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An Extended Finite Element Method with Analytical Enrichment for Cohesive Crack Modeling

First Annual Sandia National Laboratories Fracture Forum

November 2008

Jim Cox

Applied Mechanics Development (1526)

Sandia National Laboratories

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Preview

- Introduction
- PUFEM/XFEM Displacement Field Enrichment
- Numerical Formulation Issues
- Results for a Model Problem
- Extensions for Mixed mode problems
- Mixed mode Examples
- Observations and Conclusions

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Study Introduction

Objective: A "valid" means of modeling material localization in finite element analyses.

Goals:

- applicable to cohesive zone modeling => strong discontinuity
- arbitrary orientation of discontinuity relative to mesh
- "continuous discontinuity"

Approach: Develop an extended FEM (XFEM) that allows the displacement field to be enriched in the neighborhood of a strong discontinuity.

- can represent a discontinuity without mesh refinement
- can potentially represent the gradients near a surface of localization without mesh refinement

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Background

Initial related studies:

- Melenk and Babuska (1996)
  - theory for Partition of Unity FEM (PUFEM)
- Belytschko and Black (1999)
  - developed PUFEM for LEFM $\rightarrow$ XFEM
  - used asymptotic displacement fields near a crack tip for enrichment

Some Recent Related Studies

XFEM/PUFEM-Cohesive Zone Studies

- Wells and Sluys (2001)
- Moes and Belytschko (2002)
- Zi and Belytschko (2003) -- tip function addresses tip position but not the field
- Xiao and Karihaloo (2006) -- asymptotic fields
- …

GFEM

- Strouboulis, Copps, Zhang, and Babuska (2000, 2001, 2003) numerical enrichment functions -- handbook functions

History of Study

- ARL (2000-2001) -- localization in armor and penetrators
    - general formulation and algorithms
- SNL
    - HDBT/CSRF
        - enrichment functions and algorithms
    - ESRF -- assessment of PUFEM/XFEM for fracture
        - analytical enrichment functions, enrichment schemes, and mixed-mode
        - partial implementation in Tahoe
    - LDRD -- fatigue cracking
        - stress smoothing and limits of enrichment functions
    - ESRF -- ductile fracture
        - formulation for finite deformations
        - implementation in Tahoe

Past Work

Future Work

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PUFEM Displacement Field Enrichment

- Standard FEM
- PUFEM/XFEM

Global displacement approximations

$$u(x) = \sum_{i=1}^{N_\Phi} \Phi_i(x) u_i$$

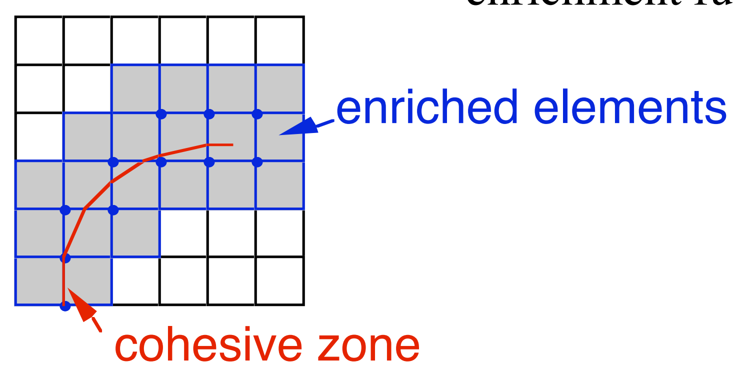
$$u(x) = \sum_{i=1}^{N_\Phi} \Phi_i(x) u_i + \sum_{j=1}^{N_\Lambda} \sum_{i=1}^{N_\Phi} \Lambda_j(x) \Phi_i(x) \alpha_{ij}$$

Element displacement approximations

$$u(x) = \sum_{i=1}^{N_N} N_i(x) u_i$$

$$u(x) = \sum_{i=1}^{N_N} N_i(x) u_i + \sum_{j=1}^{N_\Lambda} \sum_{i=1}^{N_N} \Lambda_j(x) N_i(x) \alpha_{ij}$$

enrichment functions



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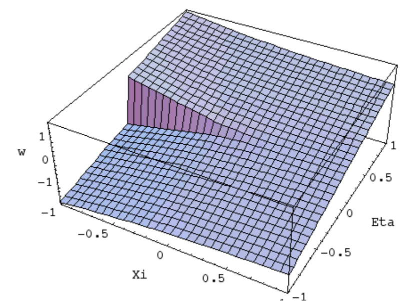
“My Path to Enrichment”

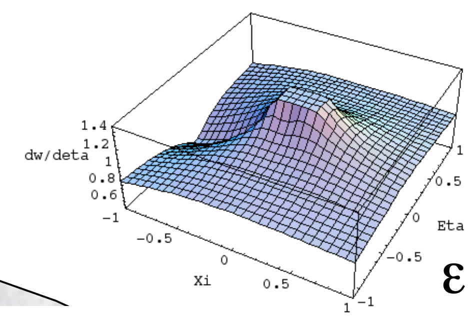
“I am not discouraged, because every wrong attempt discarded is another step forward. I have not failed. I’ve found 10,000 ways that won’t work.” – Thomas Edison

Formulated simple series that incorporated a discontinuity.



u_2

Formulated simple functions that had key features of accurate numerical results.



ε_{22}

Analytically derived enrichment functions based upon the Muskhelishvili formalism.

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Enrichment Functions: An Analytical Source

Muskhelishvili formalism

Hong & Kim (2003) obtained a series solution to the inverse problem

Zhang & Deng (2007) obtained "asymptotic solutions"

– both assumed linear elastic isotropic material (except for cohesive zone)

Additional analysis was used to:

verify the proposed solutions

extend them for field variables required by the XFEM

- Displacements

$$u_1 + iu_2 = \frac{1}{2\mu}\left\{\kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)}\right\}$$

where φ and ψ are analytic functions, and $z = x+iy$.

- Another set of analytic functions simplify $u_{i,j}$ and σ_{ij} expressions

$$\Phi(z) = \varphi'(z) \qquad \Omega(z) = \left[z\varphi'(z) + \psi(z)\right]'$$

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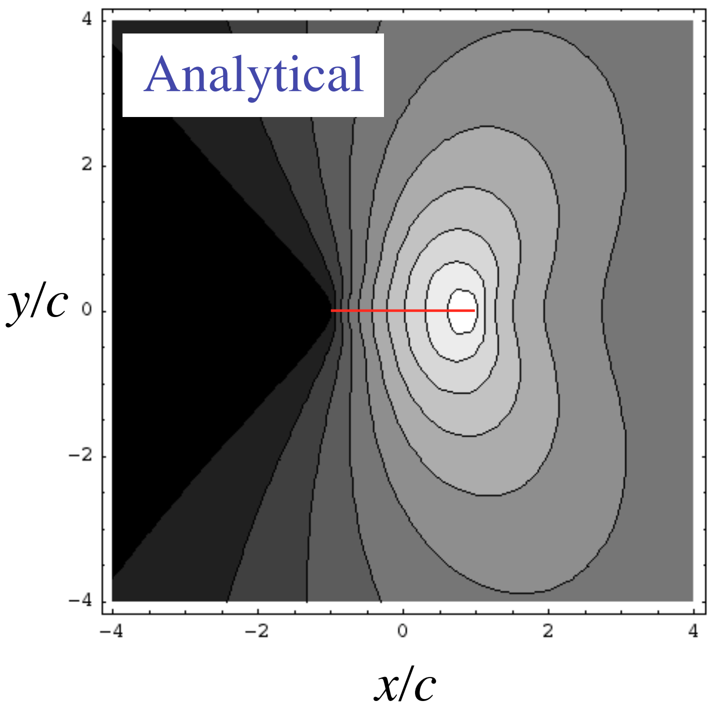
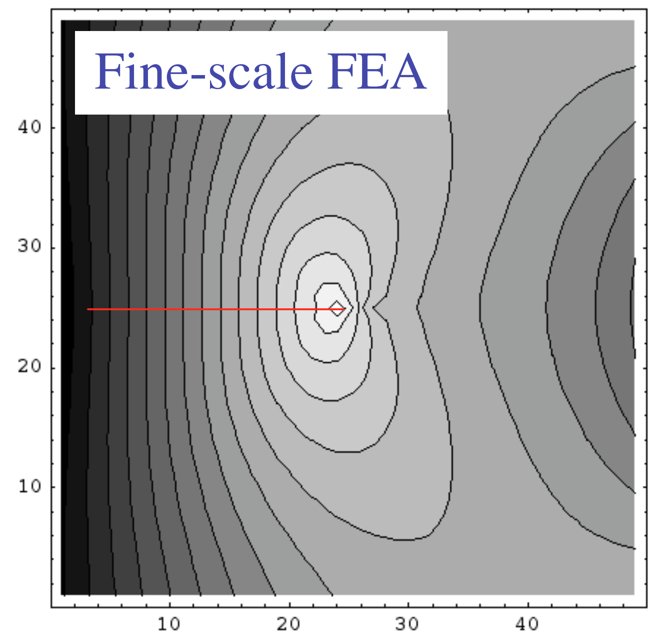
Enrichment Functions: An Analytical Source

Qualitative comparison of σ_{22} with fine-scale FEA

- Analytical ~ First terms in series for Hong & Kim solution
- "Fine-scale" FEA ~ results for finely meshed FEA with interface el.

Note: problems differ and CZ sizes are not to the same scale.





Cohesive zone length = $2c$

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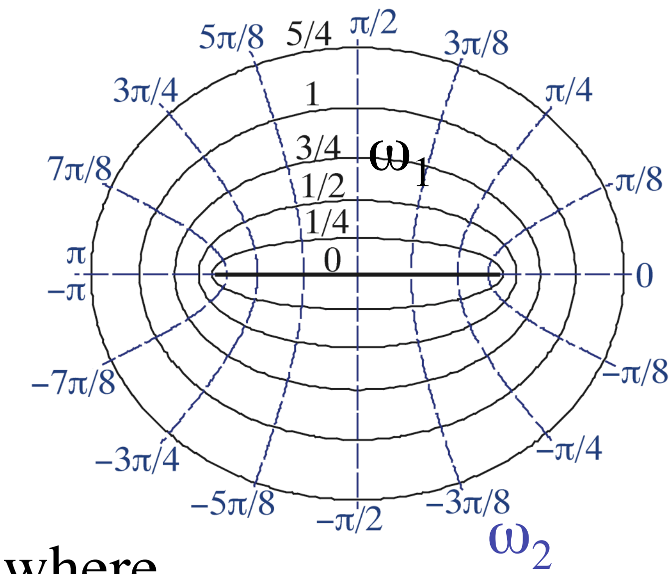
Enrichment Functions: An Analytical Source

Zhang & Deng (2007) solve the problems in terms of elliptic coordinates (ω)

$$z = c\cosh(\omega)$$



Symbolically the inverse map is give by

$$\omega = \cosh^{-1}(z/c)$$

complex analysis leads to more forms for both of these.

They adopt a Westergard stress function where

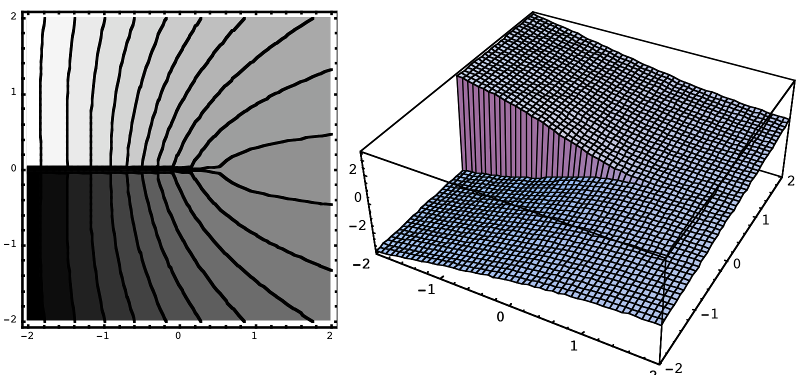
$$\Phi(z) = Z(z) \qquad \Omega(z) = Z(z)$$

$$Z(z) = -\frac{\mu B\left[(\lambda-2)e^{\lambda\omega(z)} - \lambda e^{(\lambda-2)\omega(z)}\right]}{2\lambda(\lambda-2)}$$

They are argue that λ=-1/2 Eigenvalue -> asymptotic solution.

Mode-I Enrichment Functions

- Based upon the asymptotic solutions of Zhang & Deng



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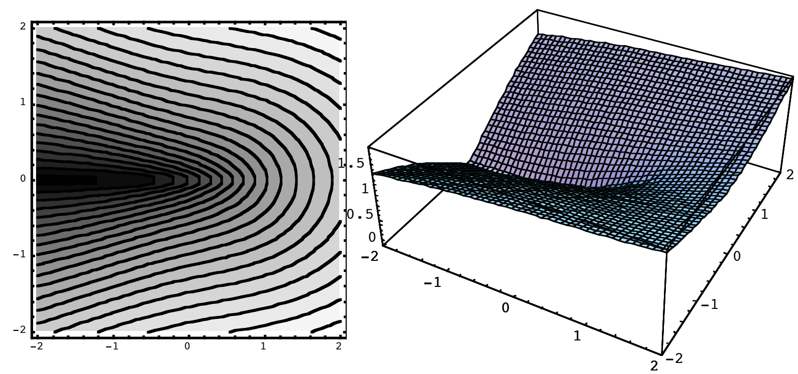
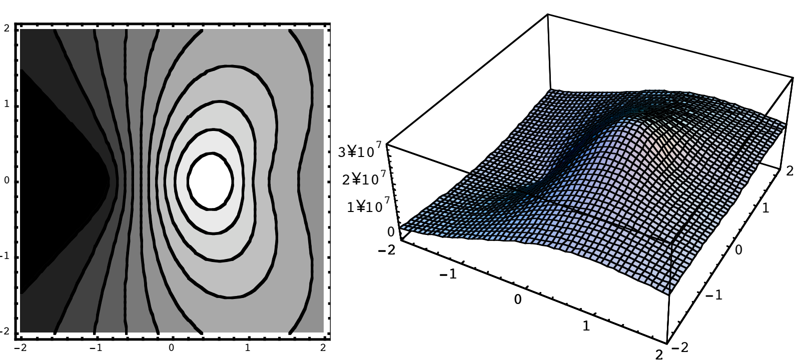
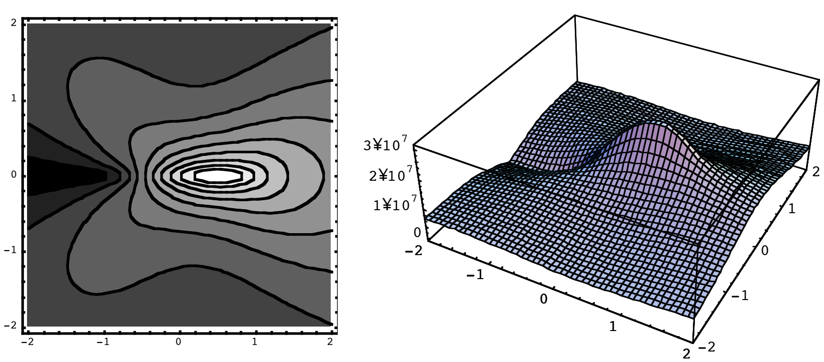


u_1



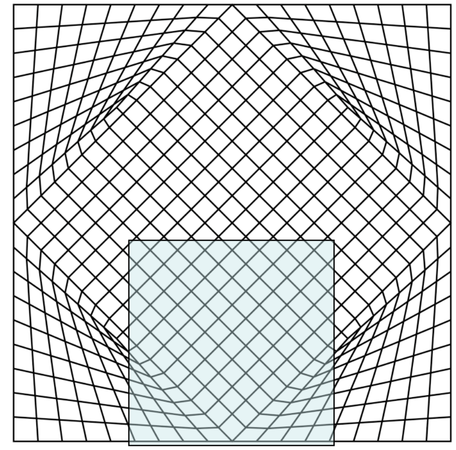
σ_{22}



σ_{11}

Neighborhood Enrichment

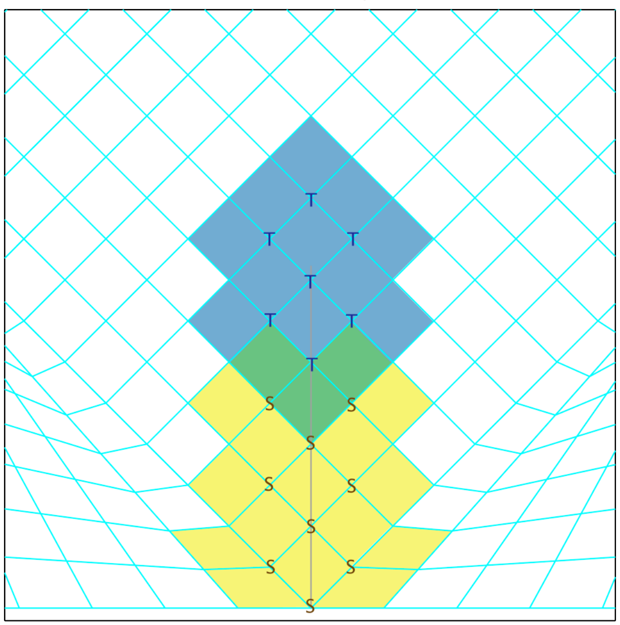
Aka the Mr. Roger's modification



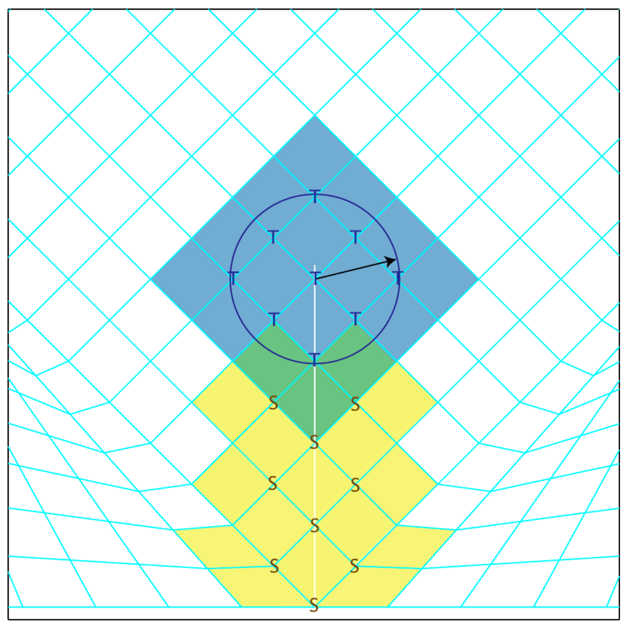
Enriches additional nodes within a user-defined neighborhood of the tip.

Done each time the tip enters a new element.

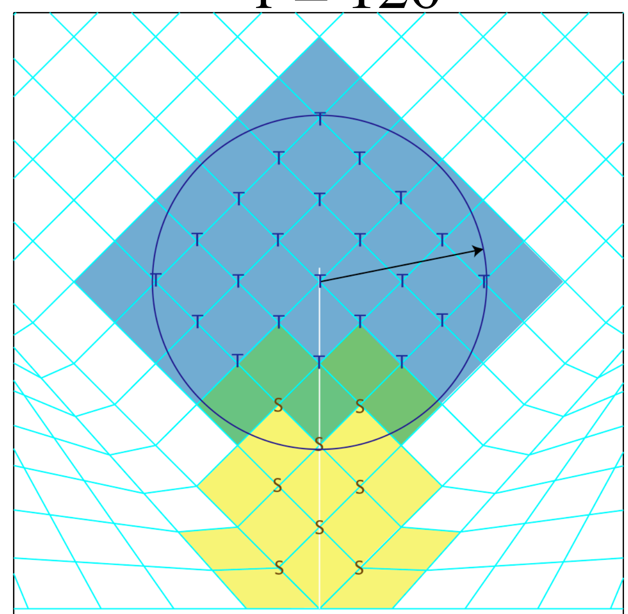
r = 0



r = 63

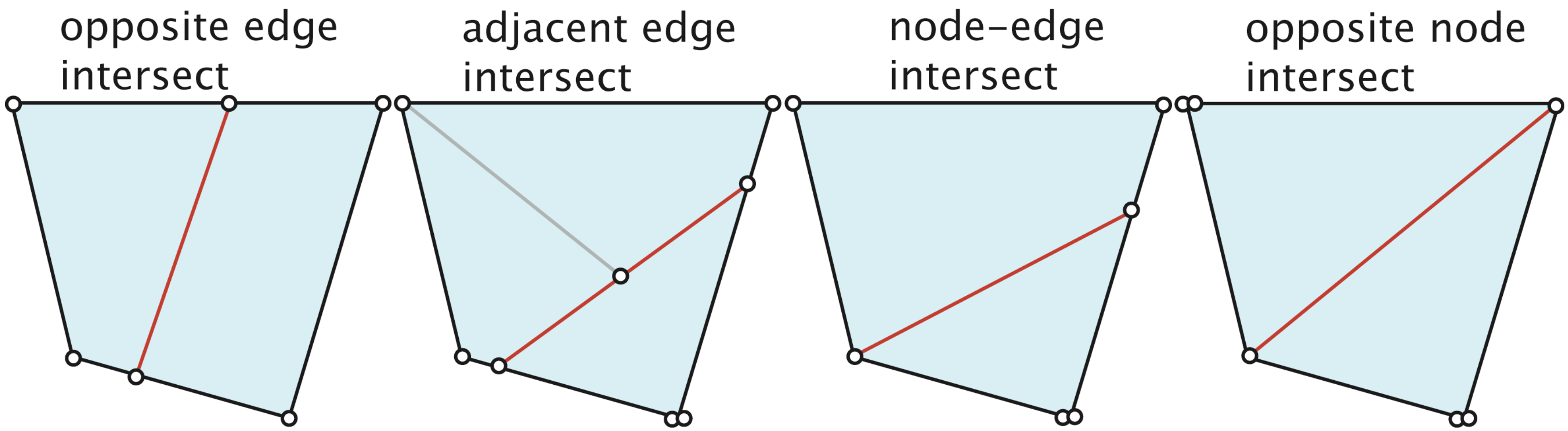


r = 126



Numerical Formulation Issues

- Element Integration



- Key solver issue: new DOFs that result from adding enrichment to nodes do not have a good initial values $\Rightarrow$ nonlinear solver can have problems.

  Solution: Penalty relaxation in a multi-level solver

Numerical Formulation Issues

```
Assign equation numbers;   Determine storage for K;
Repeat (* time increment loop *)
|    Repeat (* outer level solver loop -- aka localization loop *)
|    |    Update K & R
|    |    Reset penalty number to large value when entering a new element, else 0
|    |    …
|    |    Repeat (* penalty reduction loop *)
|    |    |    Relax the penalty number
|    |    |    Reset line search
|    |    |    Repeat (* nonlinear iteration loop *)
|    |    |    |    Factor K
|    |    |    |    Forward eliminate & back substitute to obtain dUiter
|    |    |    |    Repeat (* line search loop *)
|    |    |    |    |    Search line for dU_iter
|    |    |    |    |    …
|    |    |    |    Until (||R||<R_toler) OR (||R||<||R_old||)
|    |    |    |    …
|    |    |    Until ||R||<R_toler
|    |    Until penalty number is reduced to zero
|    |    …
|    Until localization is complete
|    U:= U + dU_step;    dU_step:= 0;    U_old:= U
|    …
Until time stepping is complete
```

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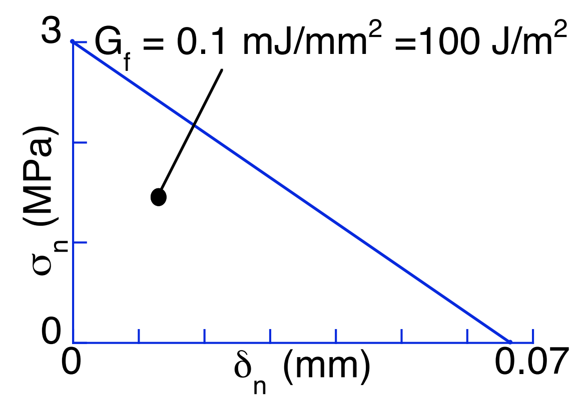
Preview of Results

- Mode-I Model Problems -- emphasis on reproducing the cracking history
  - Results for aligned meshes
  - Results for skewed meshes
- Extensions for "mixed mode"
- Mixed mode examples

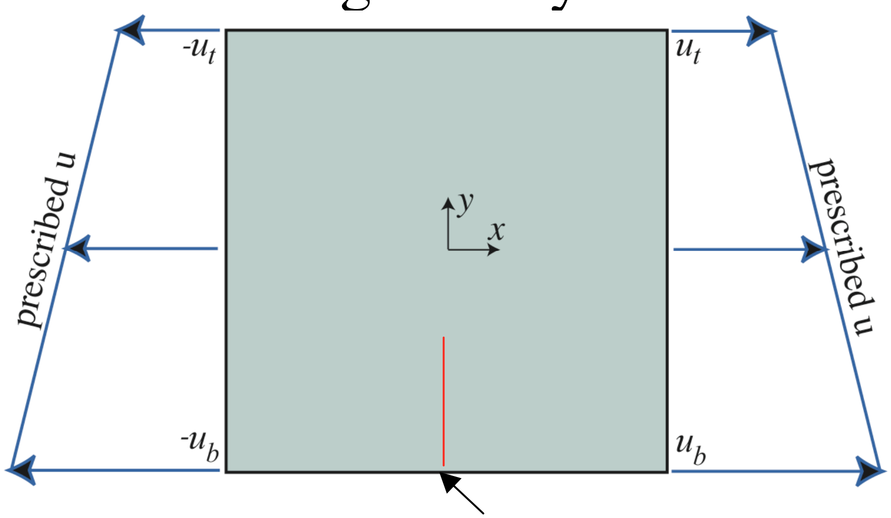
quasibrittle

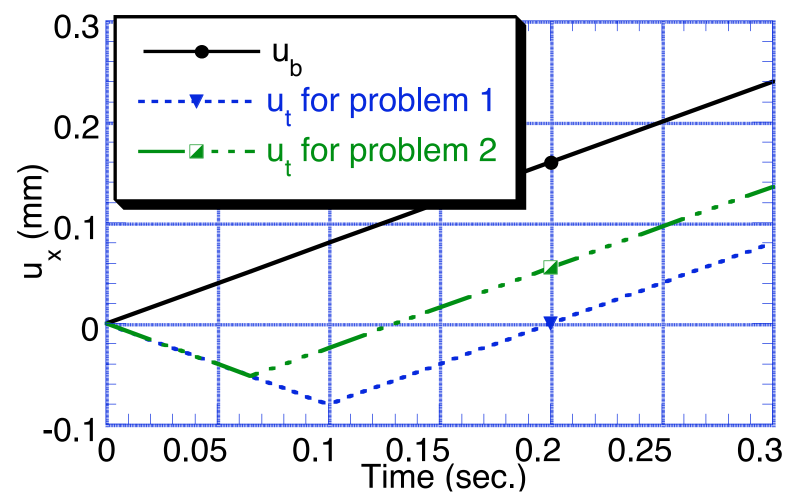
Initial Simple Test Problems

- Concrete test problems
  - relevant to HDBT
  - domain 1 m x 1 m
  - process-zone size ~ O(250 mm)
  - representative concrete tensile properties (except for simplified linear softening)
  - mode I quasistatic crack propagation



Problem geometry





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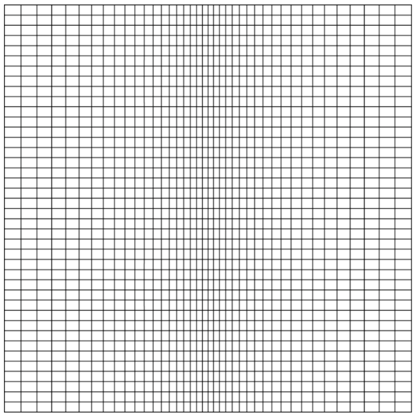
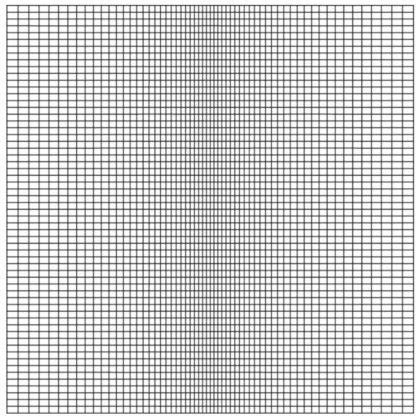
Spacial Discretizations

- Fine FEM meshes – accurate reference solution

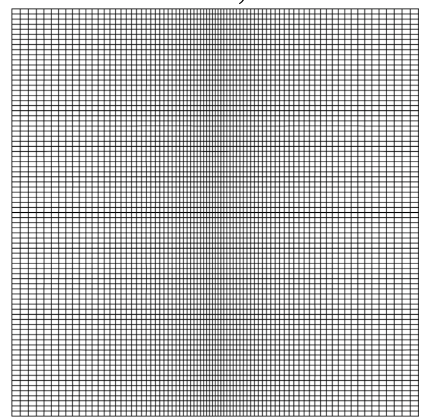
41x40 ~ 3444 dofs



61x60 ~ 7564 dofs

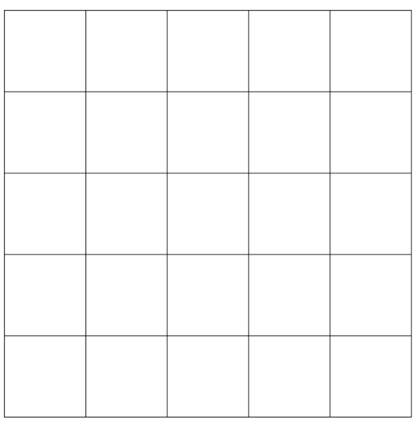


81x80 ~ 13,284 dofs

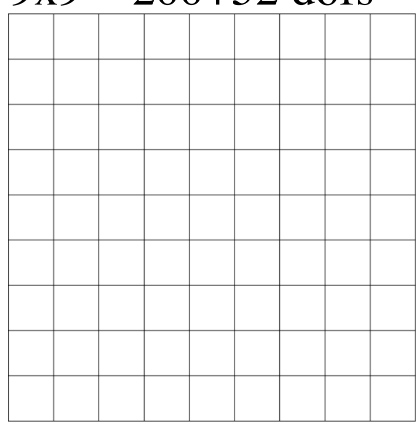


- XFEM – Aligned Meshes

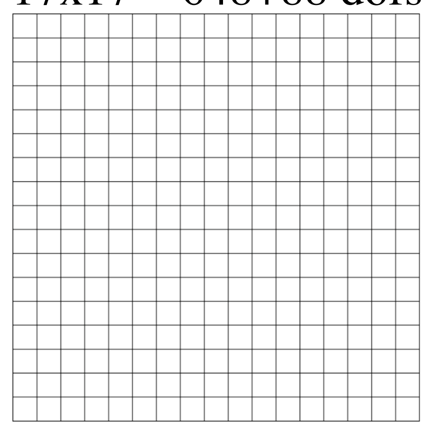
5x5 ~ 72+36 dofs



9x9 ~ 200+52 dofs



17x17 ~ 648+88 dofs



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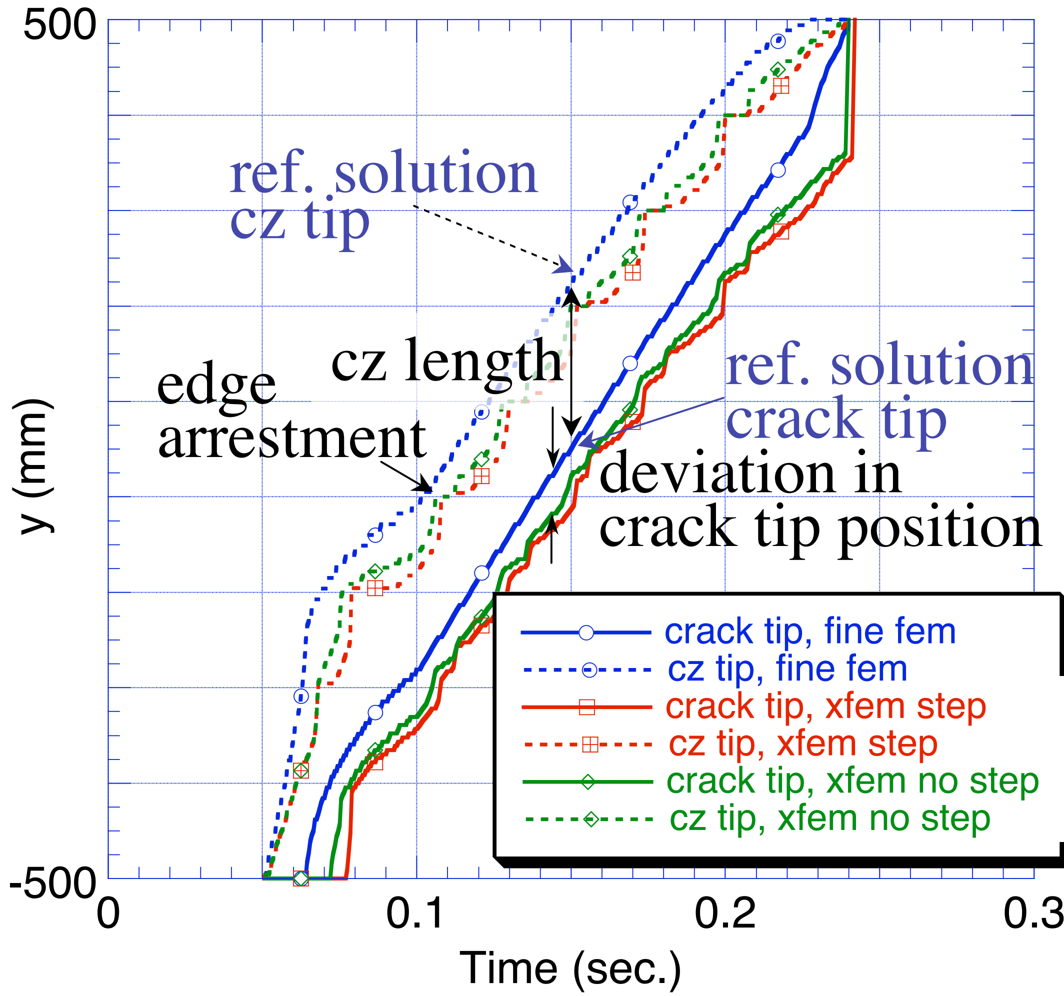
Extremes Histories



ave. dev. ~ 62 mm

ave. dev. ~ 46 mm

9x9 mesh, c = 125 mm

Transition to step is not necessary.

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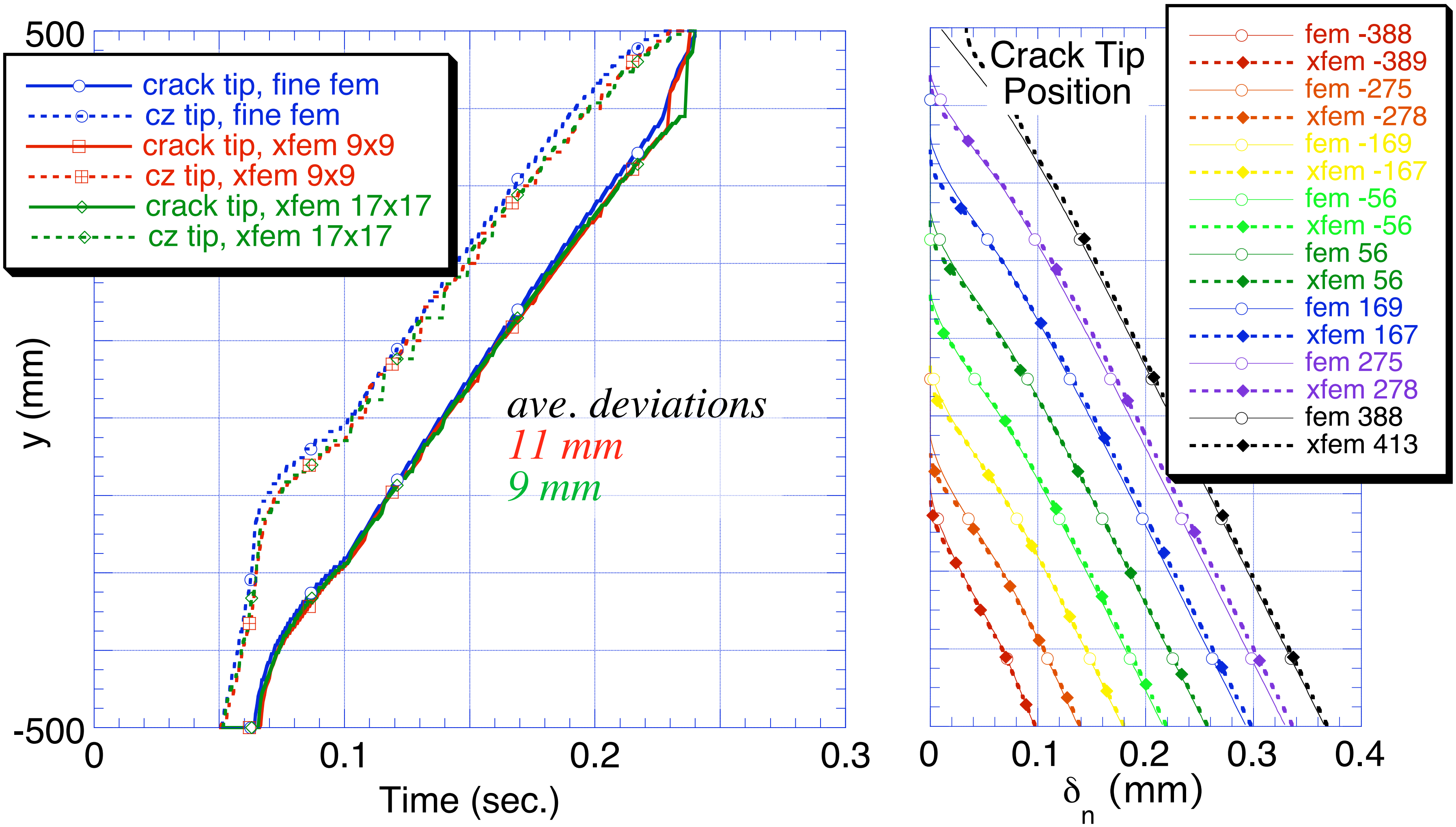
Extremes Histories and Crack Profiles



c = 50 mm

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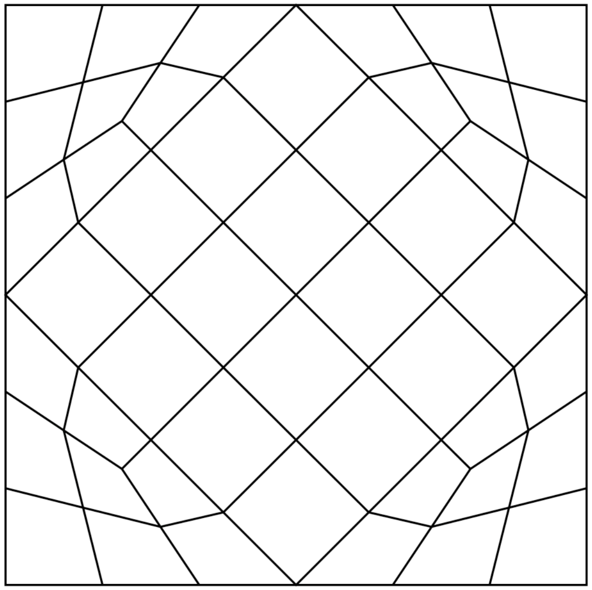
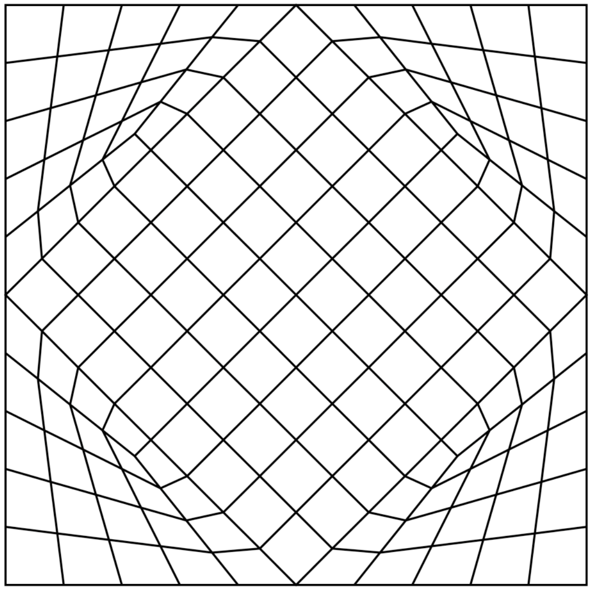
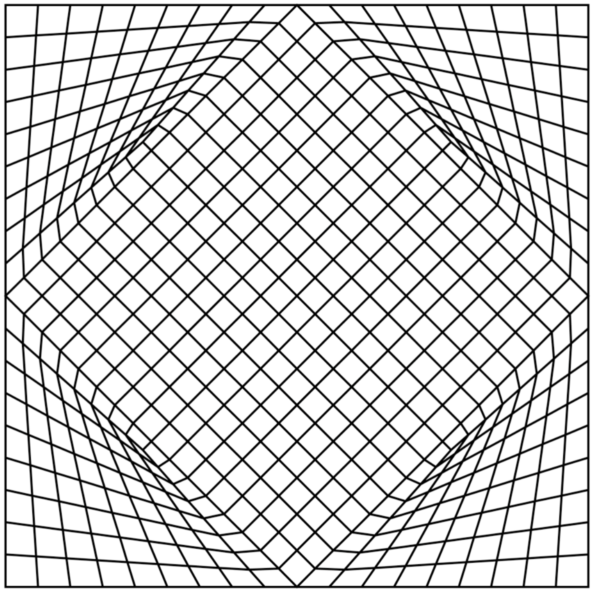
XFEM Skewed Mesh Tests



4x4 @ 45°

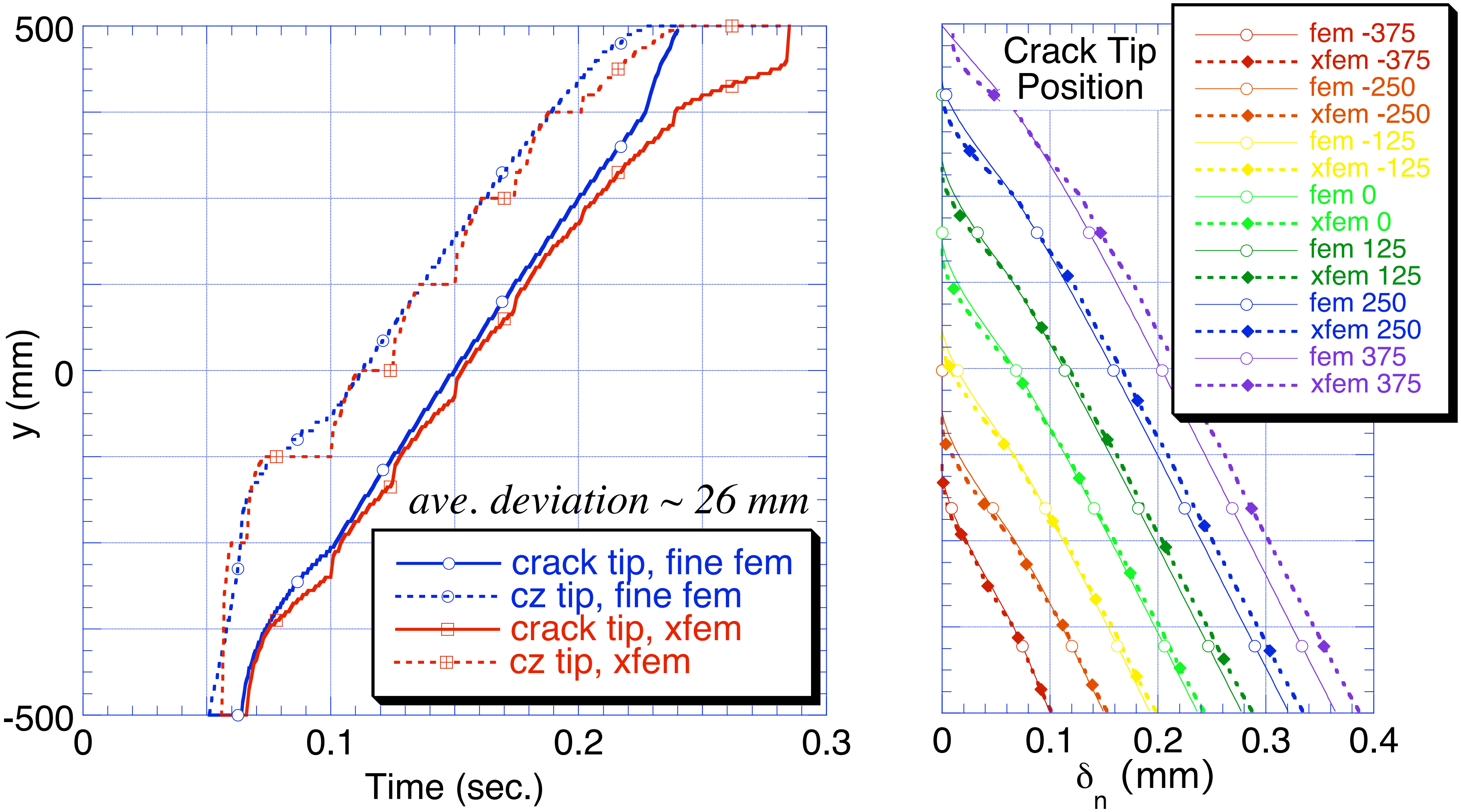


8x8 @ 45°



16x16 @ 45°

Extremes and Crack Profiles



8x8 mesh, c = 75 mm

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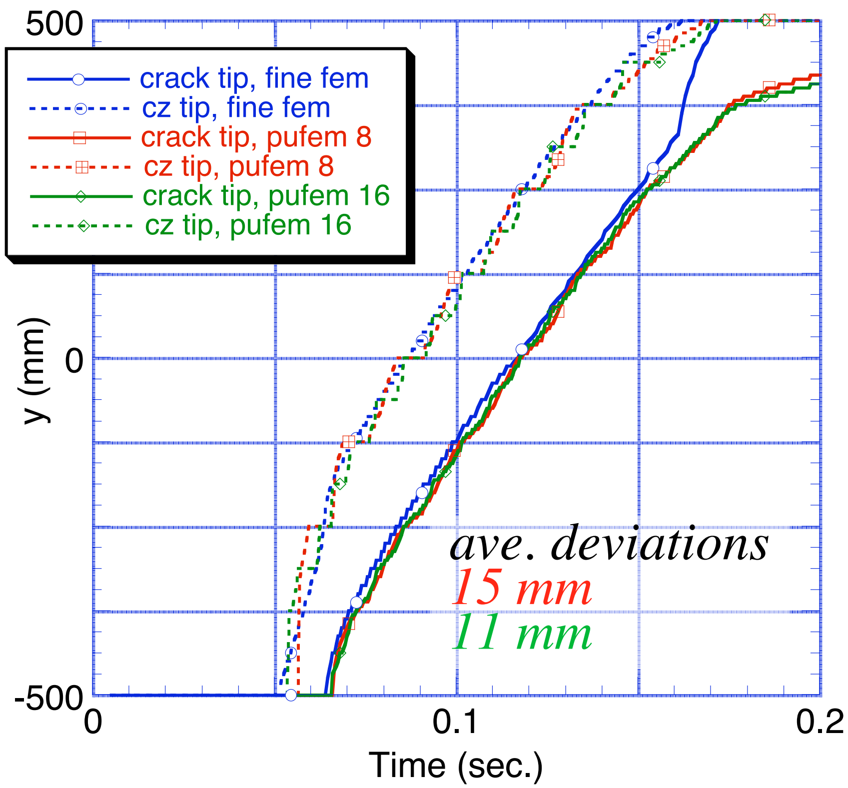
Extremes Histories



Problem 2

c = 75 mm

Crack Propagation and Direction Calculations

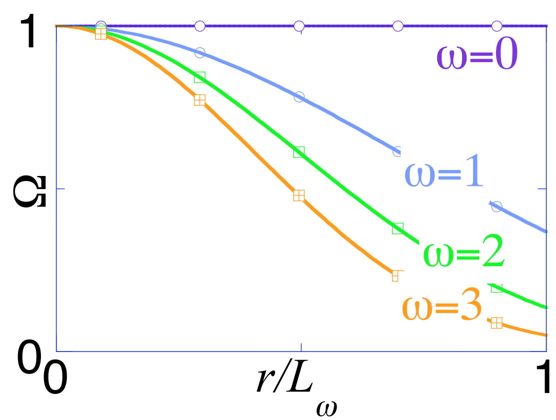
"Stress smoothing" used when a crack enters a new element

- Deviation between a polynomial approximation and the FEM approximation

$$d(\mathrm{x}) = \sigma^{p}(\mathrm{x}) - \sigma^{fem}(\mathrm{x}) = c_0 + c_1 x + c_2 y + \ldots - \sigma^{fem}(\mathrm{x})$$

- Weighting function

$$\Omega(\mathrm{x}) = \exp\left[-\omega\left(\frac{r}{L_\omega}\right)^2\right]$$

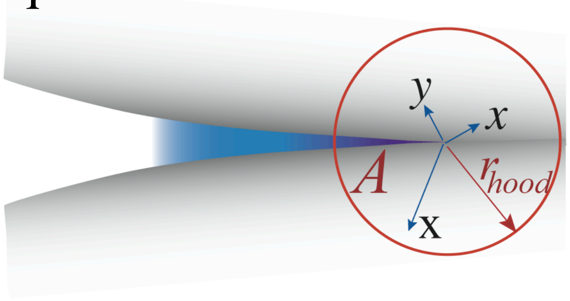


- Residual measure

$$R^2 = \int_A \left[\Omega(\mathrm{x}) d(\mathrm{x})\right]^2 dA$$

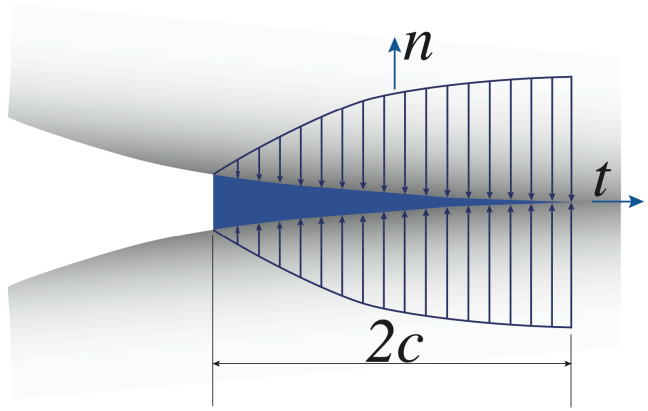
- Using the Gauss point values → weighted least squares solution

$$R^2 \approx \sum_{i=1}^{n} \left[\Omega(\mathrm{x}_i) d(\mathrm{x}_i)\right]^2 |J_i| w_i^{gauss}$$



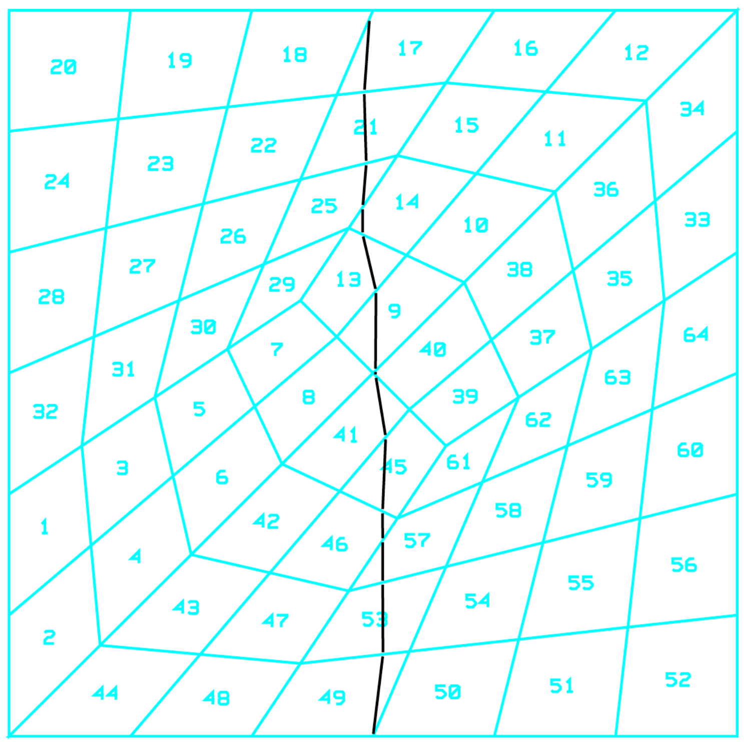
Cohesive Zone Insertion

- In theory insertion occurs when $\sigma_{max} > \sigma_t$
- Issue: residual error between continuum and cohesive zone



- Numerical criterion: $\sigma_t > \sigma_{max} > \sigma_{cz}$

Model Problem with Arbitrary Intersects



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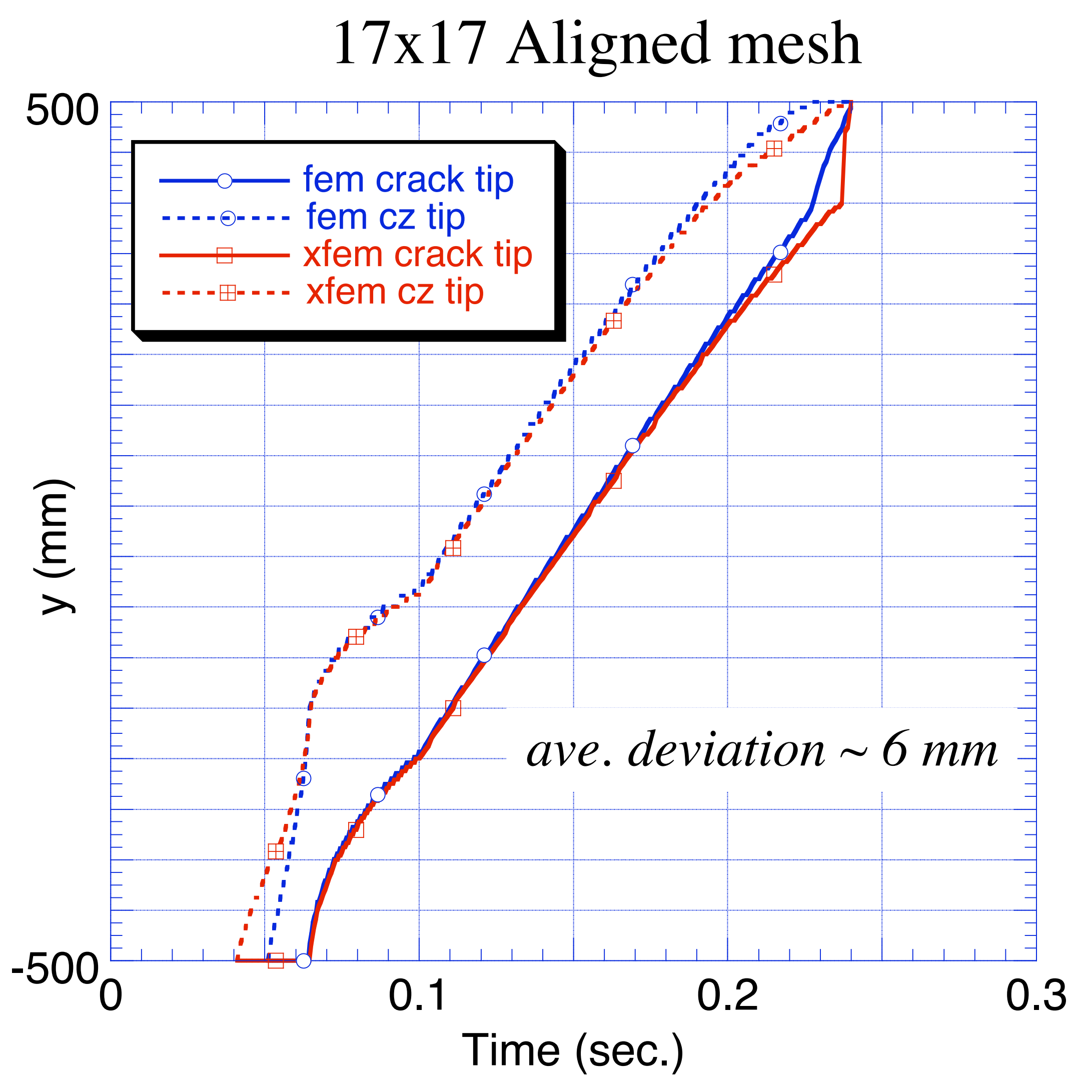
Model Problem with Stress Smoothing



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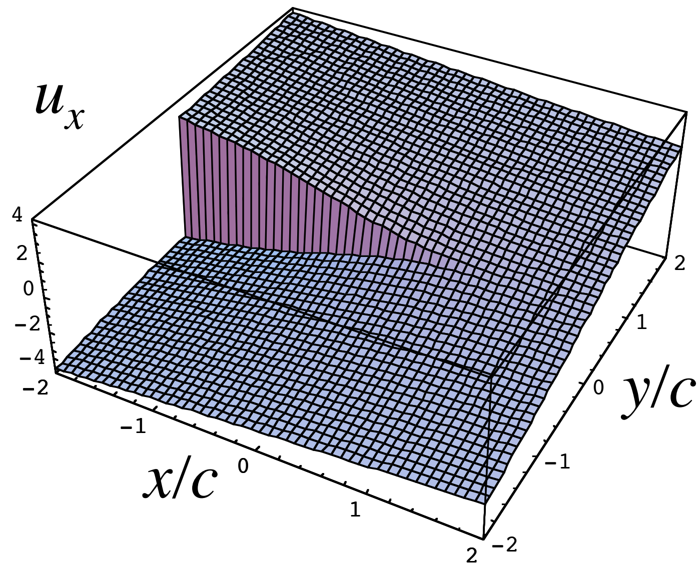
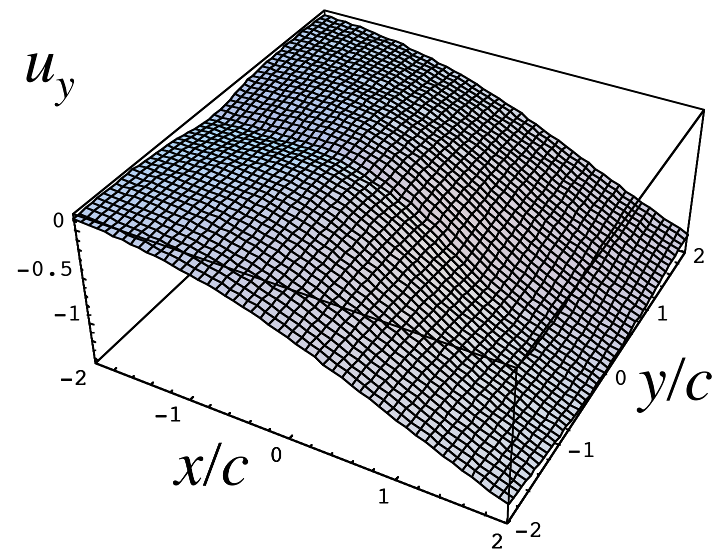
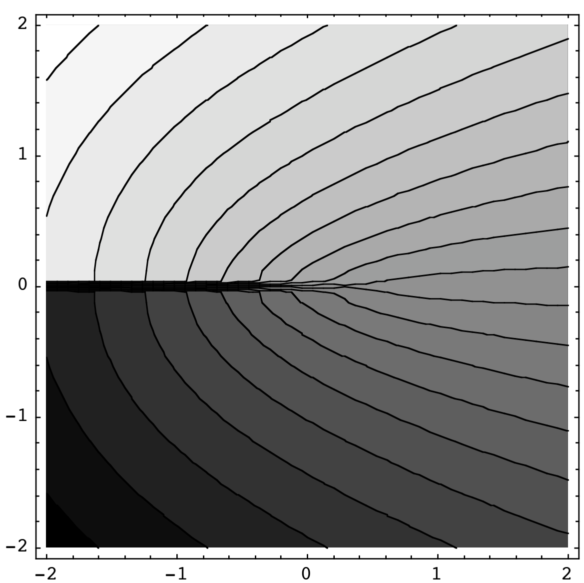
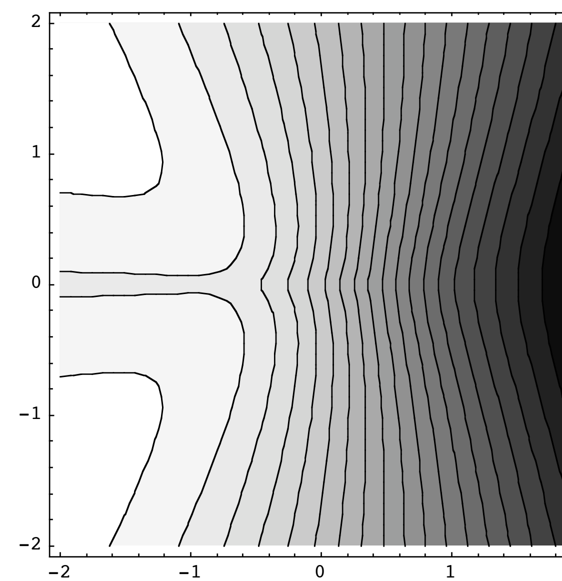
Mode II Enrichment Functions

Based on Zhang & Deng (2007)









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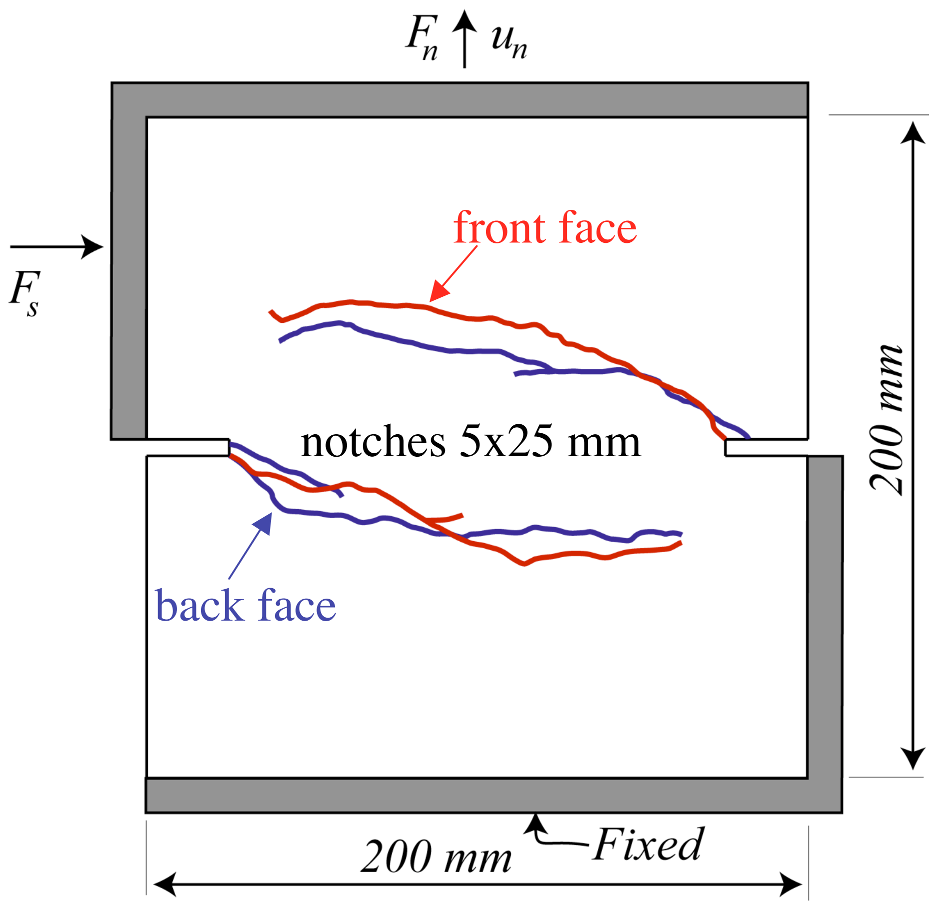
Mixed Mode Fracture Problem

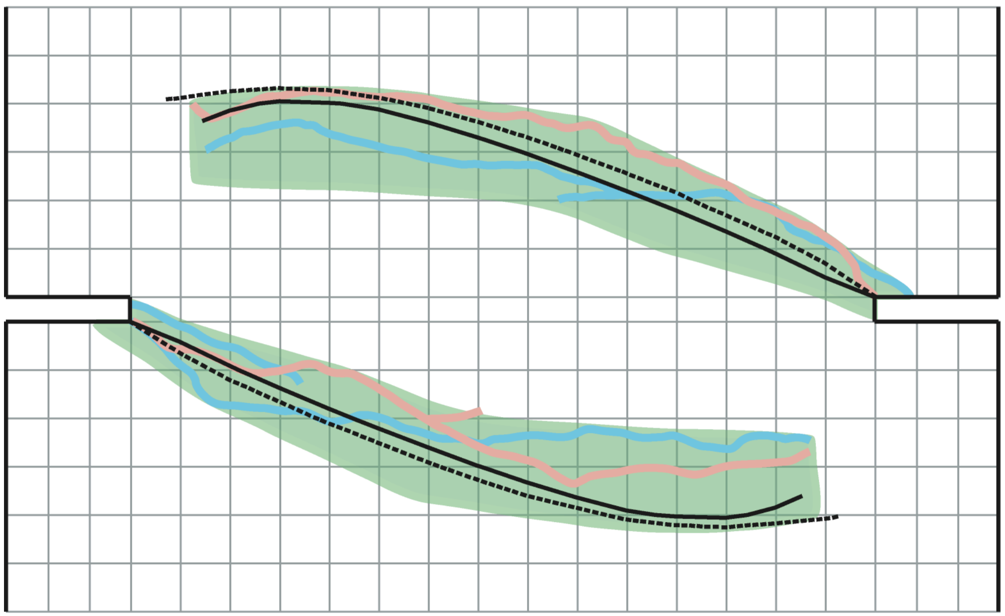
Double edge-notched specimen (Nooru-Mohamed 1992)
and experimental crack paths
Concrete square, 50 mm thick

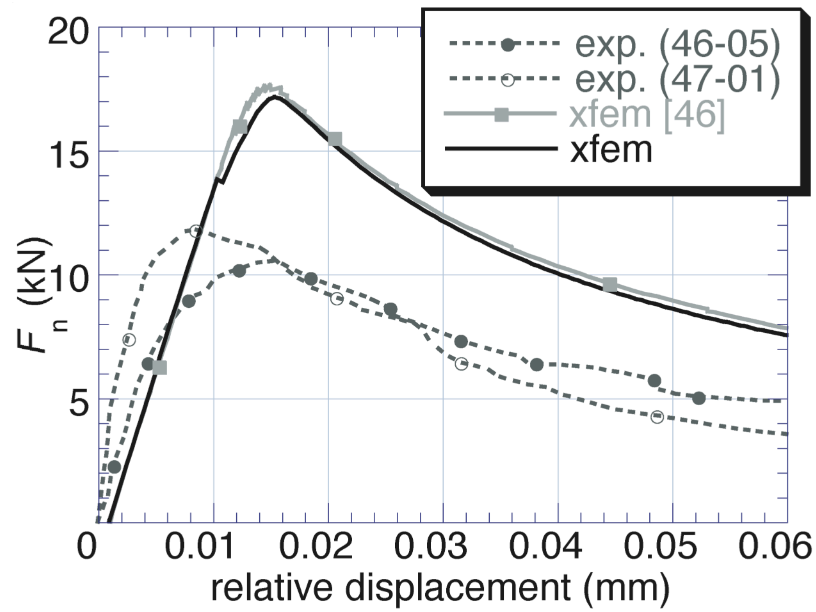
Load Path 4b:
$F_s = 10$ kN



Mixed Mode Fracture Problem

XFEM simulation results for test give crack paths within the experimental scatter.





$r_\sigma = 20$ mm ~ $2h$, n=3, ω=0, full-disk, $\sigma_{cz}/\sigma_t = 0.8$

[46] Meschke & Dumstorff (2007)

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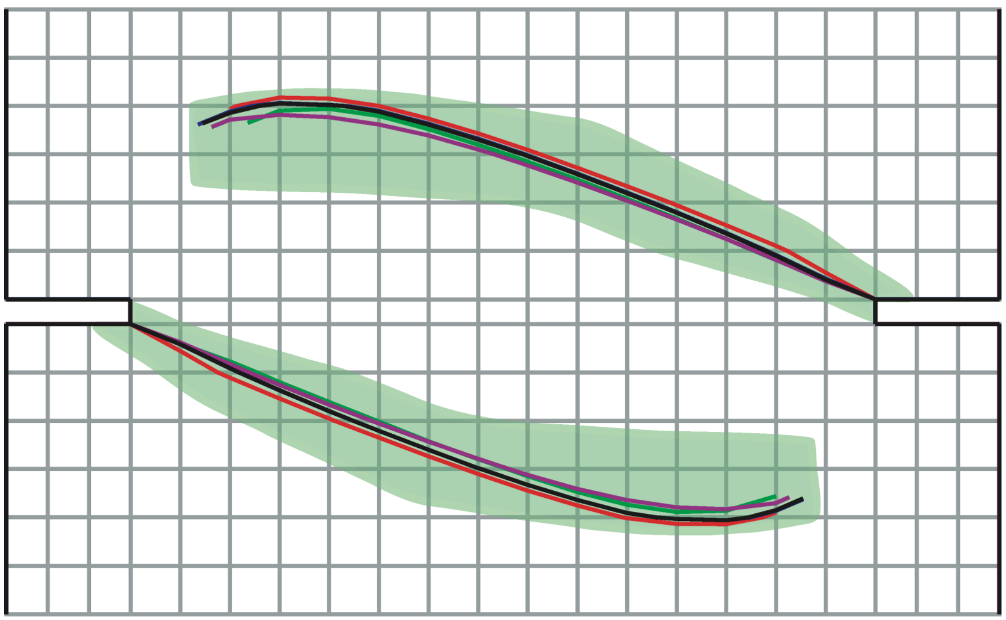
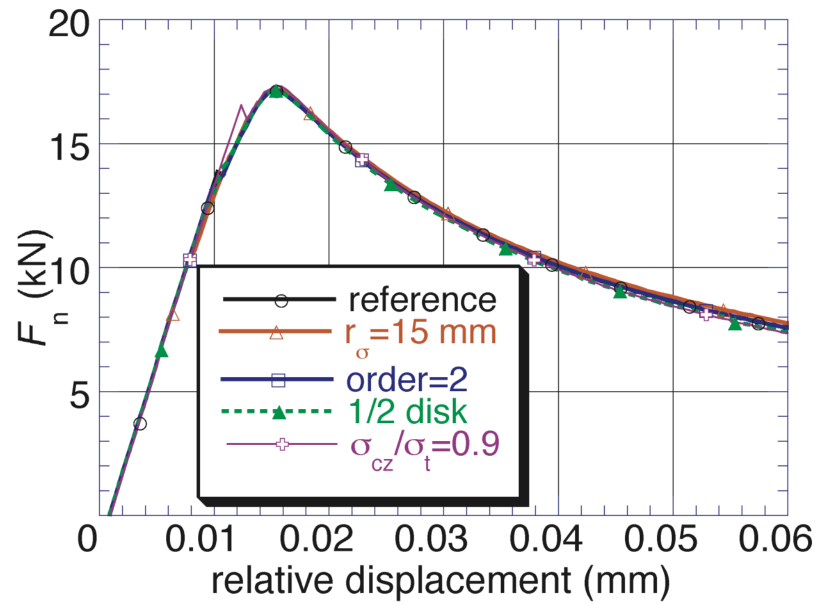
Mixed Mode Fracture Problem

XFEM simulation results varying several parameters.





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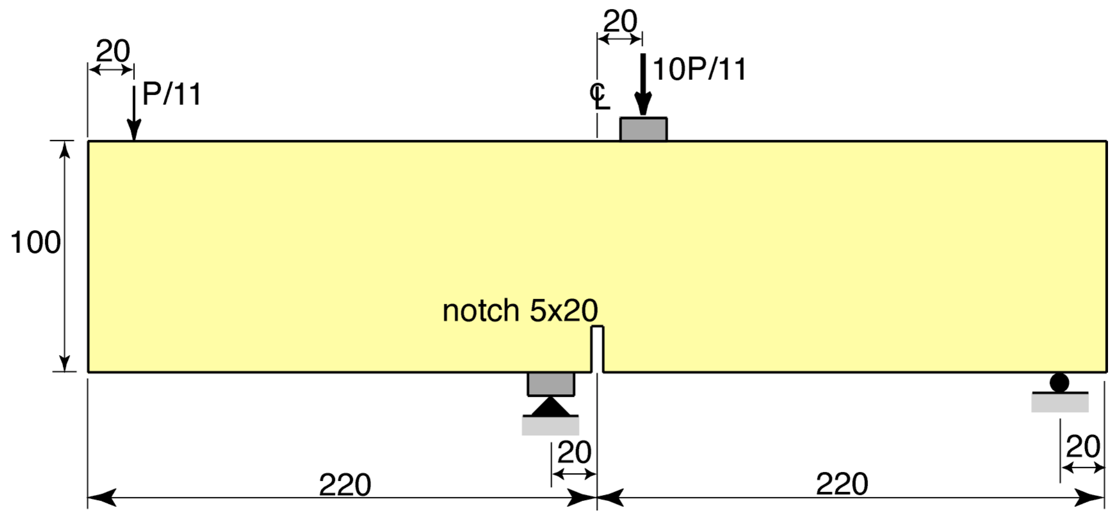
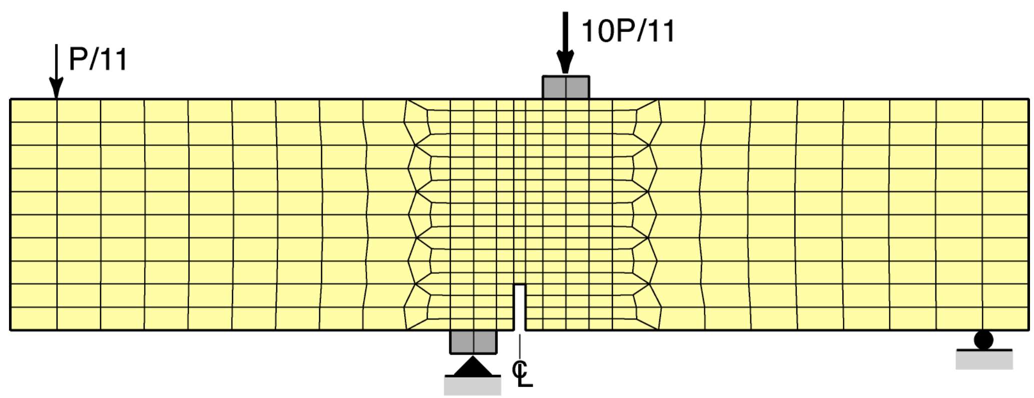
Single Edge-Notched Beam Specimen

Experimental work of Schlangen (1993)

units ~ mm, thickness = 100 mm





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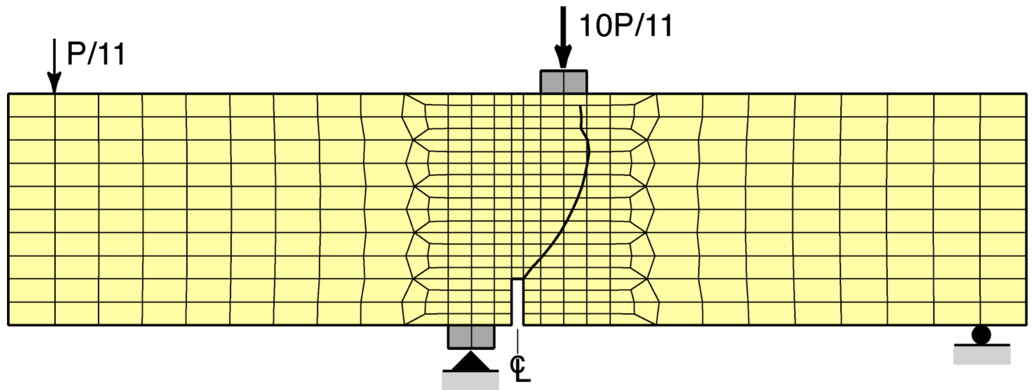
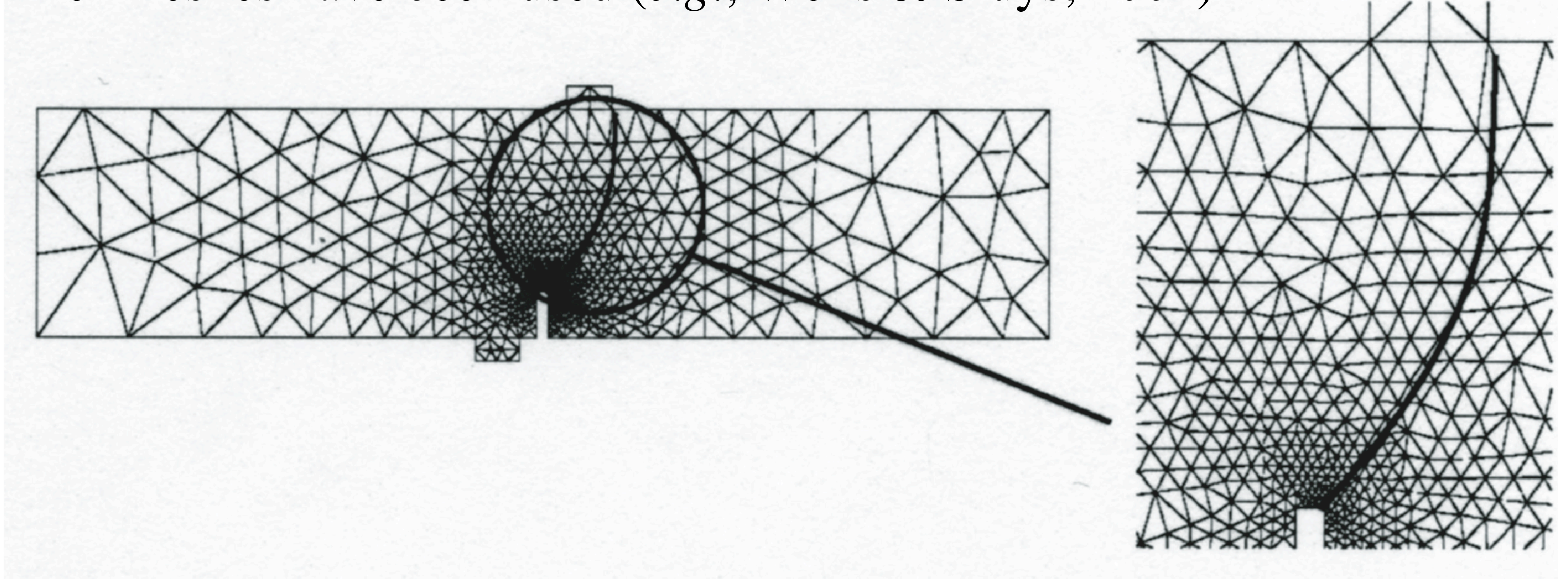
Single Edge-Notched Beam Specimen



Finer meshes have been used (*e.g.*, Wells & Sluys, 2001)



Observations & Conclusions

- No free-lunch -- algorithm complexity ↑ with analytical enrichment
- Analytically enriched XFEM for cohesive zone modeling of localization has potential.
- Not the best approach for every application
- Several open issues, *e.g.*:
  - Value of *c* and its possible adjustment
  - Can the accuracy be improved?
  - How useful is analytical enrichment for materials that are:
    - anisotropic?
    - Inhomogeneous?
    - inelastic?
    - amenable to finite deformations?
  - Could the method facilitate stochastic FEA of fracture?
  - How difficult it this to incorporate into a production code?

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Questions?

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