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Finite-Difference Numerical Simulation of 3D Seismic Wave Propagation

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a ***briefing for***

ExxonMobil Visitors

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Why Compute Synthetic Seismic and Acoustic Data?

1) **Fundamental research:** study scientific issues associated with wave propagation in earth and atmosphere environments (radiation, reflection, refraction, scattering, attenuation, dispersion, etc.).

2) **Applied research:** understand practical issues related to remote sensing and imaging with seismic and/or acoustic waves (detection, resolution, sensitivity, parameter estimation accuracy, etc.).

3) Engage in **prediction**, hypothesis testing, or simulation (ground motion, CO_2 sequestration monitoring, fluid inclusion effects, etc.).

4) Enhance **interpretation** of field-recorded seismic and /or acoustic data.

5) **Validate** data processing, analysis, interpretation, imaging, or inversion **algorithms** with *realistic* synthetic data generated from known earth and atmosphere models (Marmousi Model, SEG/EAEG Salt Model, SEAM project).

6) **Design** field or laboratory data acquisition **experiments** or **equipment** (survey planning, illumination studies, borehole tools, core sample apparatus).

7) Develop and **enhance numerical computation** capabilities (algorithm parallelization, memory reduction, execution speedup, FD operators, absorbing boundary conditions).

8) Improve **seismological education** via modern visualization capabilities.

Seismic and Acoustic Wave Propagation R&D in the SNL Geophysics Department

R&D Thrust: Development and application of advanced numerical algorithms for simulating 3D seismic and acoustic wavefields propagating within realistic geologic and atmospheric environments:

- isotropic elastic and anelastic (i.e., attenuative/dispersive) solid media.
- fixed and moving fluid (acoustic) media.
- poroelastic (fluid-saturated solid) media.
- *anisotropic (directional) media (both elastic and anelastic) under development.*

Numerical Solution Methodology: Explicit, ***time-domain finite-differencing*** of coupled systems of first-order partial differential equations, representing "full physics" mathematical characterization of continuum-mechanical wave propagation problems:

- TD FD method is simple, flexible, fast, and historically popular in petroleum industry.
- known numerical stability and dispersion properties.
- accommodates point-by-point heterogeneity in medium properties.
- readily parallelizable via spatial domain decomposition strategy.

But:

- large-scale or broadband simulations can be very expensive.
- full-physics solution may be difficult to interpret.

Elastodynamic Velocity-Stress System

$$\frac{\partial v_i}{\partial t} - \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j} = \frac{1}{\rho}\left[f_i + \frac{\partial m_{ij}^a}{\partial x_j} \right]$$ (3 equations)

$$\frac{\partial \sigma_{ij}}{\partial t} - \lambda \frac{\partial v_k}{\partial x_k}\delta_{ij} - \mu\left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right] = \frac{\partial m_{ij}^s}{\partial t}$$ (6 equations)

Nine, coupled, first-order, linear, non-homogeneous partial differential equations.

Wavefield variables:
- $v_i(\mathbf{x},t)$ - velocity vector
- $\sigma_{ij}(\mathbf{x},t)$ - stress tensor

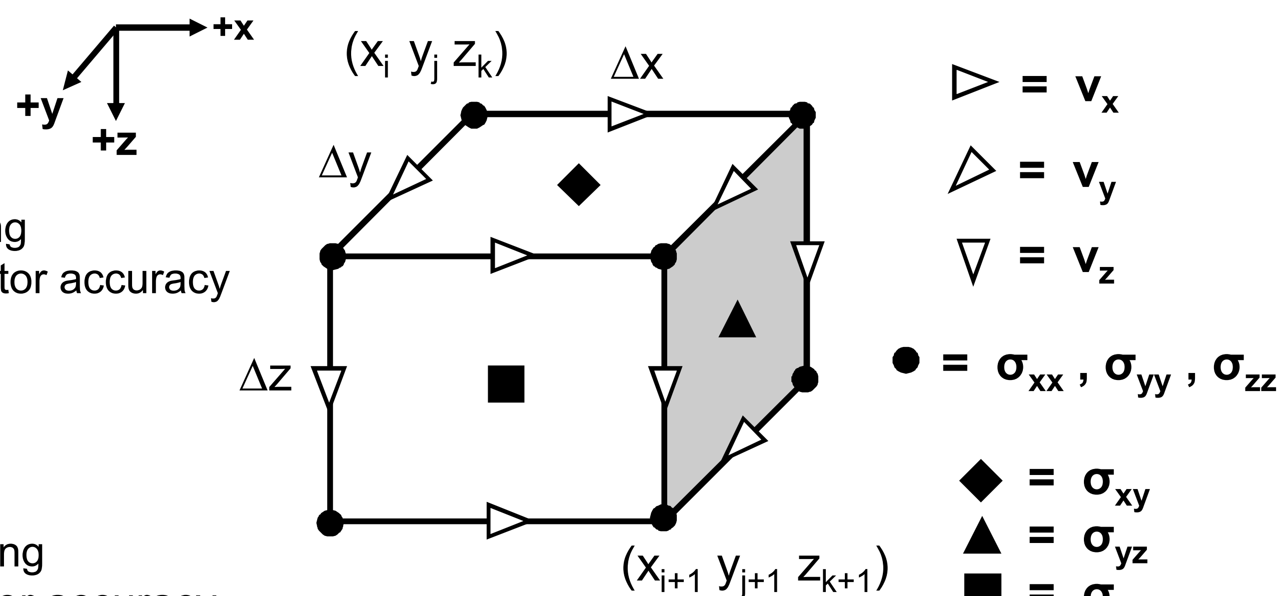
Earth model parameters:
- $\rho(\mathbf{x})$ - mass density
- $\lambda(\mathbf{x})$, $\mu(\mathbf{x})$ - elastic moduli

Body sources:
- $f_i(\mathbf{x},t)$ - force vector
- $m_{ij}(\mathbf{x},t)$ - moment tensor

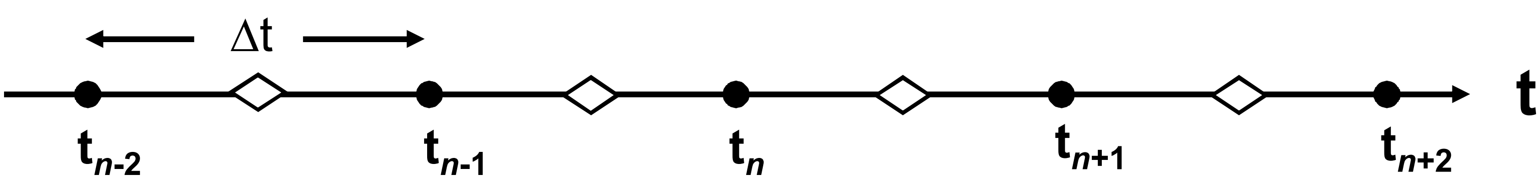
Derived from fundamental principles of continuum mechanics (conservation of mass, balance of linear and angular momentum), an isotropic elastic stress-strain constitutive relation, and linearization to the infinitesimal deformation regime.

Staggered Spatial and Temporal Storage Schemes

3D spatial staggering
⇒ high centered FD operator accuracy



1D temporal staggering
⇒ high centered FD operator accuracy



◇ = v_x , v_y , v_z
● = σ_{xx} , σ_{yy} , σ_{zz} , σ_{xy} , σ_{yz} , σ_{xz}

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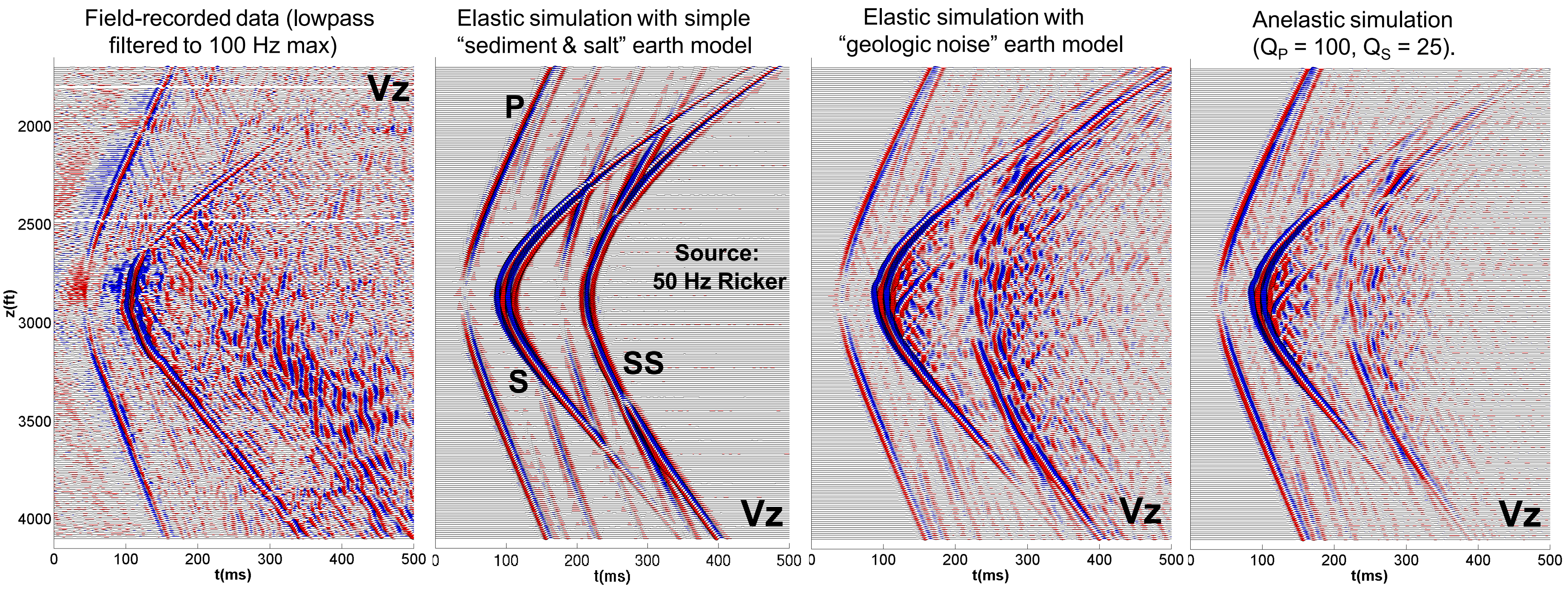
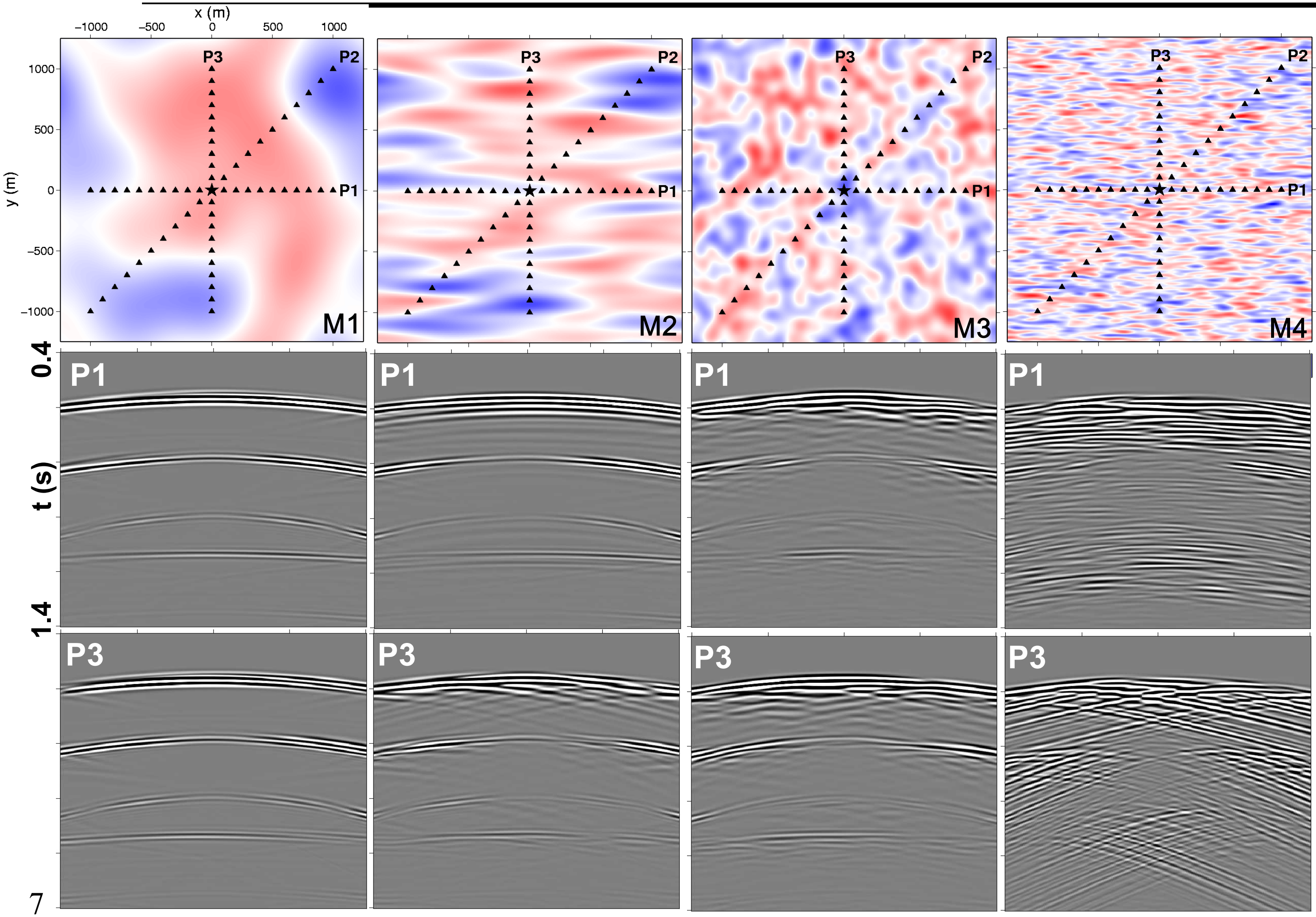
Bayou Choctaw Salt Dome Dual-Well Seismic Data



1) Field data: borehole hydraulic vibrator and 3C velocity receivers.
2) Numerous seismic events observed (well-to-well P and S, salt flank reflections, coda).
3) 3D elastic and anelastic modeling used to replicate and interpret field data:
- timing and amplitude of direct P and S; salt flank reflections; rugose salt flank creates coda; attenuation reduces amplitude of strong reflected SS phase.

Permian Basin Seismic Scattering



Horizontal cross-sections: Central Basin Platform deep carbonates.

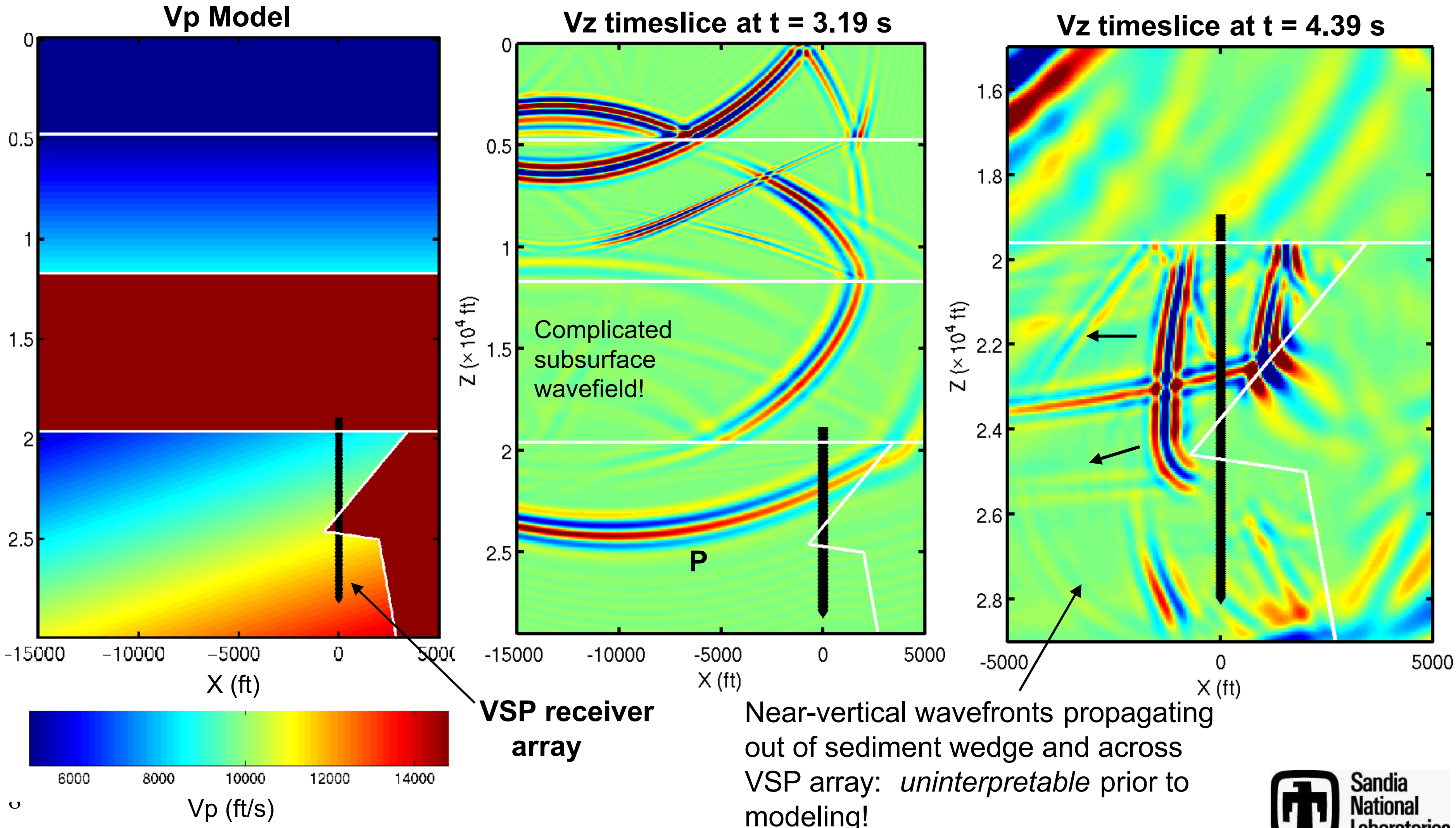
Large/small and isotropic/anisotropic perturbations in Vp.

Profile #1

Scattered seismic energy (ΔVz traces) calculated with 3D elastic FD algorithm.

Profile #3

Gulf of Mexico Marine VSP Simulation: Salt Flank Overhang Model



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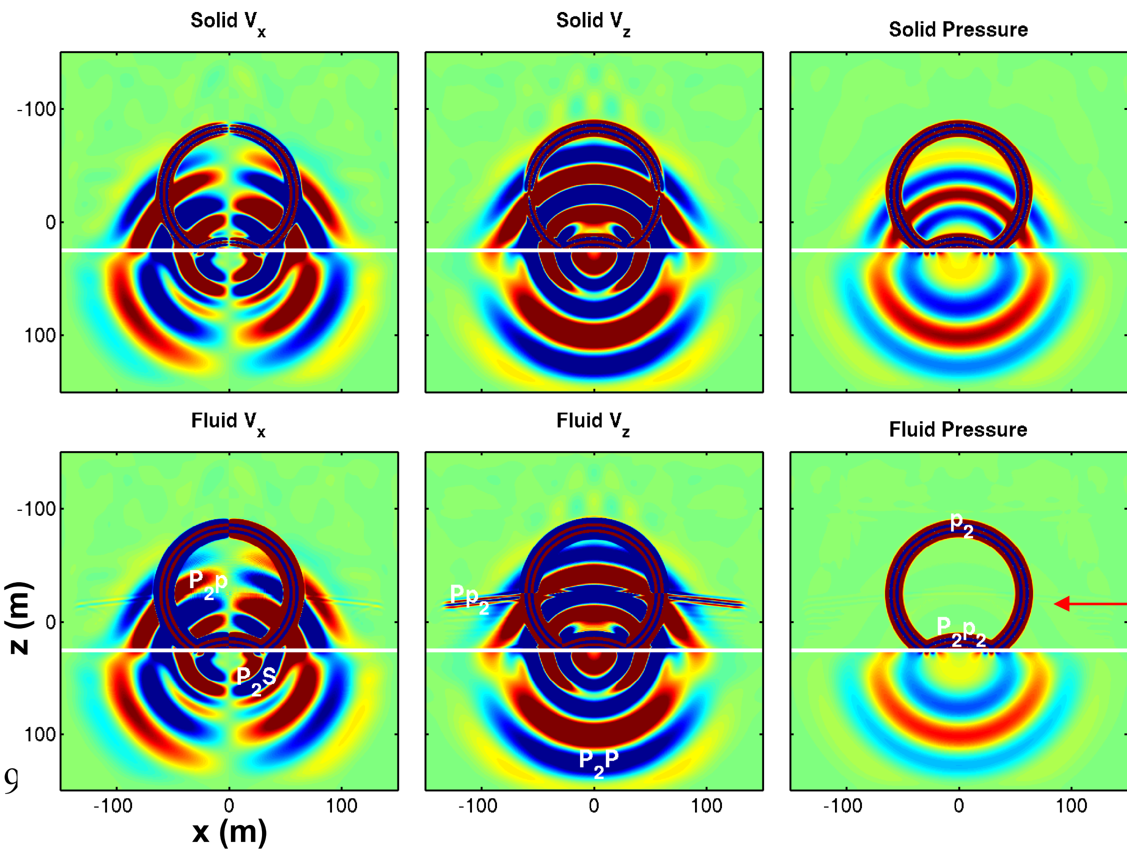
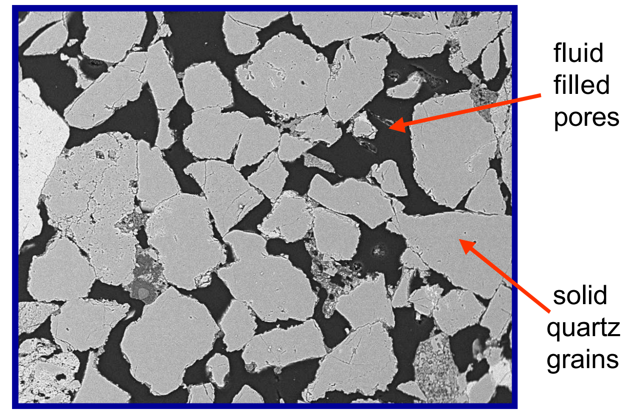
Poroelastic Wave Propagation Modeling

Velocity-stress-pressure finite-difference algorithm, based on Biot theory, simulates 3D wave propagation within a heterogeneous fluid-saturated solid.



Castlegate sandstone



porosity: ~ 28%

permeability: ~ 0.3 Darcy

Reflection, transmission, and mode conversion of poroelastic waves at gas-brine contact within saturated sandstone.

Note **slow P wave**, predicted by Biot theory, but rarely observed in field data.

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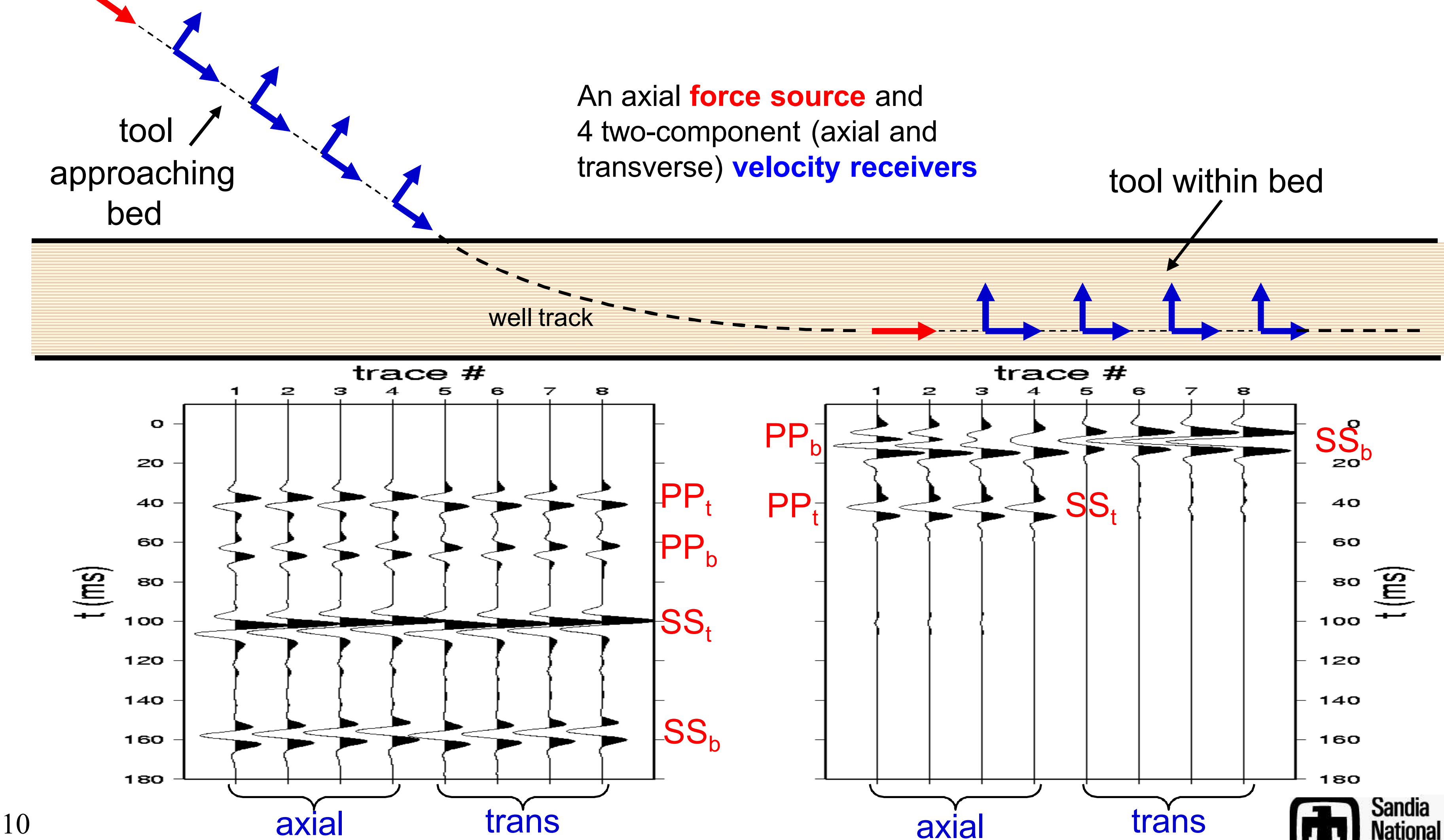
Single-Well Seismic Acquisition Tool Responses



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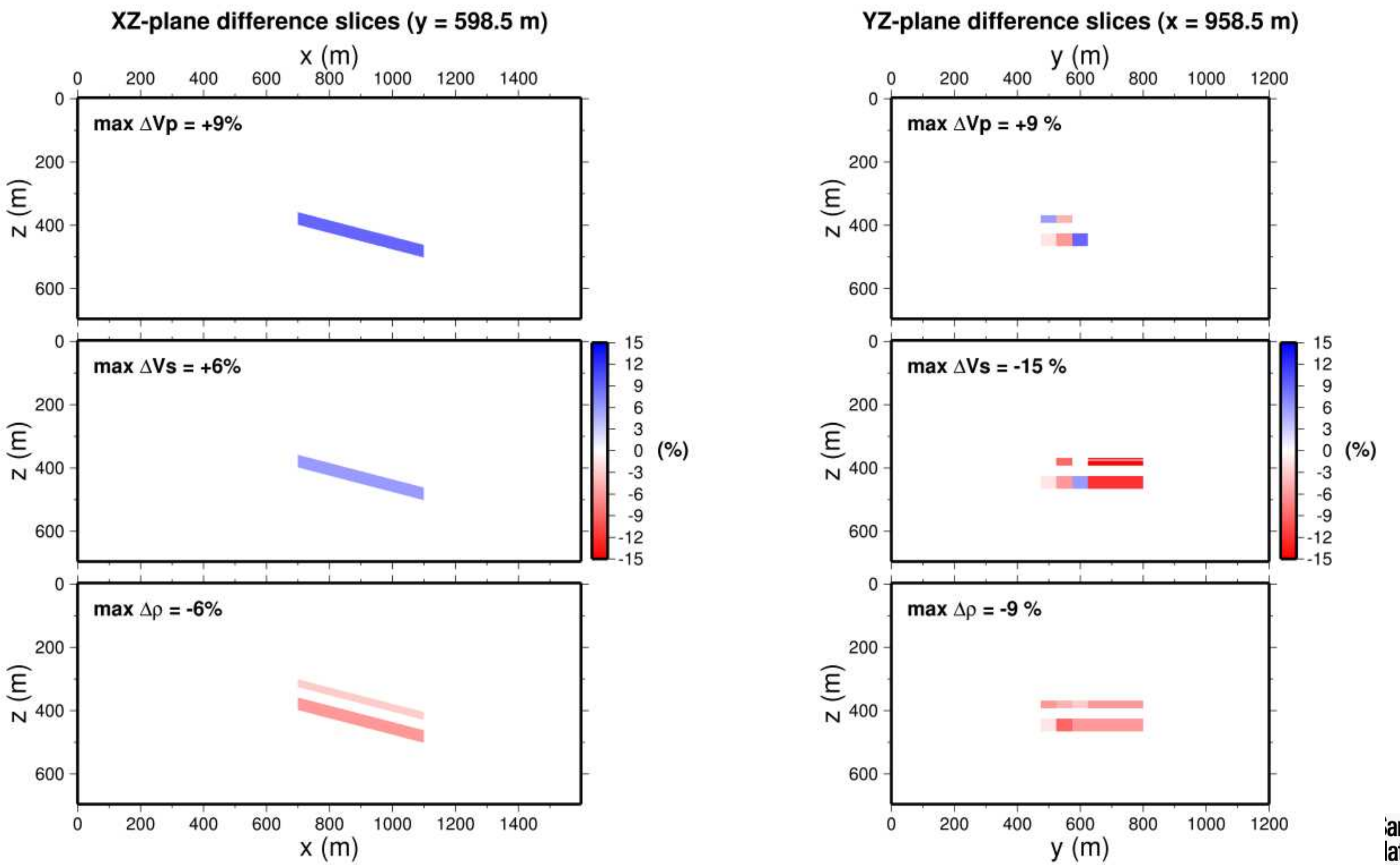
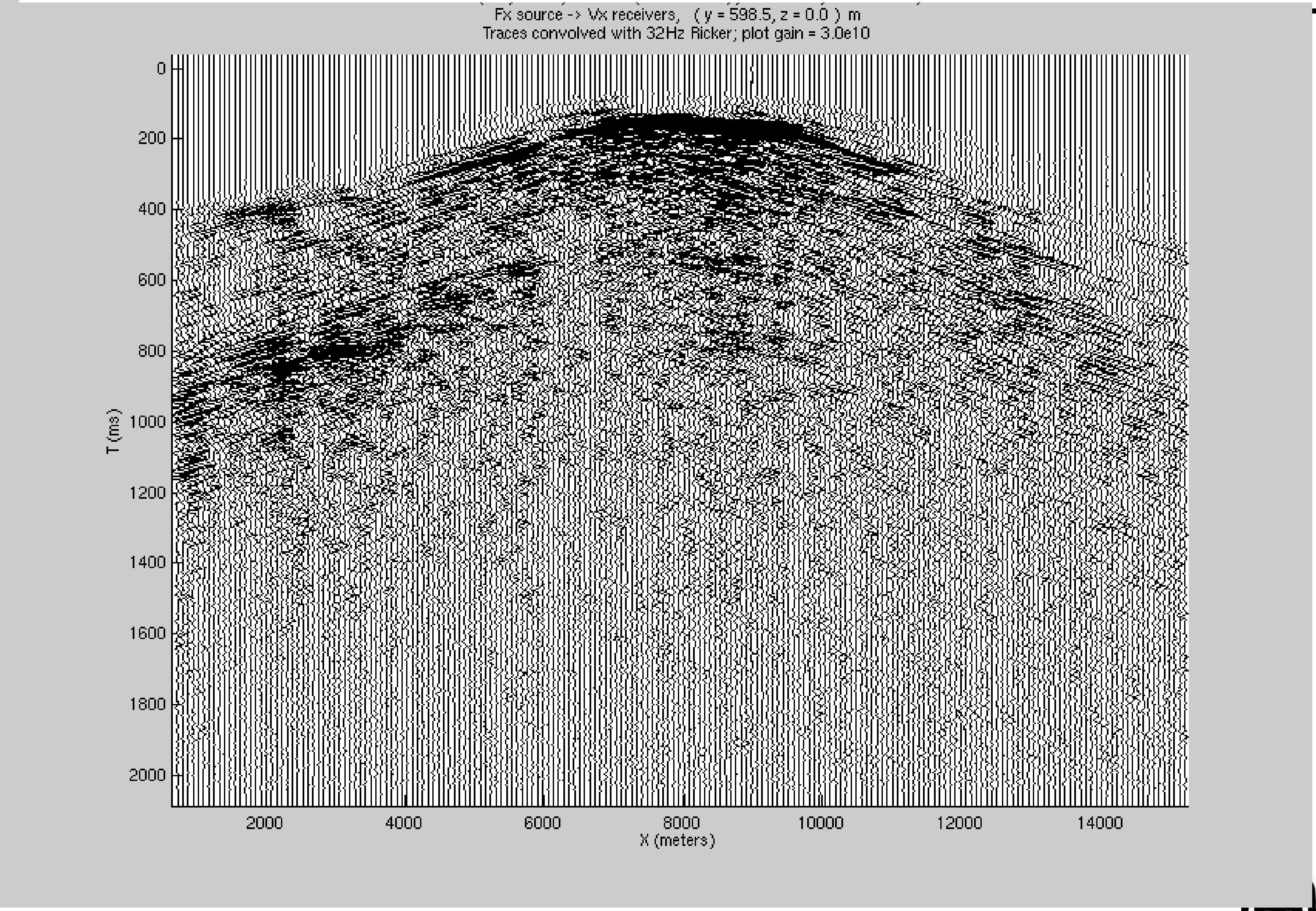
Time Lapse Seismology Modeling Example: Medium Property Differences



Synthetic Seismic Reflection Data: Pre-Injection, Buried Fx source / Surface Vx receivers



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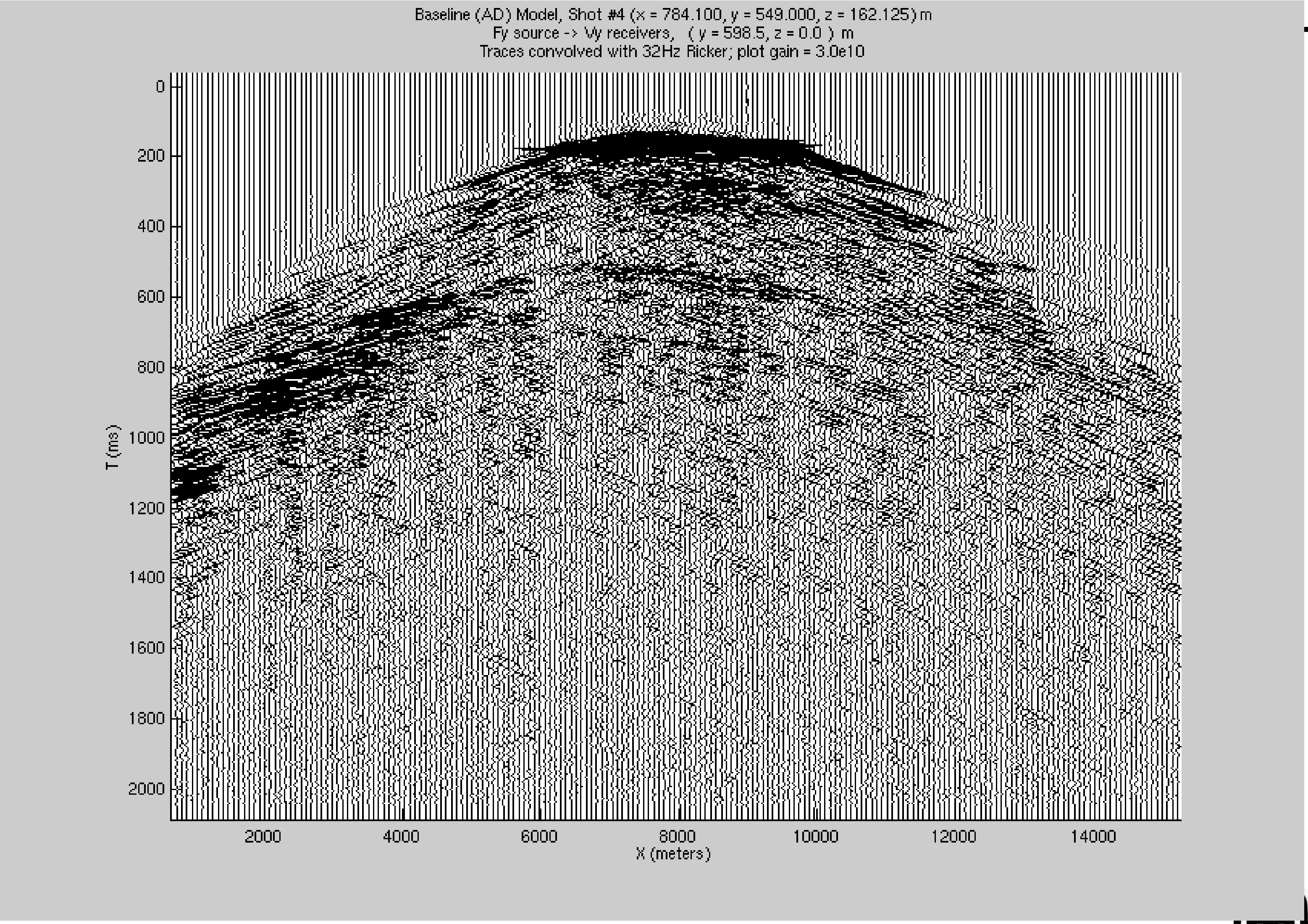
Synthetic Seismic Reflection Data: Pre-Injection, Buried Fy source / Surface Vy receivers



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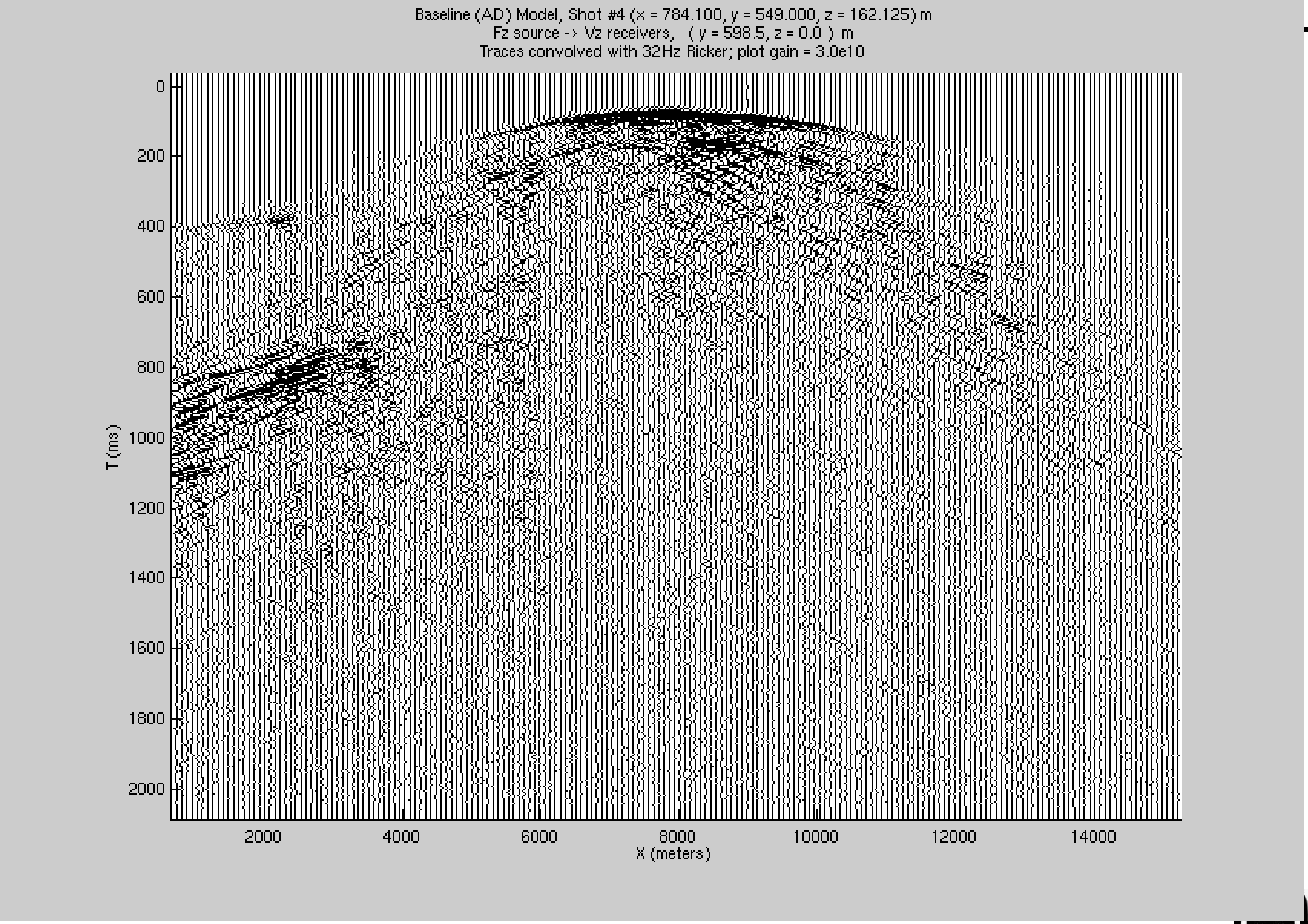
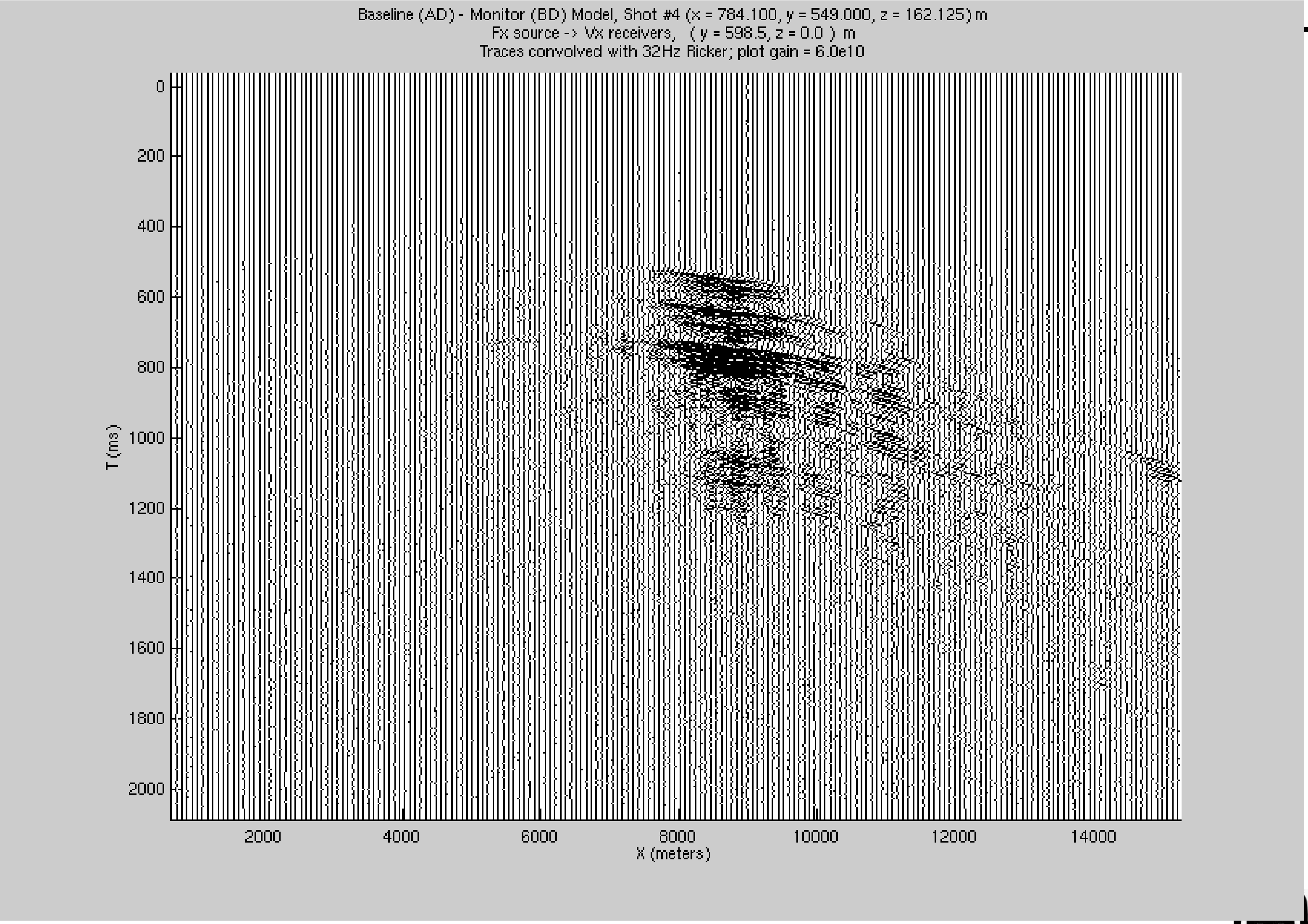
Synthetic Seismic Reflection Data: Pre-Injection; Buried Fz source / Surface Vz receivers



Difference Data: Pre- minus Post-Injection; Buried Fx source / Surface Vx receivers



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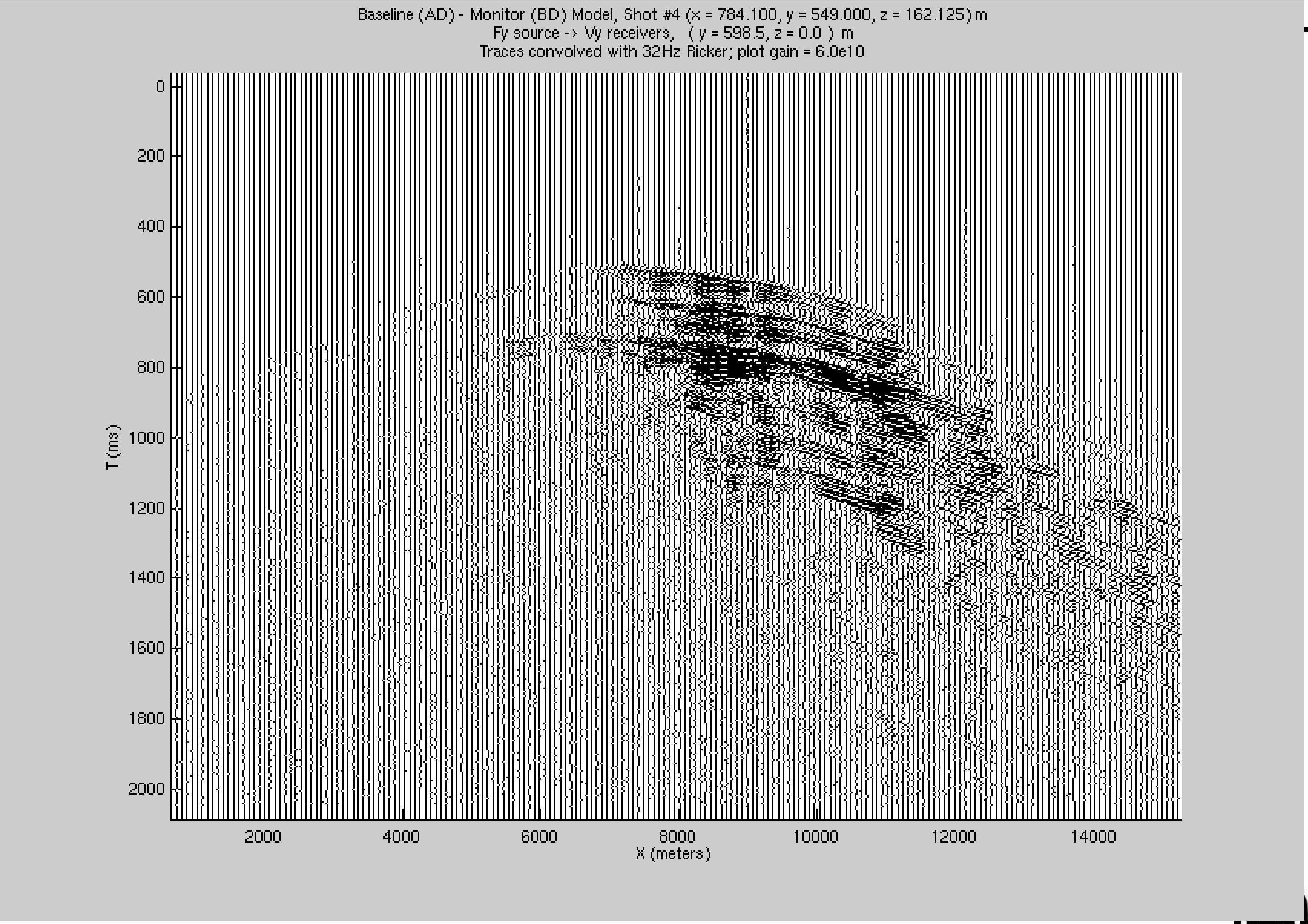
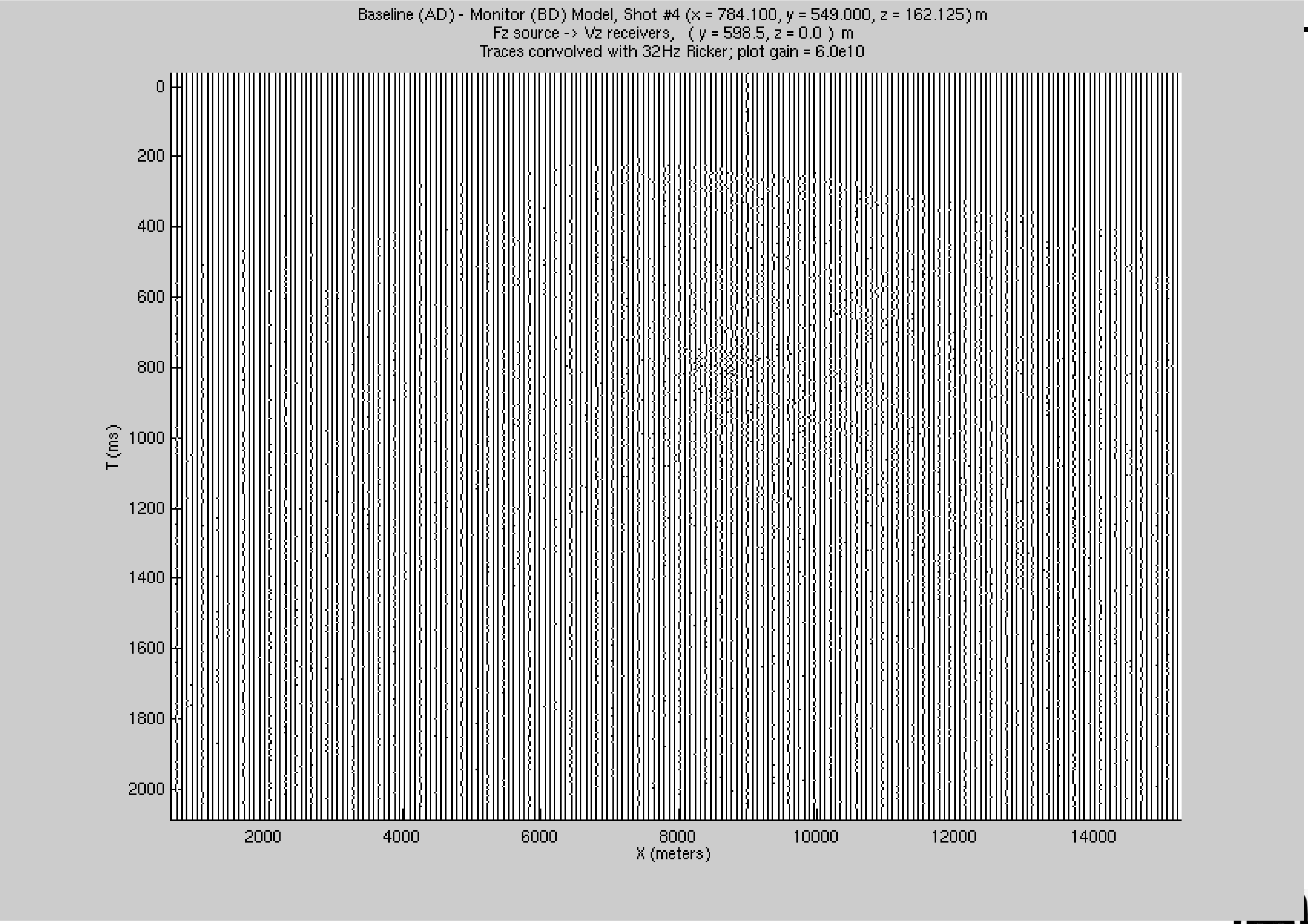
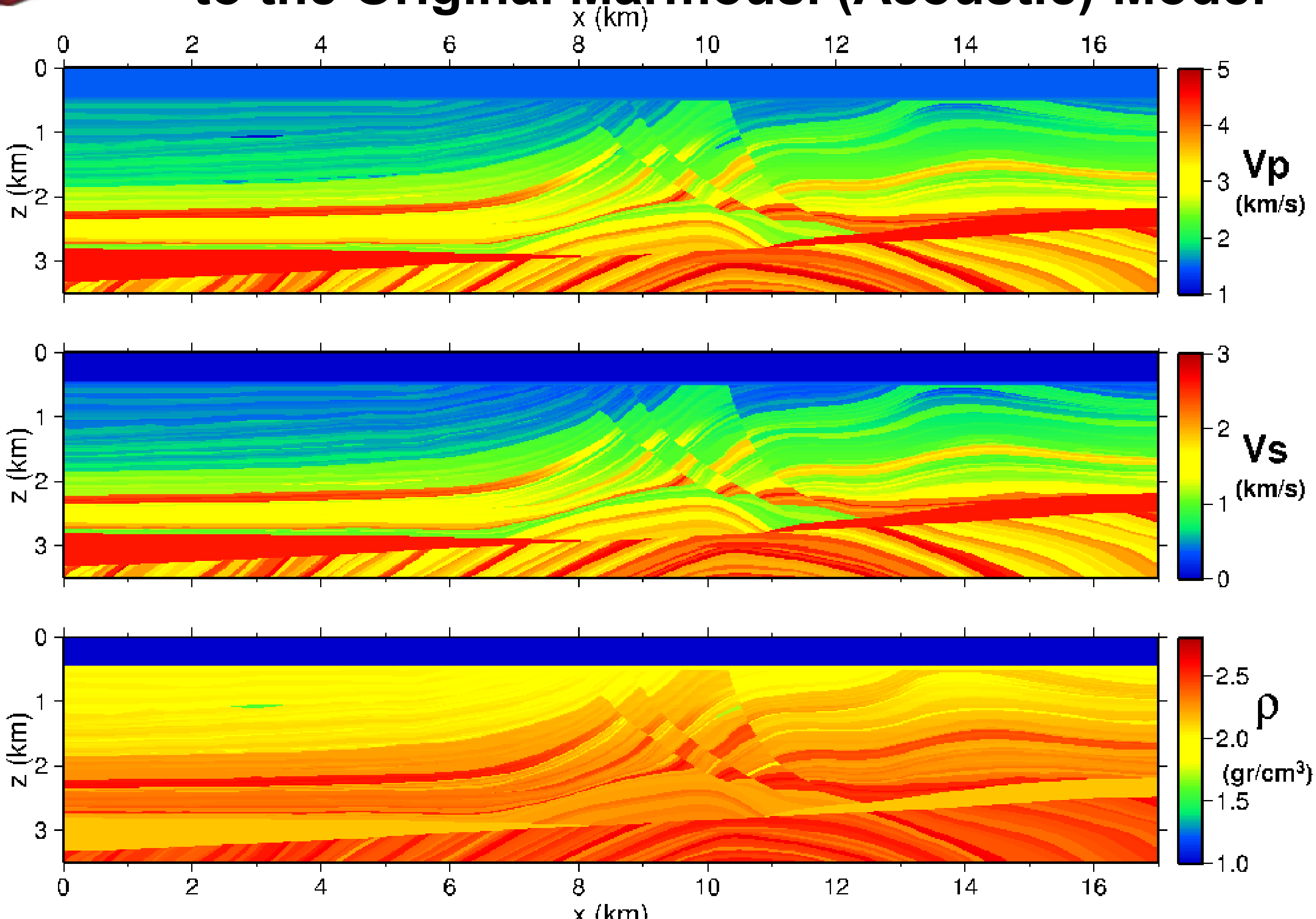
Difference Data: Pre- minus Post-Injection; Buried Fy source / Surface Vy receivers



Difference Data: Pre- minus Post-Injection; Buried Fz source / Surface Vz receivers

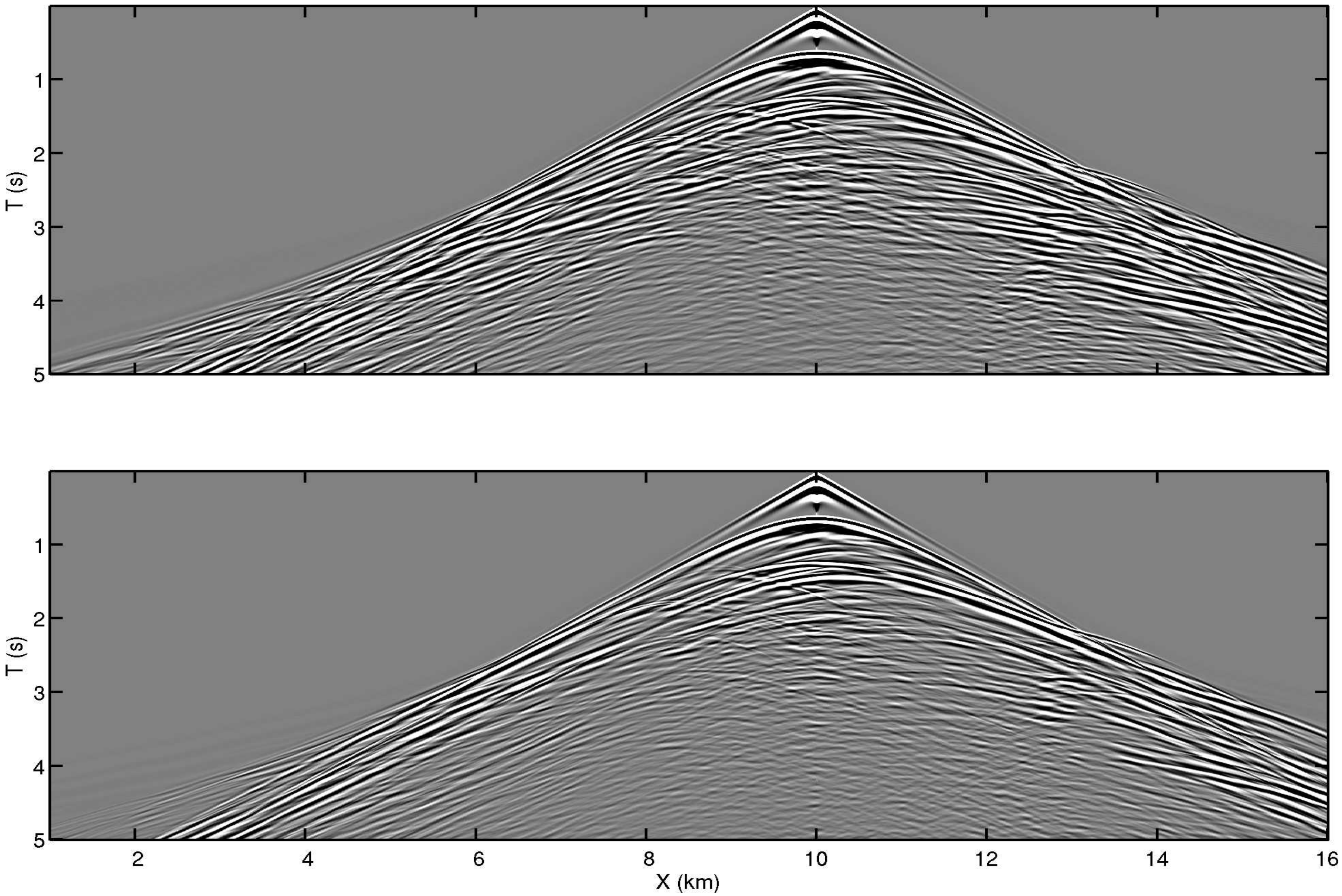


Marmousi2: An Isotropic Elastic Upgrade to the Original Marmousi (Acoustic) Model



After Martin et al.,The Leading Edge, Feb. 2006. SNL 3D elastic modeling conducted with 5 m spatial grid interval ⇒ ~1.2 billion gridpoints!

Pressure Trace Comparison

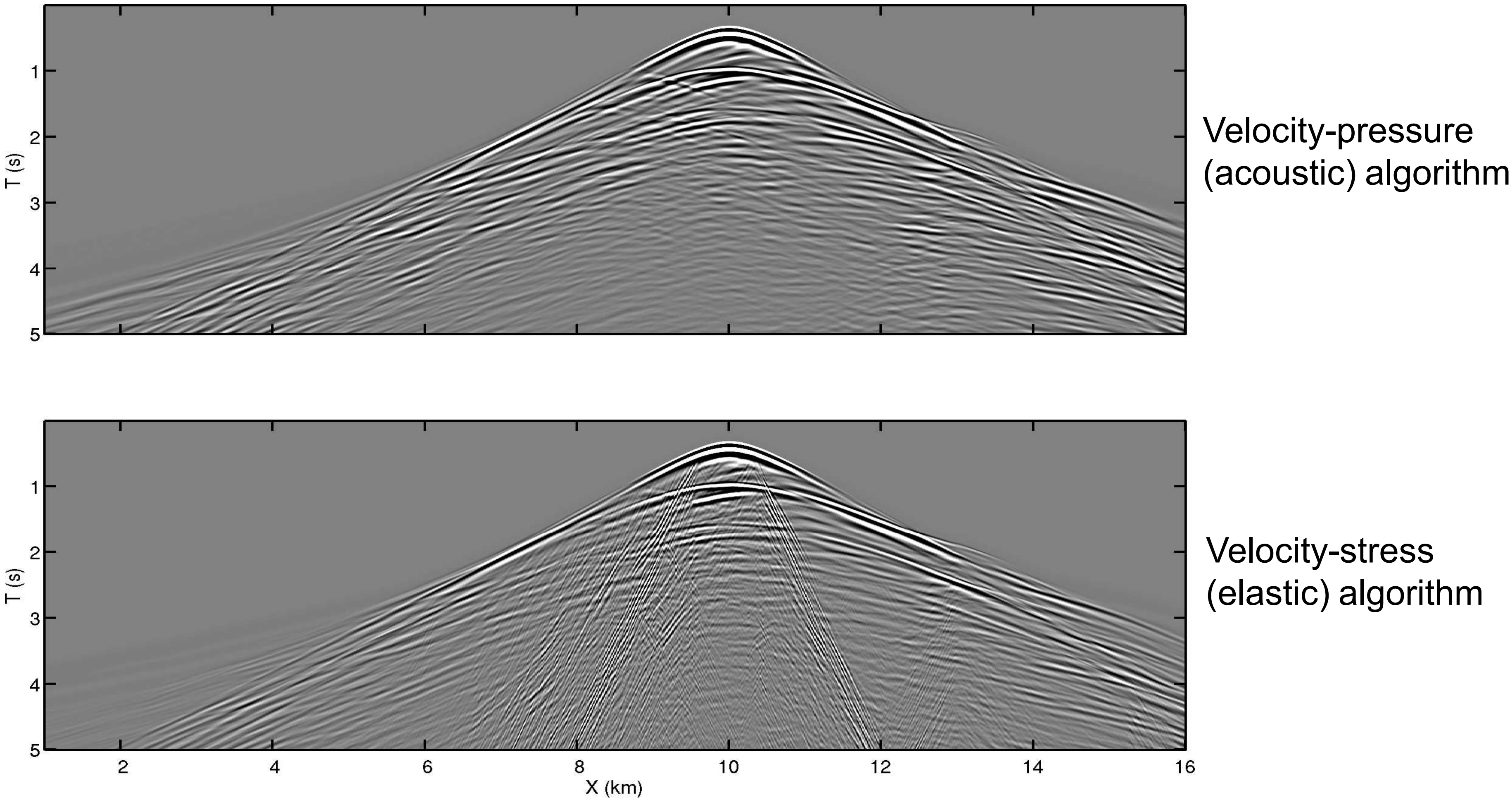


Velocity-pressure (acoustic) algorithm

Velocity-stress (elastic) algorithm

1501 hydrophones, 5 m below sea-surface, arrayed from x = 1 km to x = 16 km. Note strong similarity of calculated responses.

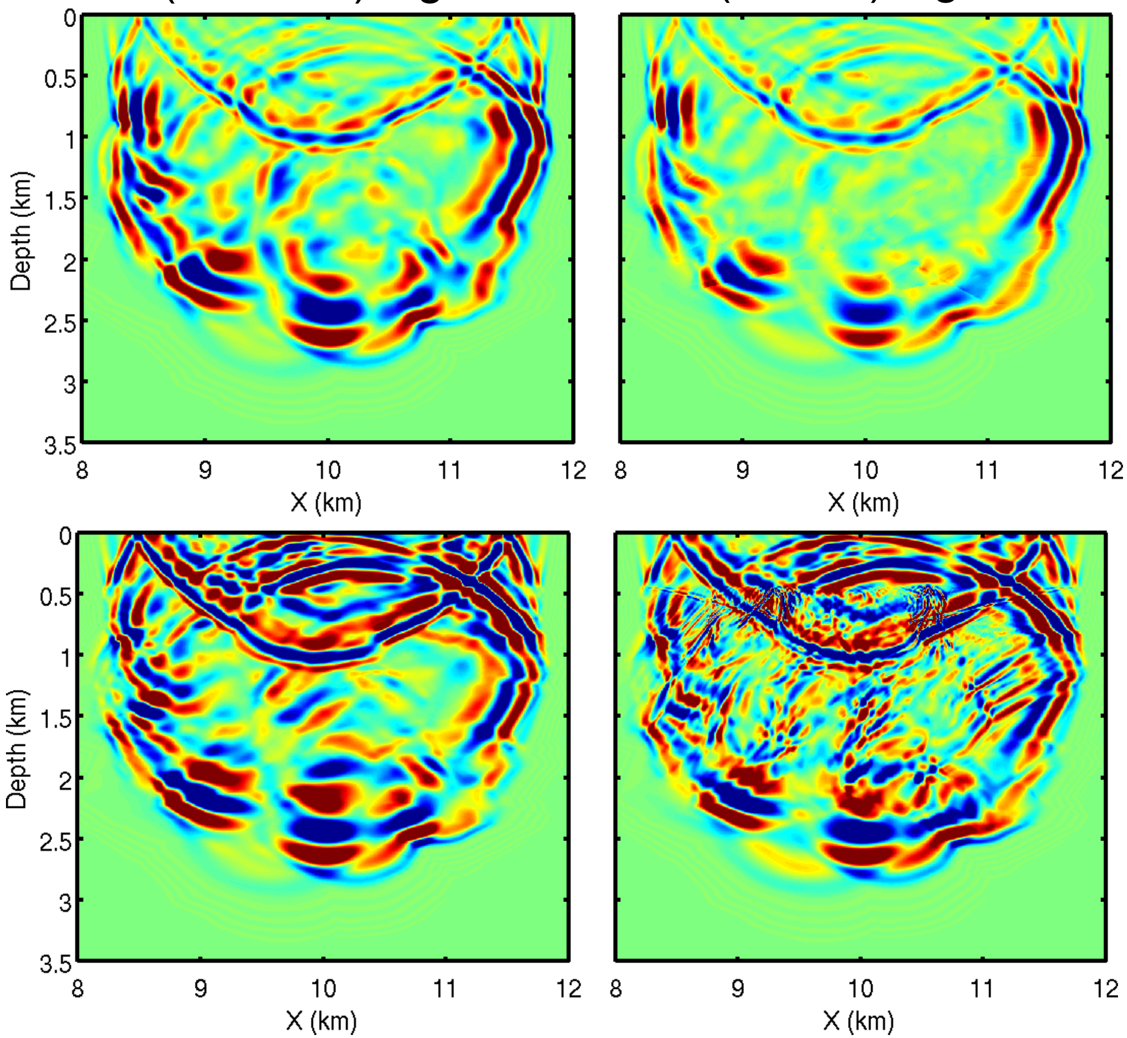
Ocean Bottom Seismometer Trace Comparison



1501 vertical component (Vz) ocean bottom seismometers, located 450 m below sea-surface, arrayed from x = 1 km to x = 16 km. Note strong differences in calculated responses.

Timeslice Comparisons: Pressure and Vz Particle Velocity

VP (acoustic) algorithm | VS (elastic) algorithm



Pressure timeslices;
t = 1.37 s.

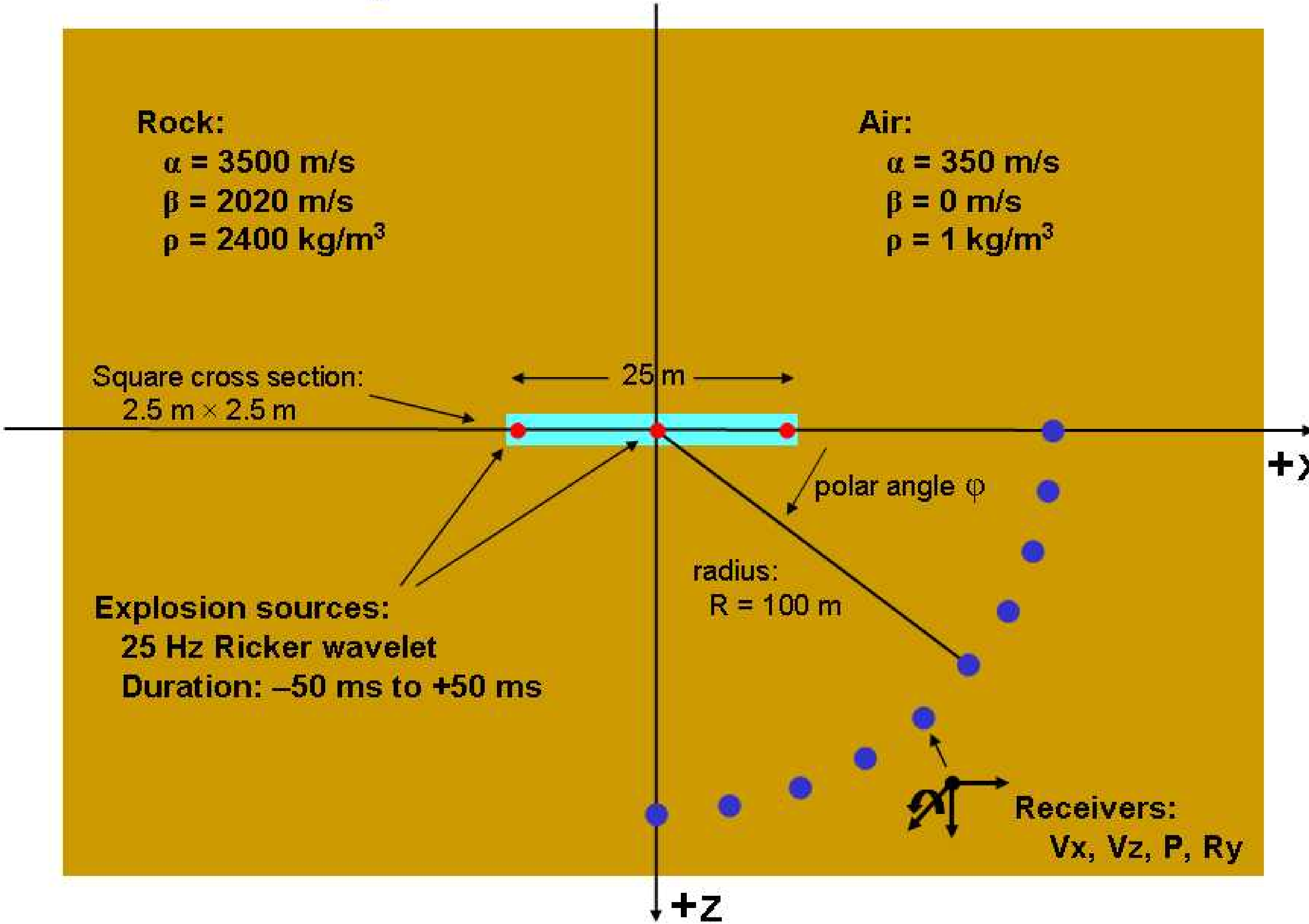
(note similarity)

Vz Velocity Timeslices;
t = 1.37 s.

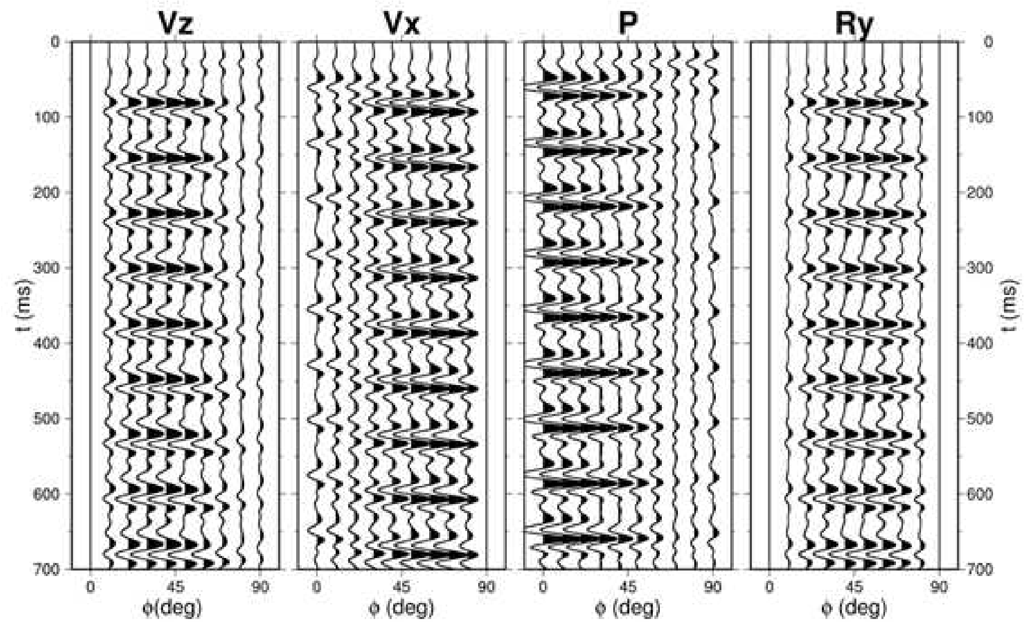
(note difference)

Underground Structure Resonances

Straight Air-Filled Tunnel Model

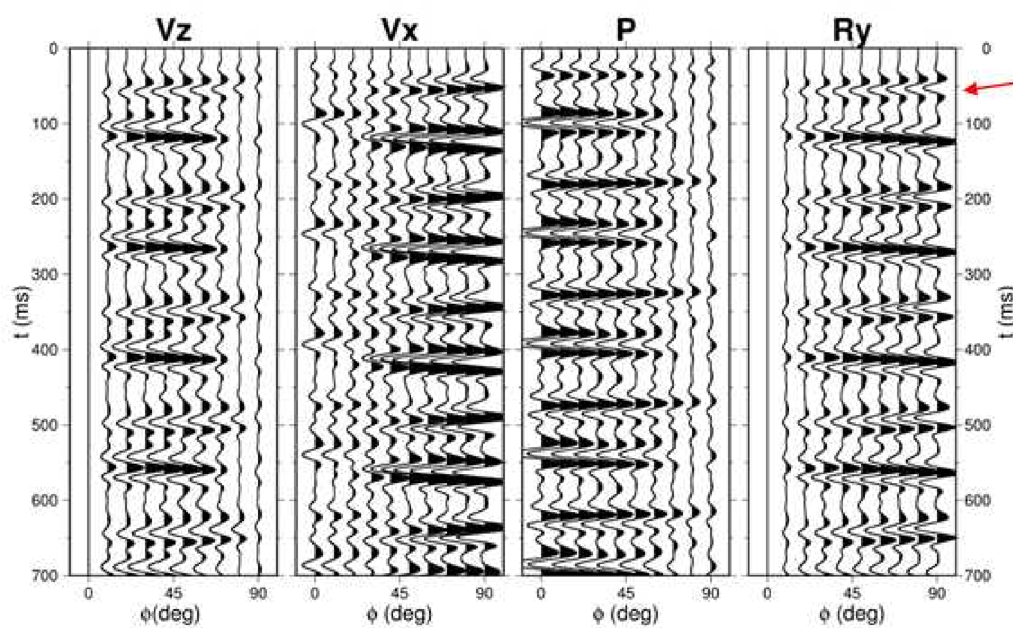


Resonant Response Comparison (Vz, Vx, P, Ry)



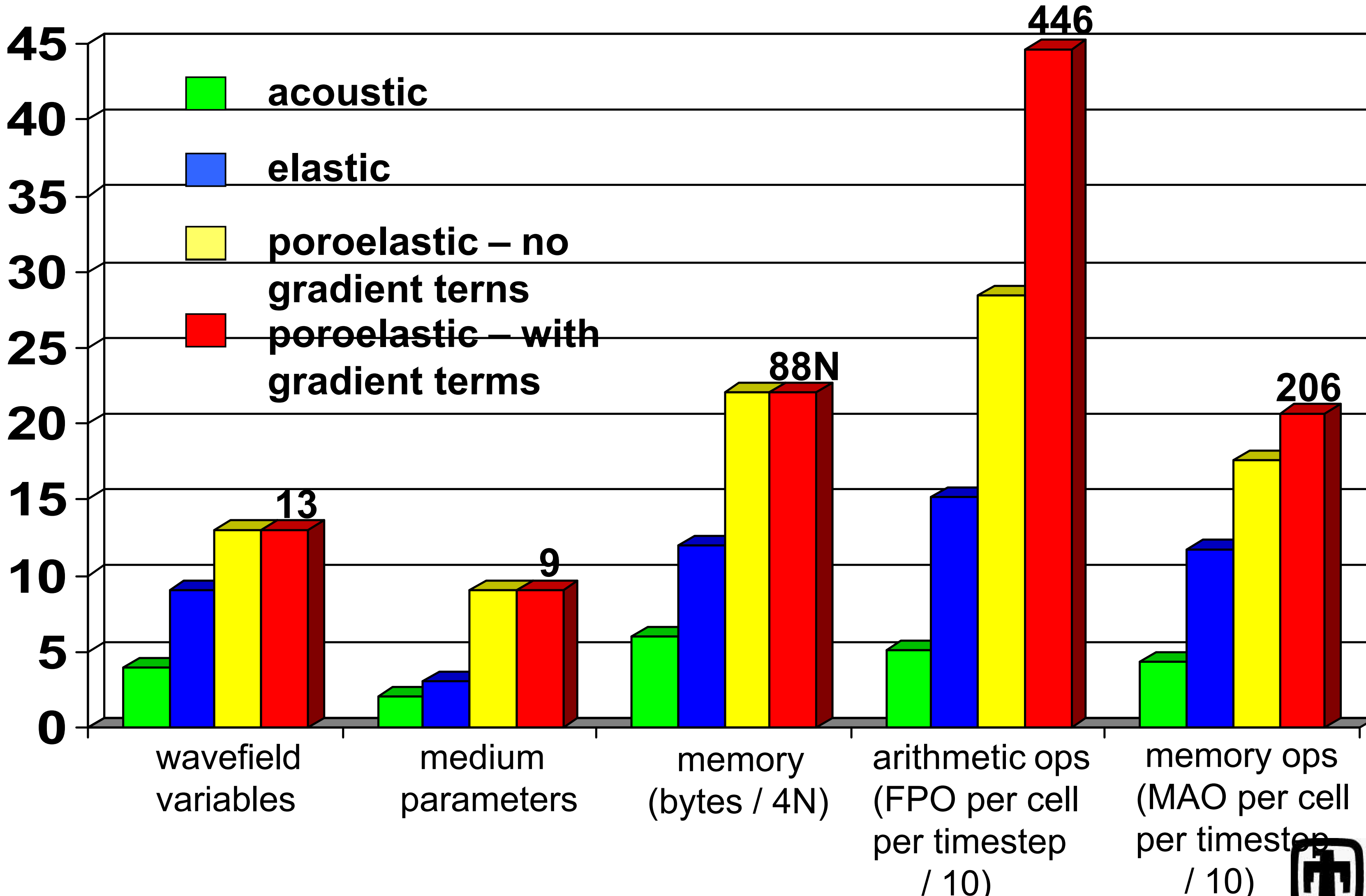
Source: (0,0,0) m
(at center of tunnel)

source pulse terminates here!



Source: (-12.5,0,0) m
(at left end of tunnel)

Time-Domain Finite-Difference Algorithm Comparisons: 3D O(2,4) temporal / spatial staggered solution of 1st-order coupled PDE systems for heterogeneous media



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Run Time Estimation Equation: The Famous "Fourth-Power Law"

$$T_{\mathrm{run}} = \left[(x_{\max} - x_{\min})(y_{\max} - y_{\min})(z_{\max} - z_{\min})(t_{\max} - t_{\min})V_{\max}\right]$$

$$\times \left[\frac{32\tau}{\eta_1 N_{\mathrm{proc}}}\right]\left[\frac{f_{\max}}{\eta_2 V_{\min}}\right]^4$$

$V_{\mathbf{min}}$, $V_{\mathbf{max}}$ **= min, max velocities**
$f_{\mathbf{max}}$ **= max frequency**
τ **= seconds / gridpoint / timestep / processor**
N_{proc} **= number of processors**

FD numerical factors:

$$\Delta t = \eta_1 \frac{\Delta h}{2V_{\max}} \qquad \Delta h = \eta_2 \frac{V_{\min}}{2 f_{\max}}$$

where Δt **= timestep,** Δh **= grid interval**

Assumptions: Uniform (and identical) grid interval in all 3 coordinate directions; identical parallel processors; perfect parallel scalability; neglects ancillary FD operations (ABCs, free-surface, source insertion, receiver interpolation, model input, data output). Ideally, $\eta_1 = \eta_2 = 1$, i.e., algorithm is run at temporal CFL and spatial Nyquist limits.

Example Cost Calculation: Gulf of Mexico Acquisition Scenario

Parameters for Algorithm Execution Time Estimation:

1) V_{min} = 500 m/s (sub-seabed shear), V_{max} = 5000 m/s (salt).
2) f_{max} = 50 Hz.
3) X = Y = Z = 10 km; T = 10 s.
4) η_1 = 1 (ideal); η_2 = 0.4 (5 Δh per λ_{min}).
5) $\tau = N_{FPO} / R$ with $N_{FPO} \approx 150$ (3D elastic VS with O(2,4) FD), and R = 2 GHz (too low?).
6) N_{proc} = 1000 (too high?).

$\Rightarrow$ $T_{run} \approx$ 130 hours!

Cost = T_{run} x N_{proc} x P = $13,000 (with P = dollars/ processor hour ~ 0.1)

10,000 source seismic survey implies $130 million total cost!

(for different parameters, just scale result using the fourth-power law!)

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Cost Sensitivities: Where are the effective improvements?

Earth Model:

$$\frac{\partial \text{Cost}}{\partial V_{min}} = -104.2 \ \$/\text{m/s} \qquad \frac{\partial \text{Cost}}{\partial V_{max}} = 2.6 \ \$/\text{m/s}$$

Simulation Parameters:

$$\frac{\partial \text{Cost}}{\partial X} = \frac{\partial \text{Cost}}{\partial Y} = \frac{\partial \text{Cost}}{\partial Z} = 1.3\ \$/\text{m} \qquad \frac{\partial \text{Cost}}{\partial T} = 1302.1\ \$/\text{s} \qquad \frac{\partial \text{Cost}}{\partial f_{max}} = 1041.7\ \$/\text{Hz}$$

Algorithm Construction:

$$\frac{\partial \text{Cost}}{\partial N_{fpo}} = 86.8 \ \$/\text{FPO}$$

!

Computational Hardware:

$$\frac{\partial \text{Cost}}{\partial N_{proc}} = 0.0\ \$/\text{m/s} \qquad \frac{\partial \text{Cost}}{\partial R} = -651.0\ \$/0.1\,\text{GHz}$$

?

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Algorithm Research and Development Issues: Faster Speed, Reduced Memory, Higher Accuracy, and Superior Seismics!

Different Media Types:

- anisotropic elastic and anelastic (attenuative/dispersive) media.
- improved treatment of poroelasticity, or "beyond Biot".

Algorithmic Issues:

- higher order temporal and spatial FD operators.
- optimized FD operator coefficients.
- better ABCs (PML?), allowing effective treatment of the 'thin model".
- efficient treatment of piecewise homogeneous or "factorized" media.

Hybrid Algorithms:

- mixed physics/math approach for multiple-media-type models.
- TD finite-integro-difference method for solid absorptive media.
- spatial FD operator order switching for models with large velocity range.

Sources and Receivers:

- multiple simultaneous sourcing for order-of-magnitude speedup.
- compressional/shear wavefield separation via pressure/rotation receivers.
- wavefield directional filtering via Poynting vector implementation.