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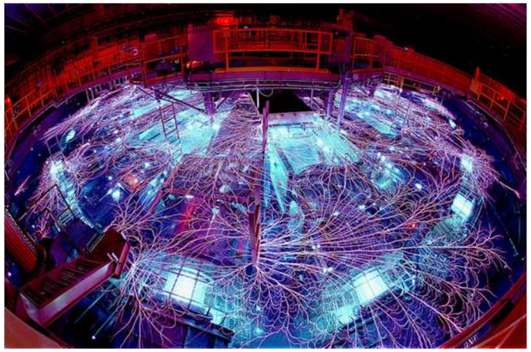

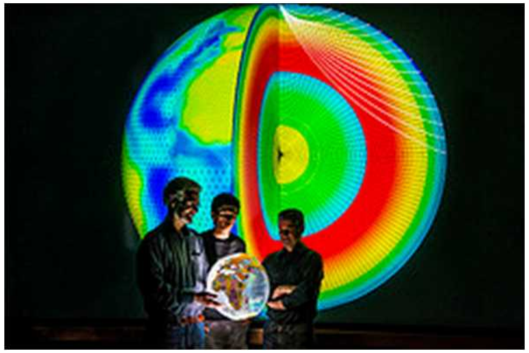

Dispersion Analysis of Acoustic Elements in Sierra/SD

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Dispersion

- Dispersion is when a wave splits into smaller waves, where some of those smaller waves travel either faster or slower than the main wave
- Causes errors in the pressure and velocity
- The dispersion changes as the time step and element size change
- Trying to find an optimum time step/element size combination that reduces errors caused by dispersion

CFL Number (Courant-Friedrich-Levy)

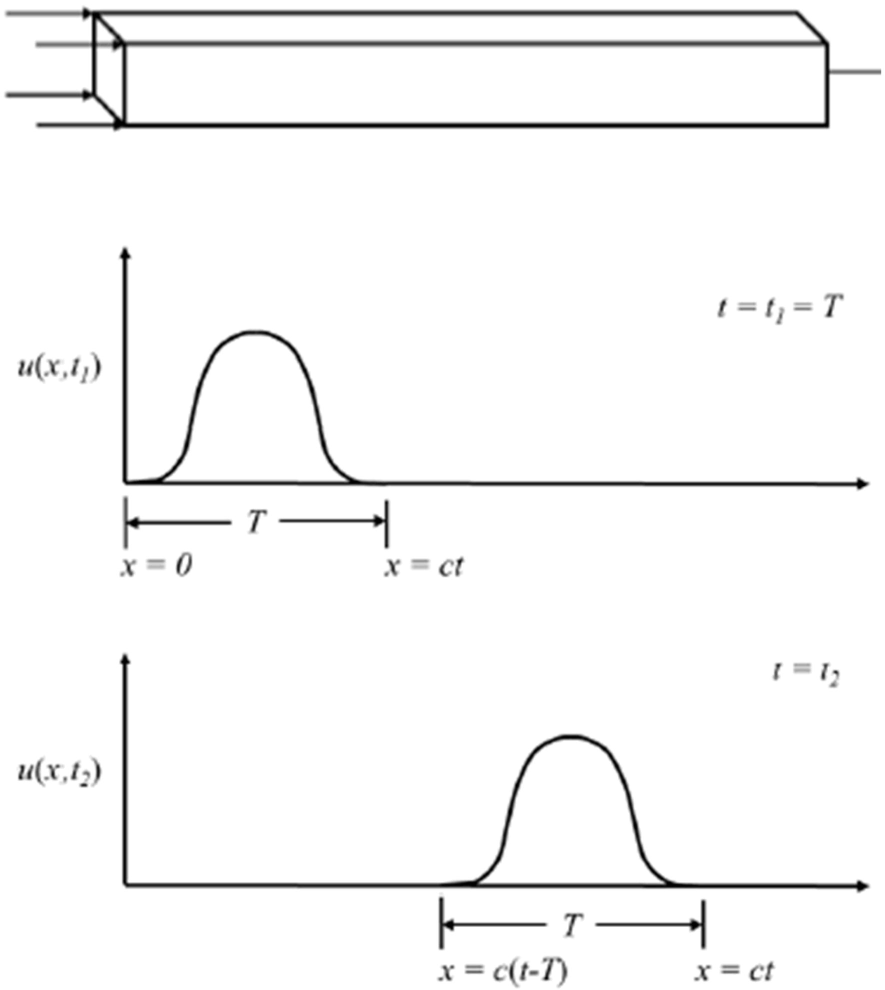
- $CFL = \frac{c\Delta t}{\Delta x}$
- c = speed of sound in medium
- Δt = time step
- Δx = element size
- Describes how long the wave takes to travel across one element

Mass Matrices

- Consistent – Full mass matrix
- Lumped – Only the diagonal terms of the mass matrix are used
- Lumped-Consistent – Higher order mass matrix (α = 0.5):

$$M = \alpha * M_{lumped} + (1-\alpha) * M_{consistent}$$

1D Problem



- Acoustic velocity applied to end of the waveguide. In one-dimension, the haversine pulse should travel unchanged across the length of the guide

1D Solution

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial u}{\partial t}$$

Momentum Equation

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial}{\partial t}[\frac{1}{2}(1-\cos(\omega t - kx))]$$

Plug in u, the haversine function

$$\frac{\partial p}{\partial x} = -\rho[\frac{1}{2}\omega \sin(\omega t - kx)]$$

Differentiate u with respect to time

$$p(x,t) = \frac{1}{2}\rho c(1-\cos(\omega t - kx))$$

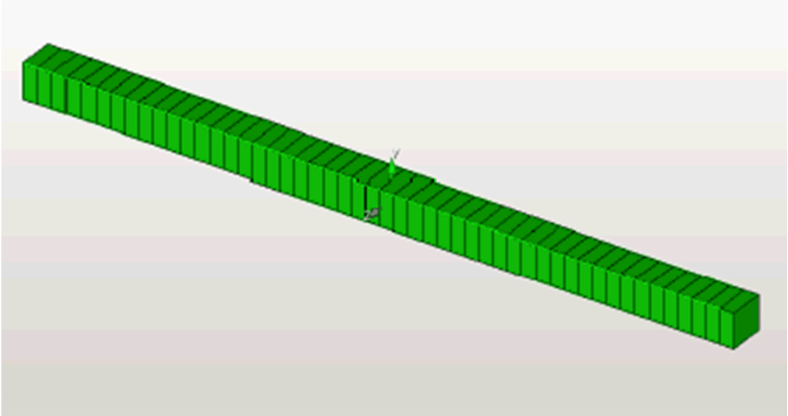
Integrate both sides with respect to x

$$p(x,t) = \rho c u(x,t)$$

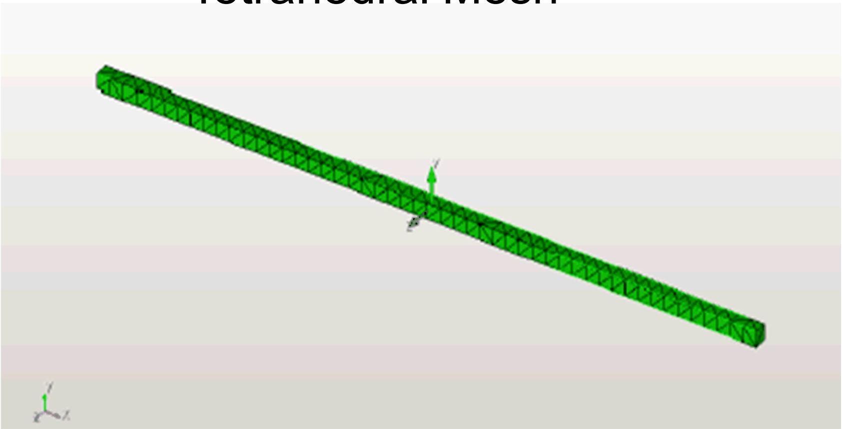
Pressure should keep same haversine form

1D Model

Hexahedral Mesh



Tetrahedral Mesh



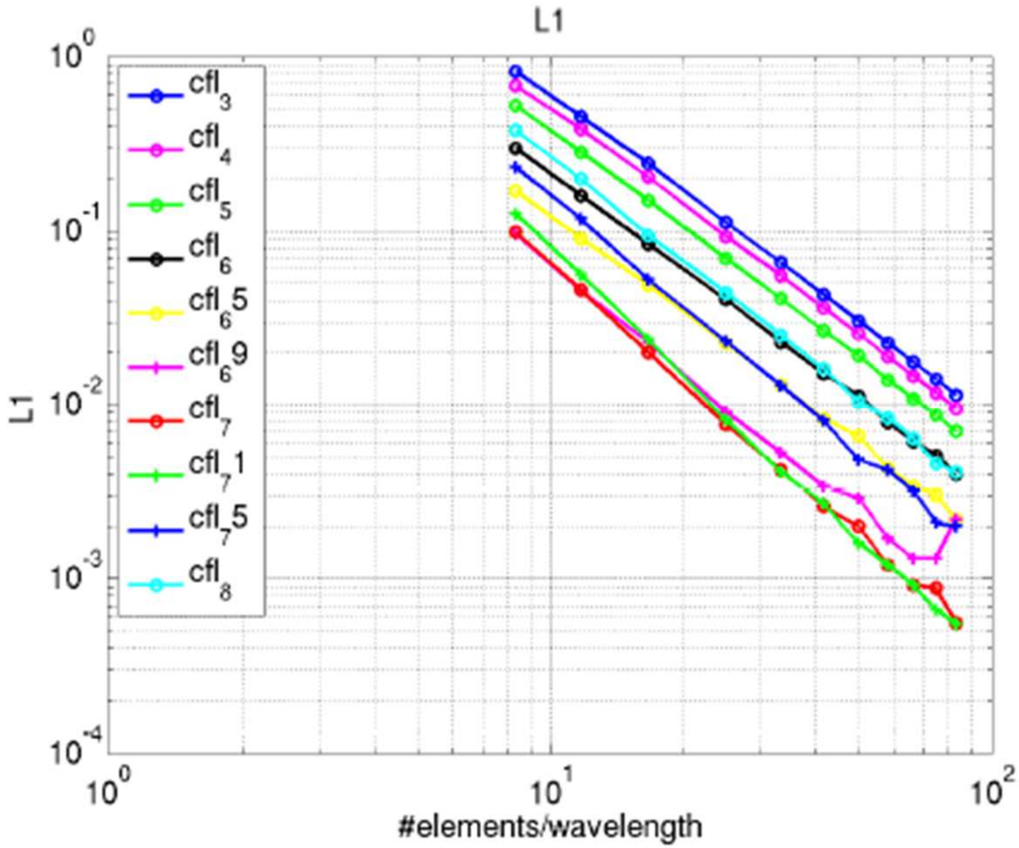
- To refine the mesh, the number of intervals specified along the length of the bar was increased from 50-1000

1D Results

Less Dispersion- 500 Elements – CFL = 0.3

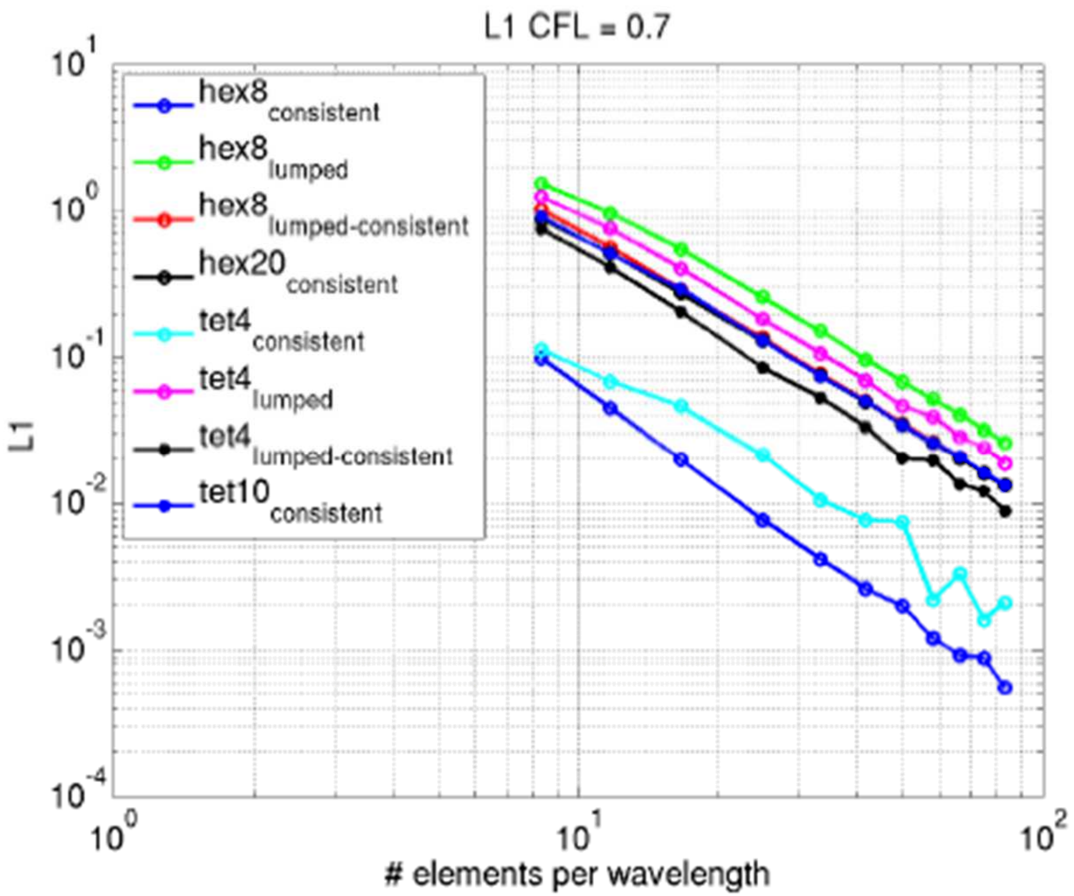
More Dispersion- 100 Elements – CFL = 0.3

1D Results



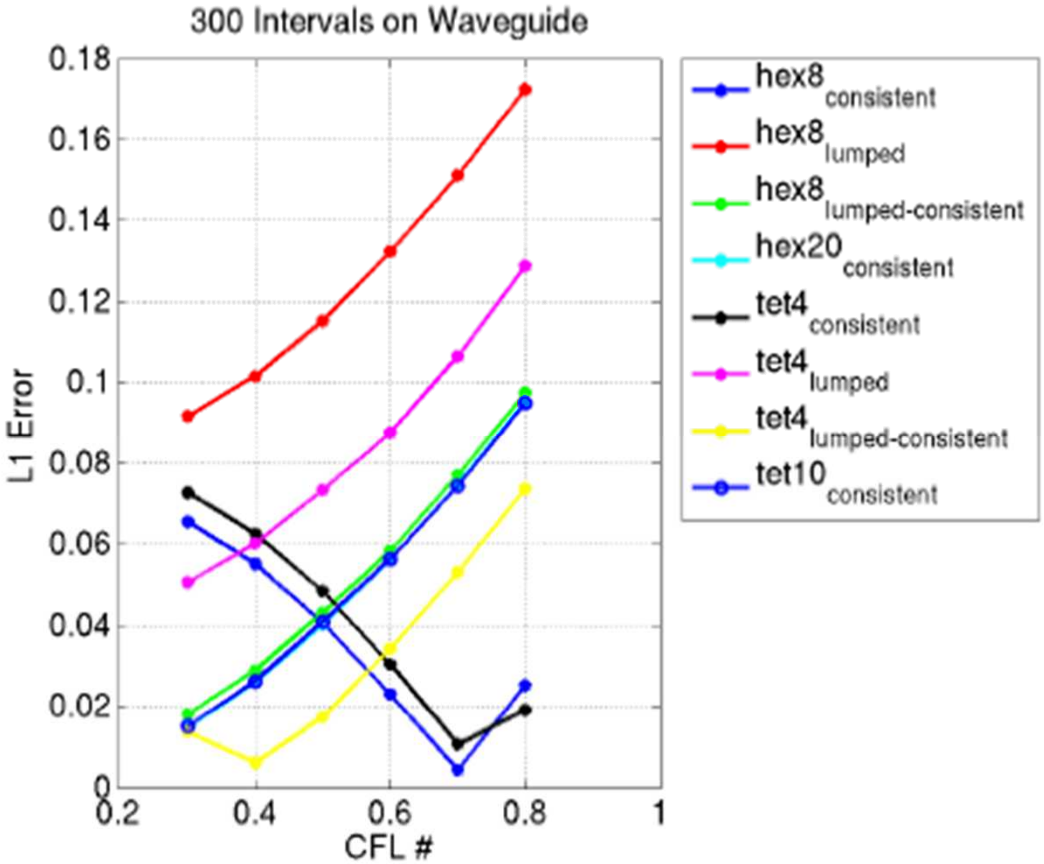
L1
cfl_3
cfl_4
cfl_5
cfl_6
cfl_6 5
cfl_6 9
cfl_7
cfl_7 1
cfl_7 5
cfl_8
10^0
10^-1
10^-2
10^-3
10^-4
L1
10^0
10^1
10^2
#elements/wavelength

1D Results



L1 CFL = 0.7
hex8 consistent
hex8 lumped
hex8 lumped-consistent
hex20 consistent
tet4 consistent
tet4 lumped
tet4 lumped-consistent
tet10 consistent
L1
elements per wavelength

1D Results



300 Intervals on Waveguide
L1 Error
CFL #
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.2
0.4
0.6
0.8
1
hex8 consistent
hex8 lumped
hex8 lumped-consistent
hex20 consistent
tet4 consistent
tet4 lumped
tet4 lumped-consistent
tet10 consistent

3D Problem

- For the analysis in 3D, a sphere with a cavity is used as the model
- Acoustic velocity applied to inner surface and the wave should propagate outwards towards the outer surface
- We expect the pressure pulse to decrease in magnitude because it spans over a larger volume as the radius increases, but the leading and lagging dispersion should not be occurring

3D Solution

$$\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

Begin with wave equation, only radial parts due to symmetry

$$p(r,t) = \frac{1}{r}F_1(r-ct) + \frac{1}{r}F_2(r+ct)$$

Solving wave equation above gives this general form of solution

$$p(r,t) = \frac{Aa}{2r}(1-\cos(\omega t - kr + ka))$$

We want our pressure to take this form of a haversine

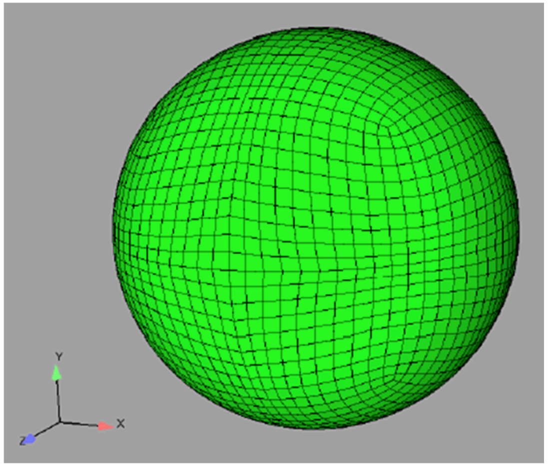
$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial r}$$

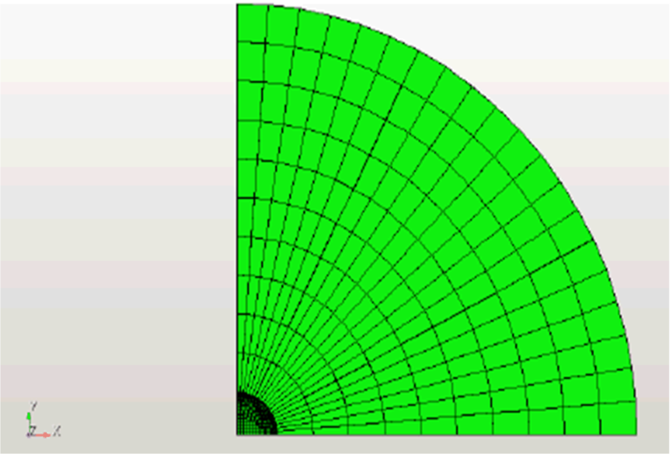
U and p are related through this linear Euler equation

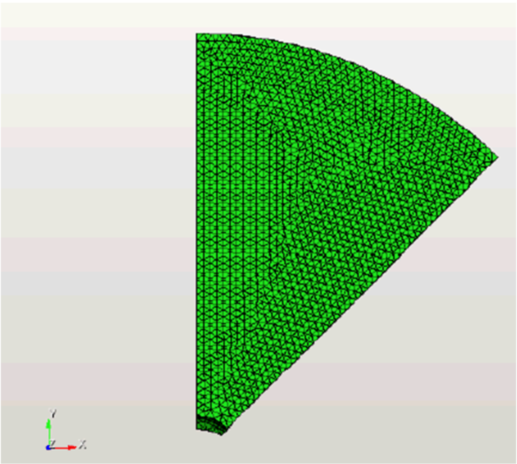
$$u(a,t) = \frac{A}{2\rho a}(t + \frac{a}{c} - \frac{a}{c}\cos(\omega t) - \frac{1}{\omega}\sin(\omega t))$$

Integrating and solving for u provides the acoustic velocity that must be applied to obtain desired pressure pulse

3D Models







Full Sphere

- Used for hex meshes with lumped/lumped-consistent mass matrices

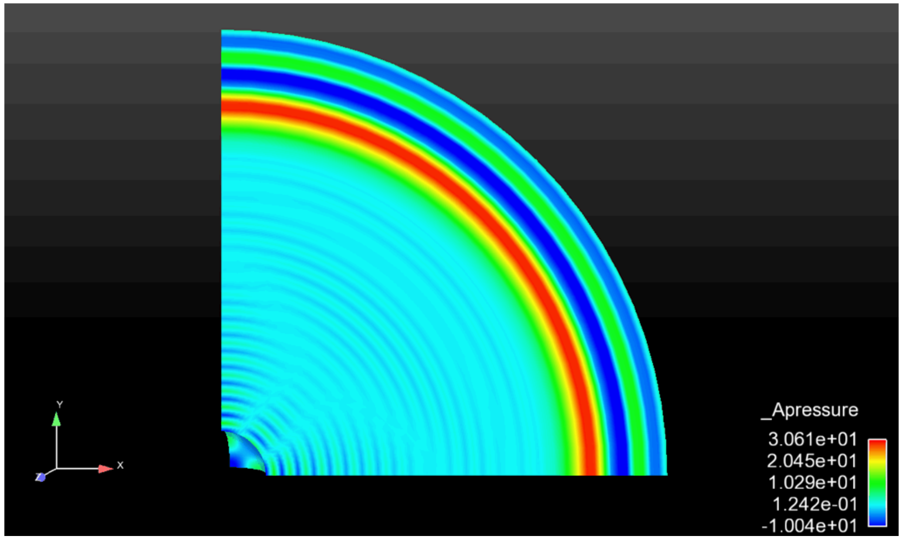
1/8 Sphere

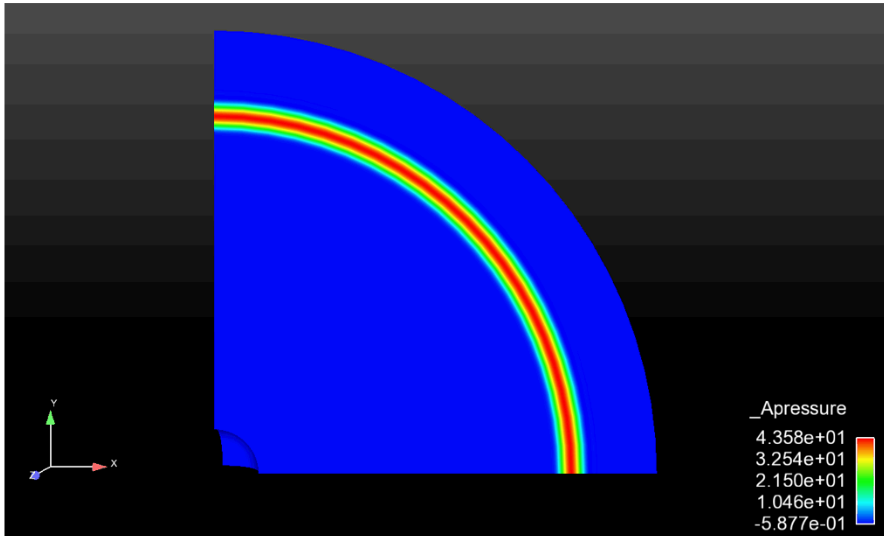
- Used for hex meshes with consistent mass matrix

1/16 Sphere

- Used for tet meshes

3D Results



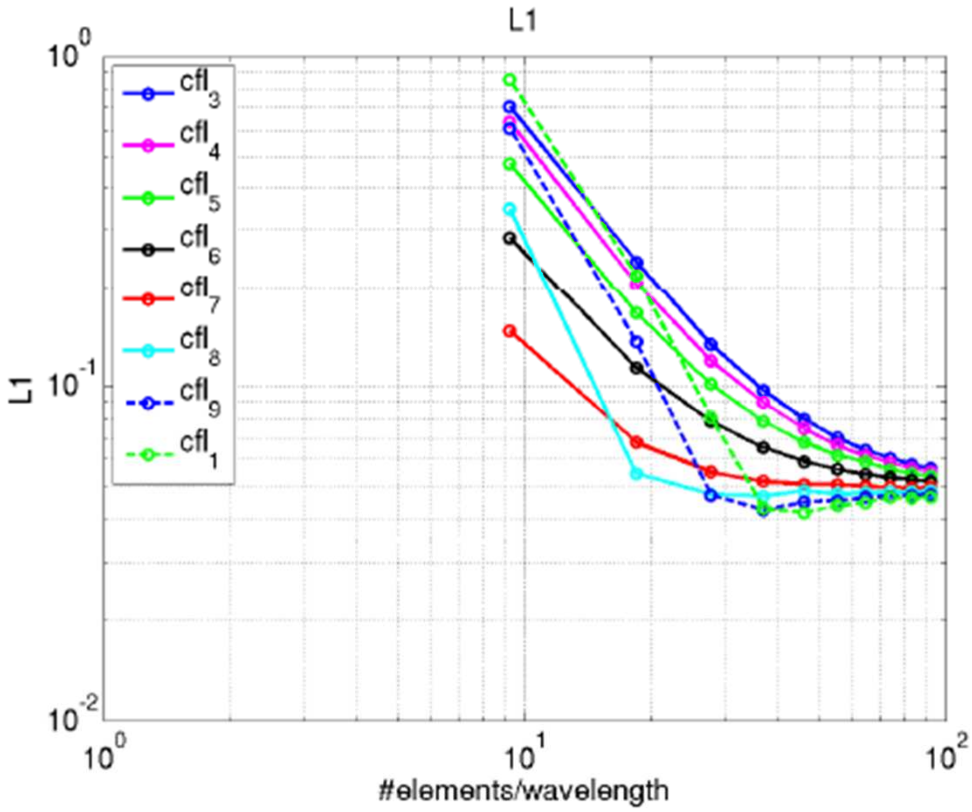


CFL = 0.3
50 Radial Intervals

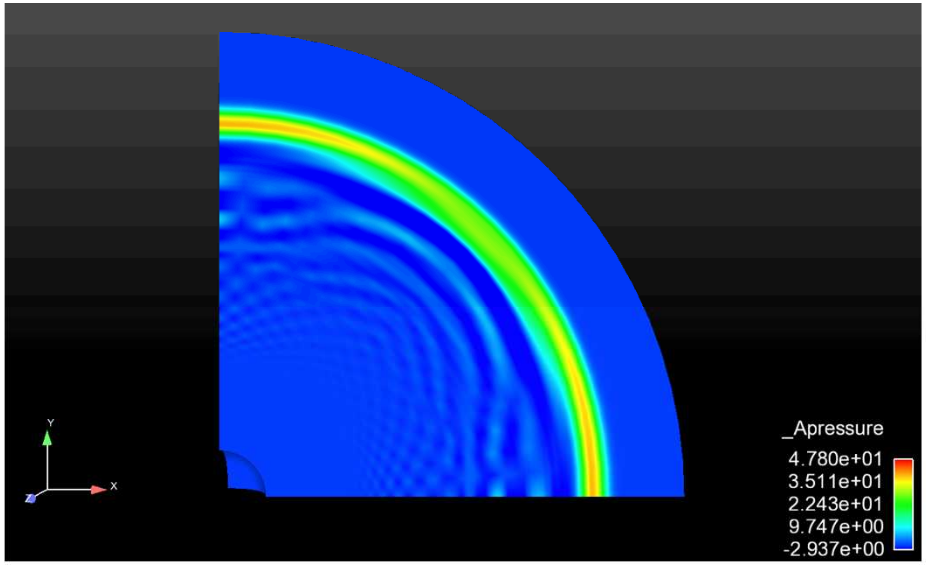
CFL = 0.3
1000 Radial Intervals

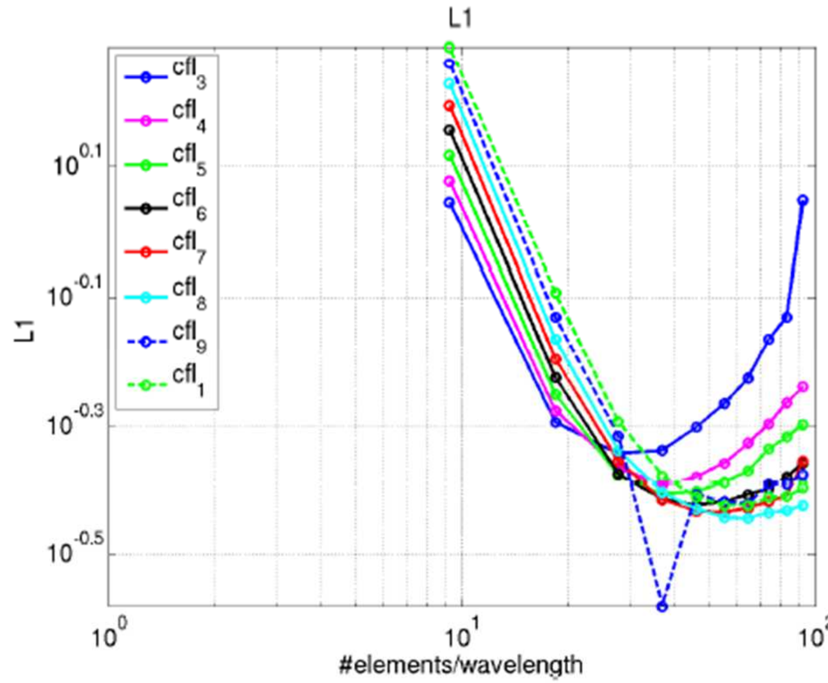
3D Results

- L1 error for hex8 elements using a consistent mass matrix
- Optimal CFL of 0.7
- Shows convergence



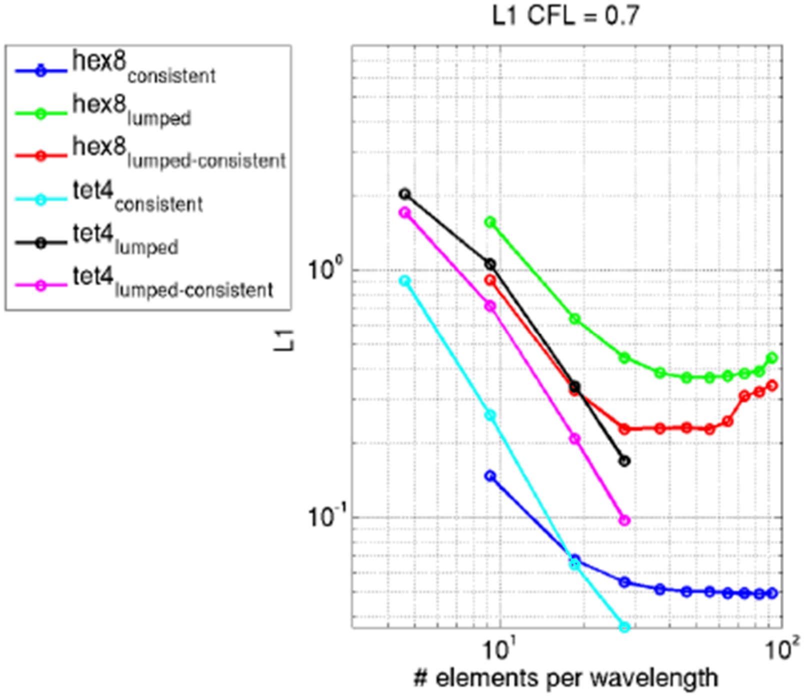
3D Results

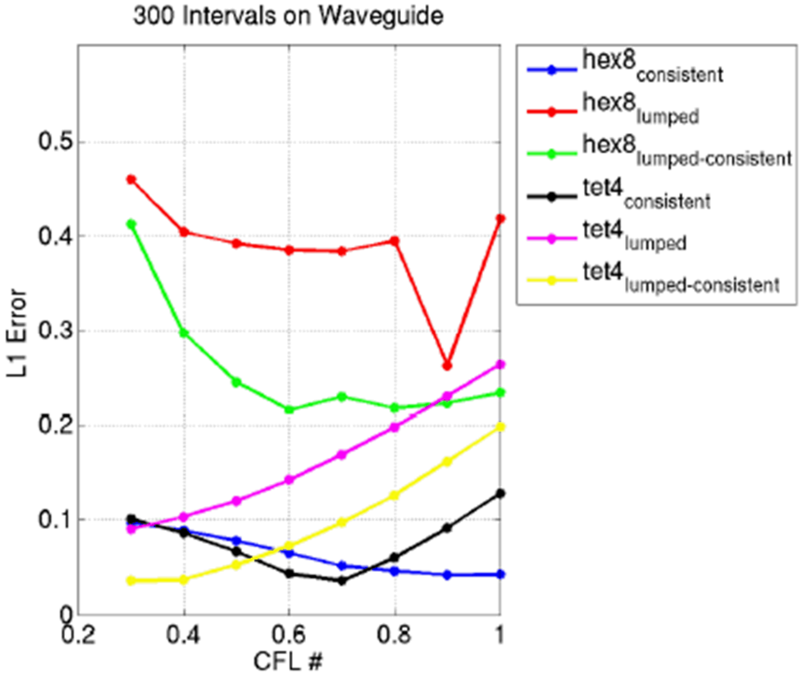




When using a lumped mass matrix, a disturbance along the edges occurred, so a full sphere was used

3D Results





- Hex8 Lumped/Lumped-Consistent show highest error