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ESP700 Pre-Lecture 3: Crash Course on Probability & Decision Theory, and Dakota

Org 1544: V&V/UQ and Credibility Processes

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3 April, 2014

U.S. DEPARTMENT OF ENERGY

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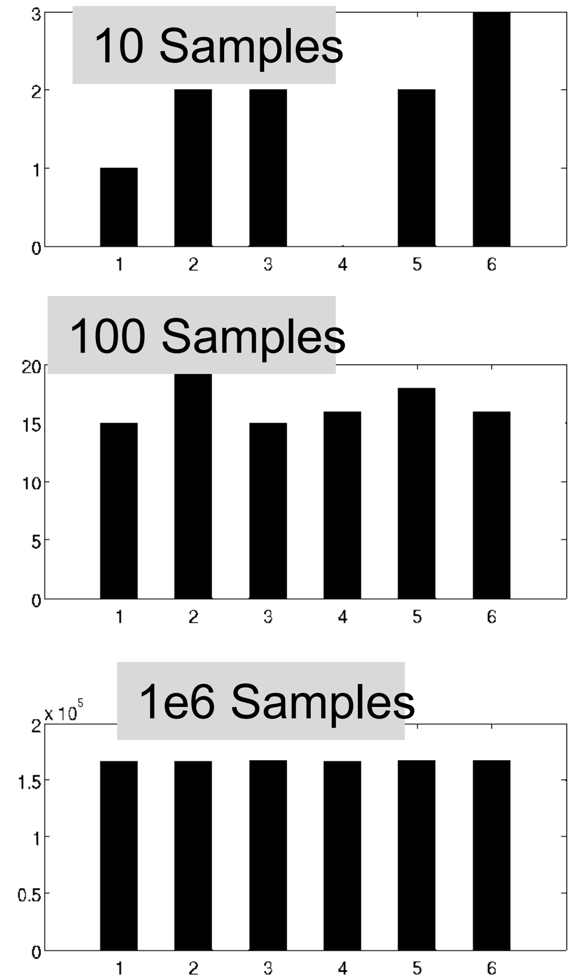
Probability & Statistics

- Statistics = analysis of past events
  - **Observe** outcomes of events
  - **Build** & test statistical model **to explain outcomes**
- Probability = prediction of future events
  - **Assume** probability model
    - Often based on a statistical model
  - **Predict** outcomes **to make decisions**

Statistics question: Is the Die fair?

- Observe data, find correlations
  → Hypothesize causation
- Statistics issues:
  - Inputs & Responses/ QoI:
    Shooter, face number before throw
    Number of tumbles, face after throw
  - Quantity & Quality of data
- Not a focus of this course



Histograms show frequencies of each *outcome* for the *response/QoI*: face number for a fair die

Probability Question: What is the next roll?

- Cannot be answered deterministically
- Identify **ALL possible outcomes** for the QoI
- Assume a model:

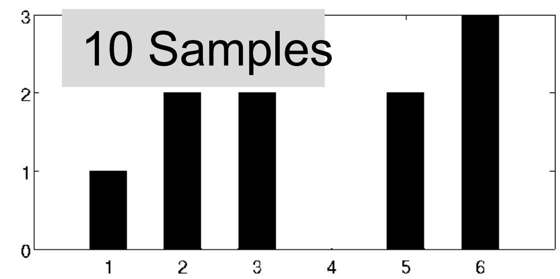
  Probability of occurrence of each outcome

→ Defines a probability distribution

- Use model to make predictions and decisions

Known unknowns, unknown unknowns?

- 10 rolls of a die
  - What is the statistical model?



- Probability: what is the probability model?

$$P(face = 4) = 0\,?$$

- Inherent weakness of models
  - They include only what you put in
  - “Garbage in, garbage out”
  - “Models are wrong, some are useful”

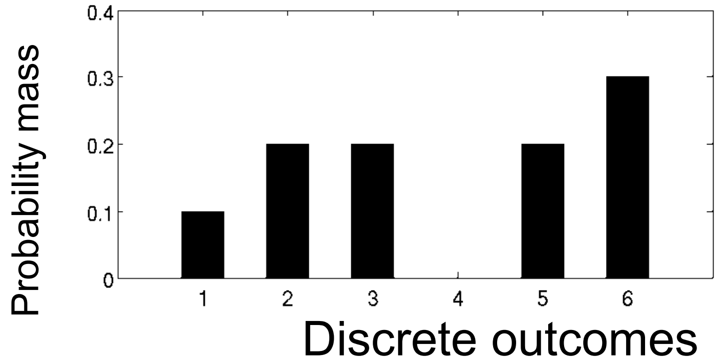
A bit of jargon

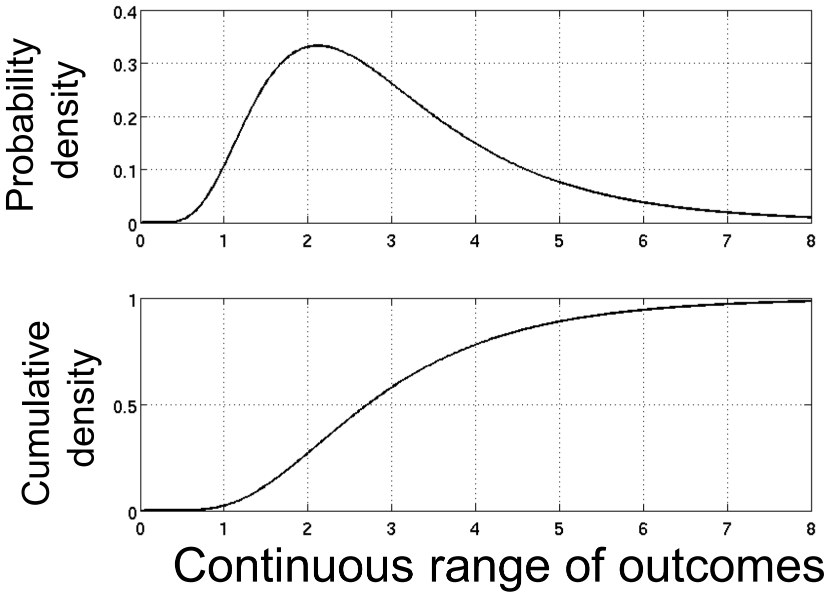
- (Real valued) Random Variable
  - A function that associates *outcomes* with *probabilities* of their occurrence
- Discrete:

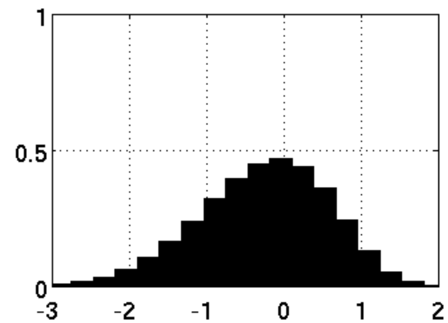
  Coin flip – *outcome* is heads, P(heads) = 1/2
- Continuous:

  Failure – *outcome* is that material will fail if subjected to impact at 1km/s, prob = ½
- Random variable ←→ probability distribution

Probability distributions



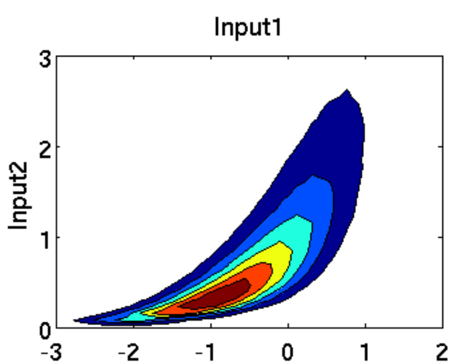


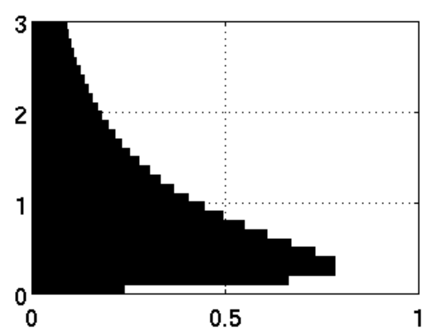


2+ dimensions
project to 1D and 2D
→ See 2D correlation

Ways to summarize a distribution:

Mean, mode, median, standard deviation, etc.





Commonly used terms

- Statistics: based on data – N datapoints
  - Sample Mean $\bar{\mathbf{x}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i$
  - Sample Variance $s_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2$
  - Variance of the mean $\mathrm{var}\left(\bar{x}\right) = \frac{s_x^{\,2}}{N}$

Estimates from data can have error

Cannot calculate variance from 1 datapoint

More samples → better estimates

- Correlation
  - Two random variables are correlated if the outcome of one changes the probabilities of the second
  - Example: Number of gate guards & Wait times at gate

Uncertainty, Probability, Decisions

- What is uncertainty? Lack of information
  - Dice example
- Mathematically describe the amount of available info
- Probability distributions are ONE way to model information and uncertainty
  - It’s JUST A MODEL
- Decision theory: formal way to choose actions when the outcomes are uncertain, using probability theory

Decision Theory

Decision = select a action

Enumerate actions

1. Actions have costs

Estimate costs & benefits

2. Actions impact the possible outcomes

Use Models

3. Outcomes have benefits
4. Value = benefit – cost

Possibility that value < desired
→ **risk**

Pick the action to maximize the predicted value, based on which outcomes you expect to happen

Probability Question: What is the next roll?

Engineering question:
Can you make money from this?

- How would you find out?
- Do these theories actually help?
  - Design of Experiments
  - Statistics
  - Probability
  - Sensitivity analysis
  - Uncertainty quantification
  - Decision theory

Die Game

- Pay $, guess the face number & win!
- Have Sandia's $$$$ to spend
- Based on available knowledge:
  - Should you play?
  - Which number to pick?
  - How much to bet?
- What is the expected value? Risk?

Layers of Decisions

- Gather more info to improve the decision?
  - **This is ALSO a decision problem**
- Examples:
  - Consider additional actions
  - Characterize the die (improve models)
  - Understand the payout (understand models)
  - See if model predictions match gameplay (validate)
- Is the added benefit worth the additional cost??

If you don't know how the top decision is made, you will not make the best sub-decisions

Questions?

- Probability Theory – Introduction to Probability (Bertsekas)
- Decision Theory – Making Hard Decisions (Robert T. Clemen)

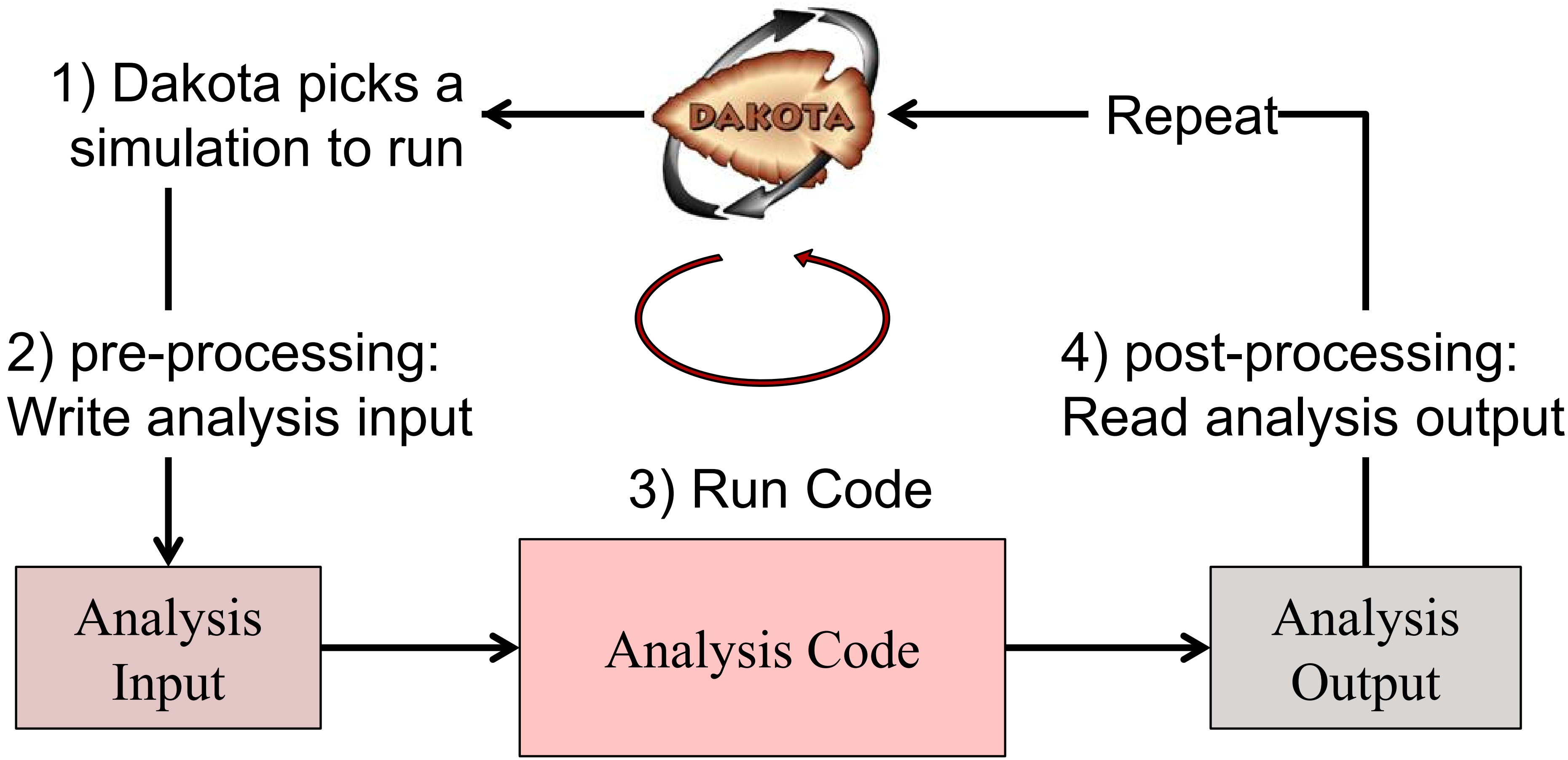
What is Dakota?

- **Automation** of model runs
  - Make it cheap to run simulations (user time, not CPU hrs)

+

- **Methods**
  - Make the runs more useful
  - Uncertainty Quantification, Surrogate Models, Optimization, Design of Experiments, Parameter Study, Sensitivity Analysis

How does it work?



Look familiar? Many analysts do this manually.
Many similar codes exist.

Comparisons

Manual

- Repetitive process
  - Costs add up
- Can make mistakes, lose track of data, simulations, etc.
- No scripts

Automation

- Requires scripting
  - Larger up-front cost
  - Minimal additional cost
- Fewer errors
- Gain access to powerful methods

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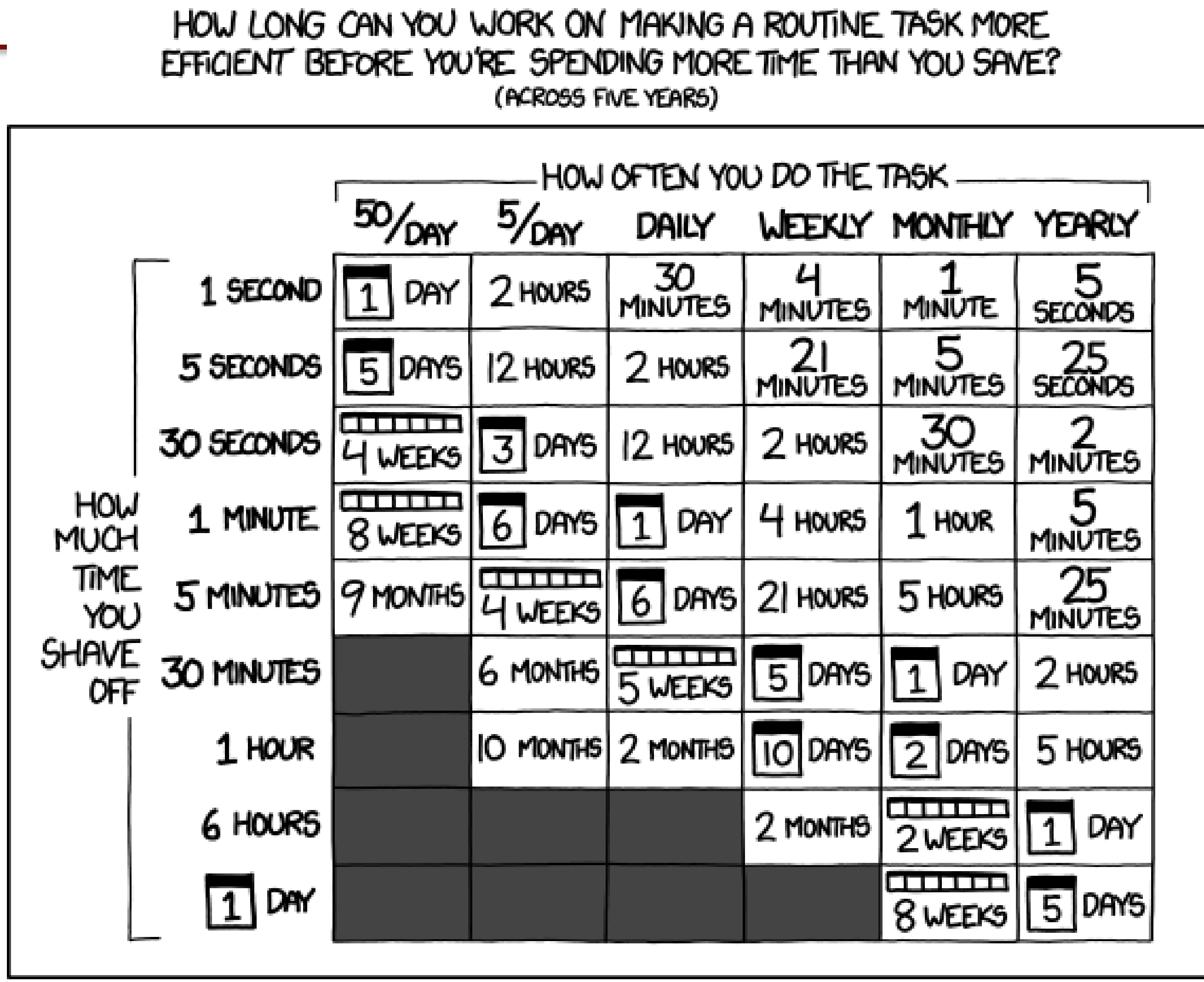

HOW LONG CAN YOU WORK ON MAKING A ROUTINE TASK MORE EFFICIENT BEFORE YOU'RE SPENDING MORE TIME THAN YOU SAVE?
(ACROSS FIVE YEARS)

HOW MUCH TIME YOU SHAVE OFF / HOW OFTEN YOU DO THE TASK	50/DAY	5/DAY	DAILY	WEEKLY	MONTHLY	YEARLY
1 SECOND	1 DAY	2 HOURS	30 MINUTES	4 MINUTES	1 MINUTE	5 SECONDS
5 SECONDS	5 DAYS	12 HOURS	2 HOURS	21 MINUTES	5 MINUTES	25 SECONDS
30 SECONDS	4 WEEKS	3 DAYS	12 HOURS	2 HOURS	30 MINUTES	2 MINUTES
1 MINUTE	8 WEEKS	6 DAYS	1 DAY	4 HOURS	1 HOUR	5 MINUTES
5 MINUTES	9 MONTHS	4 WEEKS	6 DAYS	21 HOURS	5 HOURS	25 MINUTES
30 MINUTES		6 MONTHS	5 WEEKS	5 DAYS	1 DAY	2 HOURS
1 HOUR		10 MONTHS	2 MONTHS	10 DAYS	2 DAYS	5 HOURS
6 HOURS				2 MONTHS	2 WEEKS	1 DAY
1 DAY					8 WEEKS	5 DAYS

Good news – Dakota team can help

Tools in ESP700

- See Dakota in action
  - Demonstrate capabilities/ methods
  - Simple workflow
- Learn about available resources
- What you won't see
  - Dakota tutorial
  - Scripting details
  - Complicated data analysis
- Today's examples also use Matlab
- Similar tools:
  - JMP
  - Minitab
  - R
  - Octave
  - Excel
  - Etc…

Questions?

- Dakota resources:
  - dakota.sandia.gov
  - Dakota Product Manager for 1500: Ken Hu
  - Dakota support: Adam Stephens

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ESP700 Lecture 3: Methods and Tools for Sensitivity Analysis

Org 1544: V&V/UQ and Credibility Processes

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Models are wrong, but they can be useful

Questions to ask about the model:

What parts do we need to understand?

- Sensitivity analysis: which inputs affect the response?

How well do we know the response value?

- UQ: how do uncertainties in inputs affect the response?

Do we know enough? ARE the models useful?

- V&V → how accurate / wrong is the response?

What are the costs and benefits? VALUE?

Outline

- A day in the life of a 1544 analyst
- Examples
- Sensitivity analysis
- Uncertainty quantification
- Surrogate models
- Advanced methods

Basic methods, demonstrations, interpretation, value proposition

Lecture 3 Goals

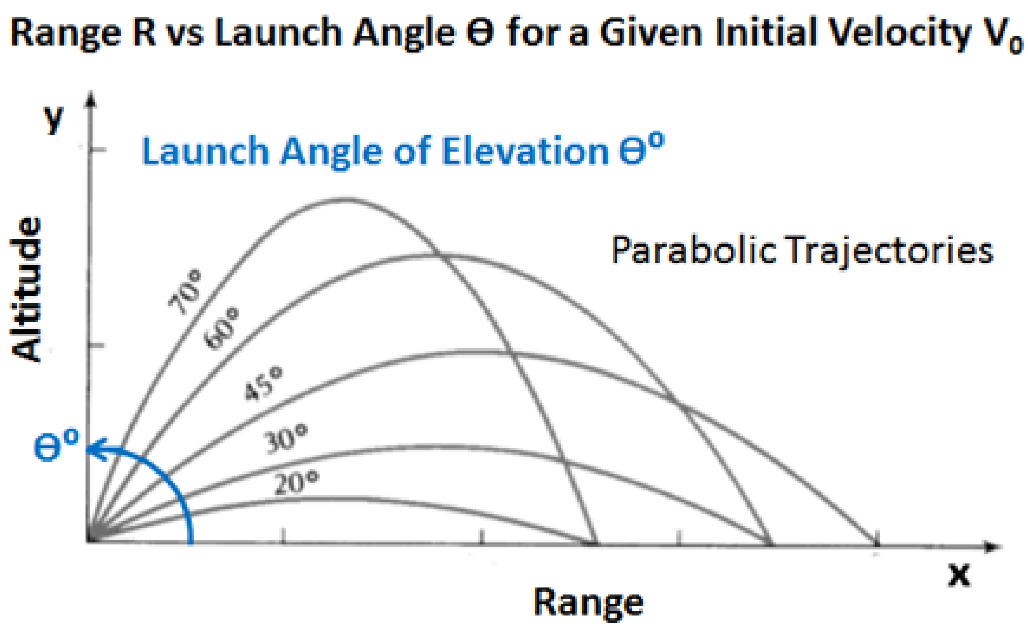
- Introduce topics at a high level
- Describe the basic methods
- Promote Dakota usage
- Demonstrate methods and tools
  - Simple example – compute ballistic trajectory
  - Case study – 3 leg structural dynamics problem
- **Leave with a basic knowledge of methods, tools, context – there is much more**

What does 1544 do?

- Project work
  - Supporting other analysts (1500 and others)
  - Sensitivity analysis, UQ, V&V, optimization
  - Tailoring methods for each project
  - Interpreting results
- Methods research & development
- Creating resources/ tools

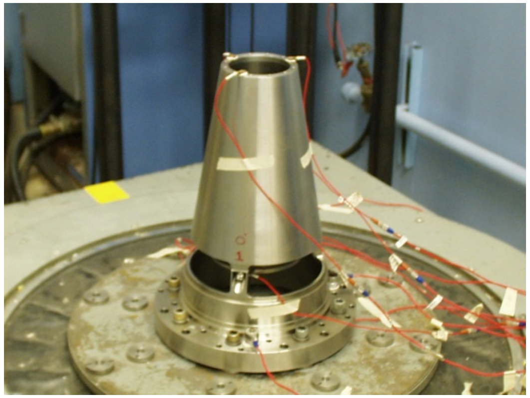
→ Enable other analysts to do what we do

Example: Ballistic Trajectory

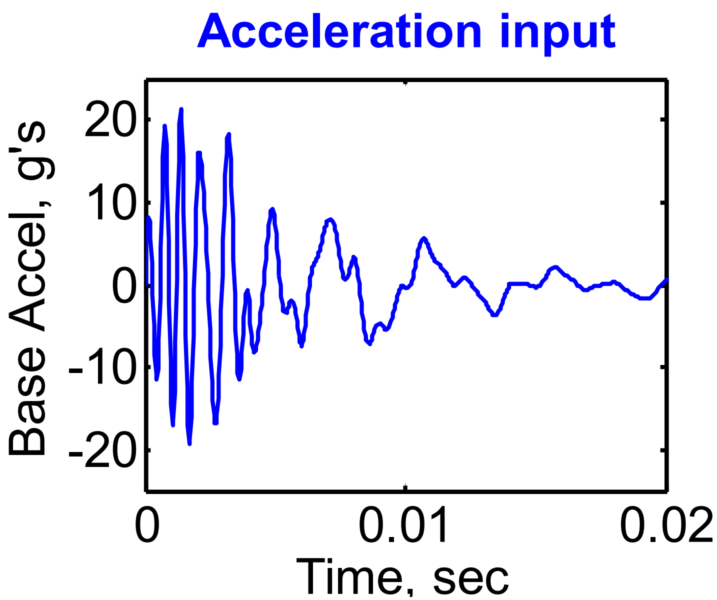


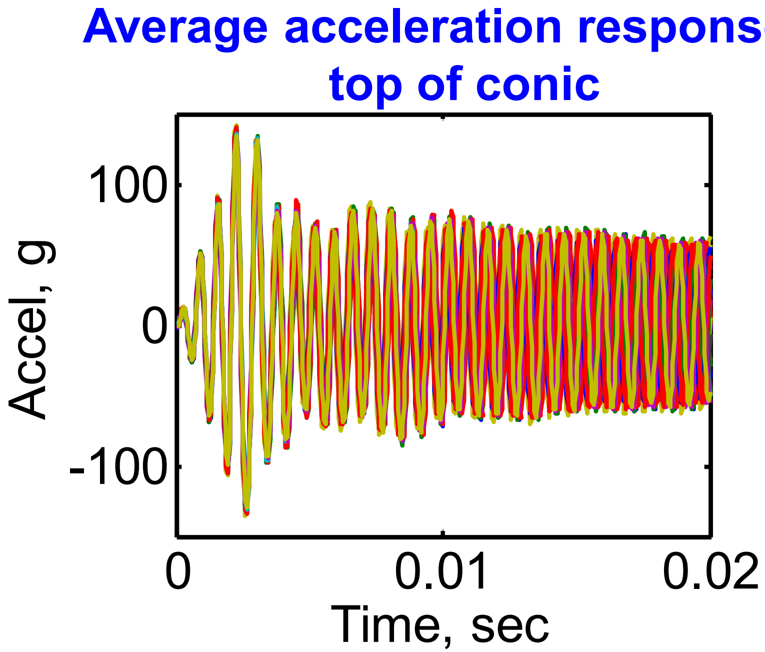
- Inputs
  - Angle, θ
  - Initial Velocity, V_0
  - Gravity
- Quantities of Interest
  - Max Height, H
  - Range, R

Case Study: 3leg model

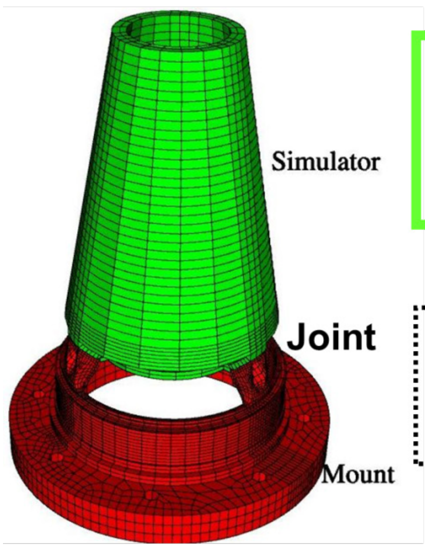


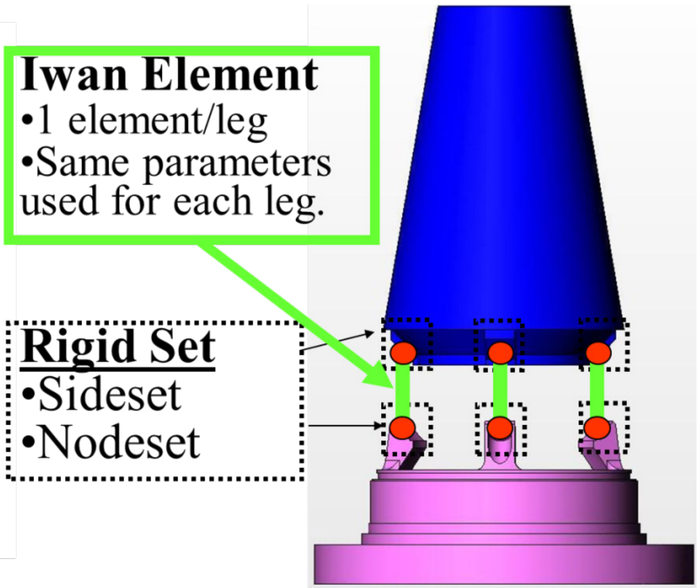
- Hardware consists of 3 top conic sections and 3 bottom sections
- 9 total combinations of top/bottoms

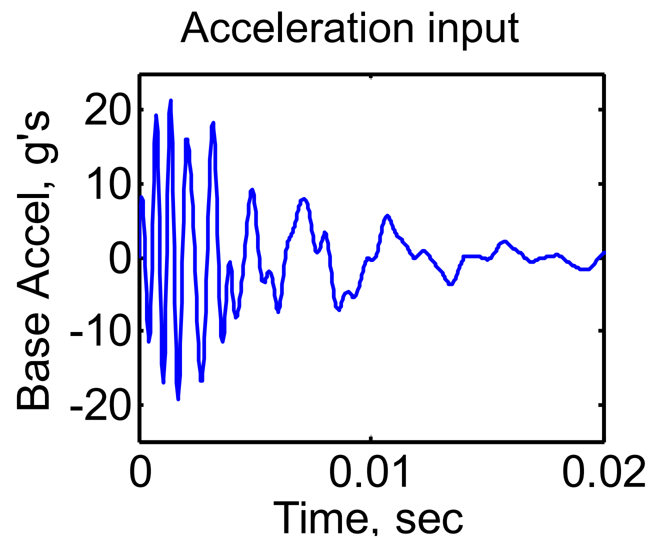


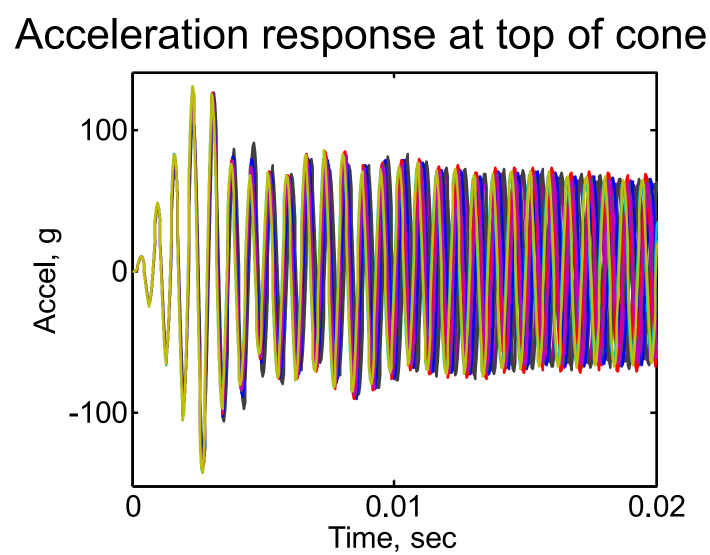


Case Study: 3leg model



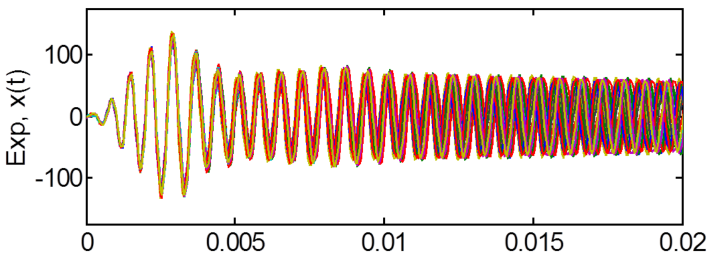


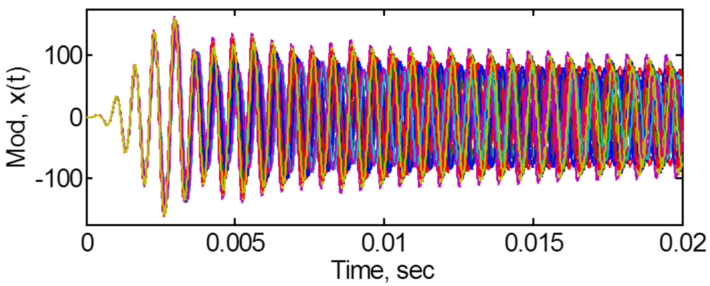


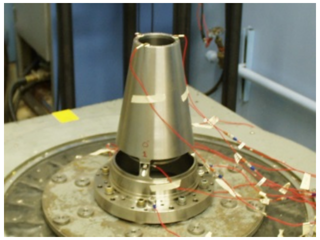


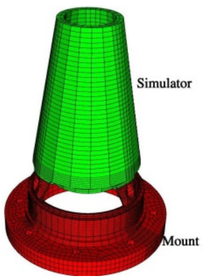
- 3D finite element model representing 3 leg hardware was created
- Bolted joints (J) are modeled using an Iwan element
- Non linear transient analysis was performed using Sierra-SD (structural dynamics)

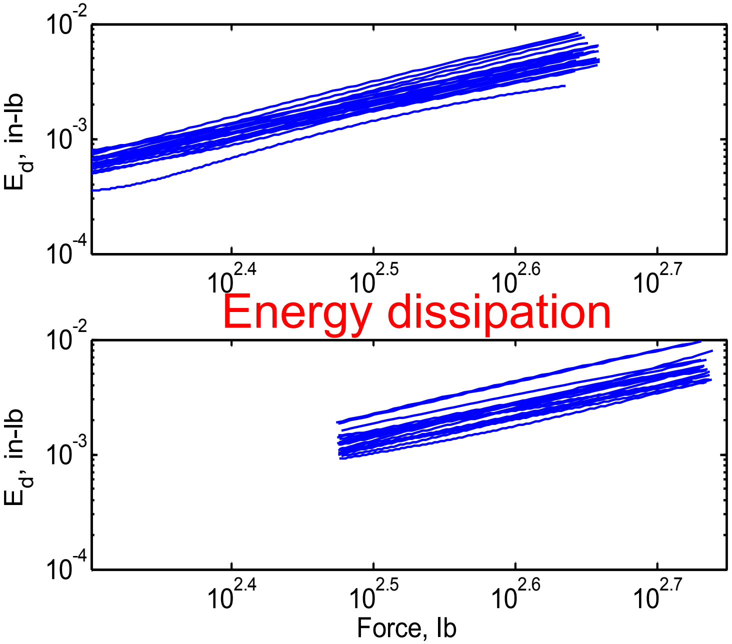
Quantities of interest



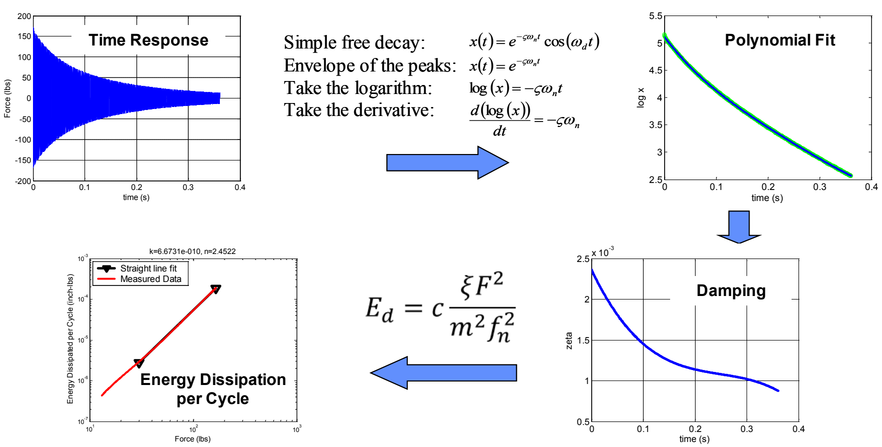








Energy dissipation per cycle from transient responses



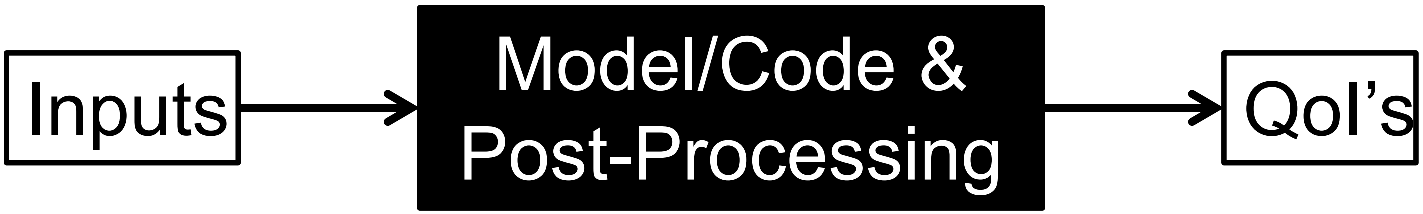
Slope
Peak Force
Max Acceleration

3leg Simulations

- Sierra SD, a.k.a Salinas
- Ran on CEE platforms, 8cores, ~20 min
- Salinas results file
- Dakota “drives” the simulations
- Automation
  - Bash scripting, Linux utilities
  - Matlab post-processing

Abstract View of Simulation

- Think of models / code as a black box
  - Inputs go in, QoI's come out



- Input = ANYTHING that changes the QoI's
  - Model parameter, code setting (solvers, tolerances)
  - Boundary conditions, external forcing, etc.
  - mesh, geometry
  - model form a.k.a model structure
  - Computational hardware

Sensitivity Analysis

- How do changes to inputs affect the response?
  - How "sensitive" is the response to each input?
  - Direction and magnitude
- Typically focus on model parameters OR other inputs
  - **Today – focus on quantitative inputs (parameters)**
  - Lecture 2: verification – Sensitivity analysis and uncertainty quantification for meshes/ codes

Example 1: Ballistic Trajectory

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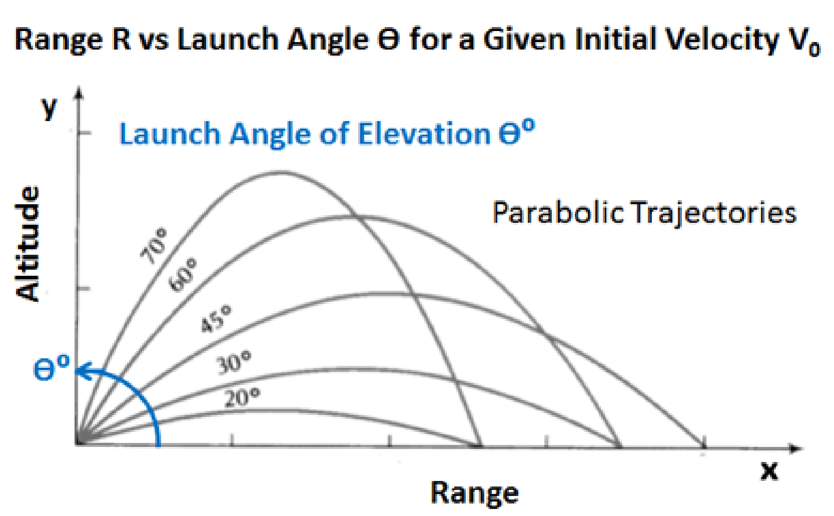


- Qualitative Sensitivity, a.k.a. "Expert Opinion"
  - V_0↑ → Height ↑ & Range ↑
  - Gravity ↑ → Height ↓ & Range ↓
  - θ↑ → Height ↑ & Range ??
  - V_0↑, θ↓ → Height ?? & Range ??

Is this enough information?

Why Do Sensitivity Analysis?

- Identify trends in responses – exploration
  - Bonus information: smoothness, robustness
- Provide a focus for future work
  - Model development
  - New experiments
  - Characterization of input uncertainty & UQ

Goal: spend resources to understand the **significant inputs** for the **important responses**

Quantitative Sensitivity Analysis

Summary

1. Vary the inputs
2. Run the model
3. See if QoI's change, compute metrics

- More samples → more information
- Methods – efficiently compute metrics
  - Efficiently gather information

Basic Quantitative Sensitivity Analysis

- Local sensitivity:
  - Metric: Partial derivative
  - Method: Finite differences
- Must pick nominal point and step sizes
- Example: Ballistic Trajectory
  - Nominal $\theta = 45^0, V_0 = 10m/s, g = 10m/s^2$
  - Step size = +10%
  - Results:

Range $$\left.\frac{\partial R}{\partial \theta}, \frac{\partial R}{\partial V_0}, \frac{\partial R}{\partial g}\right|_{nom} = [-0.00 \;\; 0.21 \;\; -0.09]$$

Height $$\left.\frac{\partial H}{\partial \theta}, \frac{\partial H}{\partial V_0}, \frac{\partial H}{\partial g}\right|_{nom} = [0.04 \;\; 0.21 \;\; -0.09]$$

Lessons

- Sensitivities: $V_0 > g > \theta$
  - On Earth: how much does gravity vary?
  - **Most sensitive ≠ most significant**
  - Must consider the possible range of values
- Compare sensitivity of height and range
  - Depends on the QoI
- Repeat at: $\theta = 1^0, V_0 = 10m/s, g = 10m/s^2$
  - Sensitivities: $\theta > V_0 > g$
  - Nominal value matters

Cheap – bare minimum of model evaluations

Limited – Estimate of main effects, no interactions

Case Study: 3leg

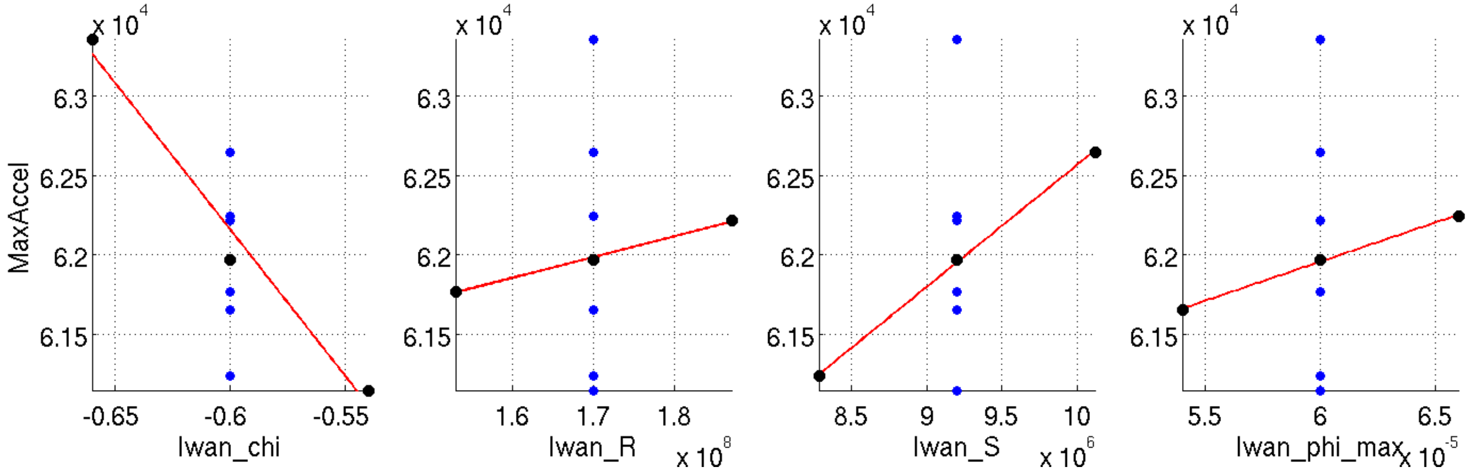
- Local sensitivity
  - Pick nominal values for 4 Iwan parameters
  - Pick range & step size = 10%
  - QoI's are most sensitive to Phi_max
- Absolute change in response over whole range
  - chi and S are most significant

Forward differences :	'chi'	'R'	'S'	'phi_max'
MaxAccelSensitivity	-2e-2	2e-10	1e-8	8e2
MaxAccelDiffOverRange	1400	250	680	280

'chi' >> 'S' >> 'phi_max' ≈ 'R'

"Projection" Plots

- 5D space – 4 parameters, 1 response
- Project onto 2D – collapse 4 parameters into 1D



Central differences

- Visualize change over ranges (2x step size)
- Absolute changes of QoI are an indicator of significance
- Limited # of runs → receive minimal information

Basic Method 2 – Sampling “LHS”

1. Define ranges for each input
2. Sample uniformly within the ranges for all inputs
3. Run model
4. Analysis: plotting + correlation coefficients

Matlab demo – Latin Hypercube Sampling

- **Concept** – local sensitivity at many nominal values
  - Average the sensitivities “globally” over parameter space
  - Need a lot of samples

No connotations of probability

Qualitative LHS Analysis

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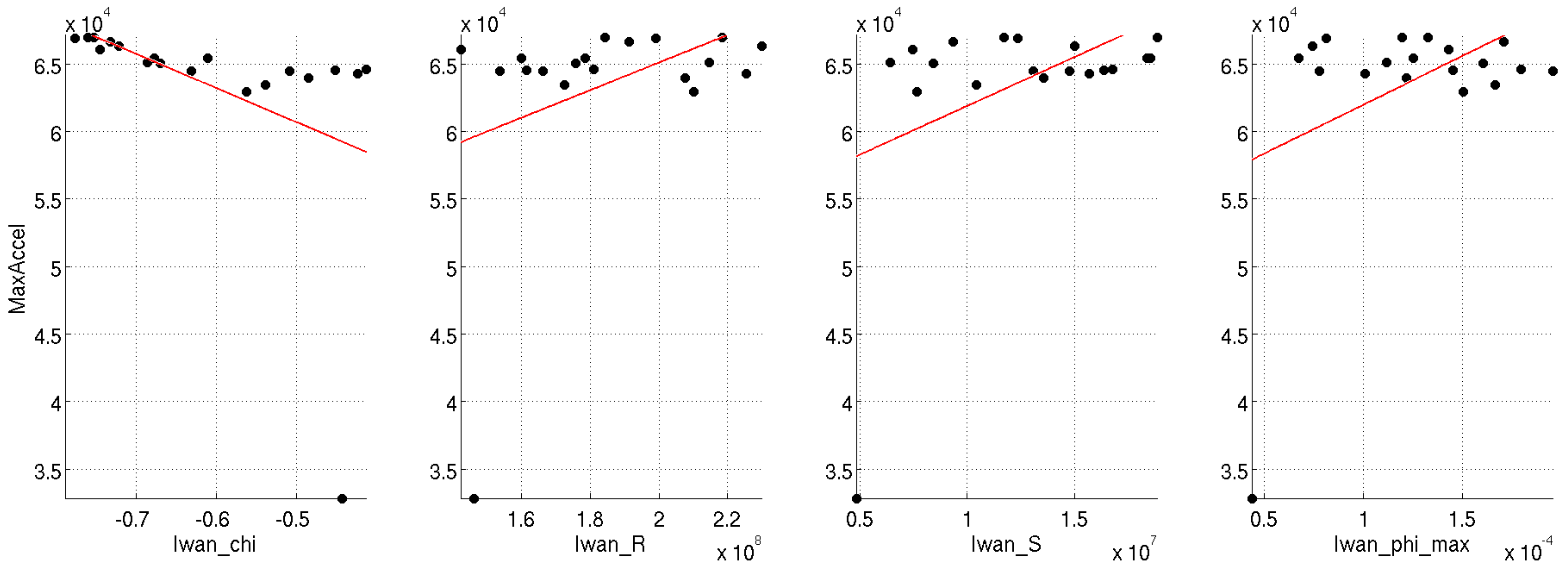
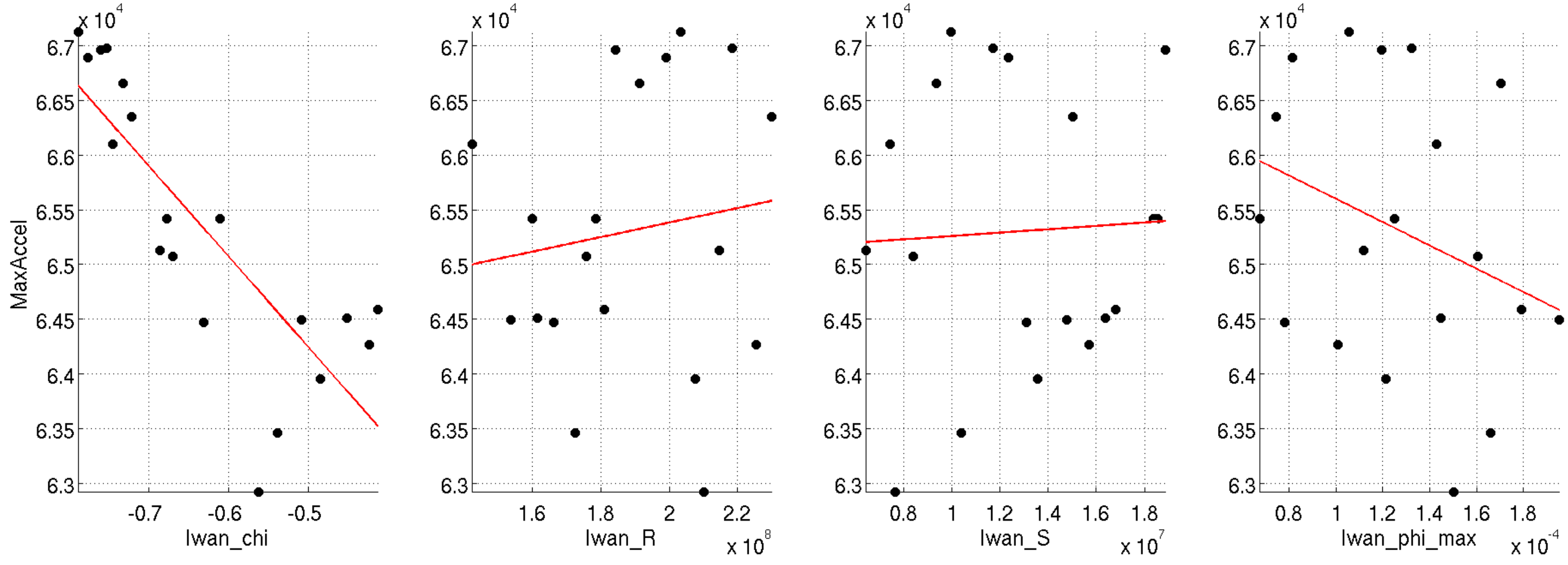
- Projection, scatterplot see nonlinear trends
- Compare vertical spread vs. trendline
  - Qualitative indicator of significance



Local Sensitivity Result:
'chi' >> 'S' >> 'R' > 'phi_max'

Qualitative LHS Analysis

- Identify outliers w/ major effect on quantitative indicators
- Investigate discrepancy between visualizations and quantitative results



Local Sensitivity Result:
'chi' >> 'S' >> 'R' > 'phi_max'

Quantitative LHS Analysis

Simple Correlation Matrix

	MaxAccel
chi	-0.43
R	0.25
S	0.52
phi_max	0.45

Partial Correlation Matrix

	MaxAccel	slope
chi	-0.56	0.46
R	0.18	0.34
S	0.64	0.25
phi_max	0.57	-0.21

- Correlation Coefficients
- Regression slope ∝ simple correlation
- Linear assumptions!
- Compare simple vs. partial
  - Difference indicates significance of interactions between inputs

Show Dakota Output –
most data analysis software will compute these also

Quantitative LHS Analysis

- Simple correlation:
  measures the strength and direction of a linear relationship between variables
- Partial correlation:
  like simple correlation but adjusts for the effects of the other variables
- Rank correlations:
  simple and partial correlations performed on “rank” of data

Correlation coefficients have range [-1 , 1]

0, no relationship

1, strong positive relationship

-1, strong negative relationship

List Sensitivity Methods

- Expert Opinion
- Local methods
  - **Finite differences**
  - Design of Computer Experiments
- Global methods
  - Morris one at a time
  - Sampling
    - Monte Carlo, Quasi-Monte Carlo
    - **Latin Hypercube Sampling (LHS)**

Partial derivatives
ANOVA

Correlation coefficients
Variance Based Decomposition
Sobol indices

Costs & benefits: you get what you pay for

- For N parameters
- Local sensitivities
  - Finite differences: N+1, 2N+1 model runs
  - Local estimates, no interactions
- Design of Computer Experiments
  - Full Factorial: 3^N (grows FAST) Curse of Dimensionality
  - Other special designs → reduced cost, need to think
  - Use ANOVA – get “main effects” a.k.a sensitivity in each dimension, plus sensitivities for 2D interactions

What about sampling

Local Sensitivities

- Known cost
- Very easy to implement
- Limited information
  - Local analysis

Sampling

- Scales better with dimensions, N > 4
- Global, nonlinear effects
- Benefit is hard to predict

Warnings on Sampling

- Quality of statistics
  - Known convergence of statistics
  - Absolute accuracy of statistics is NOT KNOWN a priori
- Do not know whether the qualitative or quantitative results will help to downselect parameters

Some Guidance

- Which method?
  - Trade off simplicity and cost vs. amount of information
  - How much info do you need for sensitivity analysis?
  - **Why choose just one?**
  - Start w/ cheap local sensitivity method, add LHS
- How many LHS samples?
  - **Rule of thumb**: 10*N → get trends, mean, variance
- Use incremental studies – N, 2N, 4N, 8N, 16N …
  - Can predict computational cost
  - When benefit stops increasing, stop analysis

3leg Sampling

- Ran 10, 20, 40 LHS samples
- See if metrics change

Simple Correlation Matrix MaxAccel			
# samples	10	20	40
chi	-0.33	-0.43	-0.46
R	-0.14	0.25	0.17
S	0.62	0.52	0.49
phi_max	0.39	0.45	0.39

Partial Correlation Matrix MaxAccel			
# samples	10	20	40
chi	-0.68	-0.56	-0.59
R	-0.59	0.18	0.15
S	0.78	0.64	0.61
phi_max	0.62	0.57	0.52

- Outlier has BIG effect w/ only 10 samples
- Check against local sensitivity result
- Check assumptions: are parameter ranges sensible?

Sensitivity Analysis Conclusion

- How many samples are needed to assess significance?
- **Recall goal: learn model, prioritize future analysis**
- Risks – will this impact the project, decision?
  - Miss significant parameters
  - Run future analysis on the wrong parameters
  - Future analyses is too expensive
- But remember…
  - This is exploratory work
  - Don’t spend too much time/effort

Time Permitting

- Sensitivity Analysis workflow
  - Automation of Salinas runs
  - Post-processing in Matlab
  - Dakota study
- Advanced Methods
  - Morris one at a time
  - Variance based decomposition w/ Sobol indices

Questions?

- "Sensitivity Analysis in Practice A Guide to Assessing Scientific Models" by Saltelli, A. and Tarantola, Stefano and Campolongo, Francesca and Ratto, Marco. John Wiley & Sons, Chichester. 2004.

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ESP700 Lecture 3: Methods and Tools for Uncertainty Quantification

Org 1544: V&V/UQ and Credibility Processes

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Models are wrong, but they can be useful

Questions to ask about the model:

What parts do we need to understand?

- Sensitivity analysis: which inputs affect the response?

How well do we know the response value?

- UQ: how do uncertainties in inputs affect the response?

Do we know enough? ARE the models useful?

- V&V → how accurate / wrong is the response?

What are the costs and benefits? VALUE?

Uncertainty Quantification

- What is uncertainty? Lack of information
- Uncertainty quantification = information quantification
  - Have a model, know the significant inputs, etc…
  - How much information do you have about QoI's?
  - What are the significant sources of uncertainty?

1. Characterize the uncertainty in significant inputs
2. Propagate
3. Interpret

Characterization of Uncertainty

- Sources of Uncertainty
  - Model parameter, code setting (solvers), mesh, geometry, model form a.k.a model structure

Focus on parameters

- Types of uncertainty
  - Epistemic and Aleatoric
  - Provide more insight into the information we have

Very confusing!
We'll return to this later

- Quantitative methods

→ parameters require mathematical description

Mathematical Description of Uncertainty

Increasing cost, complexity, information content

- None – deterministic
- Intervals
  Lower Bound |-------------| Upper bound
- Probability distributions
  - Discrete – probability mass function (pmf)
  - Continuous – probability density function (pdf)
  - Uncertainty context
    Higher mass/density → value is more probable
- Fuzzy Probability, P-boxes, Evidence Theory

Sweet spot… (opinion)

Bounds, pdf's… Where do these come from???

- Experimental Data
- Expert Opinion/ Assumptions
- Theory/ Models

Statistics: Construct models from data and assumptions

Propagation of Uncertainty

- Sampling methods – Latin Hypercube Sampling (LHS)
  - Exact same as for sensitivity analysis
  - Difference – uncertainty context
  - Based on characterization of parameter uncertainty

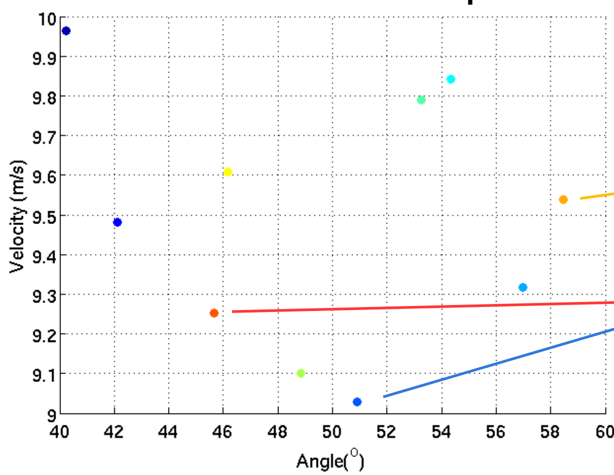
- MOST other methods can be formulated as
  - Construct a surrogate model
  - Sample the surrogate model
  - Discussed in Advanced topics

Example: Ballistic Trajectory

- If we assume θ, V_0 are uncertain → pdf's
- Then what do we know about QoI: Range?

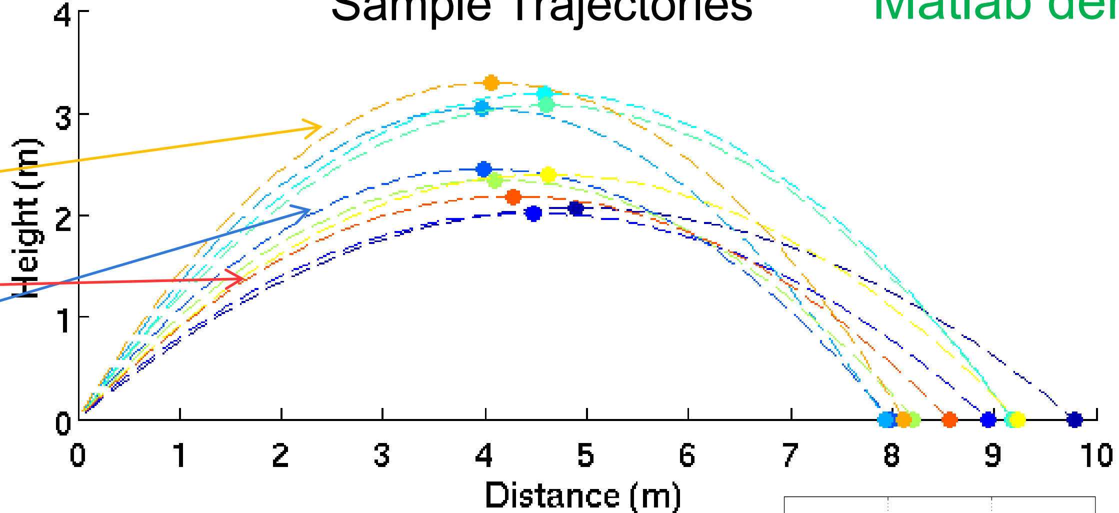
SA → neglect gravity

Parameter Samples



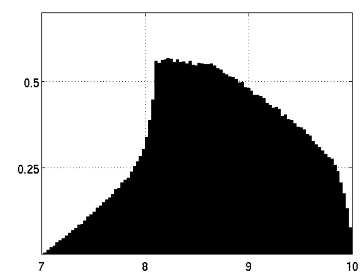
Sample Trajectories

Matlab demo



Uniform parameter pdf's

UQ result: Normalized histogram of predicted QoI:Range based on (a LOT of) parameter sampling



Non-uniform QoI pdf!

Types of Uncertainty – Definitions

- Epistemic (Reducible uncertainty)
  - Lack of knowledge about the appropriate value to use
  - Reduced through increased understanding or more data.
- Aleatoric (Irreducible uncertainty)
  - Cannot be reduced by further data
  - Variability (due to part-to-part, test-to-test variation, etc.)

Most parameters in engineering models have both aleatoric / epistemic components of uncertainty

Importance

- Epistemic vs. aleatoric distinction is subtle
- **What is the model attempting to predict?**
  - Ex: modeling a validation experiment

1. Response of a specific unit to a specific event?
2. Possible responses from a population of units, and population of events *consistent* with a scenario?

- **What do we expect to match?**
  - Only aleatoric uncertainties should match

Ballistic Trajectory Example

- Angle is determined by launcher and base
  - Launch tube creates shot-to-shot variability
  - Base is not always on level ground
- What is aleatoric vs. epistemic?
- What are we attempting to predict?

Matlab demo

To estimate WHERE the shot will hit, we don't need to decompose into aleatoric and epistemic nature of uncertainty.

The distinction provides additional insight into the *quality* of the predictions → this is important for decisions

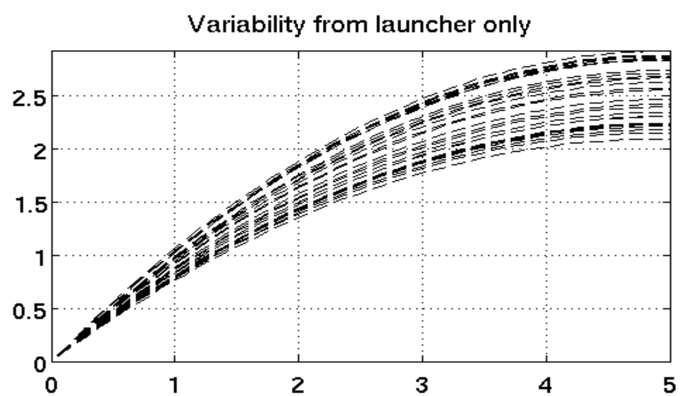
Example: Ballistic Trajectory

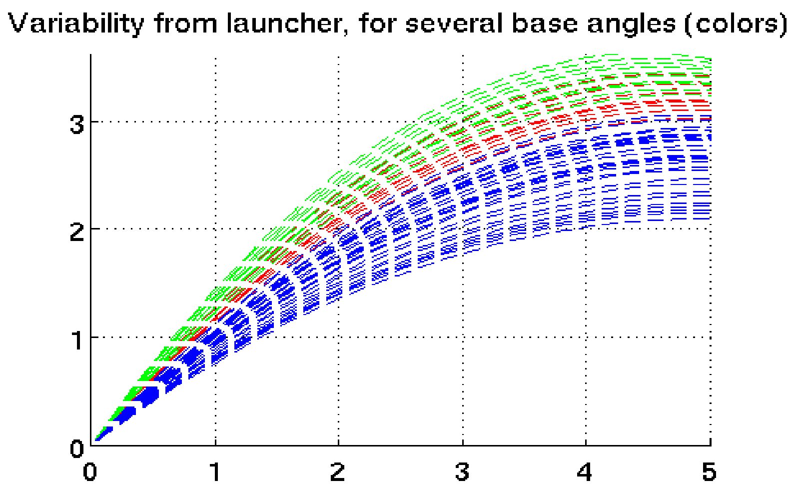
Base is fixed but unmeasured, velocity is known

Scenarios:

1. Predict range for next shot
2. Predict ranges for next dozen shots
3. Observe shots, validate model – Is our understanding of physics & uncertainty consistent w/ observed data?

Q: What is aleatoric vs. epistemic? Is the separation useful? How to use this information?



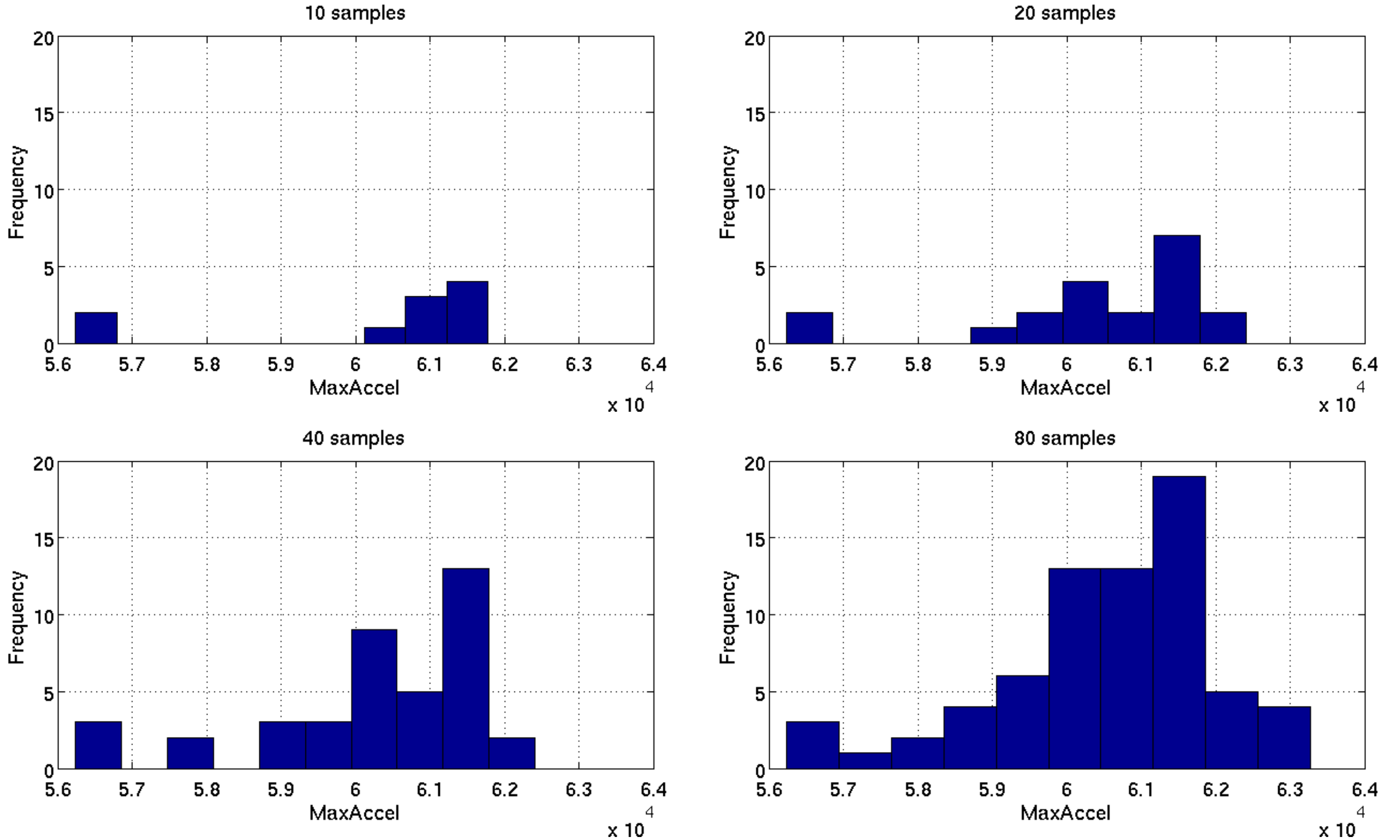


Example: 3leg Characterization

- Experimental data – 20 tests on different joints
- Iwan Model: Calibrate 4 model parameters to each
  - Result – 20 "best estimates"
  - Represents variability of joint behaviors
  - Aleatoric uncertainty
- Generalize this small set of data to a 4D joint pdf
  - Make assumptions → find a pdf that is (mostly) consistent with data
  - Example – Multivariate Gaussian, Karhunen-Loeve Expansions
- Also have epistemic uncertainty with the parameters
  - Related to assumptions
  - Related to imperfect calibration
  - Related to model form uncertainty, experimental uncertainty
  - Advanced topic – we will ignore this for now…

Example: 3leg

- 20 sets of best estimates for 4 parameters
  - Assume Gaussian distributions w/ correlations
- Propagate w/ incremental LHS: 10, 20, 40, 80 samples



Samples	Mean	Std Dev
10	60297	2131.5
20	60488	1700.4
40	60365	1558.6
80	60589	1496.3

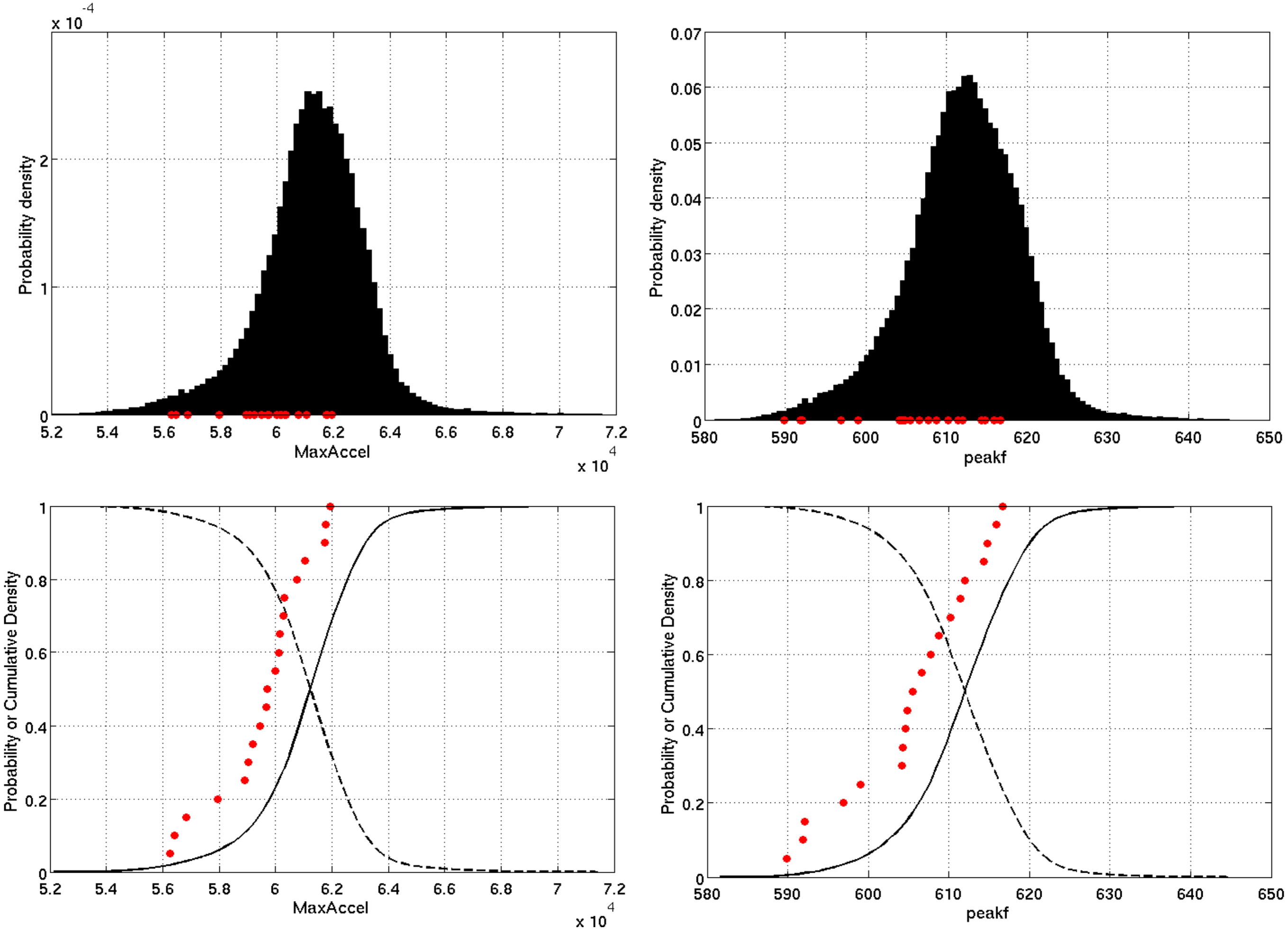
Higher moments need more samples to converge

Interpret UQ: present and assess results

- Sampling → histogram
  - With many LHS samples, normalized histogram → pdf
  - CDF – cumulative distribution function, “integrated pdf”
  - CCDF – complementary CDF
  - Many other ways to present information
- Statistics: mean, median, variance, percentiles
- Layers of information!
  - Inputs → Model → Quantity of interest
  - UQ → uncertainty / information quantification on QoI
  - Quality of UQ – convergence, data analysis on uncertainty

Example: 3leg

- Construct surrogate models from LHS
  - Gaussian Process w/ 1e6 samples



Salinas propagated best estimates

Surrogate propagated Gaussians

Same information as pdf

Different look

Surrogate Models

- Difference: mathematical form
  - Polynomials
  - Polynomial chaos expansions
  - Gaussian process
  - MARS
  - Radial basis functions
  - Neural Network
- Train from “data” = full model evaluations
  - **No physics, just fitting to data**
- Diagnostic metrics: R^2, mean absolute error, sum-squared error, cross-validation metrics
- Often the surrogate is less accurate at bounds or endpoints: use caution

Assess UQ results

- UQ → what we know about QoI's
  - Based on model, inputs, parameters
- Also need to consider the process!
  1. Characterization of parameter uncertainty
  2. Limited LHS sampling
  3. Constructing surrogates
- What to do? How to assess the effect?
  1. Verify data analysis, document assumptions
  2. Incremental LHS → check convergence of statistics
  3. Surrogate diagnostics, cross-validation, multiple surrogates

Notes on Uncertainty Quantification

- More samples → more information
- Sampling, especially LHS, gives more samples in high probability areas
  - Very good for mean, standard deviation – “bulk properties”
  - **NOT good for tails, “extreme events”**
- Same for characterization of parameter uncertainty
- Surrogate models – accurate where training data exists

- For tails: Advanced methods – reliability

Uncertainty Quantification Conclusion

- How to do UQ? Method, # samples, surrogates?
- **Recall goal: understand QoI information**
  - **Why?: QMU? PLoAS? Design study? Validation?**
- Principles
  - Fidelity of UQ should be determined by intended use
  - Always have option to do more UQ – iterate w/ application
  - Balance sources of uncertainty ("the uncertainty budget")
    - Uncertain parameters, mesh, code, model form, UQ methods, surrogate
    - Don't need high fidelity UQ when mesh is poor quality

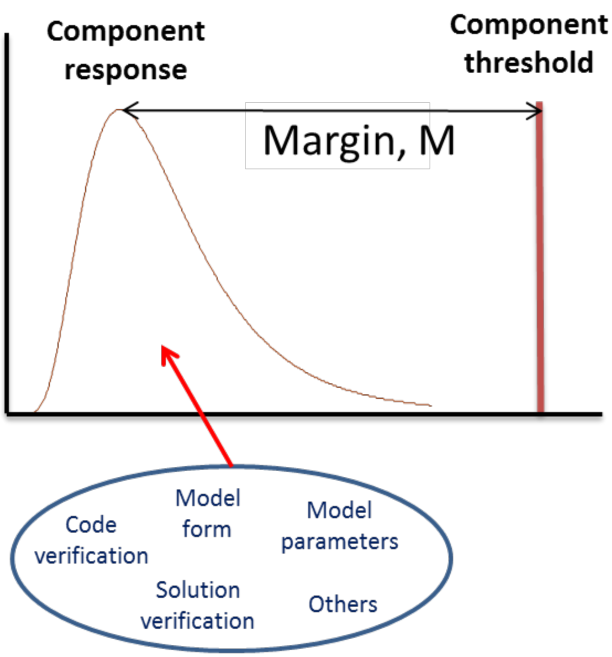
Questions?

- No "standard" texts
- Recent book:
  - http://www4.ncsu.edu/~rsmith/UQ_TIA/
- Dakota Users and Theory Manuals have many references

Big Picture

- Discussed sensitivity analysis, uncertainty quantification for model parameters
  - SA → prioritization of resources
  - UQ → pdf of QoI
- Demonstrate how each is used for an engineering project
  - 3leg example
- Understand cost and benefit
  - Uncertainty budget concept
  - Driven by the decision to be made, not math/ computer time

QMU Example



Contact

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