Uncertainty for Qualitative Variables

SAND2007- XXXX

Presented to Consequence Management Team

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- Approximate Reasoning based on Logic Evolved Decision Techniques developed at LANL by Terry Bott and Steve Eisenhawer
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- Belief/Plausibility for Fuzzy Sets for Numeric Variables: Ron Yager paper
- Adversary Defender Model Concepts
  - Peter Merkle, SNL
  - Brian Bush, LANL
- Graphical Ranking Technique
  - John Cummings, SNL

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References

- "Evaluation of Risk from Acts of Terrorism: The Adversary/Defender Model using Belief and Fuzzy Sets", SAND2006-5777, September, 2006
- "Evaluation of Risk for Acts of Terrorism using Belief and Fuzzy Sets", Journal of Nuclear Materials Management, Winter, 2007, Volume XXXV, Number 2
- "LinguisticBelief: A Java Application for Linguistic Evaluation using Belief, Fuzzy Sets, and Approximate Reasoning", SAND2007-1299, March, 2007
- "Linguistic Evaluation of Terrorist Scenarios: Example Application", SAND2007-1301, March, 2007

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Presentation Topics

- **Methodology**
- **Software Tools**

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Methodology

- Uncertainty
  - Ambiguity
    - Belief/Plausibility Measure
      - Superset of Probability Measure
  - Vagueness
    - Fuzzy Sets
      - Typically used for Numeric Variables
- Qualitative Variables
  - Fuzzy Sets
    - Purely Linguistic Fuzzy Sets
- Combination of Qualitative Variables
  - Convolution of Evidence over Linguistic Fuzzy Sets
    - Approximate Reasoning
    - Belief/Plausibility Measure for Uncertainty

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Uncertainty

- **Ambiguity**
  - Uncertainty as to what will occur in the future
    - Dow Jones Industrial Average Close on Dec. 31, 2007
      - Will be one value
      - Ambiguity as to what that value will be
- **Vagueness**
  - Uncertainty as how to categorize a known outcome
    - Dow Jones close is 13,876 on Dec. 31, 2007
      - Is this "High" ?
      - What do you mean by "High"?
  - Vagueness can be expressed Linguistically (Words)

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Ambiguity: Aleatory and Epistemic

- For a Fair Coin
  - Uncertainty is Aleatory (random)
  - Probability Heads is ½
  - Probability Tails is ½
- But if we cannot toss coin, we do not know coin is fair, we do not even knows if coin has Heads and Tails
  - May not be Fair Coin (may be Weighted for Tails)
  - May be Two-Headed or Two-Tailed Coin
  - Epistemic (state of knowledge) uncertainty
  - Insufficient information to assign Probability to Heads and Tails
  - For Total Ignorance
    - Belief/Plausibility for Heads is 0/1
    - Belief/Plausibility for Tails is 0/1
- With more information (actually tossing the coin) we can reduce Epistemic Uncertainty
  - If at least one Heads and one Tails occur in a series of tosses, we know coin has Heads and Tails
  - Many tosses needed to assess if coin is fair
- For Fair Coin we cannot reduce aleatory uncertainty

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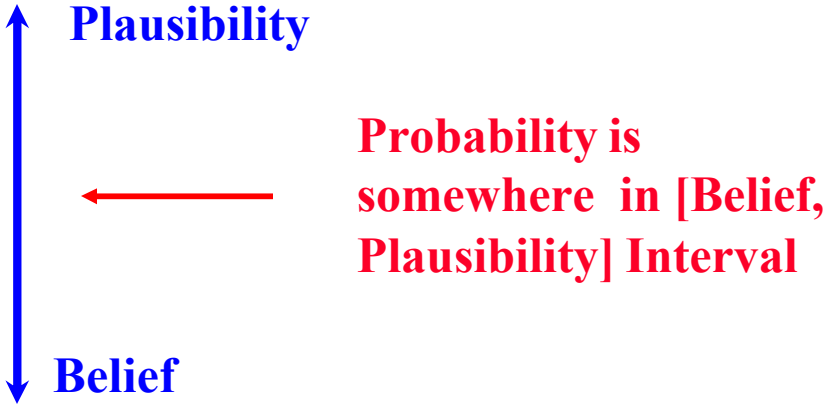
Belief and Plausibility

- Belief / Plausibility form a Lower / Upper Bound for Probability
- Belief is what probability will be
- Plausibility is what probability could be



- Similar to a Confidence Interval for a Parameter of a probability distribution; a confidence measure that parameter is in interval, but exactly where in interval is not known
- Belief/Plausibility both reduce to Probability if Evidence is Specific

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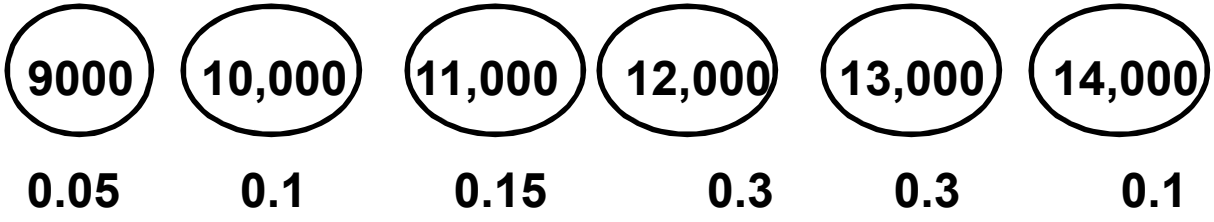
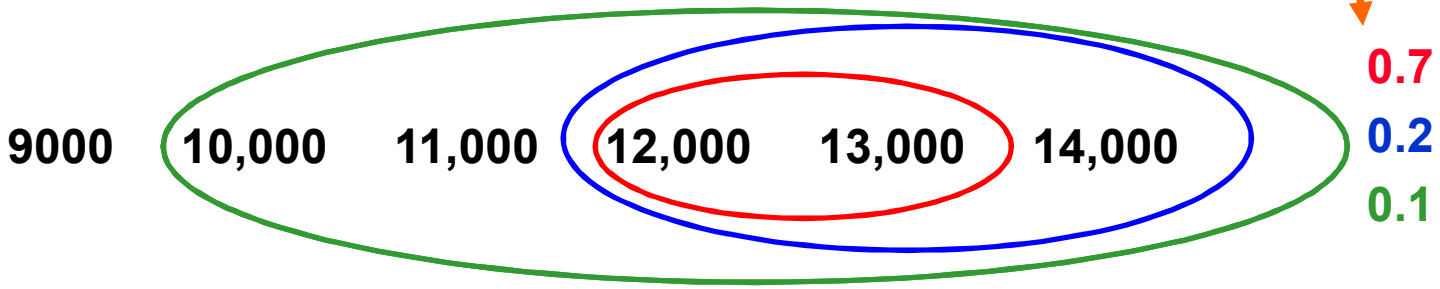
Example of Evidence: Predict Stock Market Close Dec. 31, 2007

- **Probability**



Evidence

- **Belief/Plausibility**



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Why is this useful?

- What is Likelihood of Bio-Terror attack against a Major US City?
  - Frequency of Attack (per year) somewhere in [0, 1]
  - Point Estimate? Useless, too much uncertainty
  - Probability Distribution?
    - Must assign probability to each value in [0, 1]
      - Probability for 0 attacks per year is high
      - Probability assigned to 0 affects probabilities for all other values as must sum to 1.0
        - Is P(0) = 0.01, 0.001, 0.00001, …? Don't Know

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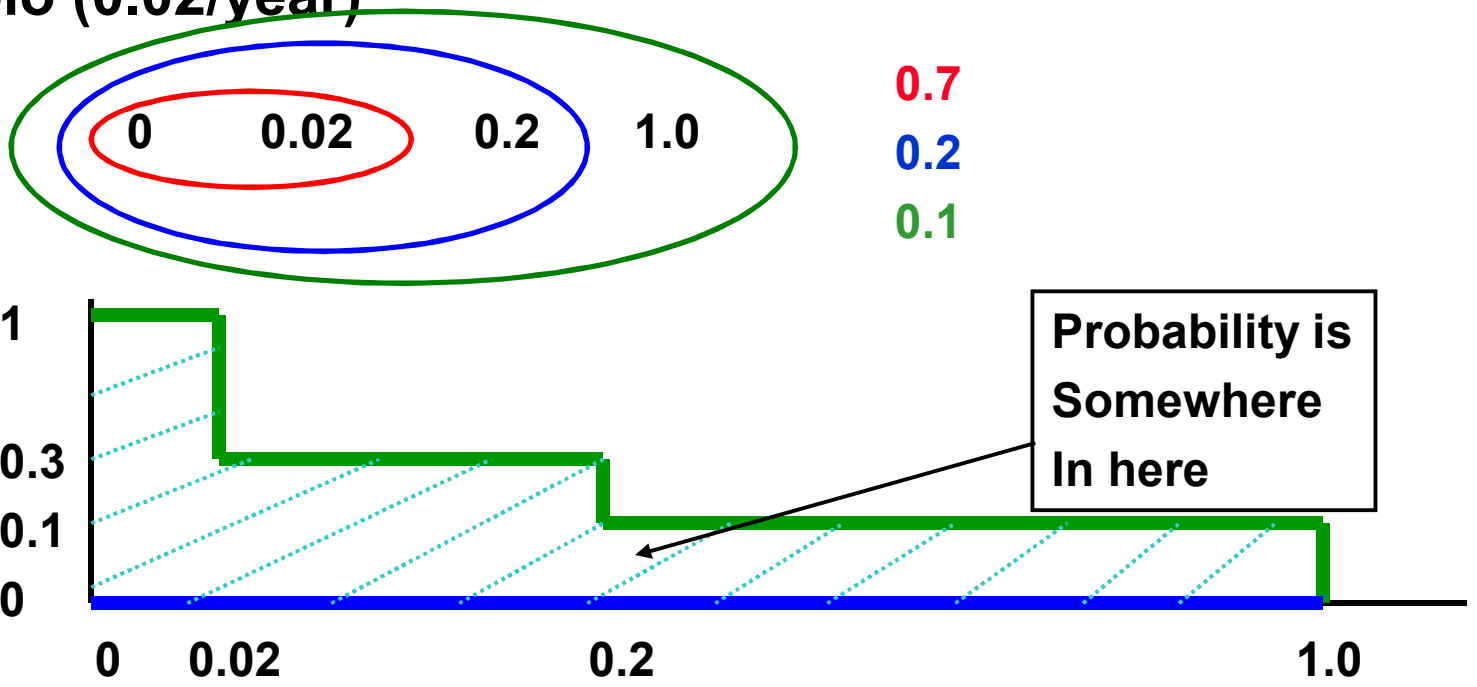
Why is this useful?

- Evidence is about 1 major Attack every 5 years (0.2/year)
  - Assume Expert Opinion is: 10% Chance Attack is bio (0.02/year)



Automated in BeliefConvolution Java code

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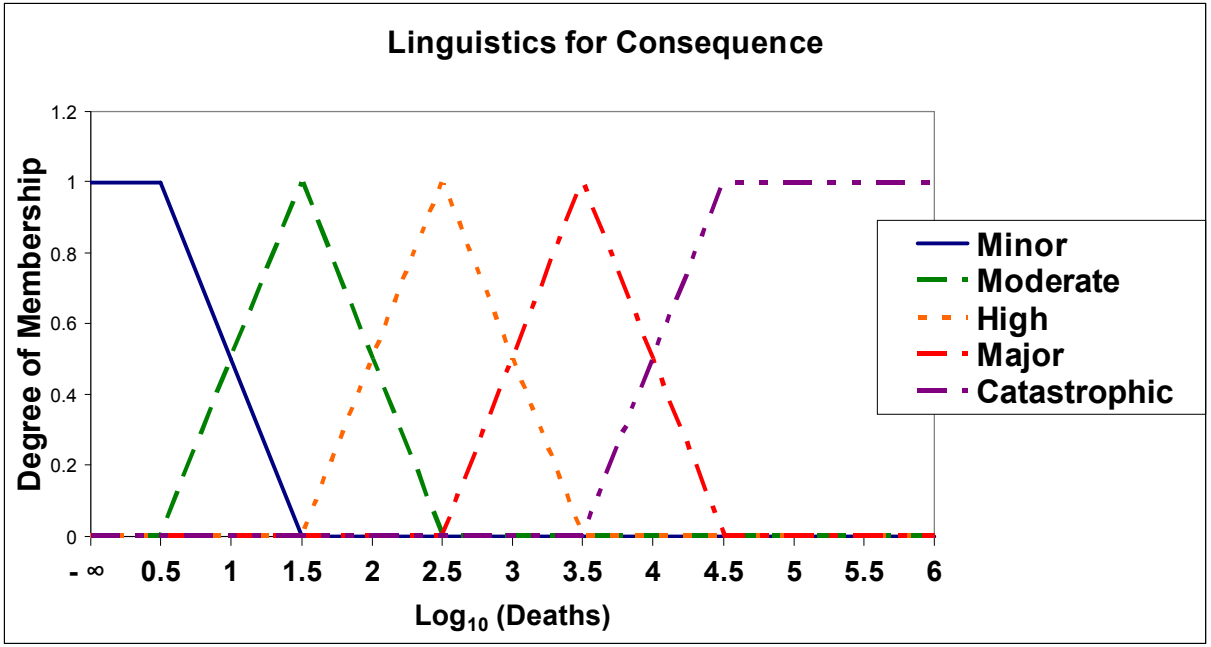
Vagueness
Fuzzy Sets for Numeric Variable

- Represent Variable with Sets to reason at Fidelity Desired. Above 30,000 deaths is "Catastrophic".
- Use Fuzzy Sets to Avoid Sharp Distinction. "Major" Deaths is Between *About* 1000 and *About* 10,000. 999 and 1001 deaths are each part "High" and part "Major".



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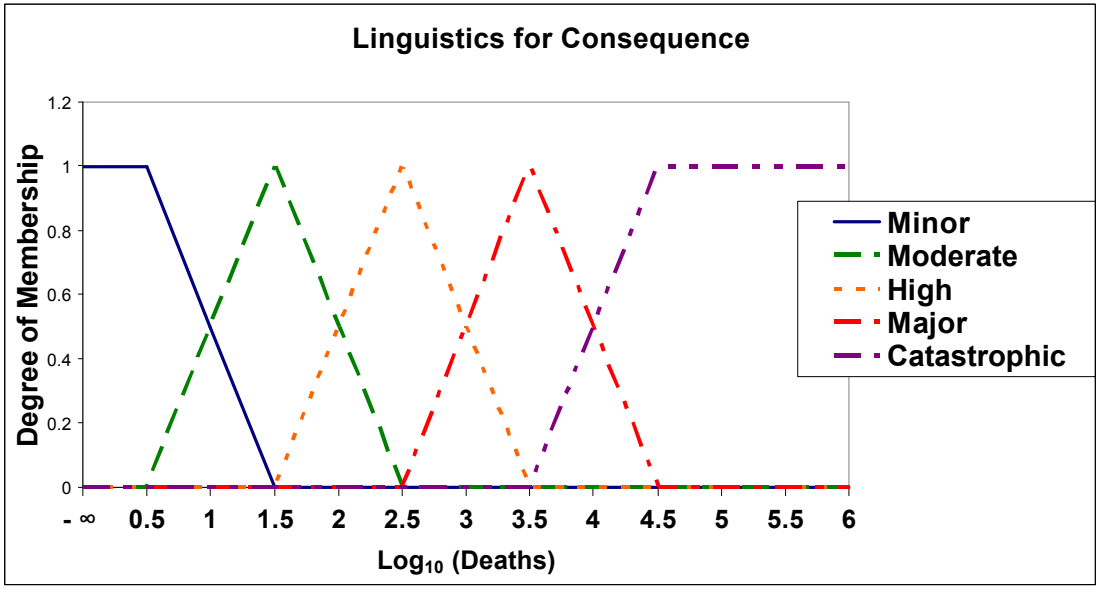
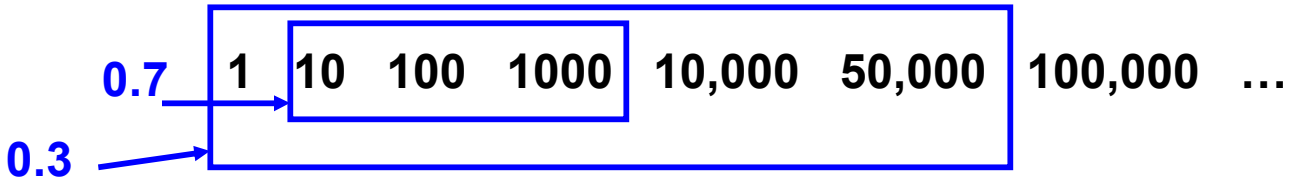
Uncertainty for Fuzzy Sets: Numeric Variable

Fuzzy Sets for Deaths



Evidence For Deaths



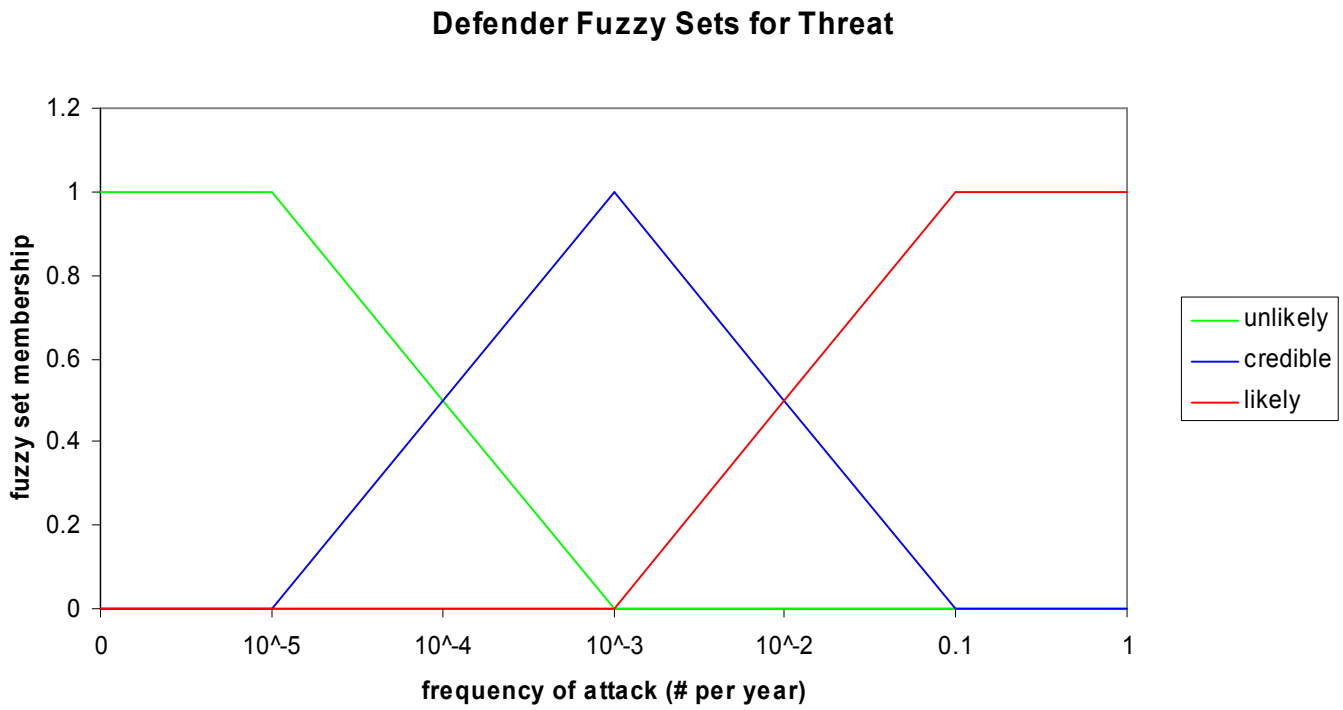
Uncertainty Distribution for Deaths:

	Minor	Moderate	High	Major	Catastrophic
Belief / Plausibility	0 / 0.65	0 / 1	0 / 1	0 / 0.65	0 / 0.3

(Calculated with BeliefConvolution code using Yager Method)

Why is this Important?

- What is Likelihood of Bio-Terror attack against a Major US City?
  - Evidence is about 1 major Attack every 5 years (0.2/year)
    - Assume Expert Opinion is: 10% Chance Attack is bio (0.02/year)
- Assume Following Fuzzy Sets for Evaluating Frequency of Attack



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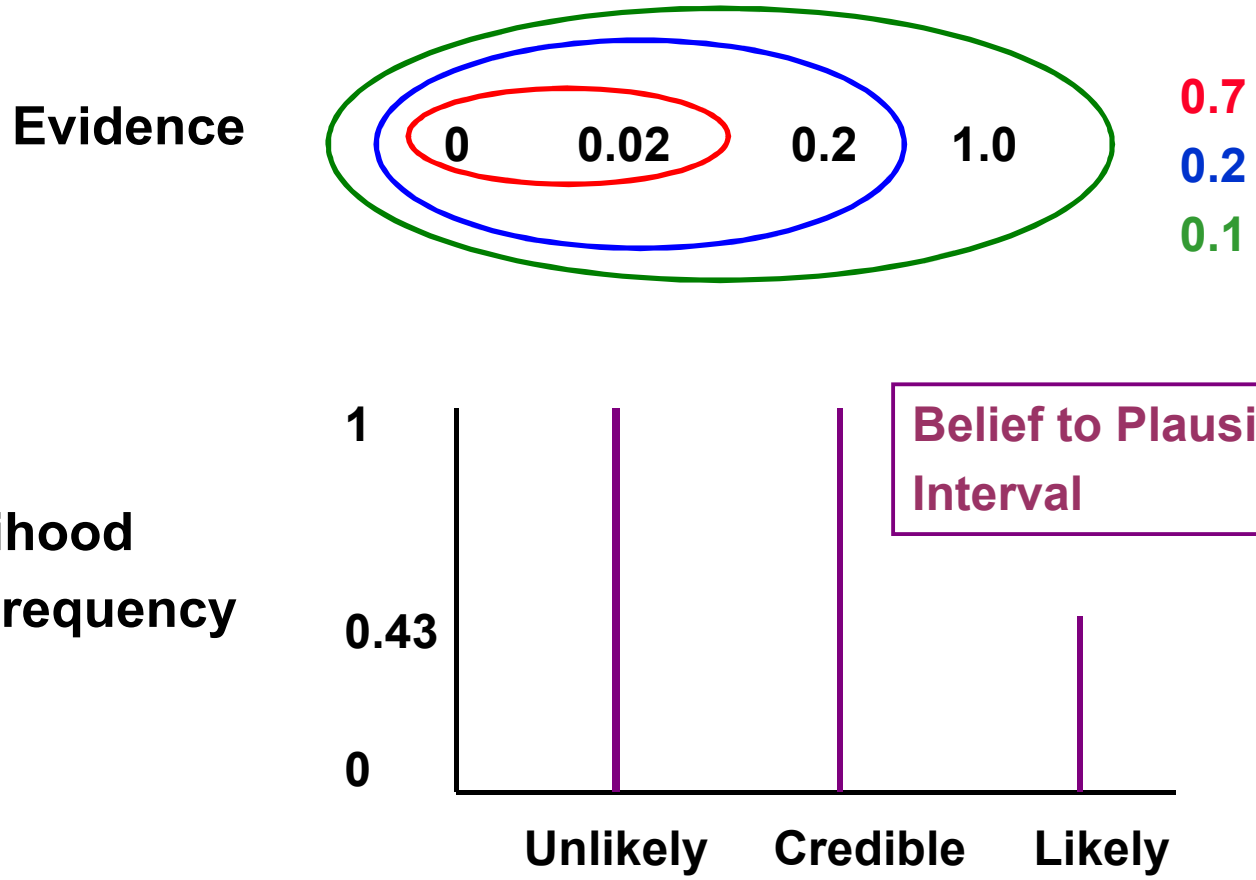
Why is this Important?



(Calculated with BeliefConvolution code using Yager Method)

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Qualitative Variables

- Variable Segregated into Purely Linguistic Fuzzy Sets
  - Variable: "Health"
  - Fuzzy Sets: "Bad", "Moderate", "Excellent"
- Why Pure Linguistics?
  - Numeric Scale is Unknown
    - Is "Health" [0, 10], [0, 10^6], [-700, square root of 42]?
  - Scaling is Un-Manageable when Combine Variables
    - Combine "Health" with "Wealth" to Evaluate "Quality of Life"
      - "Wealth" can be Numeric: [$0, $50B]
      - What is Numeric Scale for "Health"?
      - What is Numeric Scale for "Quality of Life"?

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Fuzzy Sets for Non-Numeric Variable

Adversary Level of Technical Training:

- **High School**
- **Bachelors**
- **Advanced**

Do NOT Force Numeric Measure: Requires Arbitrary Scale

Adversary Level of Technical Training:

- **High School = 1?**
- **Bachelors = 2?**
- **Advanced = 3?**

Adversary Level of Technical Training:

- **High School = 10?**
- **Bachelors = 100?**
- **Advanced = 1000?**

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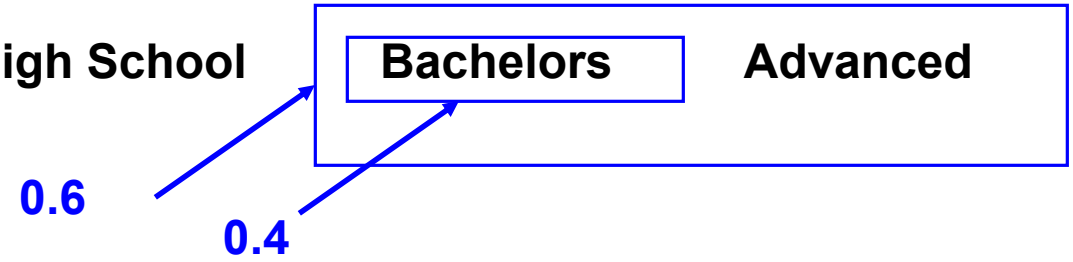
Uncertainty for Fuzzy Sets: Non-Numeric Variable

- **Fuzzy Sets for Adversary Level of Technical Training**
  - **High School Bachelors Advanced**
- **Evidence**



- **Uncertainty Distribution: Belief / Plausibility**

High School	Bachelors	Advanced
0/ 0	0.4 / 1	0 / 0.6

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Combining Variables: Convolution of Uncertainty Distributions

- **Belief/Plausibility Distributions**
  - Evidence Over Fuzzy Sets for Each Variable
- **Convolute Distributions** per the Rule Base
  - Mathematics of Belief/Plausibility
- Same Concept as Convolution of Probability Distributions
  - Mathematics of Probability

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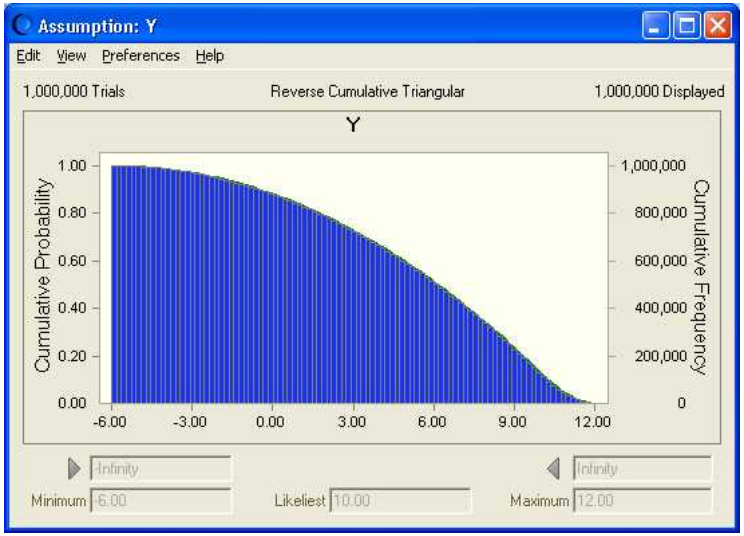
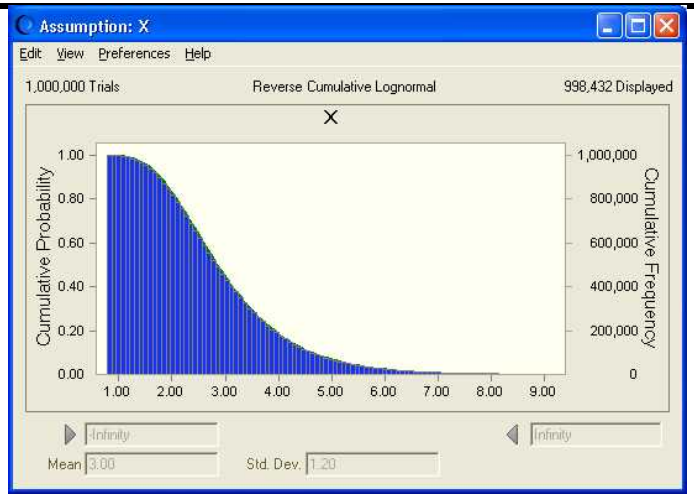
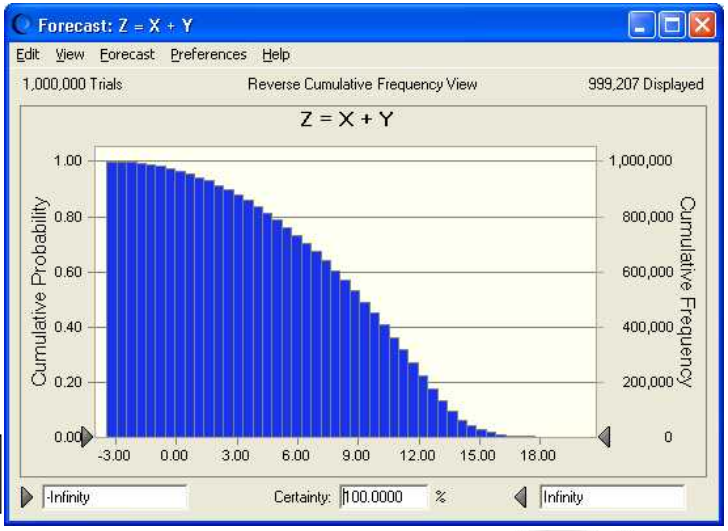
Convolute Probability Distributions: Crystal Ball Software

Z = X + Y

X and Y Independent







Mean of Z is a Point value: 8.34

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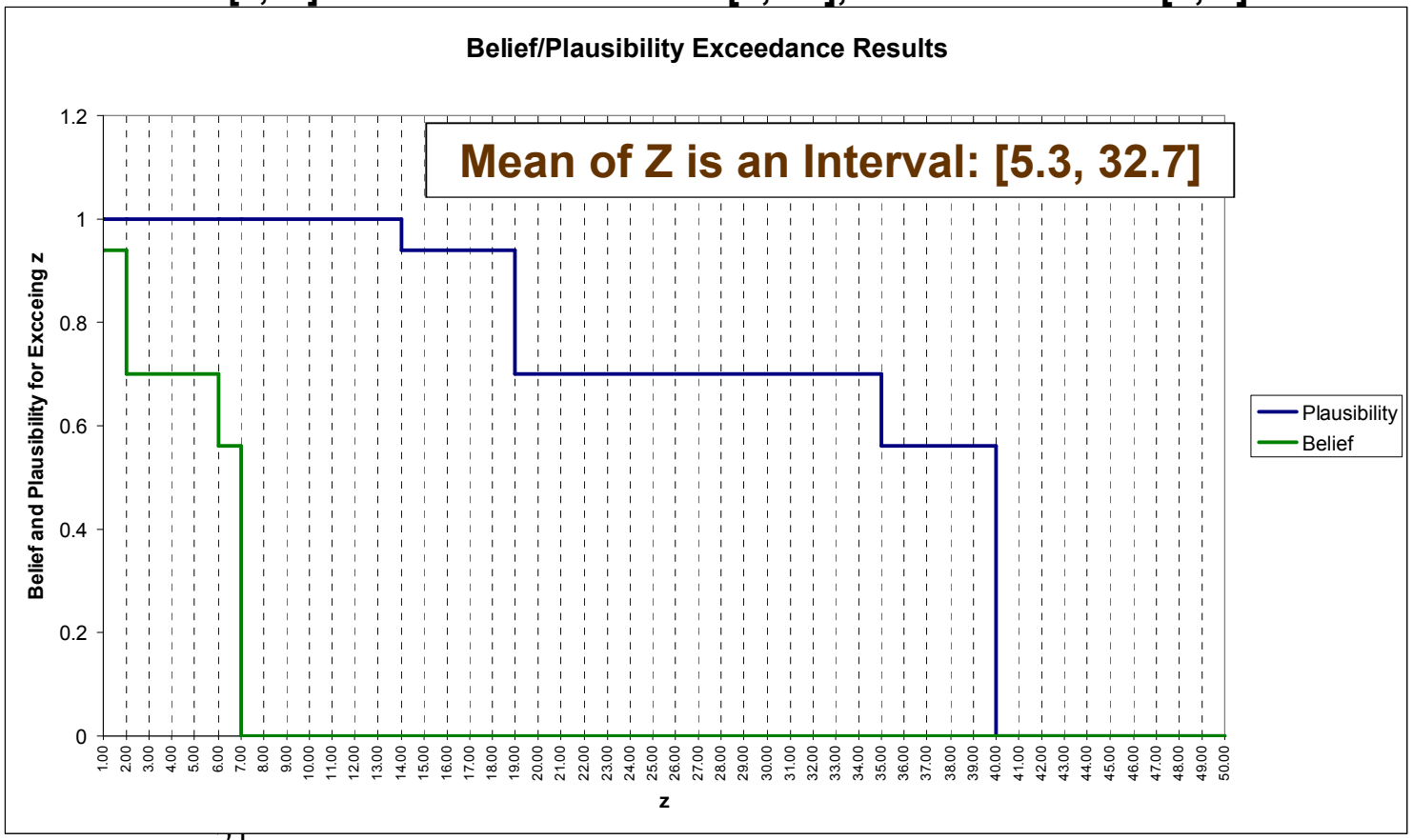
Convolute Belief/Plausibility Distributions for Numeric Variables: BeliefConvolution Software

Z = X + Y, X and Y Non-Interactive

X over [1, 20] with Evidence: 0.8 for [2, 15], 0.2 for [1,10]

Y over [0,30] with Evidence: 0.7 for [5, 25], Evidence 0.3 for [0, 4]



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Convolute Belief/Plausibility Distributions for Linguistic Variables: LinguisticBelief Software

- **Example Follows**

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Combining Qualitative Variables: Approximate Reasoning

- Mathematics for Combining Words
- If we use Words instead of numbers we need a way of combining the Words for Different Variables
- Implemented as A Rule Base for Combining Fuzzy Sets from Different Variables

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Combination of Linguistic Variables: Example

Develop the Model: Happiness for Any Individual

Define the Variables and their Fuzzy Sets

- **Basic Variables**
  - Health
    - Bad, Moderate, Excellent
  - Wealth
    - Poor, Middle Class, Rich
  - Outlook on Life
    - Pessimist, Optimist
- **Rule Based Variables**
  - Quality of Life = Heath x Wealth (x per rule base)
    - Not so Good, Good
  - Happiness = Outlook on Life x Quality of Life
    - Depressed, Accepting, Very Happy

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Combination of Linguistic Variables: Example

Develop the Approximate Reasoning Rule Base for Rule Based Variables

Quality of Life

Rules for selected RuleLinguistic

Rules for RuleLinguistic: Quality of Life

Fuzzy Set for Input Linguistic: Health	Fuzzy Set for Input Linguistic: Wealth	Output Fuzzy Set for Rule (blank if rule not set)
Bad	Poor	Not so Good
Bad	Middle Class	Not so Good
Bad	Rich	Not so Good
Moderate	Poor	Not so Good
Moderate	Middle Class	Not so Good
Moderate	Rich	Good
Excellent	Poor	Good
Excellent	Middle Class	Good
Excellent	Rich	Good

Specify Output Fuzzy Set for Selected Rule Choices Are:

Accept Rules as Shown Cancel

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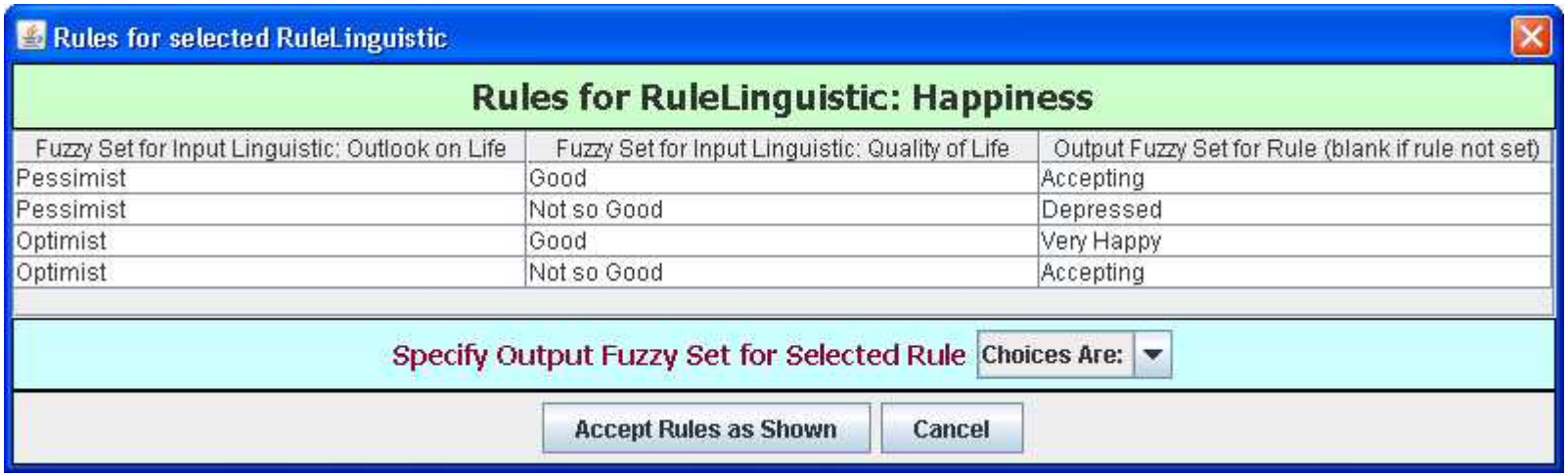
Combination of Linguistic Variables: Example

Happiness



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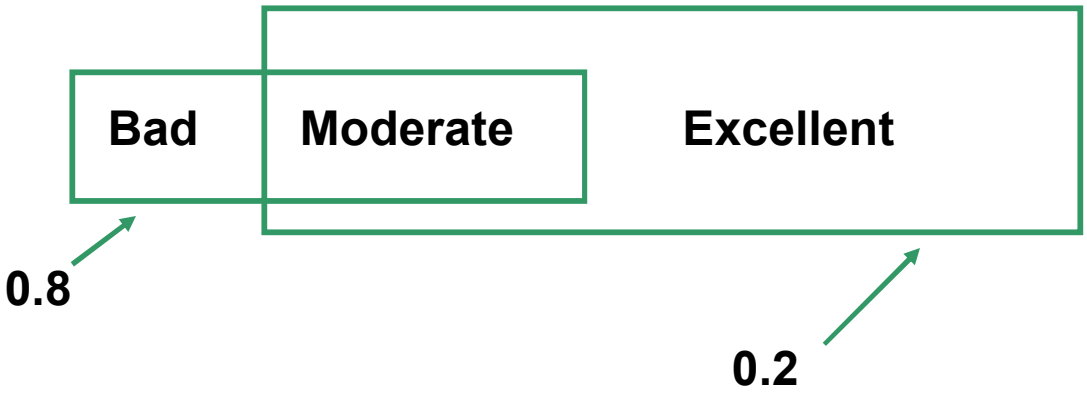
Combination of Linguistic Variables: Example

Evaluate the Model for Specific Individual: Happiness for "John"

Assign Evidence to Fuzzy Sets for Basic Variables

Health



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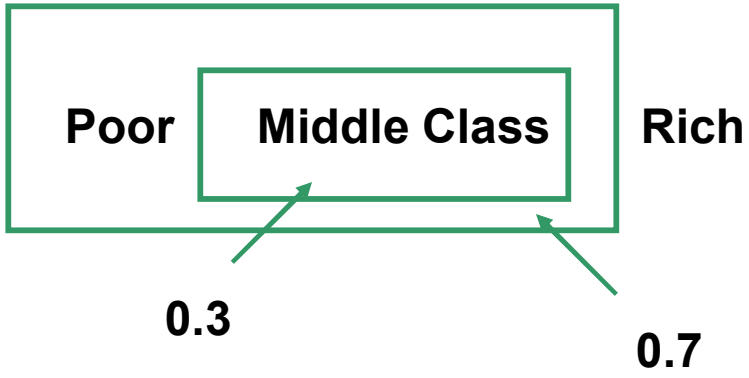
Combination of Linguistic Variables: Example

Wealth



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Combination of Linguistic Variables: Example

Outlook on Life

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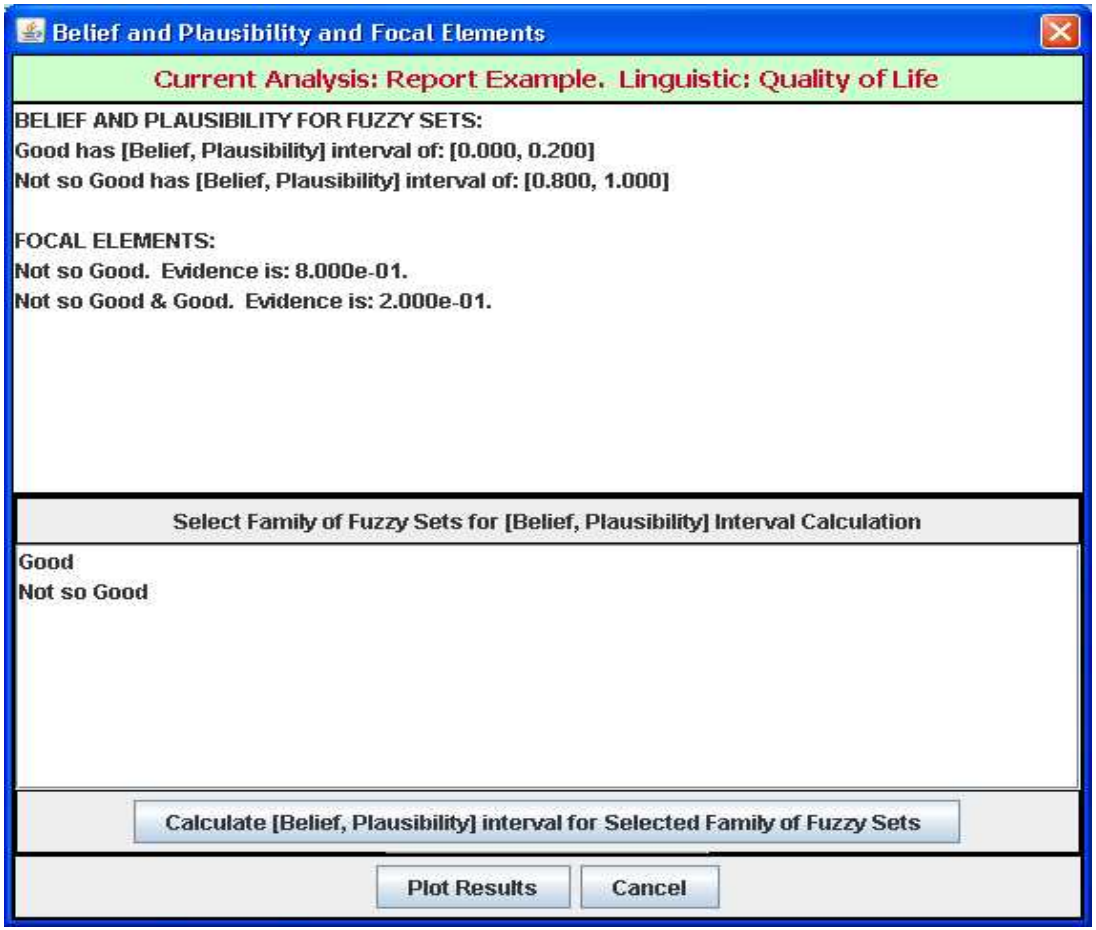
Combination of Linguistic Variables: Example

Evaluate Variable: Quality of Life for John



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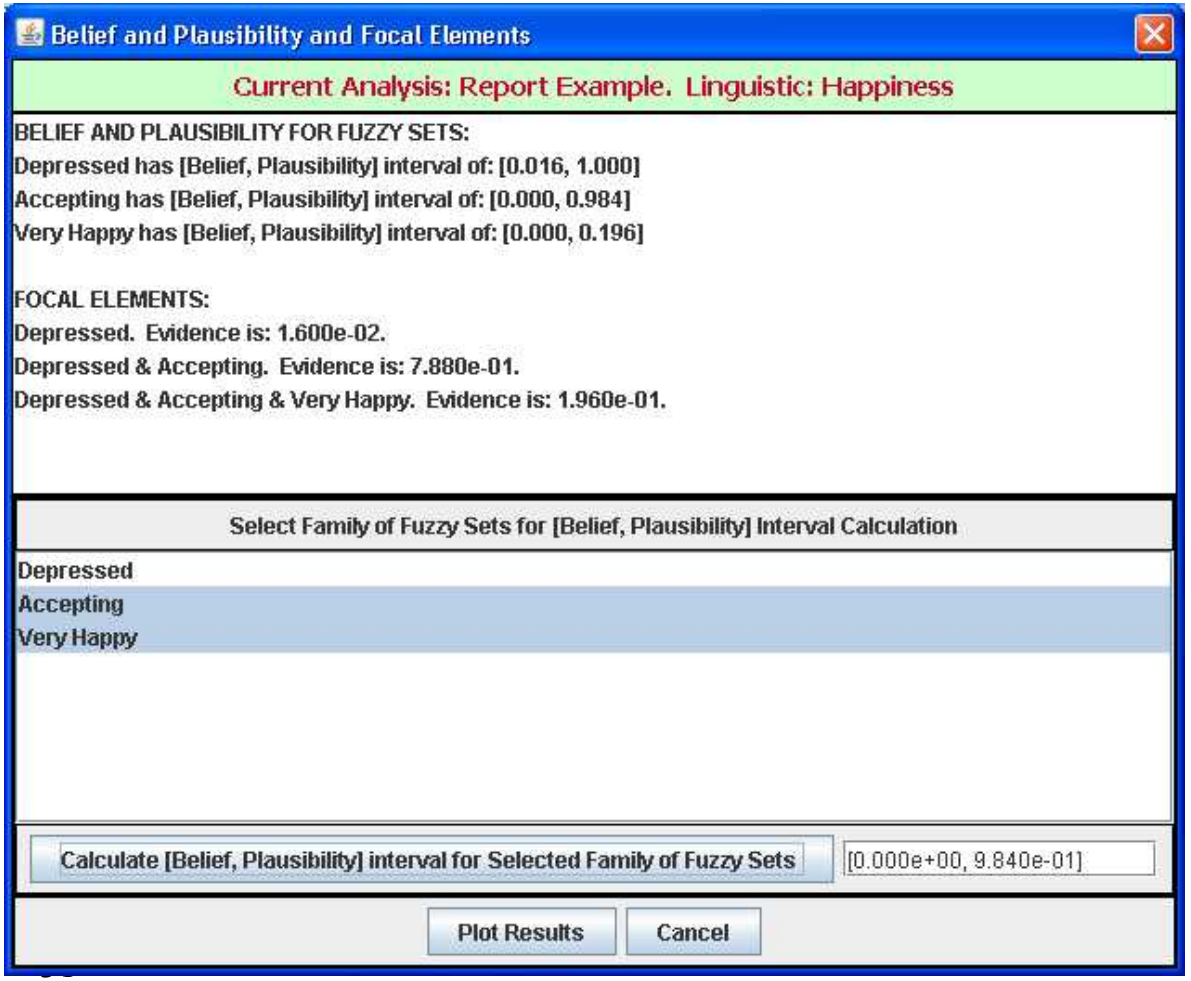
Combination of Linguistic Variables: Example

Evaluate Variable Happiness for John



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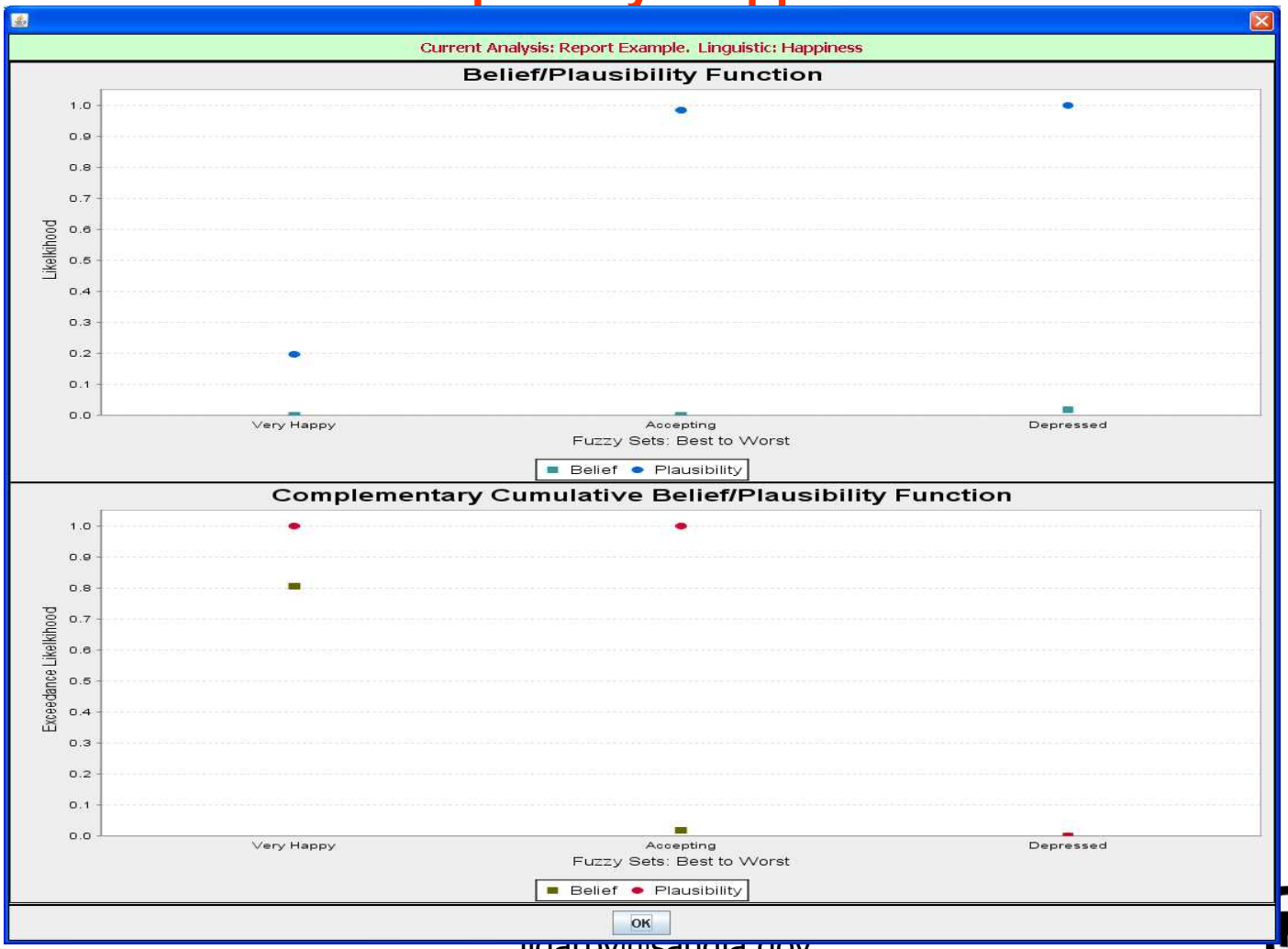
Combination of Linguistic Variables: Example

Summarize Results Graphically: Happiness for "John"



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Results Expressed as Complementary Cumulative Belief/Plausibility Functions (CCBPFs) for Linguistic Variable

- Linguistic Fuzzy Sets Ordered from "Best" to "Worst"
  - CCBPFs are Non-Increasing
- "Likelihood" of Exceeding Fuzzy Set
- "Likelihood" is Belief/Plausibility Interval
- Analogous to Complementary Cumulative Probability Function (CCPF) for Random Variable
  - CCPF Random Variable is a real number
    - discrete or continuous
  - CCBPFs Variable has linguistic fuzzy sets
    - Discrete
  - CCPF is a One Function: a Curve
  - CCBPFs are Two Functions: an Interval

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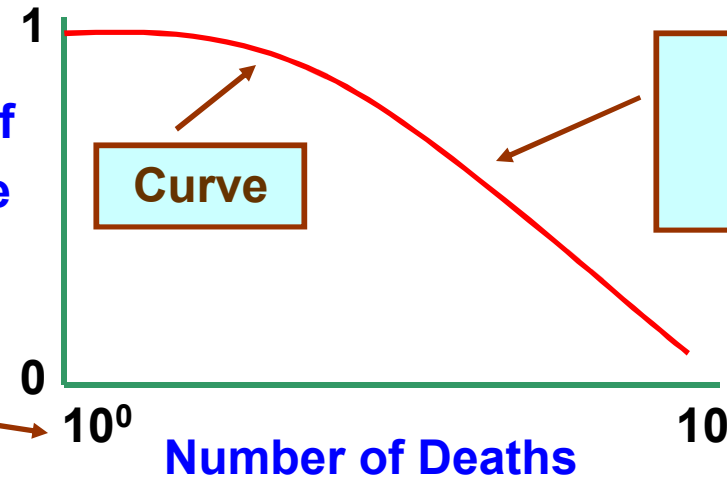
CCPF and CCBPFs

- CCPF

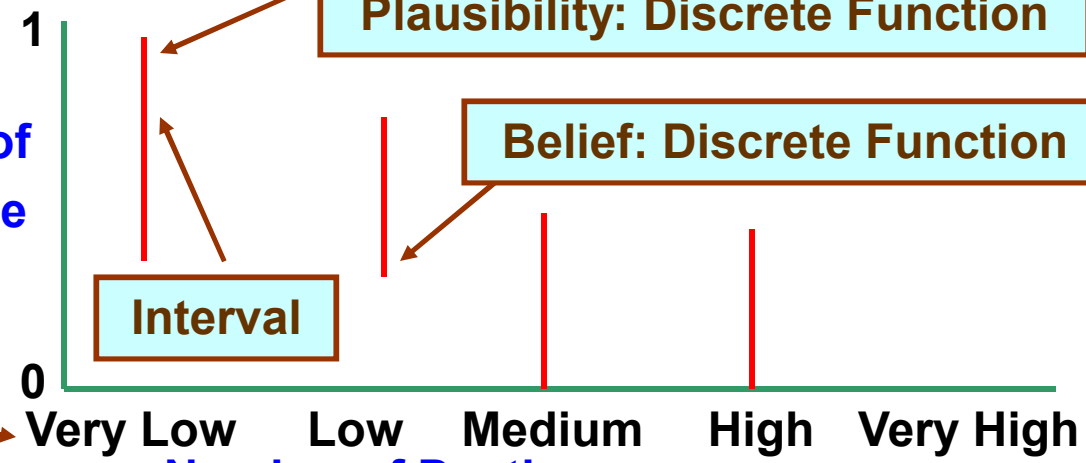
Likelihood of Exceedance



Continuous Variable: Real Number

- CCBPFs

Likelihood of Exceedance



Discrete Variable: Linguistic Fuzzy Sets

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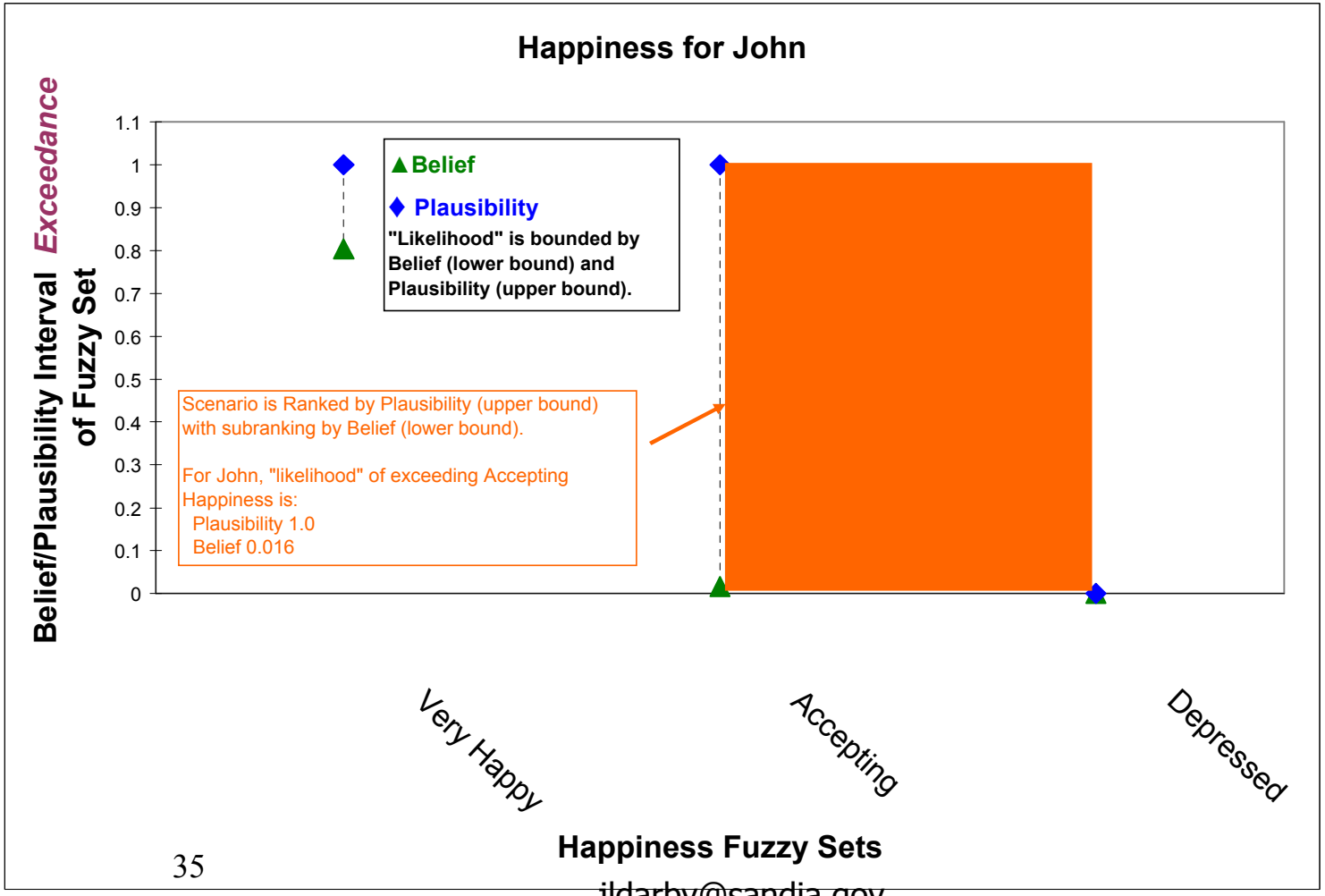
Combination of Linguistic Variables: Example

Graphical Summary for Ranking: "John"



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Custom Software Tools: Java

- BeliefConvolution
  - Convolution of Numeric Variables with Belief/Plausibility
- LinguisticBelief
  - Evaluation of Linguistic Variables
    - Linguistic Fuzzy Sets
    - Approximate Reasoning
    - Belief/Plausibility
- PoolEvidence
  - Multiple Experts provide Evidence for Variables
    - Linguistic Fuzzy Sets
  - Combine Evidence
    - Pooled Evidence for Variables
  - Input for LinguisticBelief

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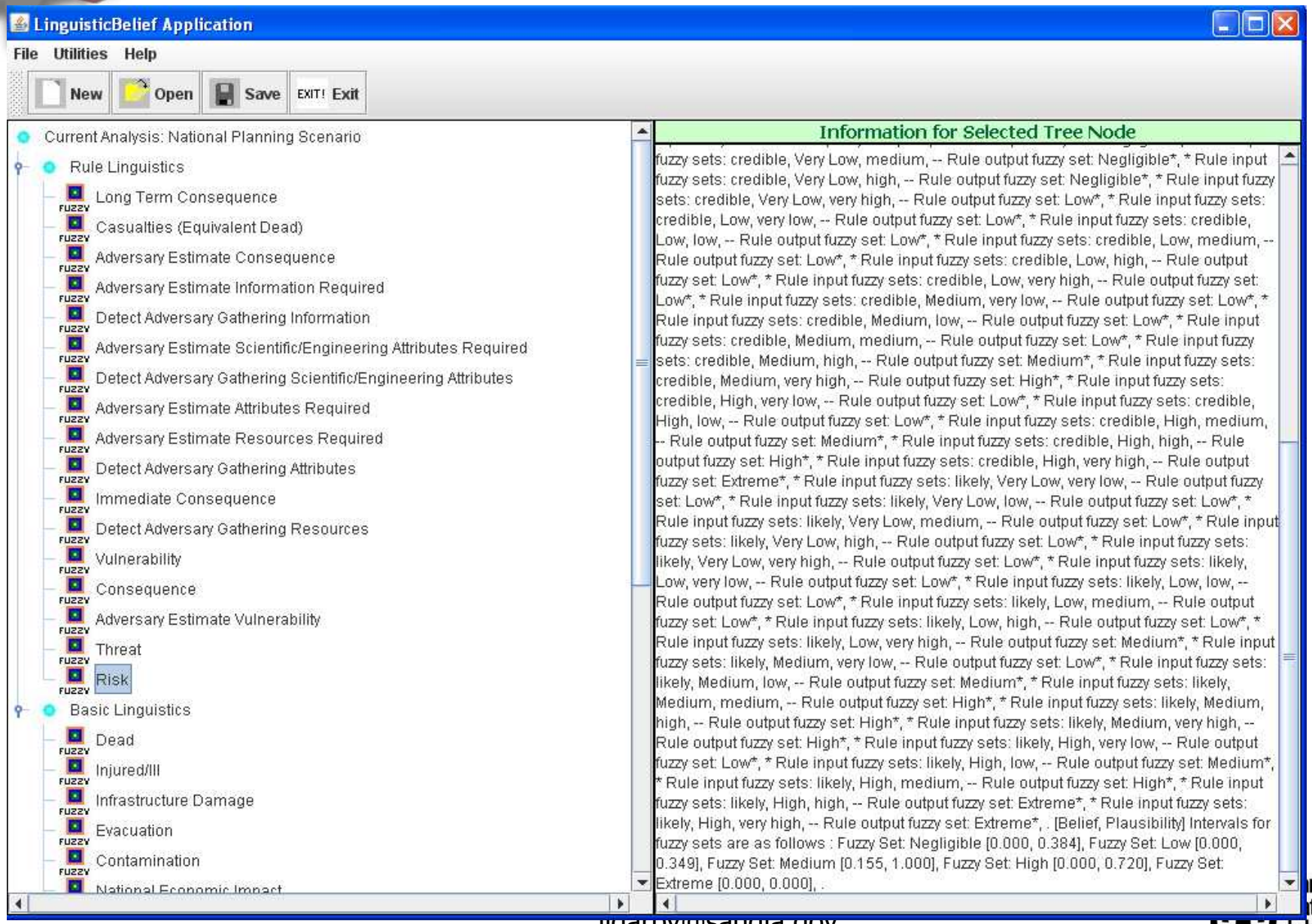
LinguisticBelief: Example Application



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Rank Order Scenarios by Risk

- Rank a scenario by the Highest Non-Zero Plausibility of Exceeding the "Worst" Fuzzy Set
  - For Scenarios with Equal Plausibility, Subrank by Highest Belief
  - Extension of "Probability of Exceedance" approach
    - Uses Fuzzy Sets instead of Numbers
    - Uses Belief/Plausibility Interval instead of Probability
  - Can be "Color Coded"
    - Shown for 3 of 5 scenarios in Following from SAND2007-1301

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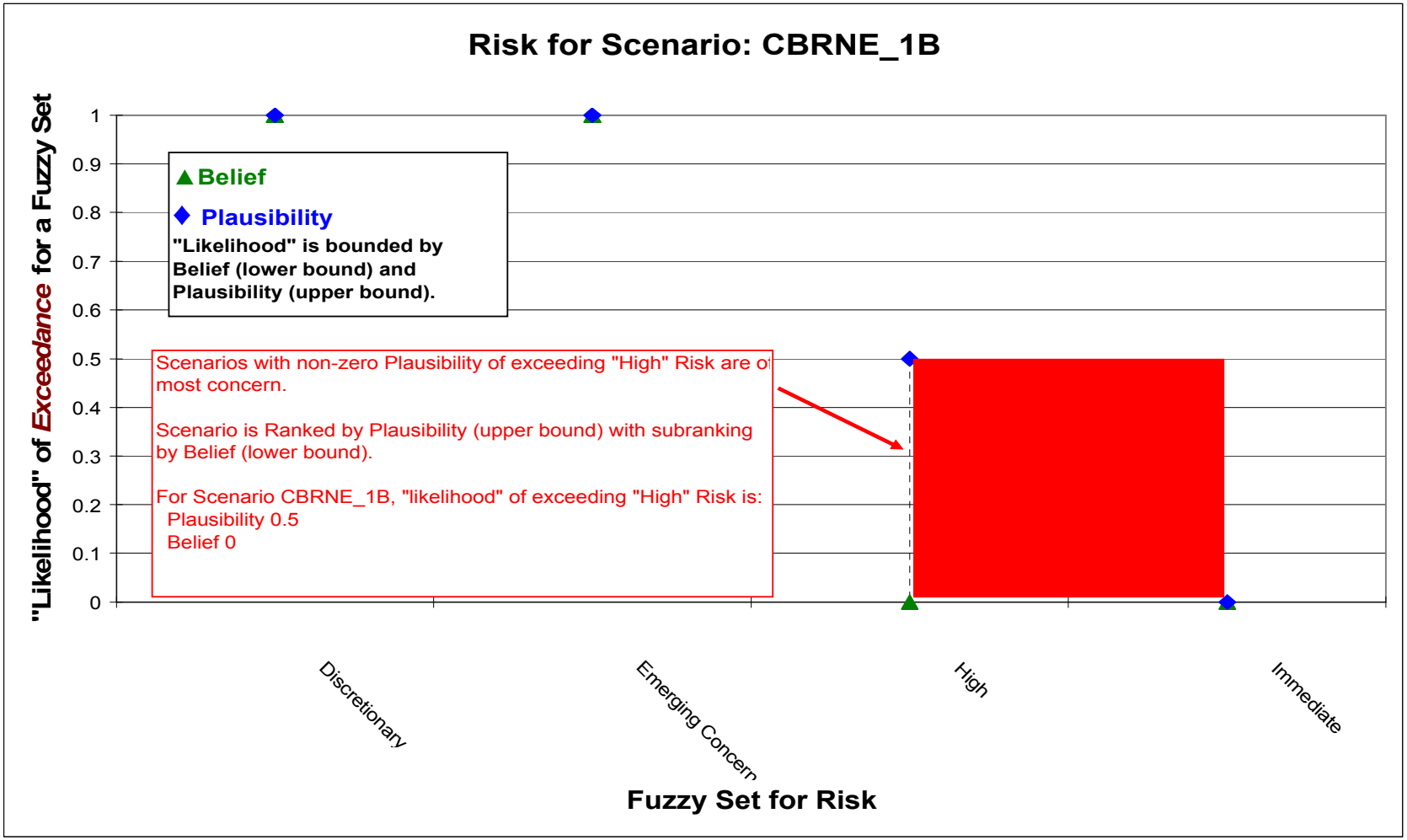
Rank Order Results



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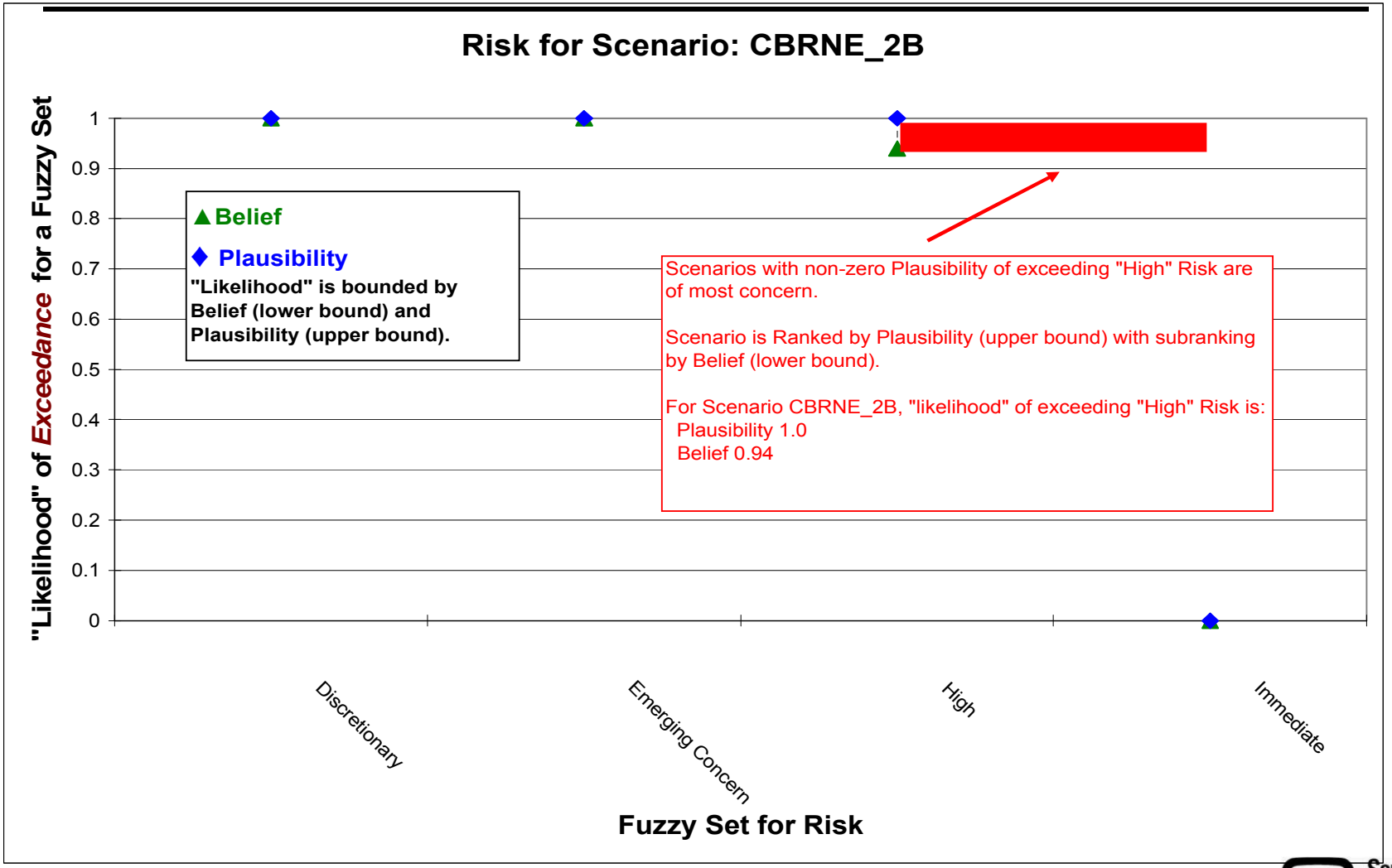
Rank Order Results



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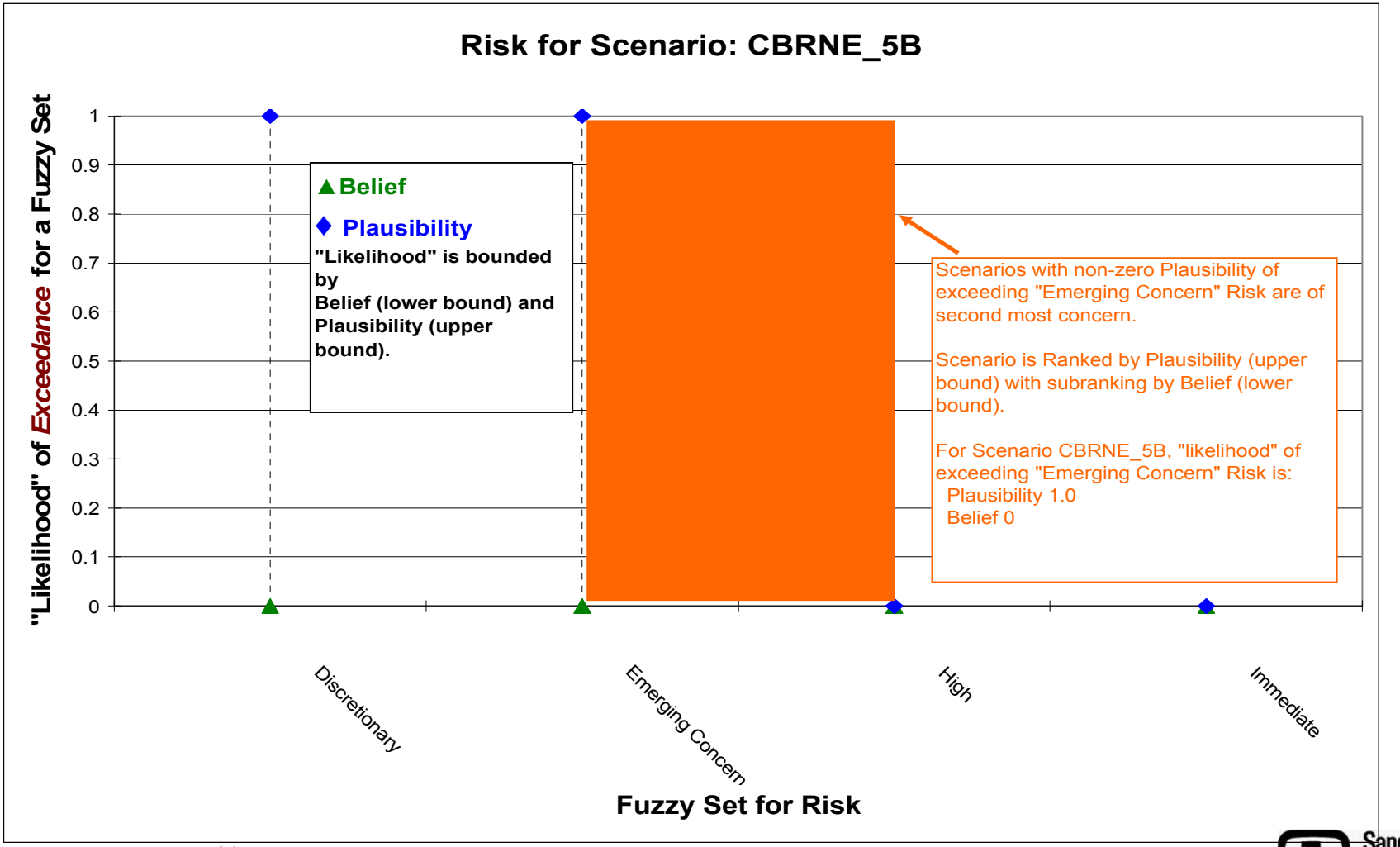
Rank Order Results



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Results of Ranking of All Five Scenarios

RANKING FOR SCENARIOS CBRNE_1B through CBRNE_5B

For *Exceeding* Fuzzy Set "High" the Scenarios rank ordered (decreasing) are:
CBRNE_2B has plausibility of exceedance of 1.0 and belief of exceedance of 0.94
CBRNE_3B has plausibility of exceedance of 1.0 and belief of exceedance of 0.77
CBRNE_4B has plausibility of exceedance of 1.0 and belief of exceedance of 0.64
CBRNE_1B has plausibility of exceedance of 0.5 and belief of exceedance of 0.0

For *Exceeding* Fuzzy Set "Emerging Concern" the Scenarios rank ordered (decreasing) (not already ranked for a worse fuzzy set) are:
CBRNE_5B has plausibility of exceedance of 1.0 and belief of exceedance of 0.0

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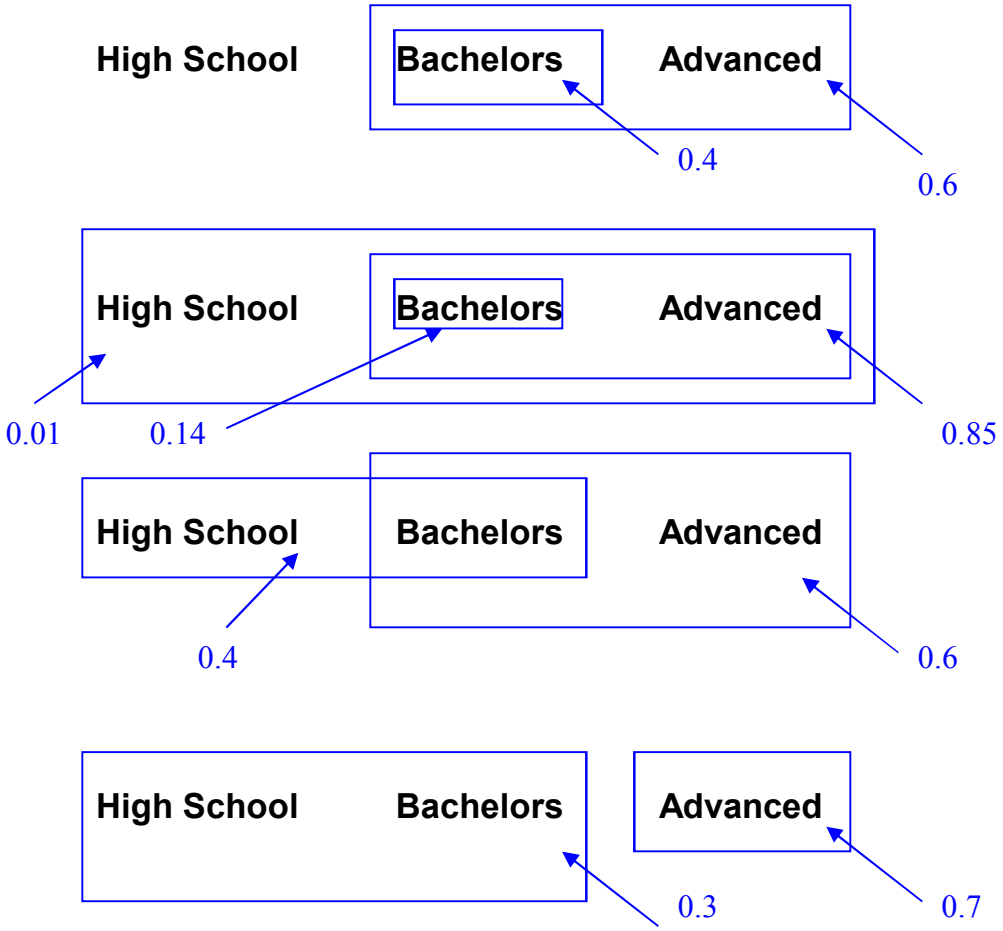
Pool Evidence from Many Experts

Adversary Level of Technical Training:

4 Experts Assign Evidence



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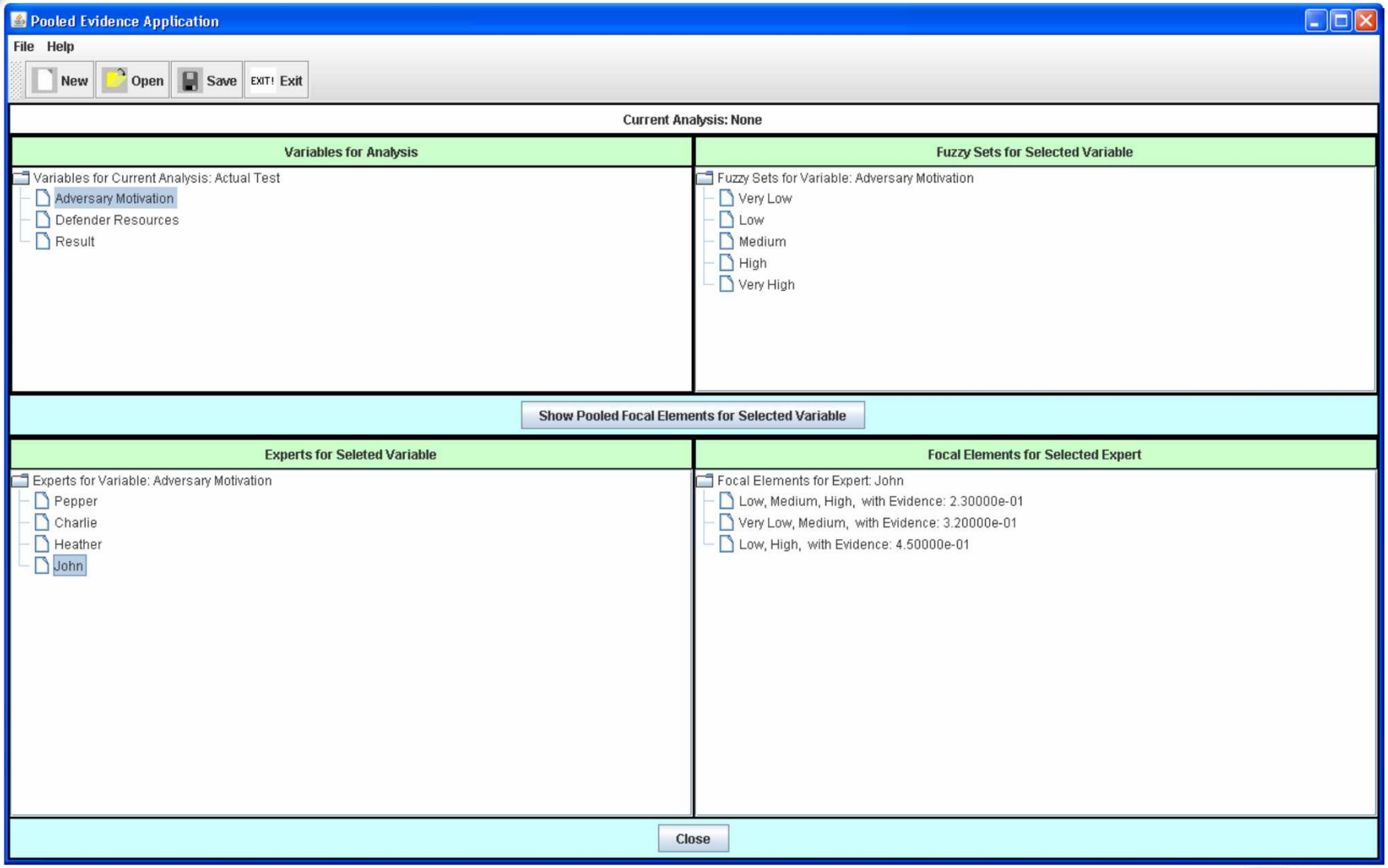
PoolEvidence: Example Application



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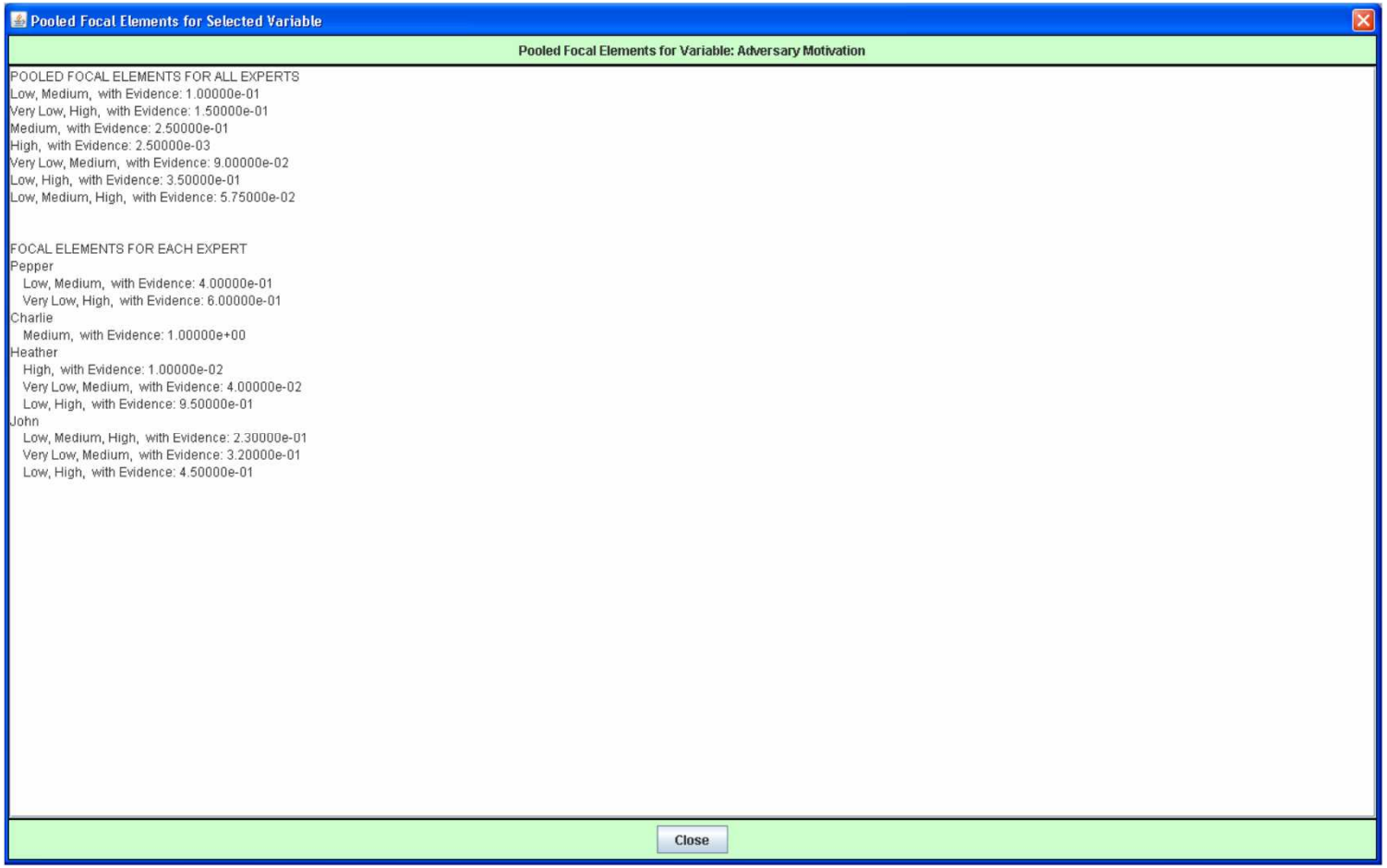
PoolEvidence



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Summary

- Uncertainty
  - Belief/Plausibility for Epistemic Uncertainty
- Qualitative Variables
  - Purely Linguistic Fuzzy Sets
- Combining Variables
  - Approximate Reasoning
  - Convolution with Belief/Plausibility
- Custom Software Tools
  - BeliefConvolution
  - LinguisticBelief
  - PoolEvidence

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