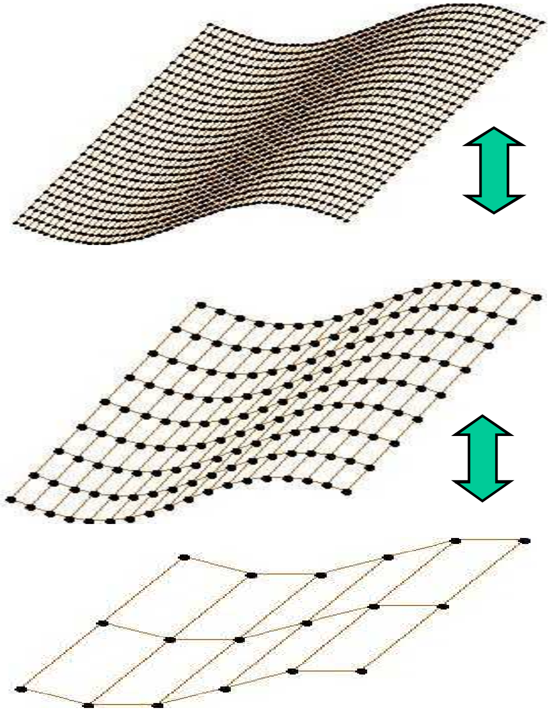
Multilevel Project

SAND2009-0126P



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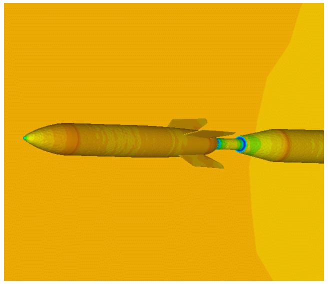
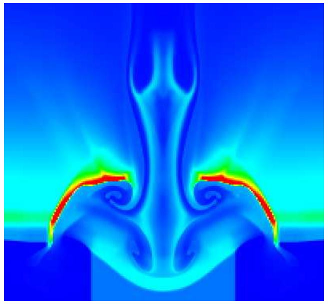
Eric Cyr (starts 1/09)

Sandia National Laboratories

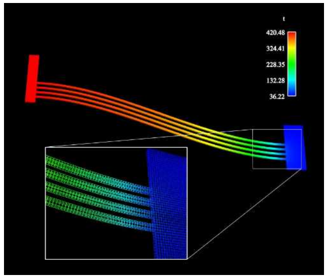
Multilevel/ML Projects

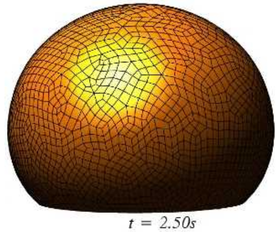
- <u>Research: areas</u>
  - » Incompressible flow
  - » Non-symmetric AMG
  - » Electromagnetics, MHD
  - » MG robustness

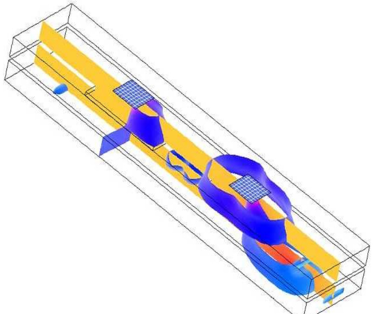


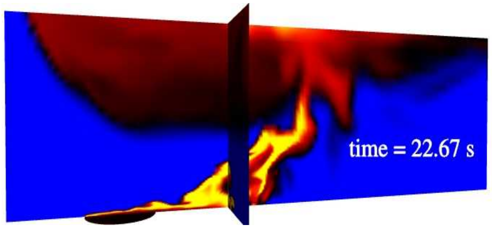


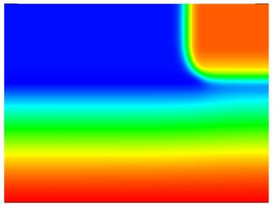
- <u>Applications: new & continuing</u>
  - » Electromagnetics (Z-pinch, rad-trans deposition, army), compressible CFD, incompressible CFD, semiconductor device modeling, combustion, MEMs, elasticity, radiation transfer, magnetohydrodynamics

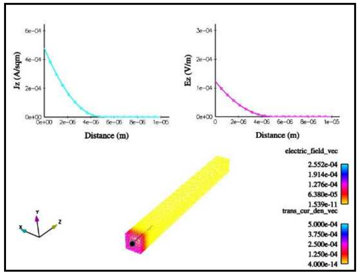




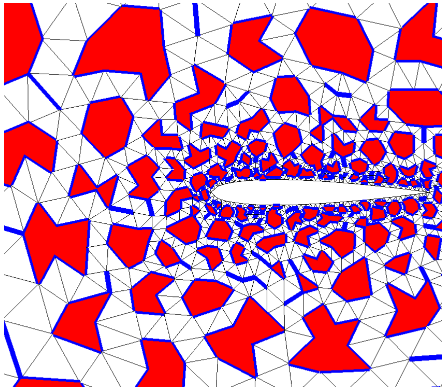








A Massively Parallel Algebraic Multigrid Solver



Smoothed Aggregation Capabilities

- Scalar & PDE systems (elliptic)
  - Symmetric, non-symmetric
  - variable dofs/block support (limited)
- Aggregation with arbitrary coarsening, load balancing, …

Smoothers

- Gauss-Seidel, polynomial, block methods, ILU, domain methods, ...

Package Leveraging

- Trilinos (Epetra, Ifpack, etc.)
- External: PETSc, SuperLU, Arpack, Parasails, kLU, ParMETIS, Zoltan, …
  - PETSc applications can construct and apply essentially any Trilinos preconditioner/iterative method, KSP solvers as smoothers

Interfaces

- C++, matlab, matlab-like, web-based

http://trilinos.sandia.gov/packages/ml

- Developer's Guide, User's Guide, MLAPI

Block factorizations & Incompressible Flow

$$\begin{bmatrix} \boldsymbol{F} & \nabla \\ \nabla\cdot & 0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \\ \nabla\cdot\boldsymbol{F}^{-1} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{F} & \nabla \\ & -\boldsymbol{S} \end{bmatrix} \qquad \Rightarrow \text{need} \quad \boldsymbol{S}^{-1} = (-\nabla\cdot\boldsymbol{F}^{-1}\,\nabla)^{-1}$$

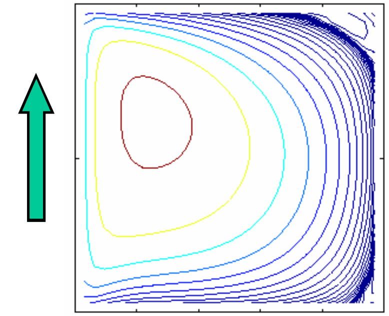
Incompressibility constraint problematic for preconditioners!

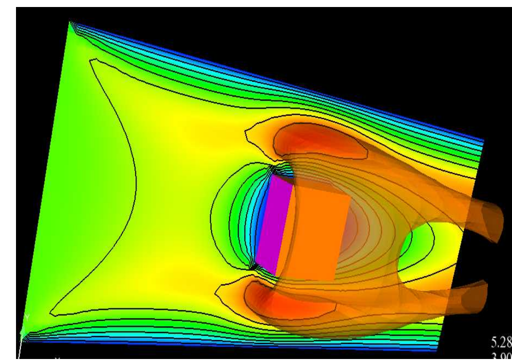
Main Idea: approx. block factorization & *standard* preconditioners to blocks

$\Rightarrow$ <u>eye toward future more complex fully implicit multiphysics</u>!!

- GMRES
- Inexact F^{-1} & A_p^{-1} via AMG
- DD uses ILUT
- Newton's Method

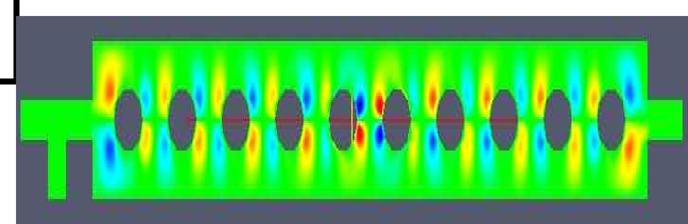
Re	Mesh	DD	PC-D	Proc
10	32x32x32	67.0 (634.6)	28.0 (803.2)	1
	64x64x64	159.8 (1507.5)	28.4 (865.2)	8
	128x128x128	356.2 (4529.3)	29.1 (1103.2)	64
100	32x32x32	61.7 (730.7)	56.0 (1232.7)	1
	64x64x64	168.5 (2131.6)	62.1 (1697.8)	8
	128x128x128	404.6 (6953.9)	67.9 (2487.3)	64



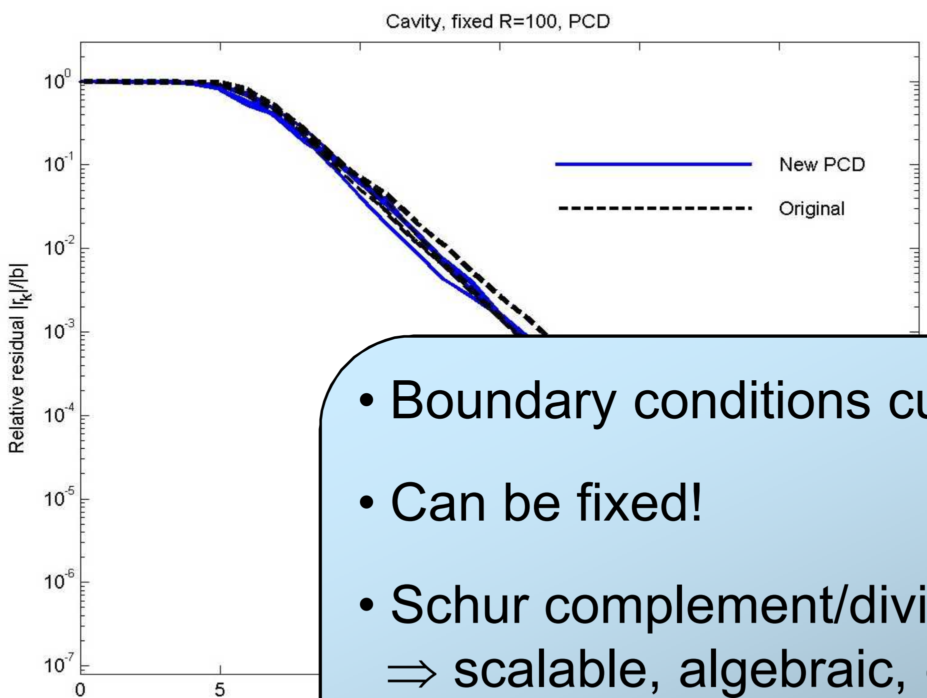


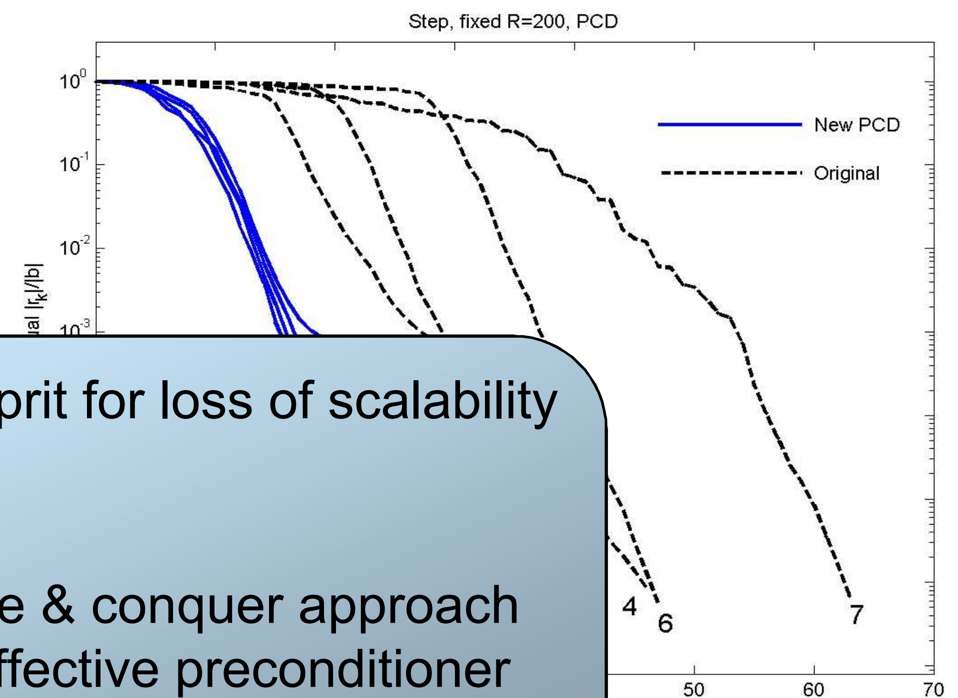
Re	Mesh	DD	PC-D	Proc
10	270K	67.2 (859.8)	20.7 (997.7)	1
	2.1 M	151.2 (2004.1)	21.7 (1507.5)	8
	16.8 M	667.2 (20908.0)	24.7 (1997.7)	64
50	270K	69.4 (889.2)	35.9 (1209.1)	1
	2.1 M	132.4 (2676.1)	38.7 (1797.2)	8
	16.8 M	637.2 (18646.0)	44.7 (2397.7)	64

Average outer iterations per Newton step. (*) is total CPU time

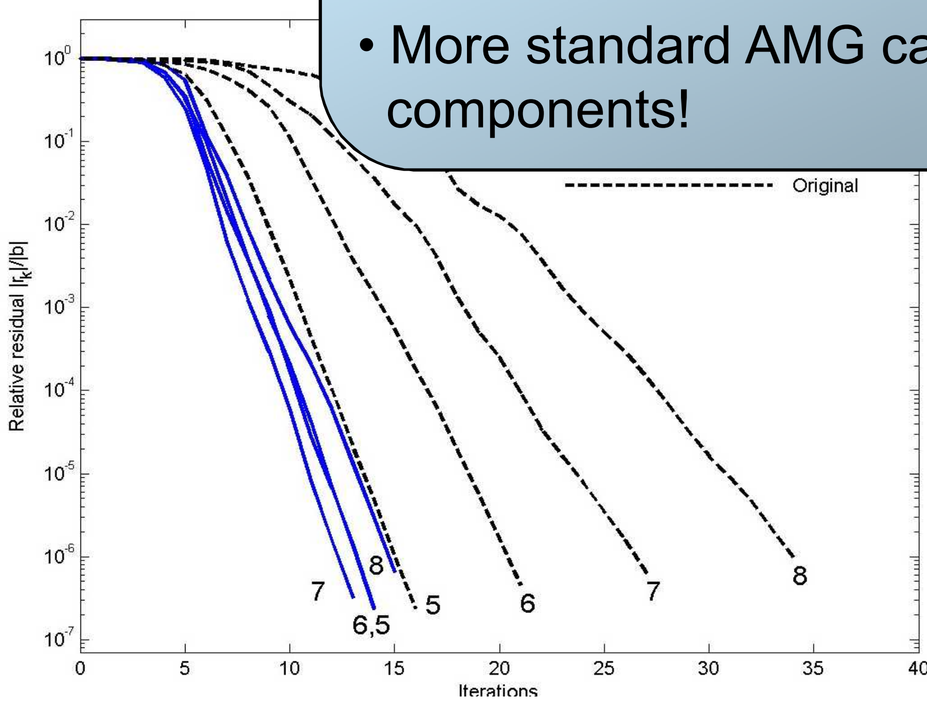


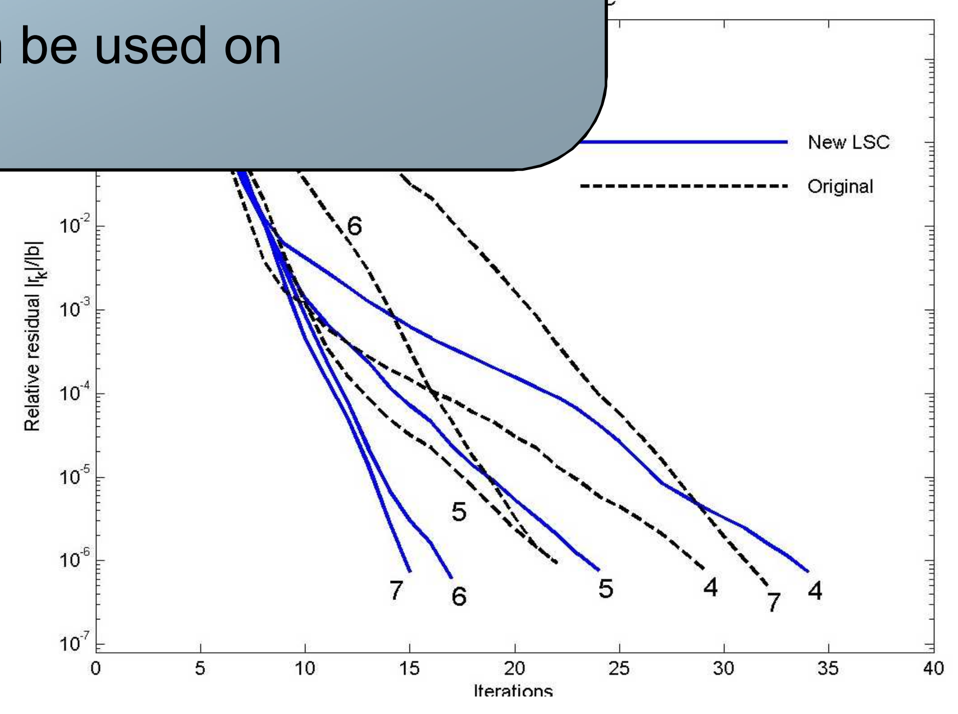
Similar results for microfluidics





- Boundary conditions culprit for loss of scalability
- Can be fixed!
- Schur complement/divide & conquer approach
  $\Rightarrow$ scalable, algebraic, effective preconditioner
- More standard AMG can be used on components!





Smoothed Aggregation

- popular AMG method for symmetric systems
  - Low cost/iteration
  - Accurate interpolation of difficult modes
  - $P = (I - \omega A)\, P_s$, $R = P^T$,
    e.g., P_s = piecewise constant
  - $A_H = R\,A\,P$, ω *minimizes* $(I - \omega A)A(I - \omega A)$

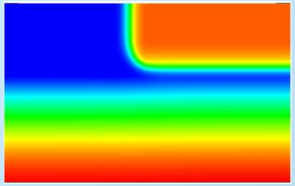
A ≠ A^T?

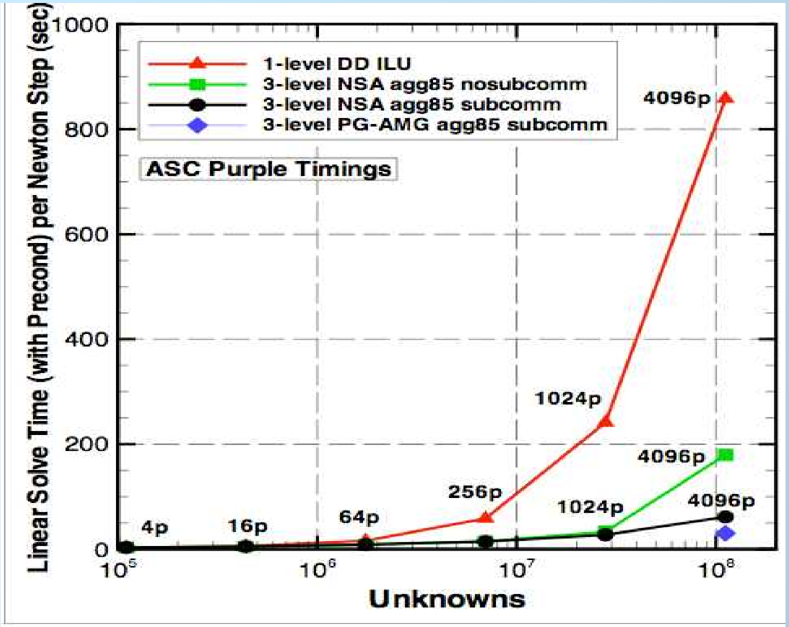
R = ? , ω = ?   Minimize what? A_H Stability?

- Replace ω with diag matrix Ω
- $R = (P_s)^T(I - \Omega A)$
- *Locally minimize based on* A^TA *norm*

⇒ New smoothed aggregation AMG for nonsymmetric systems that makes sense

NPN
BJT





DOFs	1 level	old	new P
112 M	858.0	61.4	30.2

[1] *A new Petrov-Galerkin smoothed aggregation preconditioner for nonsymmetric linear systems*, Sala & T, SISC'08.

Algebraic Multigrid for Eddy Current Equations

$$\nabla \times \nabla \times + \sigma$$

Small σ

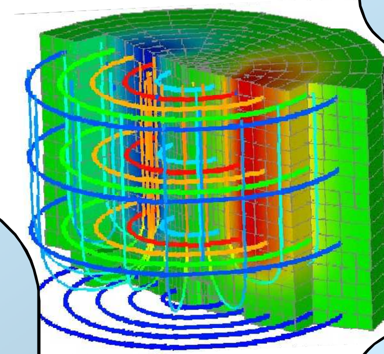
$\Rightarrow$ large near null space

$\Rightarrow$ iterative solver problems

Exact Discrete Reformulation

$$\begin{pmatrix} \Delta_v^{(e)} & \sigma\nabla \\ \sigma\nabla\cdot & \Delta^{(n)} \end{pmatrix}$$

$\Rightarrow$ Laplacian dominated

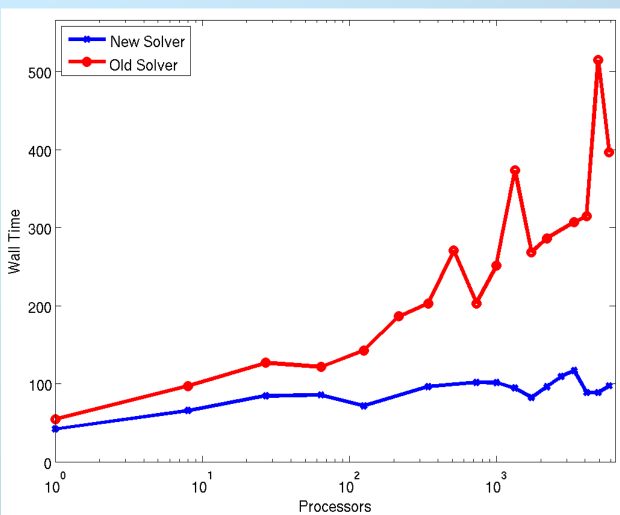


Special AMG only for finest mesh

- Special AMG on fine level
- Standard AMG on other levels
- Insensitive to jumps in σ !
- Scalable!
- Can be used with ILU
- Good behavior for indefinite problems

Results

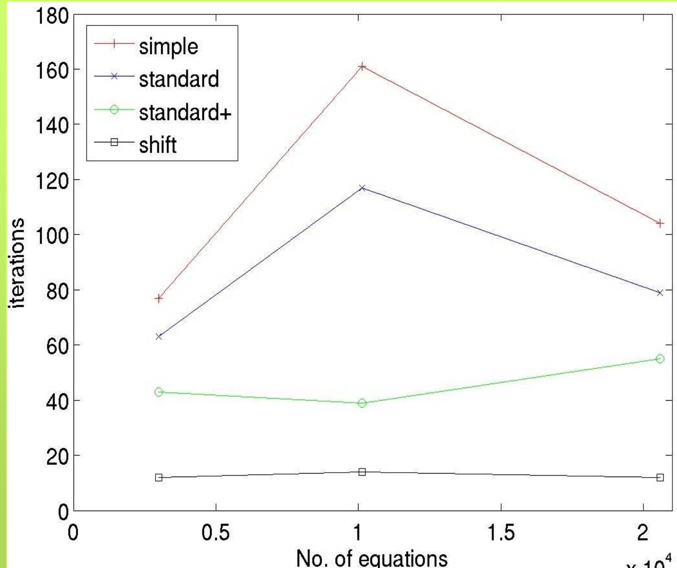
Red Storm Weak Scaling

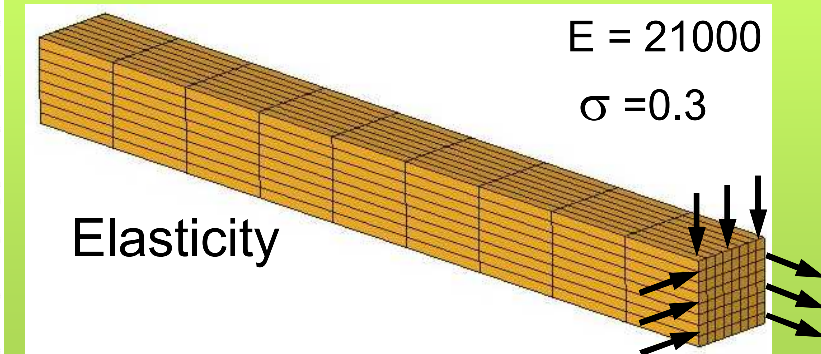


(1) *An AMG approach based on a compatible gauge reformulation of Maxwell's equations*, Bochev, H, S & T, SISC'08.

(2) *Auxiliary AMG preconditioners for Mixed Finite Element Methods, T., J. Xu, Y. Zhu, to be submitted.*

Rob





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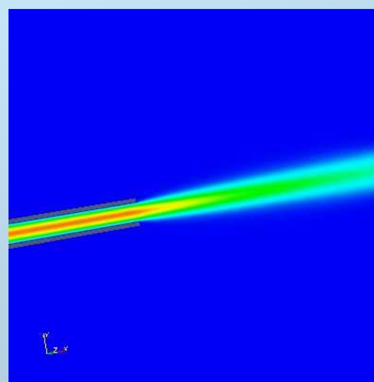
prolongator

	Old	New
its	196	22
time(s)	42.95	4.24



s

≈DOFs	coefs	ODE
2k	33	13
14k	67	13
110k	129	18

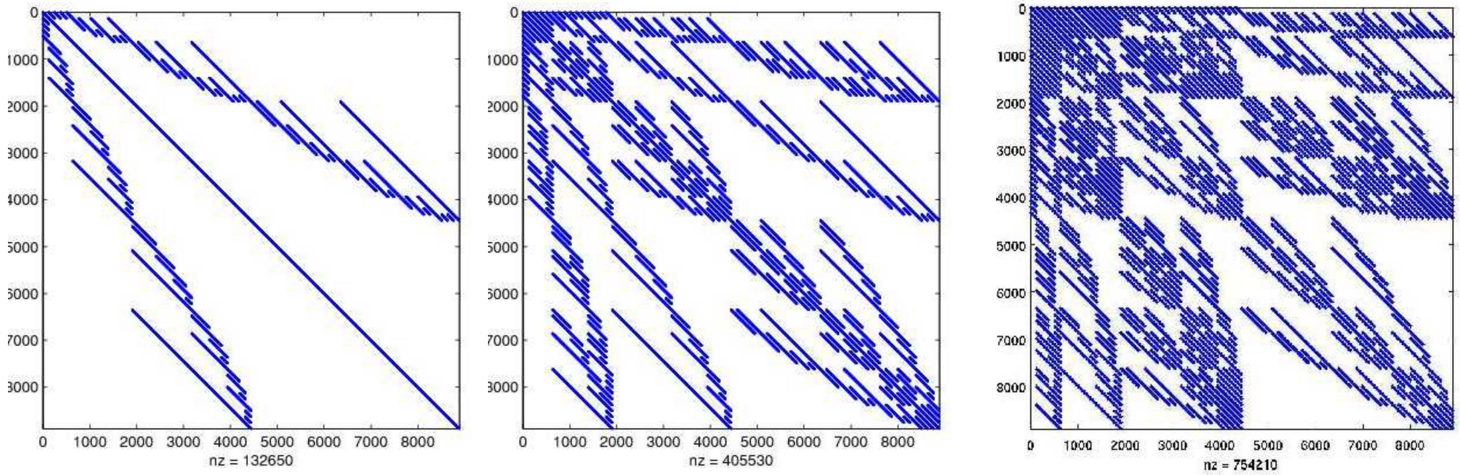
elasticity

$$z_t = - Az + \delta_i \text{ with } z(0) = 0 \Rightarrow \{$$

(1) *A Generalized Strength Measure for Algebraic Multigrid, Schroder, T, Olson,*

Future Areas

Stochastic PDEs



Physics-Based Preconditioners for MHD

MHD Pump Prototype