SAND2008-2771P

Unstructured Mesh CFD Research

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Work funded by DoE ASC program

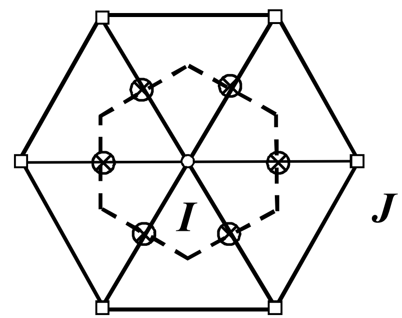
Gradient Algorithms

(Barth, T. AIAA 91-1548, Haselbacher, A. and Blazek, J. AIAA J. 2000, Schneider and Raw, Num. Heat Trans., 1987)

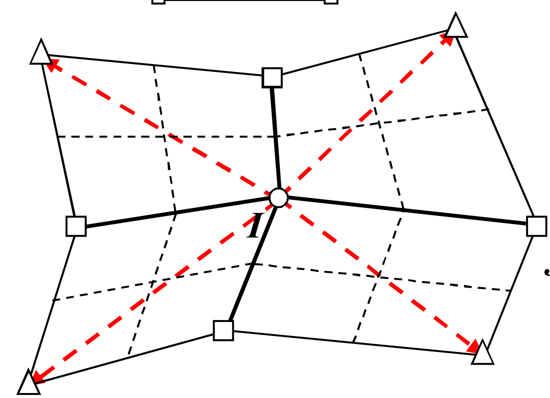
- **Green-Gauss Integral (Barth & Jespersen)**
  - Exact gradients for linear functions on tets LP (trapezoid rule)
  - Less accurate on hexahedral elements (midpoint rule)

$$\nabla u_I = \frac{1}{\Delta V_I}\int_{S_I} u\mathbf{n}\,dS \approx \frac{1}{\Delta V_I}\sum_{J\in NE(I)}\frac{1}{2}\left(u_I+u_J\right)\Delta\mathbf{S}_{IJ}$$

$$\nabla u_I^b = \frac{1}{\Delta V_I}\int_{S_I} u\mathbf{n}\,dS \approx \frac{1}{\Delta V_I}\sum_{J\in NE(I)}\frac{1}{2}\left(u_I+u_J\right)\mathbf{n}_{IJ}\left|\Delta\mathbf{S}_{IJ}\right| + u_I^b\mathbf{n}_I^b\left|\Delta\mathbf{S}_I^b\right|$$



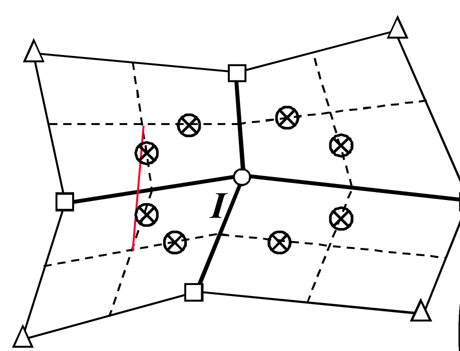
- **Least-Squares (Barth, Haselbacher & Blazek)**
  - Exact derivatives for linear functions (LP)
  - Store six weights at each node
  - Requires no special treatment at boundaries
  - Include virtual edges for non-simplex types
  - Construct linear equation for each node: $u_J = u_I + \xi_{IJ}\nabla u_I \cdot \Delta\mathbf{r}_{IJ}$
  - Inverse weighting: $\xi_{IJ} = \left|\Delta\mathbf{r}_{IJ}\right|^{-p}$



- **CVFEM**
  - Exact derivatives for linear functions (LP)
  - Element assembly
  - Integrate exact dual mesh surface

$$\nabla u_I = \frac{1}{\Delta V_I}\int_{S_I} u\mathbf{n}\,dS \approx \frac{1}{\Delta V_I}\sum_{elem(I)}\sum_{k\in edge(I)}\bar{u}_k\Delta\mathbf{S}_k \qquad \bar{u}(\mathbf{x},t) = \sum_{i=1}^{n} u_i N_i$$

$$\nabla u_I^b = \frac{1}{\Delta V_I}\int_{S_I} u\mathbf{n}\,dS \approx \frac{1}{\Delta V_I}\sum_{elem(I)}\sum_{k\in edge(I)}\bar{u}_k\Delta\mathbf{S}_k + \sum_{faces(I)}\bar{u}_I^b\Delta S_I^b$$



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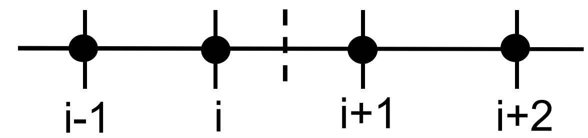
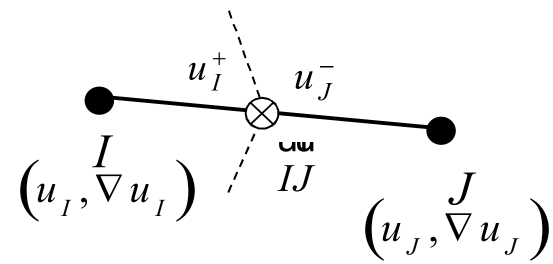
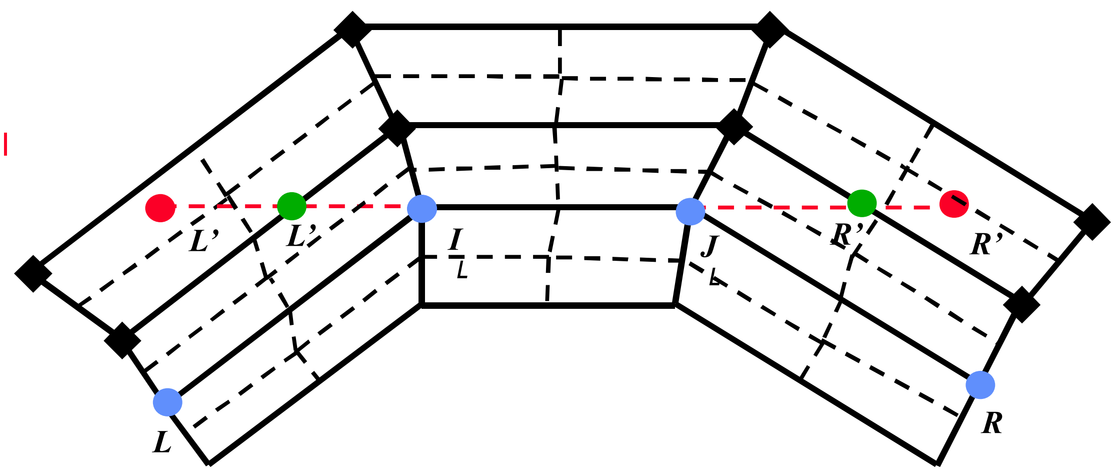
Reconstruction Techniques for Node Centered Discretizations

(Barth, T., AIAA 91-1548; Weatherill, N. et al. AIAA 93-0341; Lyra, P. et al., IJNMF, 1994)

- **Structured grid stencil for upwind schemes**
  - (i-1,i,i+1,i+2)
  - Local data support
- **Gradient based extrapolation** $(\mathbf{u}_I, \nabla \mathbf{u}_I, \mathbf{u}_J, \nabla \mathbf{u}_J)$
  - MUSCL
  - Gradient limiters required for monotonicity
  - Boundary gradients are error prone
- **Collinear edge stencil (L,I,J,R)**
  - Implement almost any flux scheme
  - Straightforward parallelization
  - Assumes edge aligned flow direction
  - In general, edges are neither co-aligned or the same length
- **Element interpolation stencil (L’,I,J,R’)**
  - Edges co-aligned and equal length
  - Any element type
  - Can result in non-adjacent element stencil
  - Difficult parallelization
- **Modified element interpolation stencil**
  - (L’,I,J,R’)
  - Edge must lie in adjacent element
  - Compact stencil
  - Straightforward parallelization







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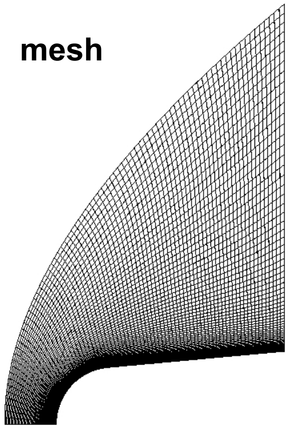
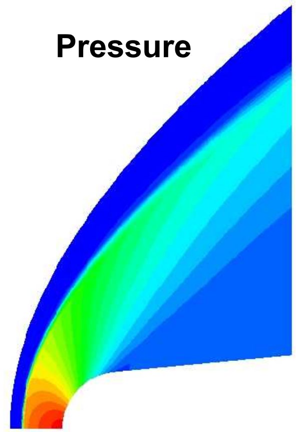
Supersonic Inviscid Flow: Blunt Wedge

mesh



Pressure



Flow Conditions:
M=3.0
Mesh:
H-mesh 81x75x3
Solver settings:
CFL=1 - 10^5
MPF=1
f_A=1.01
ILU(0)

Solver	η forcing	ILU(#)/ ILU(#)	k inner its.	Δt	Controller	CPU (sec.)	CPU ratio
MFNK/ILU(0)	.01	0		GTS		3,890	2.25
MFNK/ILU(0)	.01	0		LTS		3,549	2.06
MFNK/ILU(0)	.1	0		LTS		3,140	1.82
SI			5/.1	LTS		1,726	1.0
SI			10/.1	LTS		1,947	1.13
SI			10/.01	LTS		2,776	1.61
S1_2	.01	/0	10/.1	LTS	$\|\|R\|\|/\|\|R\|\|_0=10^{-2}$	660	0.38
S1_3	.01	/0	10/.1	LTS	$\|\|R\|\|/\|\|R\|\|_0=10^{-3}$	530	0.31
S1_4	.01	/0	10/.1	LTS	$\|\|R\|\|/\|\|R\|\|_0=10^{-4}$	545	0.32

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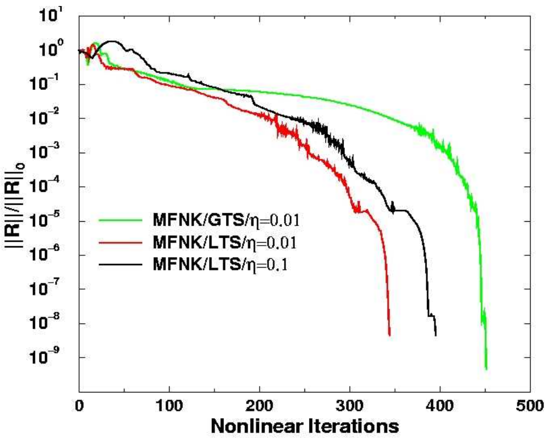
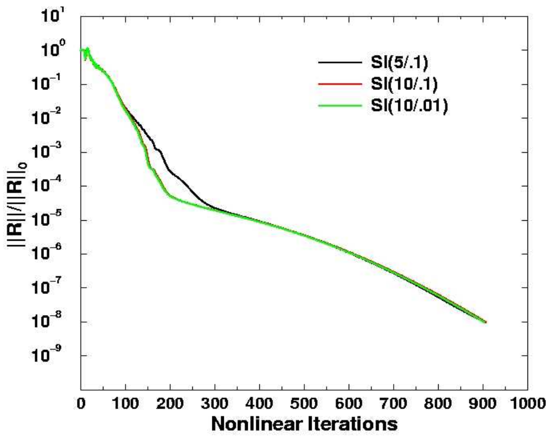
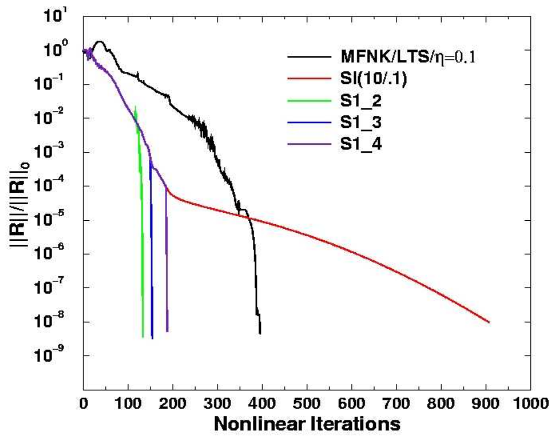
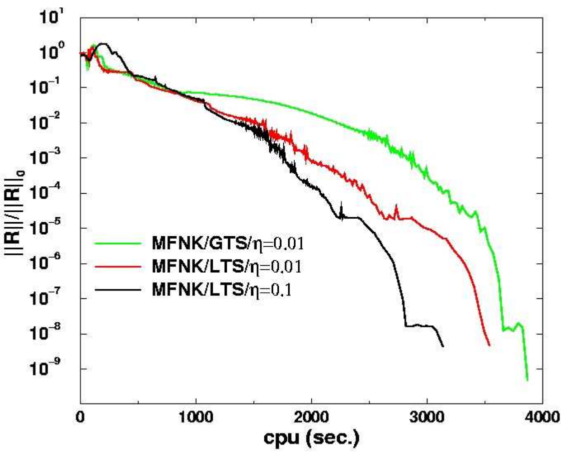
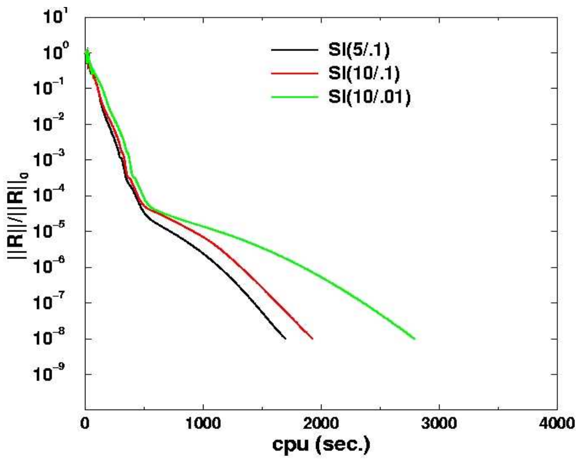
Inviscid Supersonic Flow: Blunt Wedge

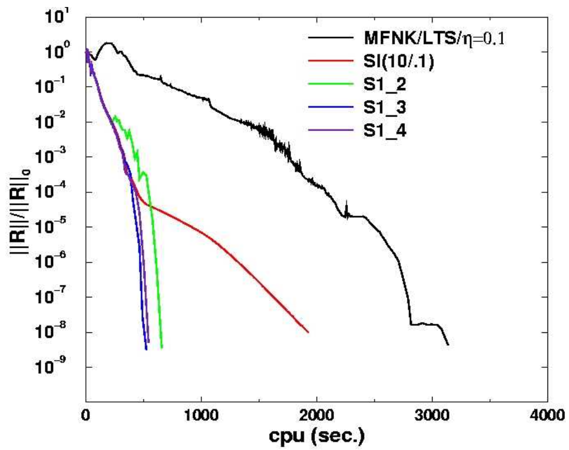












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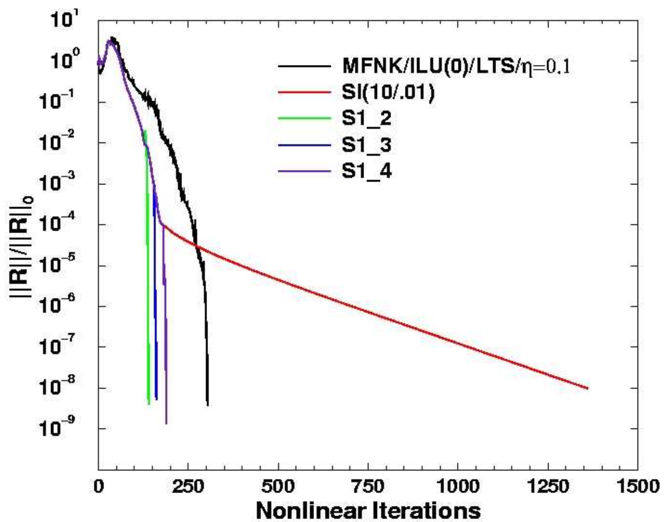
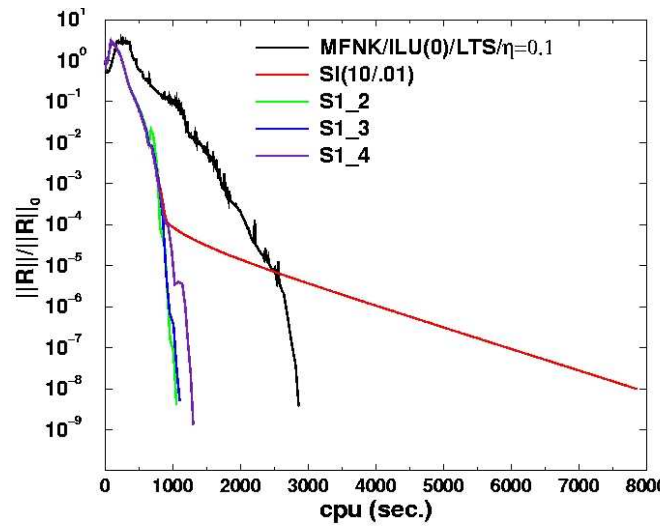
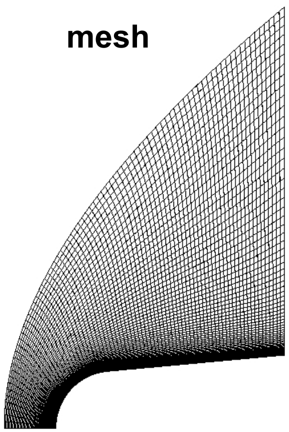
Laminar Supersonic Flow: Blunt Wedge

Flow Conditions:
M=3.0
Re=2.4E5
Mesh:
H-mesh 81x75x3
Solver settings:
CFL=1 - 10^5
LTS
MPF=1
f_A=1.01
ILU(0)

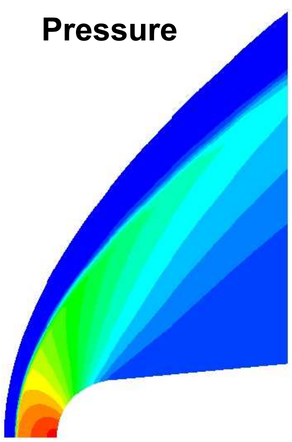




mesh



Pressure



Solver	η forcing	ILU(#)/ILU(#)	k inner its.	CPU (sec.)	CPU ratio
MFNK	.1	0		2,837	1.0
SI(10/.1)			10/.1	7,718	2.72
SI(10/.01)			10/.01	7,835	2.76
S1_2	( )/.01	/0	10/.01	1,042	0.37
S1_3	( )/.01	/0	10/.01	1,085	0.38
S1_4	( )/.01	/0	10/.01	1,292	0.46

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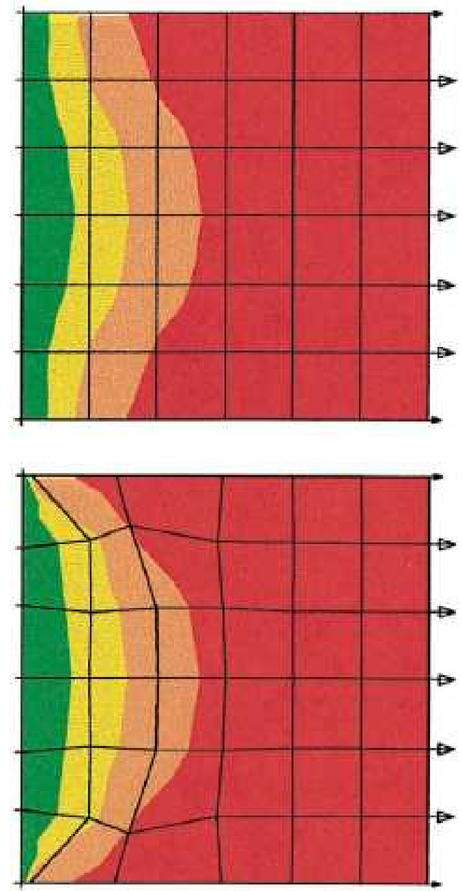
Advanced Unstructured Grid Adaptivity

- R-Adaptivity:
  - Move nodes to improve solution.
  - Mesh adjusts to solution anisotropy.
- Elliptic/Hyperbolic PDE Approach:
  - Take advantage of FEM machinery.
  - Algorithms “target” element-quality.
  - Research needed to more rationally couple physics to element-quality.
  - Variational ALE (VALE/CF) approach is a good candidate. Minimizes a functional where solution variables include node position.
  - Solver strategies needed for resulting highly non-linear systems.



Linear elasticity with
(a) fixed mesh (b) VALE
(Mueller et al., 2002)

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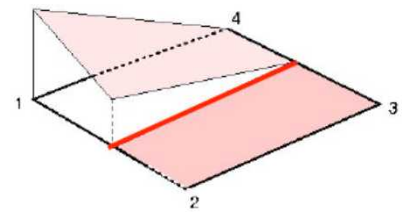
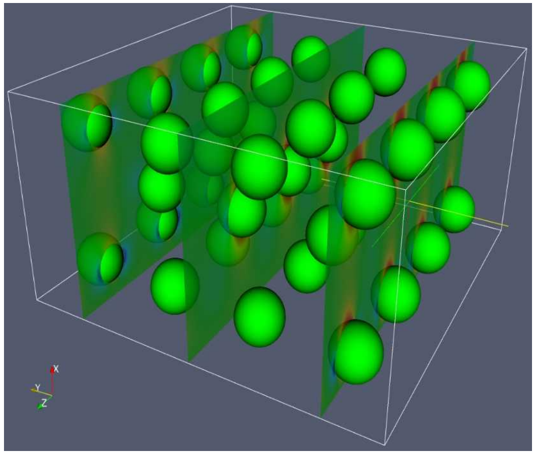
Advanced Unstructured Grid Adaptivity

- P-Adaptivity:
  - Increase accuracy via increase in span of basis.
  - Standard FEM approach requires specialized elements and careful treatment at transitions.
- Partition Of Unity (POU) Approach:
  - Derived from standard (low-order) FEM-basis.
  - “Enrichments” added in a consistent fashion - retains (at least) convergence of FEM-basis.
  - If enrichments are C0 (C1) then strong (weak) intra-element discontinuities may be captured.
  - Traditionally used for solid-mechanics - recently applied to shocks (Chessa, 2006).
  - We are interested in developing this technology for both material and shock discontinuities: solid-mechanics and hydrodynamics. Current work on explicit methods but exploring explicit/implicit algorithms. Interested in LBB for mixed-forms.

$$\phi_1(\mathbf{x}) \cdot H(\mathbf{x})$$



Bi-linear FEM basis modified with C0 enrichment.



Mechanical loading of soft-hard matrix. Inclusions modeled via POU methods.

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