.0 and belief of exceedance of 0.77
CBRNE_4B has plausibility of exceedance of 1.0 and belief of exceedance of 0.64
CBRNE_1B has plausibility of exceedance of 0.5 and belief of exceedance of 0.0

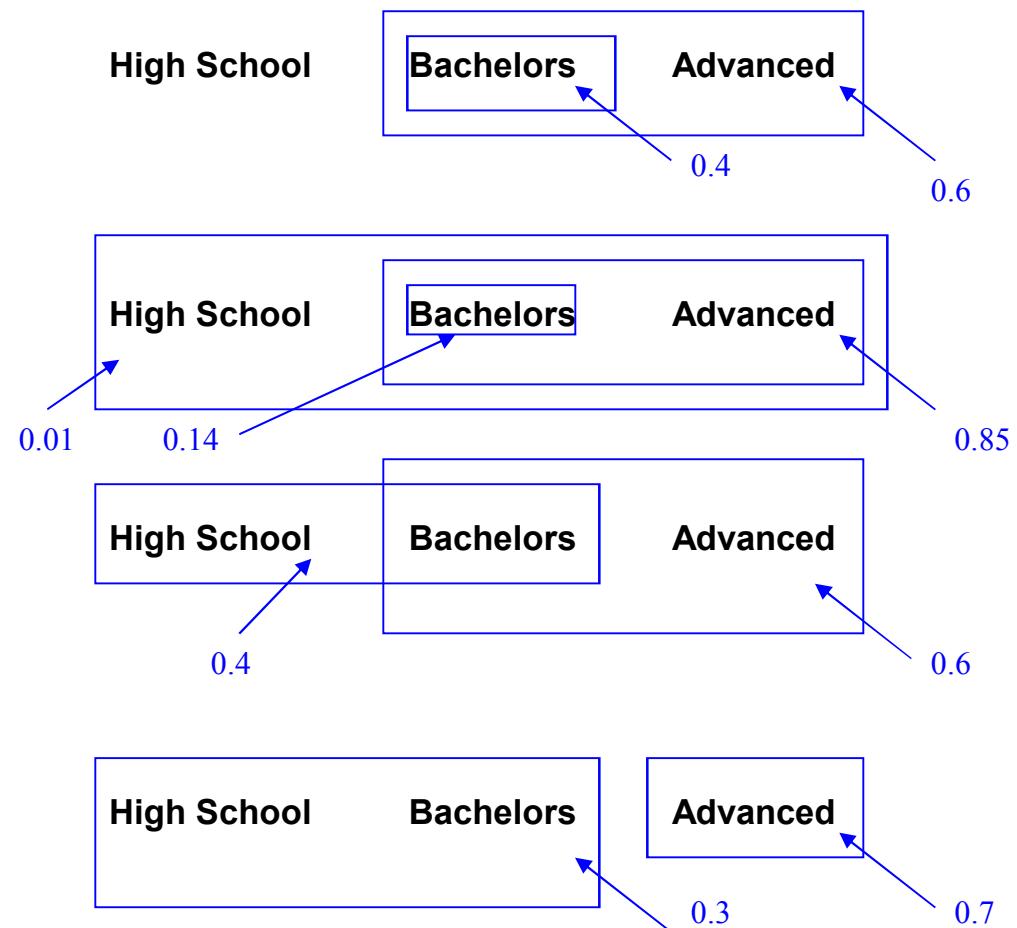
For *Exceeding* Fuzzy Set “**Emerging Concern**” the Scenarios rank ordered (decreasing) (not already ranked for a worse fuzzy set) are:

CBRNE_5B has plausibility of exceedance of 1.0 and belief of exceedance of 0.0



Pool Evidence from Many Experts

Adversary Level of Technical Training:
4 Experts Assign Evidence



PoolEvidence: Example Application

Pooled Evidence Application

File Help

New Open Save EXIT! Exit

Current Analysis: None

Variables for Analysis	Fuzzy Sets for Selected Variable
Variables for Current Analysis: Actual Test <ul style="list-style-type: none">Adversary MotivationDefender ResourcesResult	Fuzzy Sets for Variable: Adversary Motivation <ul style="list-style-type: none">Very LowLowMediumHighVery High

Show Pooled Focal Elements for Selected Variable

Experts for Selected Variable	Focal Elements for Selected Expert
Experts for Variable: Adversary Motivation <ul style="list-style-type: none">PepperCharlieHeatherJohn	Focal Elements for Expert: John <ul style="list-style-type: none">Low, Medium, High, with Evidence: 2.30000e-01Very Low, Medium, with Evidence: 3.20000e-01Low, High, with Evidence: 4.50000e-01

Close

PoolEvidence

 Pooled Focal Elements for Selected Variable

Pooled Focal Elements for Variable: Adversary Motivation

POOLED FOCAL ELEMENTS FOR ALL EXPERTS

Low, Medium, with Evidence: 1.00000e-01
Very Low, High, with Evidence: 1.50000e-01
Medium, with Evidence: 2.50000e-01
High, with Evidence: 2.50000e-03
Very Low, Medium, with Evidence: 9.00000e-02
Low, High, with Evidence: 3.50000e-01
Low, Medium, High, with Evidence: 5.75000e-02

FOCAL ELEMENTS FOR EACH EXPERT

Pepper

Low, Medium, with Evidence: 4.00000e-01
Very Low, High, with Evidence: 6.00000e-01

Charlie

Medium, with Evidence: 1.00000e+00

Heather

High, with Evidence: 1.00000e-02
Very Low, Medium, with Evidence: 4.00000e-02
Low, High, with Evidence: 9.50000e-01

John

Low, Medium, High, with Evidence: 2.30000e-01
Very Low, Medium, with Evidence: 3.20000e-01
Low, High, with Evidence: 4.50000e-01



Summary

- **Uncertainty**
 - Belief/Plausibility for Epistemic Uncertainty
- **Qualitative Variables**
 - Purely Linguistic Fuzzy Sets
- **Combining Variables**
 - Approximate Reasoning
 - Convolution with Belief/Plausibility
- **Custom Software Tools**
 - BeliefConvolution
 - LinguisticBelief
 - PoolEvidence