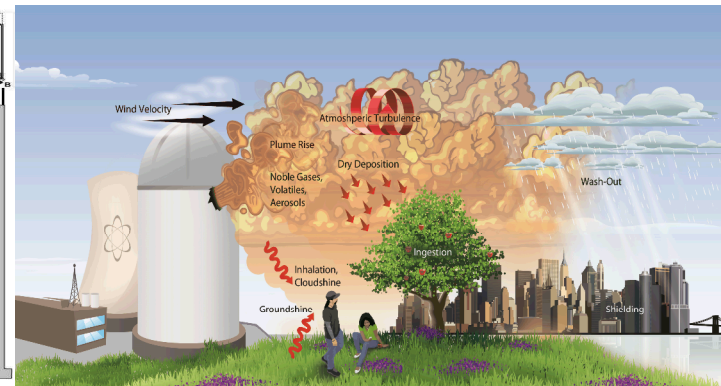
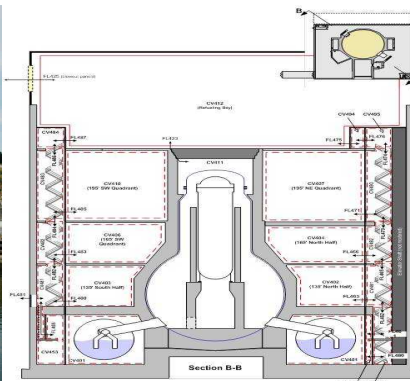


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SOARCA project uncertainty Analysis

General Purpose, uncertainty characterization and propagation

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Plan

- Uncertainty treatment in complex system
- General approach
- Uncertainty characterization
- Uncertainty propagation



Peach Bottom 10 and 20 mile analysis areas

- Uncertainty treatment is nowadays widely recognized as an important and necessary step of any complex analysis.
- The quality of an analysis equates the quality of the least accurate part: as a result, the effort spent on uncertainty is inversely proportional to the lack of accuracy it generates, compared to other aspects of the system (numerical error/accuracy, appropriateness of the physical phenomena modeled)

Consequence at decision maker/general public level

- Dealing with uncertainty means that instead of presenting a single result, the results will be presented as distribution.
- It is important for the decision maker to understand why going from a single value to a distribution is an improvement, as it informs about the risk of selecting one option vs. another
- For the same reason, when results are presented to a wide non expert audience, the effort required to educate the general audience and explain the results should not be minimized.
- The message that needs to go through is that **adding uncertainty does not mean we reduce our knowledge but that we recognize and quantify our current state of knowledge**

Parameter, inputs and model uncertainty

Uncertainty often appears in many aspects of complex systems analyses:

- Uncertainty in the inputs used in the system (the most commonly acknowledged and characterized uncertainty)
- Uncertainty in the parameters calibrating the models used. They can come from use of simplified models, not well known context of use, calibration uncertainty ...
- Uncertainty in which model to use
- Uncertainty on occurrence and strength of external events that may affect the system
-

Aleatory and Epistemic Uncertainty

- **Aleatory uncertainty:** (Perceived) randomness in the occurrence of future events
- **Epistemic uncertainty:** Lack of knowledge w.r.t. the appropriate value to use for a quantity that has a fixed, but poorly known, value in the context of a specific analysis.
- Probability usually used to characterize both aleatory uncertainty and epistemic uncertainty
- Alternatives to probability to the representation of epistemic uncertainty
 - Interval analysis
 - Possibility Theory
 - Evidence Theory

A simple example



- One want to know the likelihood of having a five when rolling a (possibly biased) die.
- By rolling more and more the same die and recording the result, one can estimate this probability.
- The more the die is rolled, the better the estimate will be. The uncertainty over this probability is due to **lack of knowledge** therefore **epistemic**
- But even if the probability to have a five or any number is well known, this knowledge (or rolling more the die) will not be useful to know what will be the result in the next roll, or at the 10th future roll... the outcome of the roll represent **randomness** or **aleatory uncertainty**

SOARCA Uncertainty Analysis Objectives

1. Identify the uncertainty in the input and parameters used in the SOARCA deterministic “best estimate,” and
2. Develop insight into the overall sensitivity of the SOARCA results to uncertainty in key modeling inputs
 - Assess key MELCOR and MACCS2 modeling uncertainties in an integrated fashion to quantify of the relative importance of each uncertain input on the potential consequences

SOARCA Uncertainty Analysis description

- Focus is on epistemic (state-of-knowledge) uncertainty in input parameter values
 - Model uncertainty addressed to the extent that some parameters represent or capture alternate model effects or in separate sensitivity analyses
 - Aleatory (random) uncertainty due to weather is handled in the same way as the SOARCA study
- Peach Bottom unmitigated, long-term station blackout scenario selected
- Scenario definition not changed after Fukushima
 - A separate qualitative discussion was included in an appendix
- Looking at uncertainty in key model inputs
 - MELCOR parameters
 - MACCS2 parameters

Risk Analysis in concept

1. Kaplan Garrick triplet

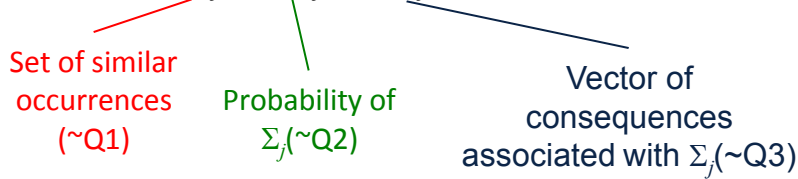
- Risk analysis: More than probability times consequence
- Risk analysis as the answers to three questions

- Q1, What can happen ?
- Q2, How likely is it to happen ?
- Q3, What are the consequences if it does happen ?

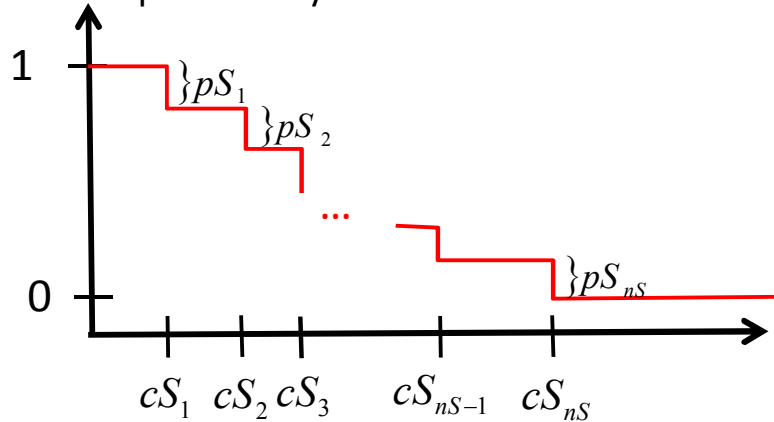
} *Aleatory uncertainty*

- Risk analysis as a set of ordered triples

$$(\Sigma_j, pS_j, \mathbf{cS}_j), j = 1, 2, \dots, nS, \Sigma_j \cap \Sigma_k = \emptyset \text{ for } j \neq k$$



- Risk analysis as a complementary cumulative distribution function (CCDF)

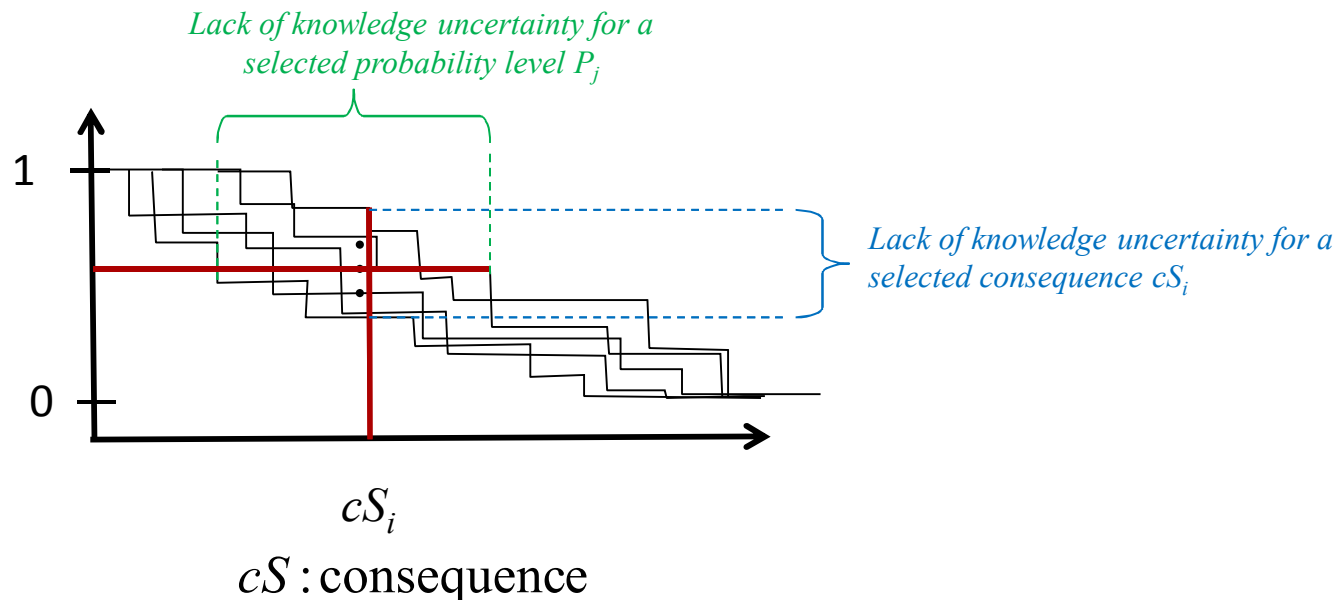


Assumed ordering

$$cS_1 < cS_2 < \dots < cS_{nS}$$

2. Addition of Lack of Knowledge uncertainty

- A fourth question underlying risk analysis
 - Q4, What is the uncertainty in the answers to the first three questions (i.e., Q1, Q2, Q3)?Epistemic uncertainty
- Implication to the fourth question: Multiple CCDFs, with each one arising from different realization of epistemic uncertainty



Analysis Possibilities

- Aleatory uncertainty only
 - Event trees
 - Random sampling
 - Importance sampling
 - Stratified sampling
 - Numerical Integration

- Epistemic uncertainty
 - Differential analysis
 - Response surface methodology
 - Sampling-based (Monte Carlo) methods
 - Variance Decomposition

- Aleatory and epistemic uncertainty
 - Some combination of above
 - Often requires careful computational planning

Uncertainty Propagation: Sampling Based method

- Sampling based method is a widely used approach to propagate uncertainty in a complex system.
- It consists in generating input sets by sampling a value for each uncertain input variable to define an input set realization and run the code for each realization.
- When both aleatory and epistemic uncertainty is considered, a nested loop structure is used (usually aleatory uncertainty using the inner loop and epistemic uncertainty as the outer loop)

Sampling based approach

General approach

- Method used follow classical sampling based method:
 1. Selection of metrics or figures of merit from the output data (\mathbf{y})
 2. Selection of uncertain parameters (definition of vector \mathbf{x})
 3. Characterization of uncertainty in \mathbf{x} (i.e., definition of D_1, D_2, \dots, D_{nX}) **using probability distributions**
 4. Generation of sample from \mathbf{x} (i.e., generation of $\mathbf{x}_k, k = 1, 2, \dots, nS$, in consistency with D_1, D_2, \dots, D_{nX})
 5. Propagation of sample through analysis (i.e., generation of mapping $[\mathbf{x}_k, \mathbf{y}(\mathbf{x}_k)], k = 1, 2, \dots, nS$)
 6. Performing uncertainty analysis (i.e., Determination of the uncertainty in analysis results that derives from the collective uncertainty in analysis inputs)
 7. Determination of sensitivity analysis results (i.e., Determination of the contribution of the uncertainty in individual analysis inputs to the uncertainty in analysis results)

Determination of outputs and inputs

Determination of Metrics (SOARCA)

- Analysis of source term releases including Cesium and Iodine release over time
- Latent cancer fatality risk and prompt fatality risk using LNT dose-response model

Key uncertain input parameters were identified

- Description of most influential uncertain parameters in study
- Guidance solicited from peer reviewers on chosen parameters and distributions; feedback from Advisory Committee on Reactor Safeguards

Uncertainty Characterization: Expert Review

- $\mathbf{e}=[e_1, e_2, \dots, e_{nE}]$ vector of epistemically uncertain analysis inputs
- Derive distribution D_i for each element e_i of \mathbf{e}
- Correlations and other restrictions between elements of \mathbf{e} possible
- Usually obtained through expert review process that draws on
 - Observational and experimental data of varying levels of relevance
 - Knowledge of relevant processes and physics
 - Requires care, thought and thorough documentation
 - Not an individual expounding with feet on desk !
- Definition of distribution D_i for element e_i of \mathbf{e}
 - Recommended: Specify minimum, maximum and selected quantiles
 - Not recommended: Specify named distribution and associated parameters
 - Specify any appropriate correlations and restrictions
 - Document !

Uncertainty Characterization: Maximum Entropy

- Entropy $H(x)$ measure of uncertainty inherent in a distribution for x

$$H(x) = \begin{cases} -\sum_{i=1}^n p_i \log p_i & , \text{ discrete : } p_i \text{ probability of } x_i \\ -\int_{\Xi} d(x) \log[d(x)] dx & \text{continuous : } d(x) = \text{density for } x \text{ on } \Xi \\ & (\text{convention: } 0 \log 0 = 0) \end{cases}$$

- $H_1(x) < H_2(x) \Rightarrow$ more inherent uncertainty in distribution producing $H_2(x)$ than in distribution producing $H_1(x)$
- Maximum entropy can be used to complete the definition of incompletely defined probability distributions with the least incorporation of additional information
- Examples of maximum entropy distributions
 - Only range $[a,b]$ known \Rightarrow uniform $[a,b]$
 - Only quantiles known \Rightarrow piecewise uniform between quantiles
 - Only mean and variance known \Rightarrow normal
 - Only range $[a,b]$, mean and variance known \Rightarrow beta on $[a,b]$
 - Only range $[0,\infty)$ and mean known \Rightarrow exponential

Uncertainty Characterization: Bayes Theorem

- Start with initial uncertainty characterization (Prior distribution)
- Obtain new information (experiment, observation)
- Update uncertainty characterization with Bayes theorem (Posterior distribution)

Possible value for x out of i possible values: x_1, x_2, \dots, x_n

Posterior probability For x_i

Prior probability For x_i

Likelihood: probability of observing information I if x_i is correct value for x

$$p_{\text{pos}}(x_i | I) = \frac{p_{\text{pr}}(x_i) p_L(I | x_i)}{\sum_{j=1}^n p_{\text{pr}}(x_j) p_L(I | x_j)}$$

New information from experiment or observation

Note: Bayes theorem with continuous probability distributions involves density functions and integrals rather than probabilities and sums

Uncertainty Characterization: Bootstrap

- Start with initial data set of size n
- Generate R new bootstrap samples of size n by sampling with replacement from original data set
- Estimate quantity x of interest for each of the R bootstrap samples (i.e., obtain x_i for $i=1,2,\dots,R$)
- Use the results x_1, x_2, \dots, x_R to estimate uncertainty in x

Uncertainty Characterization in SOARCA

- Core team of staff from SNL and NRC with expertise in probability and statistics, uncertainty analysis, and MELCOR and MACCS2 modeling for SOARCA
- Subject matter experts (SMEs) provided support in reviews of data and parameters
- Approach is based on a formalized PIRT (phenomena identification, and ranking table) process
- Focus on confirming that the parameter representations appropriately reflect key sources of uncertainty, are reasonable, and have a defensible technical basis
- Attempt to obtain contribution from uncertainty across the spectrum of phenomena operative in the analyses, through a balanced depth and breadth of coverage

Uncertainty propagation: SRS vs. LHS vs. Importance sampling

- Simple Random Sampling (SRS), sometimes called Monte Carlo, is the simplest approach. Values are sampled directly from the distribution. It has the advantage that sample can be increased easily and provides unbiased estimates of moments and quantile and allows to correlate input variables
- Latin Hypercube Sampling (LHS) applies a dense stratification and therefore provides a better coverage for each individual variable. It is as fast as SRS and allows correlation, and leads to better estimate of statistics (with less variance) but the sample size cannot be increased as easily and a sub-sample of an LHS is not an LHS
- Importance sampling (IS) can be very powerful when unlikely events needs to be studied and considered. However it usually requires good knowledge of the system to apply efficiently importance sampling. It is therefore recommended as supplemental analysis

Uncertainty propagation in SOARCA-UA: Generation of input sample and uncertainty propagation

- Uncertainty in inputs parameters propagated in two steps:
 - A set of source terms generated using MELCOR model using **3** Simple Random Sampling of original size 300.
 - A distribution of consequence results generated using MACCS2 model via LHS sample of size matching the number of converged runs from MELCOR. Finally another LHS run of size matching the sum of the MELCOR converging results has been created for a more accurate distribution of consequence results.

- Simple Random Sampling was preferred to LHS for MELCOR results for two reasons
 - having a valid sample when non converged results are not included
 - having the possibility to increase the sample size if the statistics where unstable.