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Investigation of possible nonlinear mechanisms of enhanced Transport and particle heating in a high beta plasma of a Z-pinch and in laboratory astrophysics experiments

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Road Map

- Experimental observations

- Theoretical model

- Linear dispersion relation result

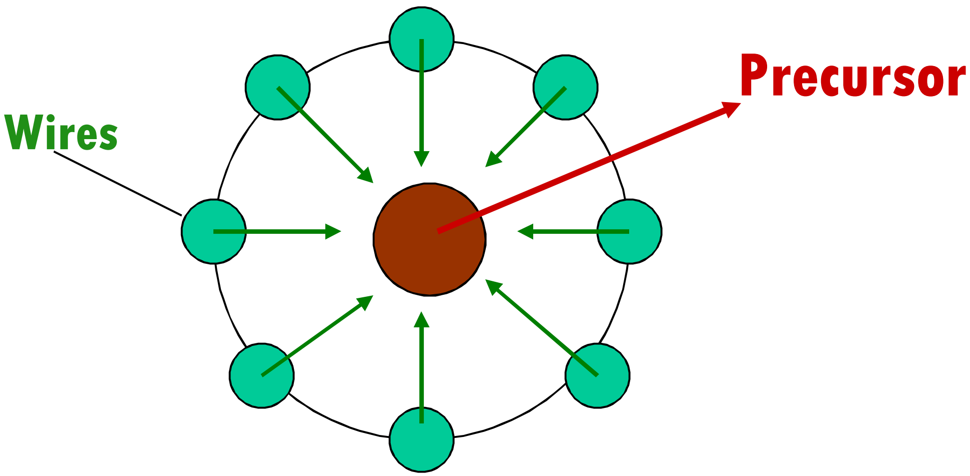
- Implementation in nonlinear Flute code
    - Numerics: two-step predictor-corrector scheme
    - Arakawa method for Poisson brackets modules
    - Initial conditions

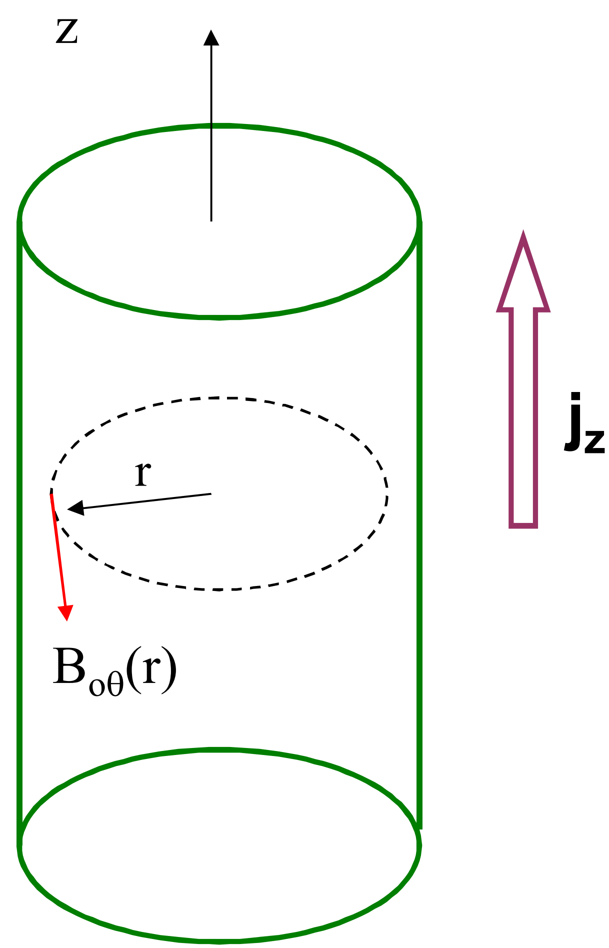
- Nonlinear results

- Future work: more exact formulation easy to implement because of modular nature of FLUTE code

Wire array imploding experiments at NTF

Current in the wire array ~ 1 MA





Plasma parameters in a precursor plasma

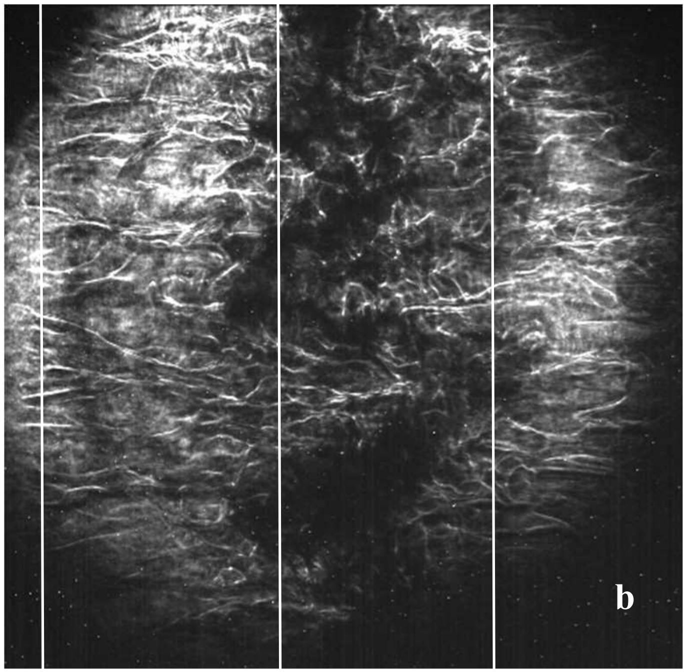
$$n_{0e} = 2.0\times10^{19}\ \mathrm{cm}^{-3},\ n_{0i} = 2.0\times10^{18}\ \mathrm{cm}^{-3},$$

$$T_i \sim T_e = 50\ \mathrm{eV},\ \ B_0 = 0.3\ \mathrm{MG}$$

$$\beta = 0.55 \quad \Omega_{cAl} = 1.1\times10^{10}\ s^{-1} \quad \omega_{pAL} = 1.4\times10^{13}\ s^{-1}$$

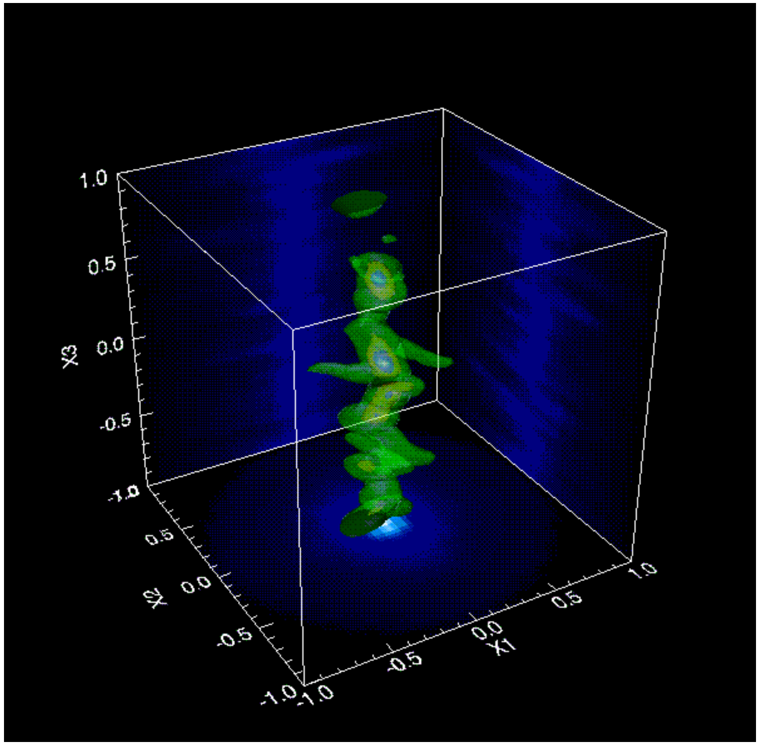
Global MHD modes in the precursor

Cathode



Anode

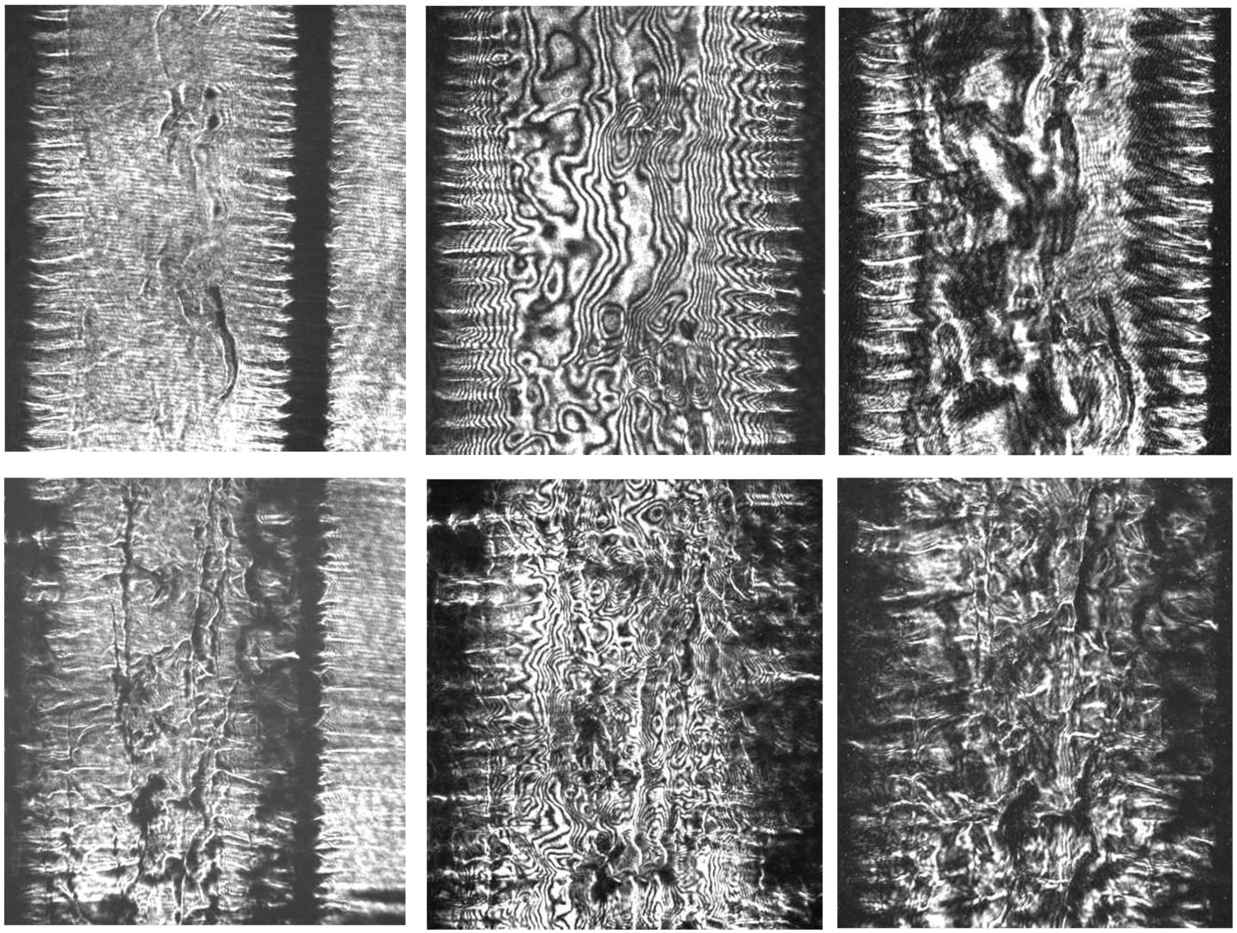
Experiment



Hybrid simulations

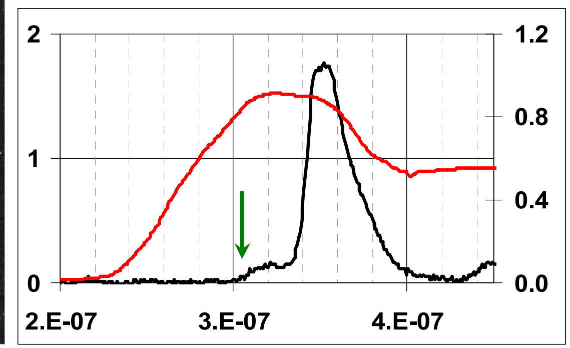
Development of plasma turbulence in the precursor

Al 8 x 15 μm wire arrays

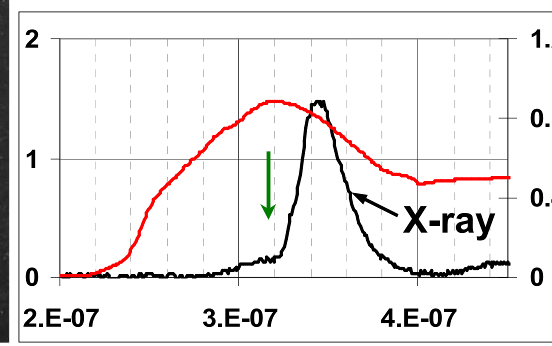


Shadowgram | Interferometry | Dark field schlieren diagnostic

Shot 454

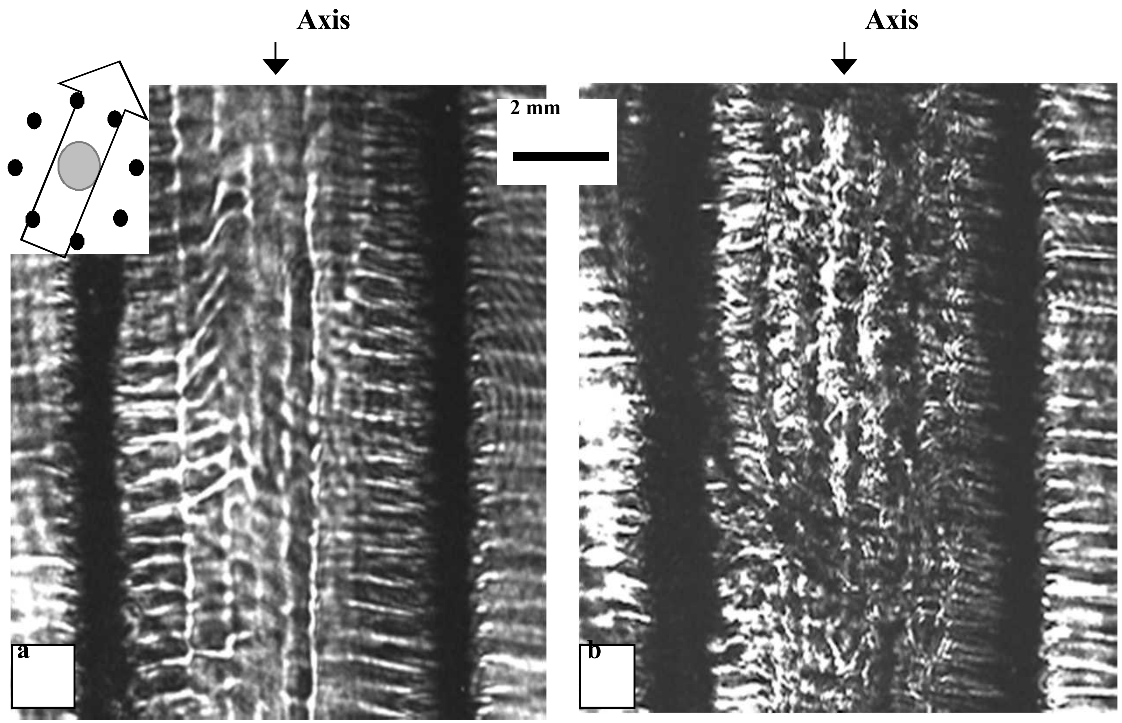


Shot 453



Ivanov et al., TPS IEEE, V 34, N 5, 2006

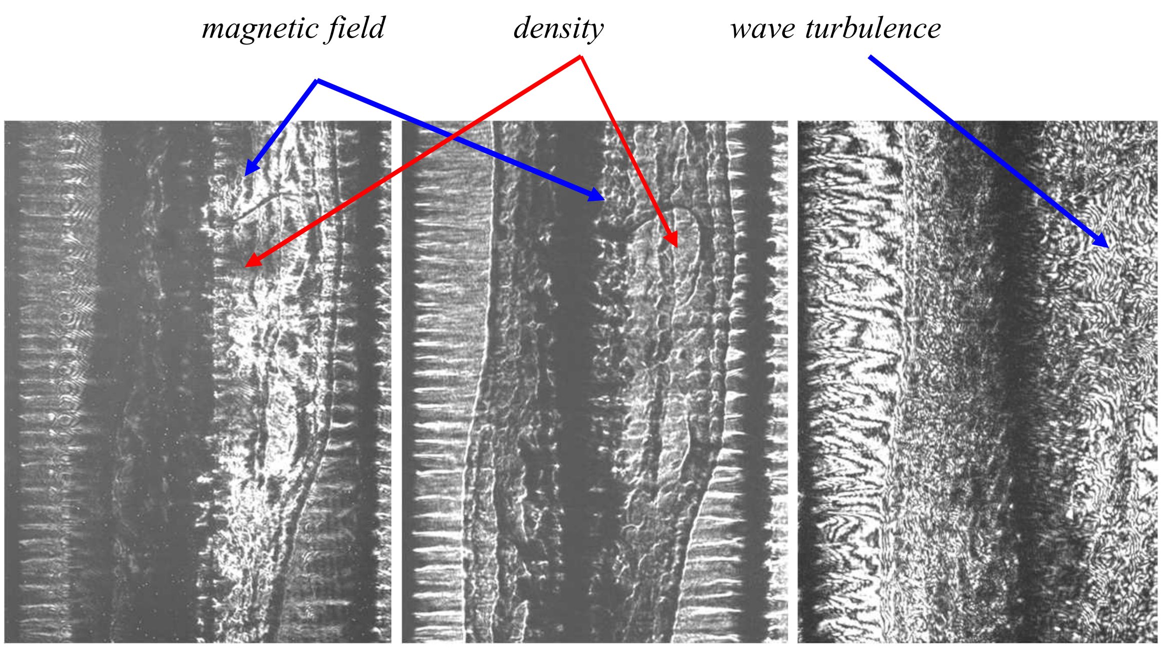
Evolution of the excited wave spectrum



Ivanov et al., TPS IEEE, V 35, N 4, p. 1170, 2007

Laser probing and Faraday rotation experimental results

Experimental results indicate non MHD behavior of excited wave spectrum

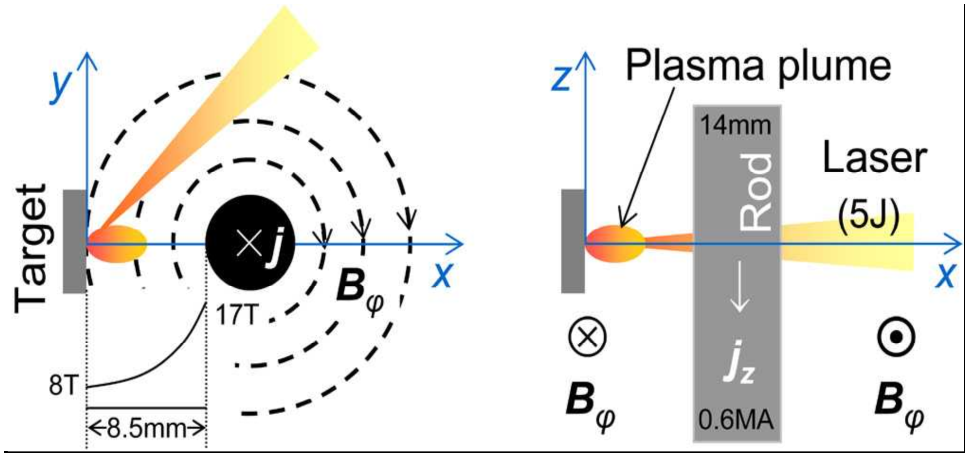


Faraday channel | Shadowgraphy | Interferometry

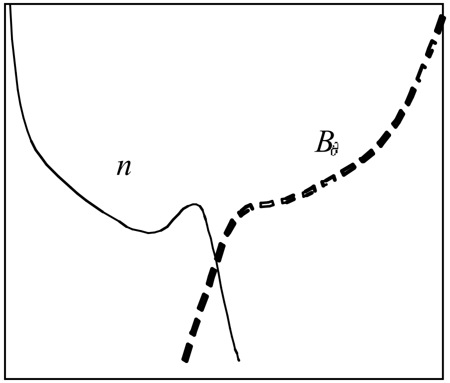
Main experimental results

- Characteristic wavelengths of excited waves ~ 0.1 - 1 mm
- Typical rise time of excited waves ~ 20 ns
- Development of large scale cells on nonlinear stage
- Wave spectrum cascading towards short scales

Laboratory Astrophysics Experiments



Experimental set-up



Experimental density profile and adjusted magnetic field radial profile

Presura et al., Astrophys. Space Sci., 2006

Sotnikov et al., Astrophys. Space Sci., 2006

Flute instability

small density perturbation $\delta n \sim \cos(\omega_k t - k_z z)$

⇓

perturbation of "gravitational" force $\delta F_r \sim g\delta n$

⇓

ions drift in axial z-direction

⇓

separation of charge

⇓

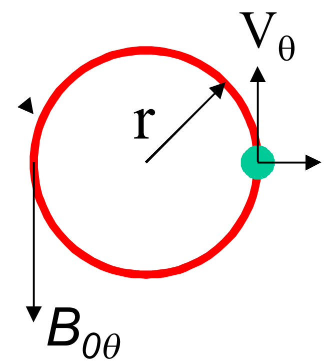
electric field

⇓

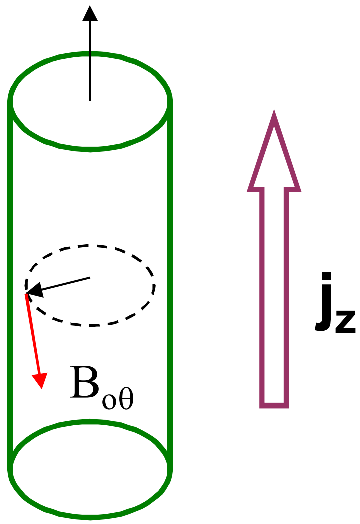
drift motion in *r*-direction brings particles from denser plasma core

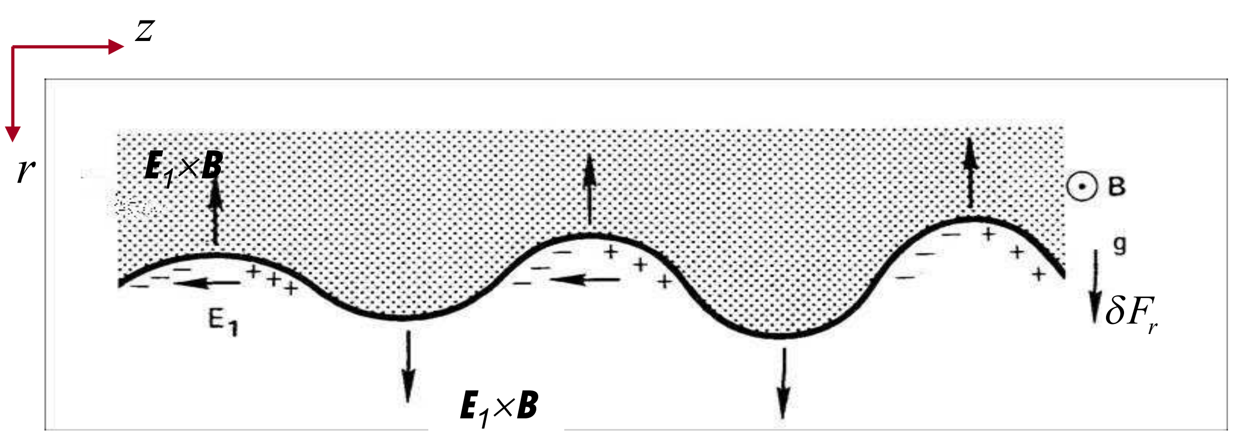
⇓

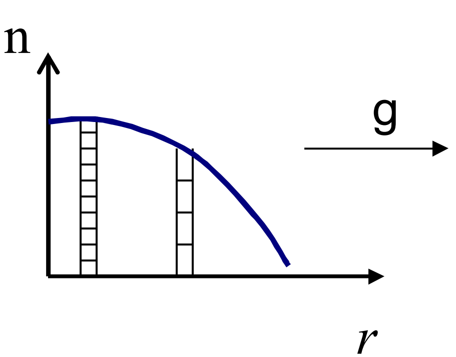
instability



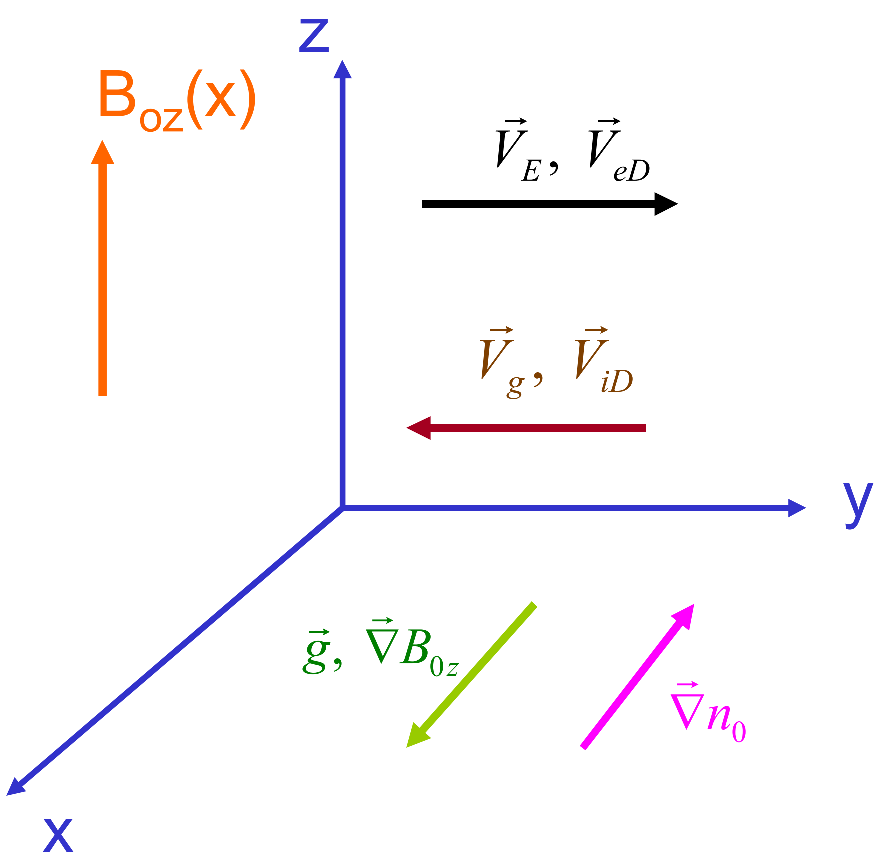
$$g = \frac{V_\theta^2}{r} \approx \frac{V_T^2}{r}$$

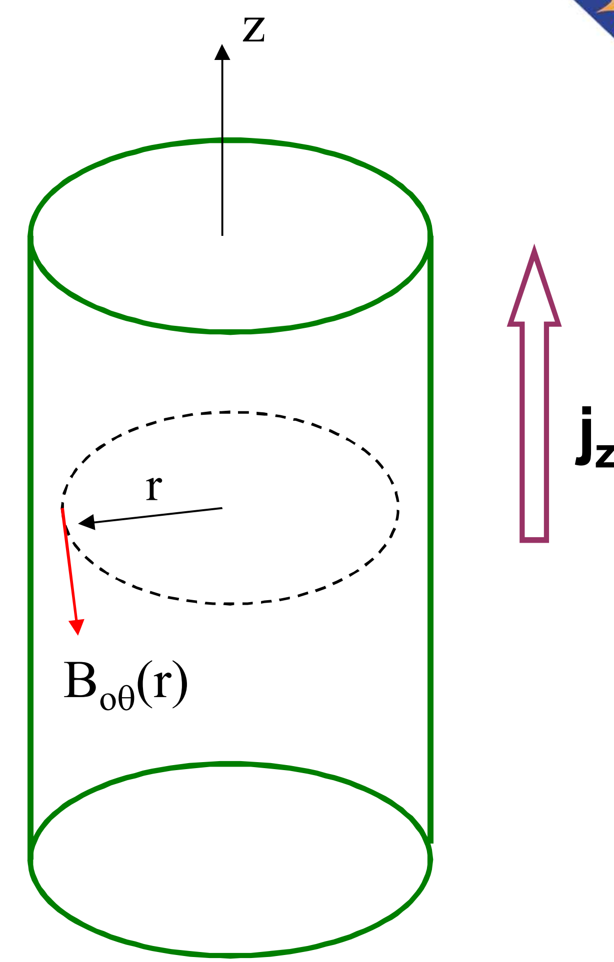






Geometry





x	y	z
↓	↓	↓
r	-z	θ

Flute modes in finite beta plasma

For flute modes the contributions of density fluctuations and finite Larmor radius effects are significant.

In high beta z-pinch plasmas contribution due to electromagnetic effects should also be included.

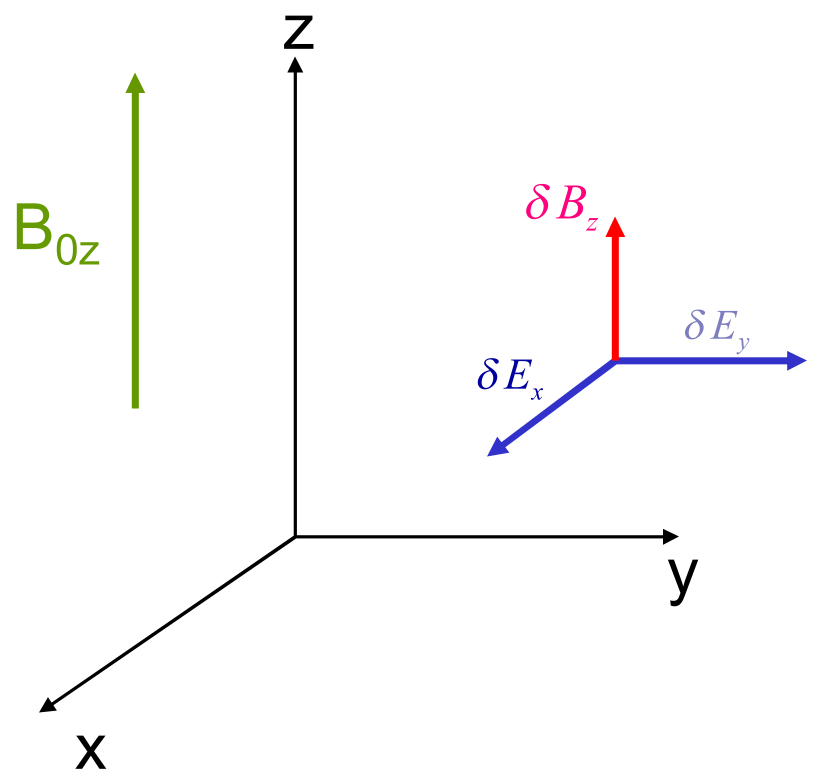
$$\frac{\omega}{\Omega_i} << 1, \quad k_{\parallel} = 0, \quad \Phi(x,y,t), \quad N(x,y,t), \quad A_x(x,y,t), \quad A_y(x,y,t)$$

$$\vec{E} = -\vec{\nabla}\Phi(x,y,t) - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \qquad \vec{B} = rot\ \vec{A}$$

$$N(x,y,t) = n_0(x) + \delta n(x,y,t) \qquad \vec{B}(x,y,t) = [B_{0z}(x) + \delta B_z(x,y,t)]\,\vec{e}_z$$

Sotnikov et al., TPS IEEE, V 34, N 5, 2006

Configuration of excited wave fields



$$\delta B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

$$\delta E_x = -\frac{\partial \Phi}{\partial x} - \frac{1}{c}\frac{\partial A_x}{\partial t}$$

$$\delta E_y = -\frac{\partial \Phi}{\partial y} - \frac{1}{c}\frac{\partial A_y}{\partial t}$$

$$\frac{\delta B_z}{B_{0z}} = -\frac{1}{2}\beta\frac{\delta n}{n_o}$$

Basic equations

$$div\,\vec{j} = 0$$

$$\frac{\partial n_e}{\partial t} + div\; n_e \vec{v}_e = 0$$

$$\frac{\partial B_z}{\partial y} = \frac{4\pi}{c} j_x$$

$$n_e = Z n_i$$

Nonlinear equations for flute modes in high beta plasma

$$\left(\frac{\partial}{\partial t}+\vec{V}_i\cdot\nabla\right)\vec{V}_i=\frac{Ze}{M_i}\left(\vec{E}+\frac{\vec{V}_i}{c}\times\vec{B}\right)-\frac{1}{M_i n_i}\vec{\nabla}P_i-\frac{1}{M_i n_i}\vec{\nabla}\cdot\widehat{\Pi}_i$$

The viscosity tensor Π_i contains only gyroviscosity components

$$\pi_{xx}=-\pi_{yy}=-\frac{1}{2}\frac{n_i T_i}{\Omega_{ci}}\left(\frac{\partial V_x}{\partial y}+\frac{\partial V_y}{\partial x}\right) \qquad \pi_{xy}=\pi_{yx}=\frac{1}{2}\frac{n_i T_i}{\Omega_{ci}}\left(\frac{\partial V_x}{\partial x}-\frac{\partial V_y}{\partial y}\right)$$

Nonlinear equations for flute modes in finite beta plasma

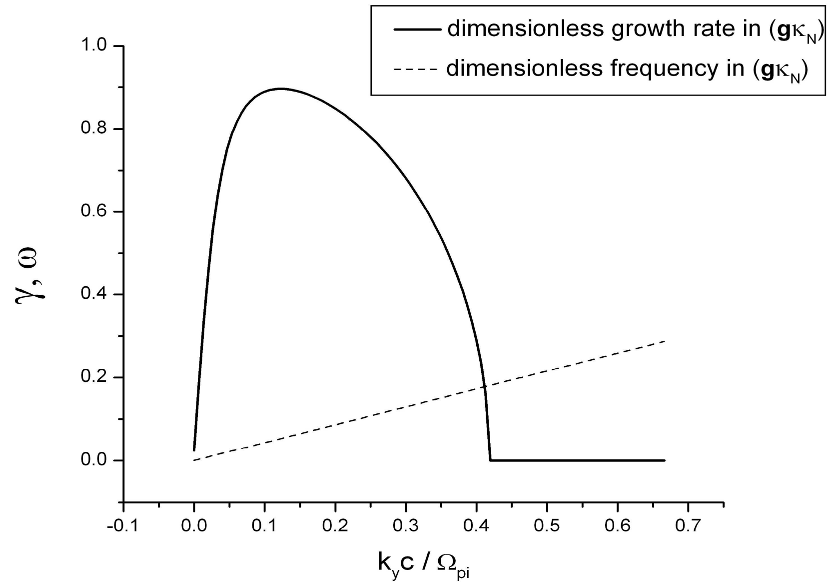
$$\frac{\partial}{\partial t}\Delta_\perp \Phi + \frac{T_i c}{2ZeB_{0z}}\kappa_B \frac{\partial}{\partial y}\Delta_\perp \Phi + \frac{T_i}{Zen_{0i}}\frac{\partial}{\partial t}\Delta_\perp \delta n_i - \frac{T_i}{2Ze}\frac{\partial}{\partial t}\Delta_\perp \frac{\delta B_z}{B_{0z}} + \frac{B_{0z}}{n_{0i}}\frac{g}{c}\frac{\partial \delta n_i}{\partial y} - \frac{g}{c}\frac{\partial \delta B_z}{\partial y} =$$

$$= \frac{cT_i}{2ZeB_{0z}}\{\Phi, \Delta_\perp (\frac{\delta B_z}{B_{0z}})\} - \frac{cT_i}{ZeB_{0z}n_{0i}}\nabla_\perp \cdot \{\Phi, \nabla_\perp \delta n_i\} - \frac{c}{B_{0z}}\{\Phi, \Delta_\perp \Phi\}$$

$$\frac{\partial \delta n_e}{\partial t} - \frac{cT_e}{eB_{0z}}\kappa_B \frac{\partial \delta n_e}{\partial y} + \frac{n_{0e}c}{B_{0z}}(\kappa_B + \kappa_N)\frac{\partial \Phi}{\partial y} - \frac{n_{0e}}{B_{0z}}(\frac{\partial \delta B_z}{\partial t} + \frac{cT_e}{eB_{0z}}\kappa_N \frac{\partial \delta B_z}{\partial y}) =$$

$$= \frac{c}{B_{0z}}\{\delta n_e, \Phi\} + \frac{cn_{0e}}{B_{0z}^2}\{\Phi, \delta B_z\}$$

Growth rate and frequency of flute modes



Experimentally observed plasma parameters

$$n_{0e} = 2.0\times10^{19}\ \text{cm}^{-3},\ n_{0i} = 2.0\times10^{18}\ \text{cm}^{-3},\ T_i \sim T_e\text{e}= 50\ \text{eV},\ \ B_0 = 0.3\ \text{MG}$$

$$R \approx 1\div 3\ mm \qquad g \approx \frac{2V_A^2}{R}$$

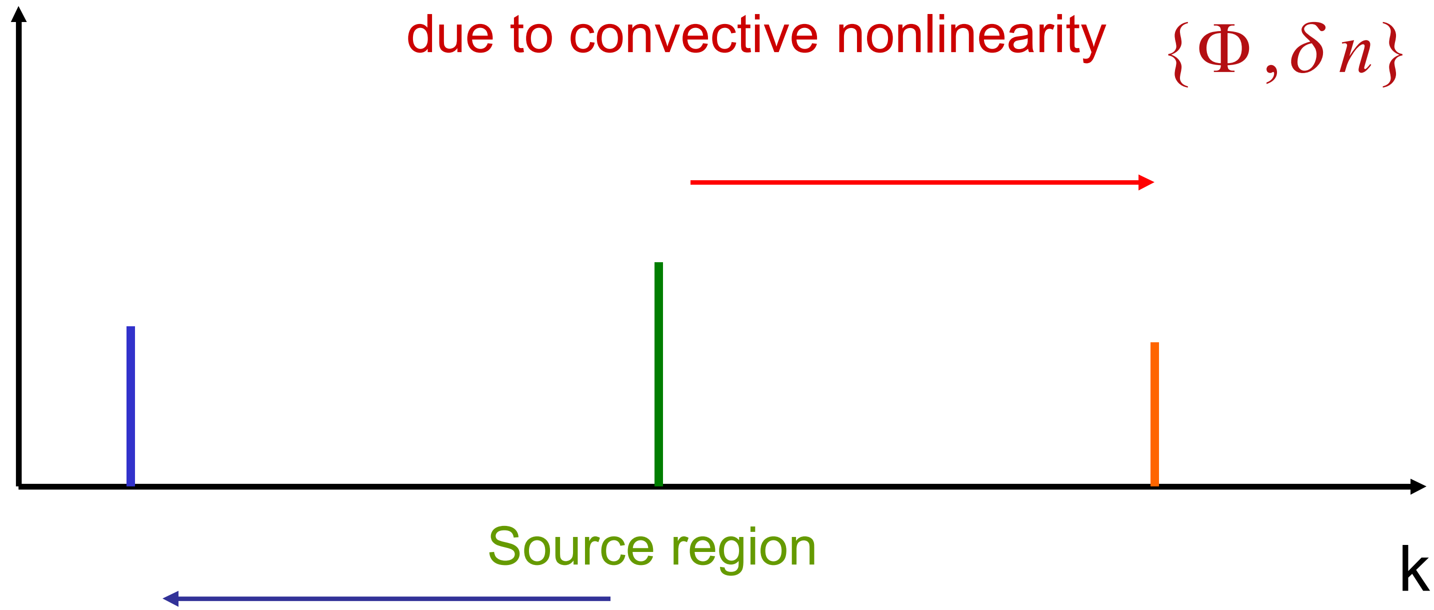
$$\frac{\omega_{pAl}}{c} = 4\times10^2\ cm^{-1} \qquad \frac{kc}{\omega_{pAl}} \approx 0.3 \qquad \lambda \approx 0.5\ mm$$

$$\gamma \approx 3\times10^7\ s^{-1}$$

$$\beta = 0.55 \qquad \Omega_{cAl} = 1.1\times10^{10}\ s^{-1} \qquad \omega_{\text{pAL}} = 1.4\times10^{13}\ s^{-1}$$

Nonlinear wave cascades

Wave energy cascade to small wavelengths
due to convective nonlinearity $\{\Phi, \delta n\}$



Source region

k

Wave energy cascade to large wavelengths and excitation of large scale structures due to polarization drift nonlinearity $\{\Delta_\perp \Phi, \Phi\}$

and diamagnetic component of polarization drift nonlinearity $div\{\vec{\nabla}_\perp \Phi, \delta n\}$

Kodama and Pavlenko, PRL 1988; Newman et al., Phys. Fluids B, 1993

Andrushchenko and Pavlenko, PoP 2002; Sandberg et al., PoP 2005.

Fluid Model of 2D Flute Turbulence

- Numerical Solutions of Nonlinear Flute Equations
- Pseudo-spectral spatial representation
- Two-step predictor-corrector time advance
- Arakawa method used throughout to treat Poisson bracket nonlinear terms
- Viscous and biharmonic dissipations advanced explicitly in time

Nonlinear equations in dimensionless form

$$\frac{\partial}{\partial \tau}\Delta_\perp\Phi - \frac{T_i}{T_e}\frac{V_n}{2+\beta}\frac{\kappa_B + (2+\frac{1}{2}\beta)\kappa_N}{\kappa_N}\frac{\partial}{\partial y}\Delta_\perp\Phi - \frac{1+\frac{1}{4}\beta}{1+\frac{1}{2}\beta}\frac{T_i}{T_e}V_n\frac{\frac{1}{2}\beta\kappa_N - \kappa_B}{\kappa_N}\frac{\partial}{\partial y}\Delta_\perp\delta n + (1+\frac{1}{2}\beta)V_g\frac{\partial \delta n}{\partial y} =$$

$$= (1+\frac{1}{2}\beta)\frac{T_i}{T_e}div\{\nabla_\perp\Phi, \delta n\} - \frac{1}{4}\beta\frac{T_i}{T_e}\{\Delta_\perp\Phi, \delta n\} + Z\{\Delta_\perp\Phi, \Phi\} + \mu\Delta_\perp(\Delta_\perp\Phi) - \frac{T_i}{T_e}(1+\frac{1}{4}\beta)D\Delta_\perp^2\delta n$$

$$\frac{\partial \delta n}{\partial \tau} - \frac{ZV_n}{1+\frac{1}{2}\beta}\frac{\kappa_B - \frac{1}{2}\beta\kappa_N}{\kappa_N}\frac{\partial \delta n}{\partial y} + \frac{ZV_n}{1+\frac{1}{2}\beta}\frac{\kappa_B + \kappa_N}{\kappa_N}\frac{\partial \Phi}{\partial y} = Z\{\delta n, \Phi\} + D\Delta_\perp\delta n$$

$$\kappa_N = -\frac{1}{n_0}\frac{dn_0}{dx} > 0 \qquad \kappa_B = \frac{1}{B_{0z}}\frac{dB_{0z}}{dx} \qquad \mu = \frac{3}{10}\frac{T_i\nu_i}{M_i\Omega_{ci}} \qquad D = \frac{m_e}{e^2}\frac{T_e\nu_e}{B_{0z}^2}$$

FLUTE Code Numerics: Time

- Two-step time integration scheme (Taking density equation as an example):

-Predictor (From time step n to n+1/2)

$$n_p^{n+1/2} = \langle n \rangle^n + \frac{\Delta t}{2} F_n^n + \frac{\Delta t}{2} D_n (\nabla_\perp^2 n)^n$$

Optional Lax spatial average $\langle n \rangle = \frac{1}{4}(n_{i+i,j} + n_{i-1,j} + n_{i,j+1} + n_{i,j-1})$

Otherwise $\langle n \rangle = n_{i,j}$

- Corrector (From time step n to n+1)

$$n_c^{n+1} = n^n + \Delta t F_{n_p}^{n+1/2} + \Delta t D_n (\nabla_\perp^2 n)^n$$

Diffusivity treated explicitly in time

FLUTE Code Numerics: Space (2D x-y Slab)

$$F_n = -C_n^1 \frac{\partial n}{\partial y} - C_n^2 \frac{\partial \phi}{\partial y} + C_n^3 \{n, \phi\}$$

- Space-centered derivatives

$$\frac{\partial n}{\partial y} = \frac{(n_{i,j+1} - n_{i,j-1})}{2\Delta y}$$

- Nonlinear advective terms in Poisson brackets handled using Arakawa's conservative method

$$\{n, \phi\} = \frac{\partial n}{\partial x}\frac{\partial \phi}{\partial y} - \frac{\partial n}{\partial y}\frac{\partial \phi}{\partial x}$$

Akio Arakawa, JCP 1, 119-143 (1966), reprinted as JCP 135, 103-114 (1997);
Also W. Horton, R.D. Estes, D. Biskamp, Plasma Physics 22, 663-678 (1980).

FLUTE Code Numerics: Space (2D x-y Slab)

- Nonlinear advective terms in Poisson brackets handled using Arakawa's conservative method (Continued)

$$\{n,\phi\} = \frac{\partial n}{\partial x}\frac{\partial \phi}{\partial y} - \frac{\partial n}{\partial y}\frac{\partial \phi}{\partial x} = J$$

$$J_{i,j}(n,\phi) = -\frac{1}{12\Delta x \Delta y}[(\phi_{i,j-1} + \phi_{i+1,j-1} - \phi_{i,j+1} - \phi_{i+1,j+1})$$

$$(n_{i+1,j} - n_{i,j}) + (\phi_{i-1,j-1} + \phi_{i,j-1} - \phi_{i-1,j+1} - \phi_{i,j+1})$$

$$(n_{i,j} - n_{i-1,j}) + (\phi_{i+1,j} + \phi_{i+1,j+1} - \phi_{i-1,j} - \phi_{i-1,j+1})$$

$$(n_{i,j+1} - n_{i,j}) + (\phi_{i+1,j-1} + \phi_{i+1,j} - \phi_{i-1,j-1} - \phi_{i-1,j})$$

$$(n_{i,j} - n_{i,j-1}) + (\phi_{i+1,j} - \phi_{i,j+1})(n_{i+1,j+1} - n_{i,j})$$

$$+(\phi_{i,j-1} - \phi_{i-1,j})(n_{i,j} - n_{i-1,j-1}) + (\phi_{i,j+1} - \phi_{i-1,j})$$

$$(n_{i-1,j+1} - n_{i,j}) + (\phi_{i+1,j} - \phi_{i,j-1})(n_{i,j} - n_{i+1,j-1})]$$

FLUTE Code Numerics: Space (2D x-y Slab)

- Fourth order derivative of density in dissipative term of vorticity equation treated as biharmonic operator

(Abramovitz and Stegun, Handbook of Mathematical Functions, p. 885)

$$\nabla^4 n_{i,j} = \frac{1}{(\Delta x)^2 (\Delta y)^2} [20 n_{i,j} - 8(n_{i+1,j} + n_{i,j+1} + n_{i-1,j} + n_{i,j-1})$$

$$+2(n_{i+1,j+1} + n_{i+1,j-1} + n_{i-1,j+1} + n_{i-1,j-1})$$

$$+(n_{i,j+2} + n_{i+2,j} + n_{i-2,j} + n_{i,j-2})]$$

- Potential ϕ obtained from vorticity $U = \nabla_\perp^2 \phi$ using Fast Fourier Transforms (FFT2 or FFTW)

$$\phi_k = -\frac{1}{k^2} U_k$$

FLUTE Code Numerics

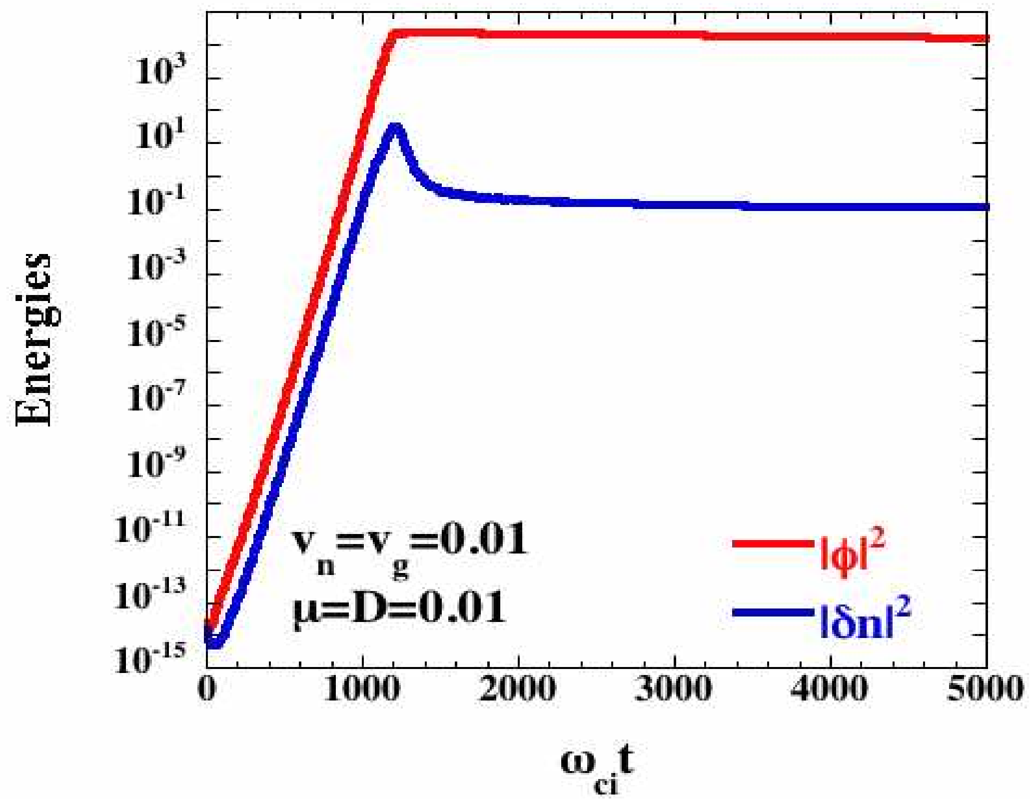
- Ease of FORTRAN programming facilitated by use of modular subroutines or templates (e.g. predictor step of density equation).

```
c
c  density equation
c
      do j=1,ncy
      do i=1,ncx
      densav(i,j)=0.25*(
     1  dens(ir(i),j)+dens(il(i),j)
     1 +dens(i,jr(j))+dens(i,jl(j))
     1  )
      end do
      end do
c
c linear terms
c
      call derivy(fy,dens,ncx,ncy,1.0)
      call add(densp,densav,fy,ncx,ncy,-dth*cofn1)
c
      call derivy(fy,pot,ncx,ncy,1.0)
      call add(densp,densp,fy,ncx,ncy,-dth*cofn2)
```

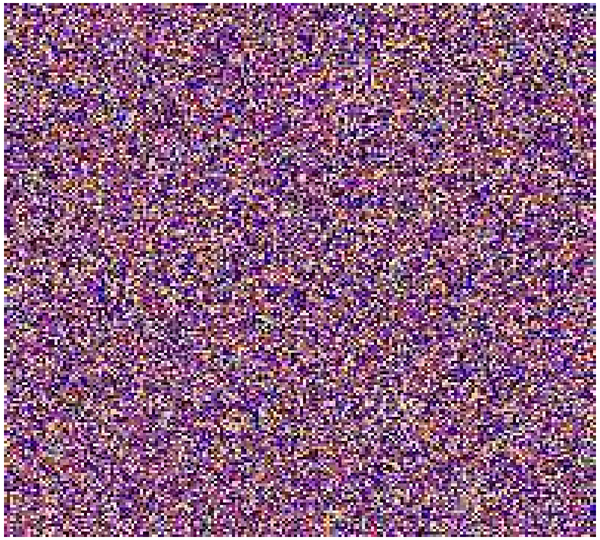
```
c
c  nonlinear terms
c
      if(nonlin.eq.1) then
      call arak(fx,dens,pot,ncx,ncy,1.0)
      call add(densp,densp,fx,ncx,ncy,dth*cofn3)
      end if
c
c  explicit viscosity
c
      if(ndisp.eq.1) then
      call dlap(fx,dens,ncx,ncy,1.0)
      call add(densp,densp,fx,ncx,ncy,dth*cofn4)
      end if
```

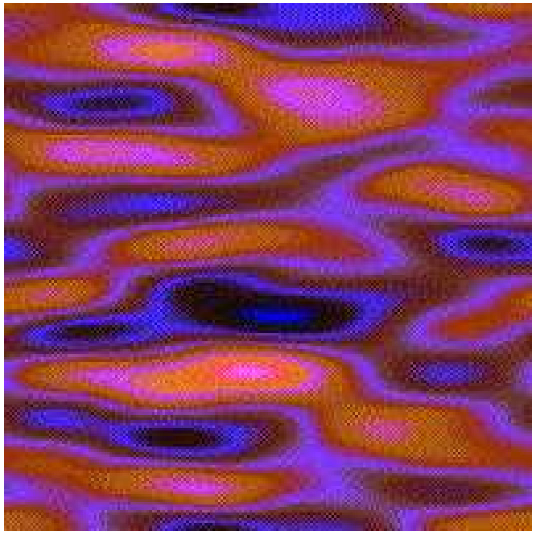
- These templates are convenient for solving different sets of equations.

Time dependence of energy



Density in linear stage

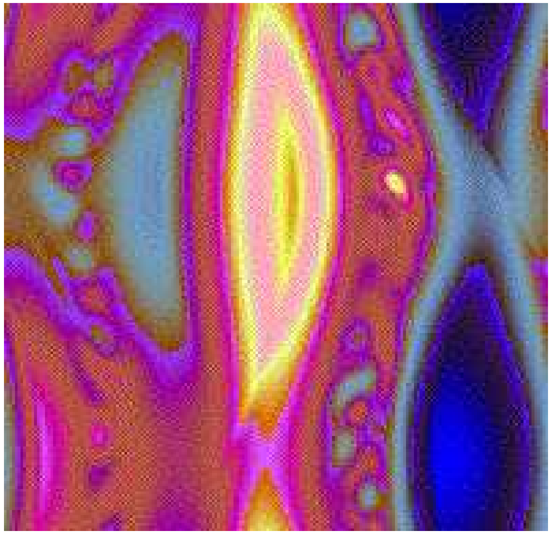


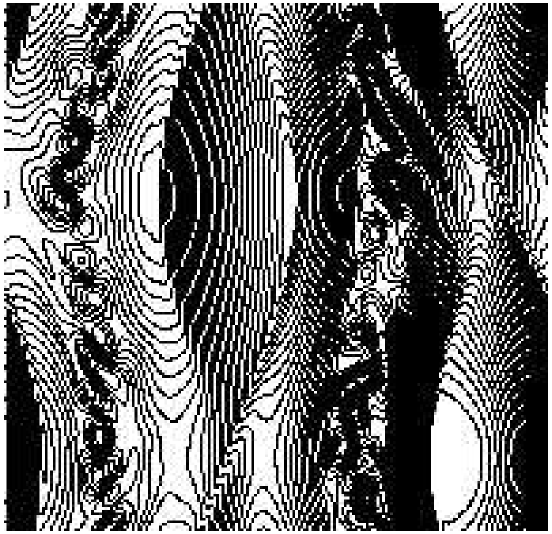


$\Omega_i t = 0$

$\Omega_i t = 1400$

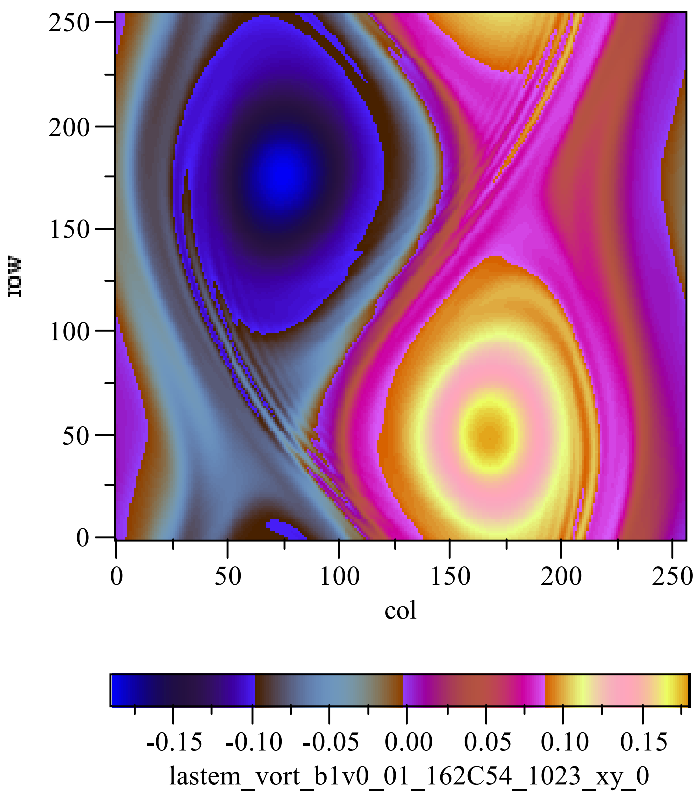
Density in nonlinear stage

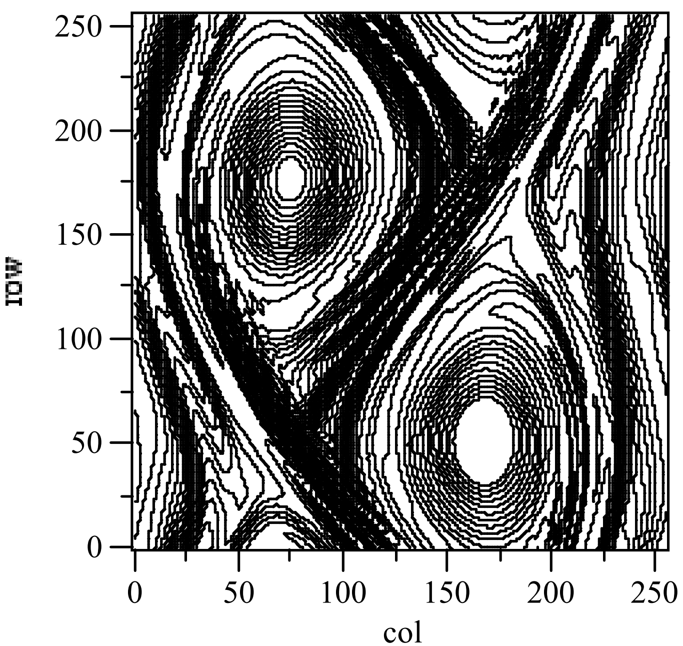




$$\Omega_i t = 4600$$

Vorticity in nonlinear stage





$$\Omega_{ci} t = 4600$$

Conclusions

- Wave activity experimentally observed in the central region of imploding wire array, where significant part of the current is concentrated, can be connected with excitation of flute-type compressible electromagnetic oscillations in finite beta plasma.

- Flute-type instability can be observed in laboratory astrophysics experiments on interaction of laser ablated plasma flow with strong magnetic field.

- Linear dispersion equation for electromagnetic flute-like mode instability has growing solution even in a finite beta plasma. Typical growth time and spatial scales of excited waves are in agreement with experimental data.

- Fluid model of flute turbulence can describe nonlinear dynamics of large scale density and magnetic field structures as well as wave energy cascading towards the short scales observed in experiment.

Equilibrium

$$\left(1+\frac{ZT_e}{T_i}\right)\kappa_N + \frac{g}{V_{Ti}^2} = \frac{2}{\beta_i}\kappa_B$$

$$\kappa_N = -\frac{1}{n_0}\frac{dn_0}{dx} > 0 \qquad \kappa_B = \frac{1}{B_{0z}}\frac{dB_{0z}}{dx}$$

$$V_{iy}^{(0)} = -\frac{g}{\Omega_{0i}} - \kappa_N \frac{V_{Ti}^2}{\Omega_{0i}} \qquad V_{ey}^{(0)} = \kappa_N \frac{V_{Te}^{*2}}{\Omega_{0i}} \qquad V_{Te}^{*2} = \frac{T_e}{M_i}$$