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Solution of Multivariate Inverse Radiation Transport Problems

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Outline

- About Sandia, my organization, and our work
- About inverse radiation transport problems
- Solution of an example problem
- Constraining the solution using multiple complementary measurements
- Neutron multiplicity counting and inverse transport problems
- Summary, ongoing and future work

Sandia National Laboratories

- Department of Energy National Laboratory
- Government-owned, contractor-operated

- Established in 1949 to support US nuclear weapon program
- Today Sandia develops a broad range of technologies that support US national security

- Main lab campuses in Albuquerque NM and Livermore CA
- Facilities in New Mexico, Nevada, Texas, and Hawaii
- More than 8,500 full-time staff & 2,200 contractors
- About 1,500 PhD & 2,700 MS/MA staff
- Annual budget of about $2.4 billion (FY2007)

Sandia Mission Areas

- Nuclear weapons
- Defense systems
- Homeland security
- Nonproliferation
- Energy & infrastructure
- Science, technology, and engineering

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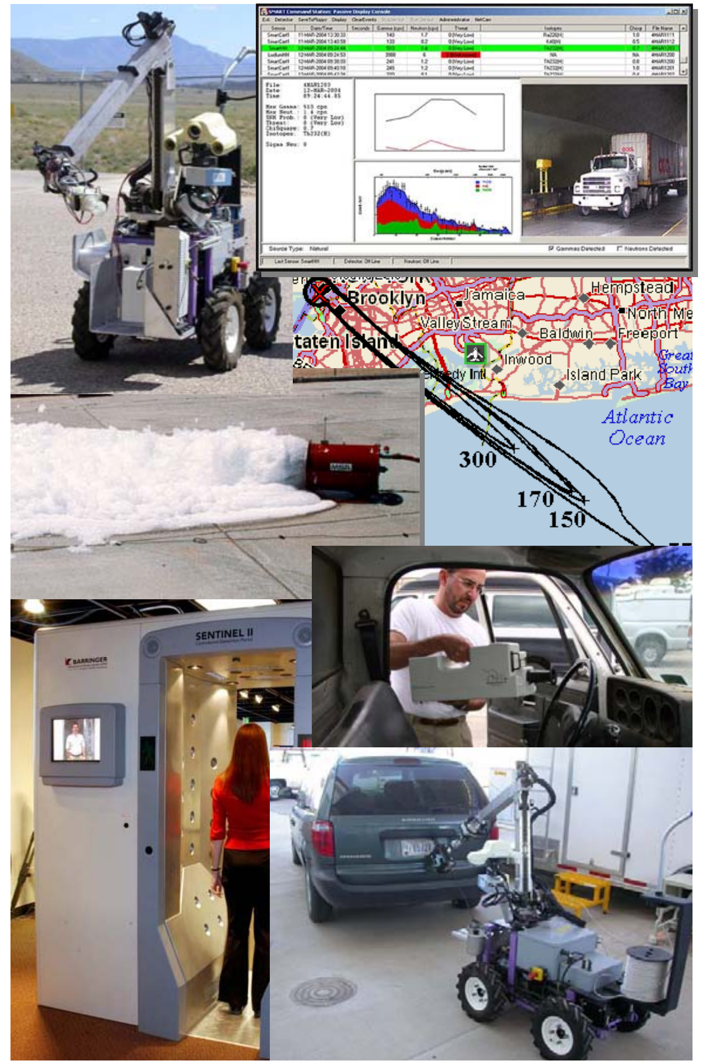
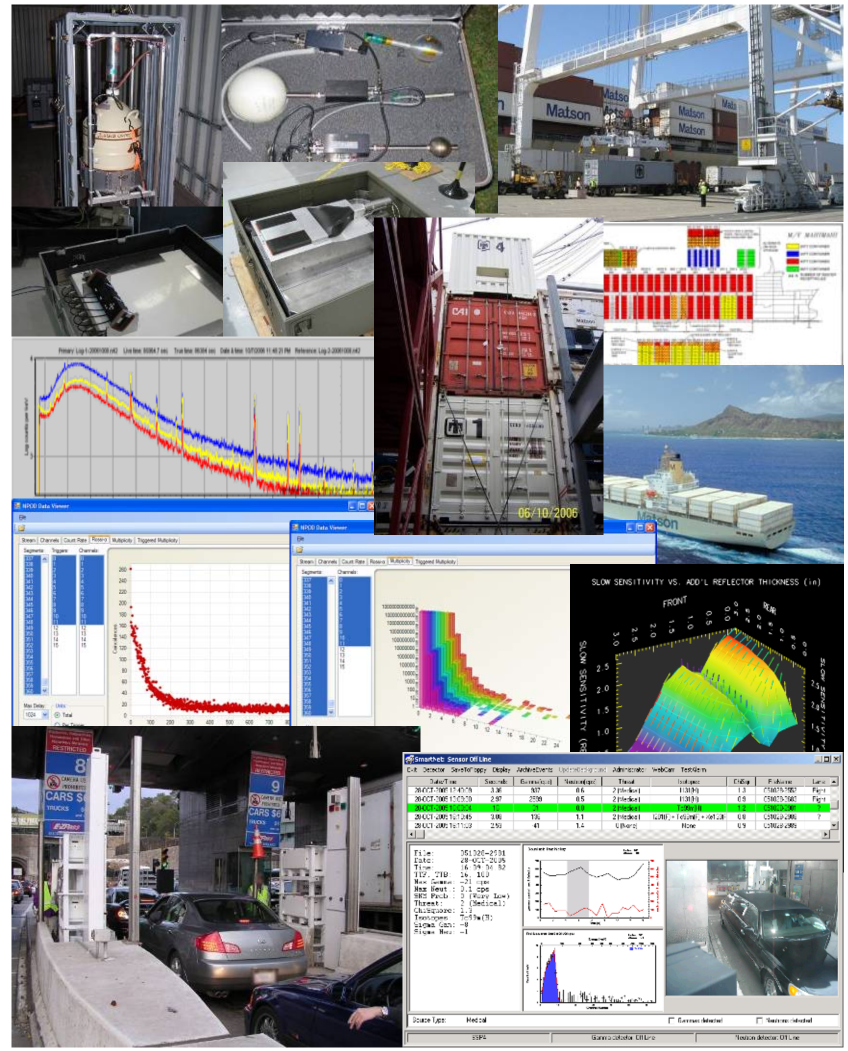
Sandia's HE/Rad/Nuc Countermeasures Program

- Explosive and radiological observables & signatures
- Detection technologies
- Analytical methods & services
- Systems integration
- Field deployment & testing
- Effects modeling
- Protection methods
- Disablement systems
- Mitigation / containment
- Performance validation
- Training & consulting



Radiation Detection and Analysis R&D

- Basic physics measurements
- Radiation transport modeling
- Analysis software development
- Sensor development & design
- Sensor deployment & testing
- Radiation signature analysis



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Inverse Radiation Transport Problems

- Objective: infer configuration of an unknown radiation source from its measured radiation signatures

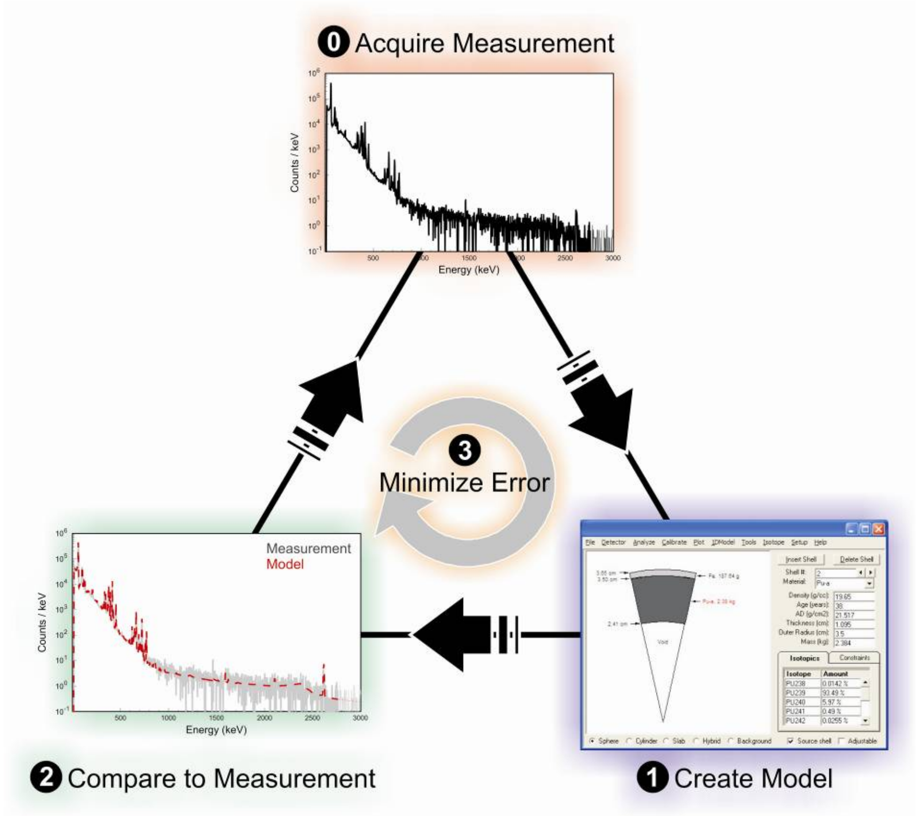
- Source features
  - Isotopic composition
  - Fissile mass & multiplication
  - Geometric arrangement of radiating and shielding materials

- Signatures
  - Gamma spectrometry
  - Neutron time-correlation and multiplicity counting

- Applications
  - Nonproliferation
  - Counterterrorism
  - Emergency Response

Solution Method

- Start from initial estimate of model parameters
- Solve forward transport models to compute radiation field
- Fold radiation field with detector response model to estimate radiation signatures
- Compute error between predicted and observed signatures
- Iteratively follow gradient in error to minimum



Solution Requirements

- Forward computations must be accurate
- Minimize bias in solution due to systematic errors in model

- Forward computations must be fast
- Minimize time required for iterations to find solution

- Model must have finite number of numeric parameters
- Minimize degrees of freedom / dimensionality of solution

- Accuracy requires high-fidelity spectral synthesis: coupled neutron/electron/photon transport calculations
- Speed requires explicit solution of transport problem: deterministic transport
- Tractable problems do not have arbitrary geometry

Radiation Observables

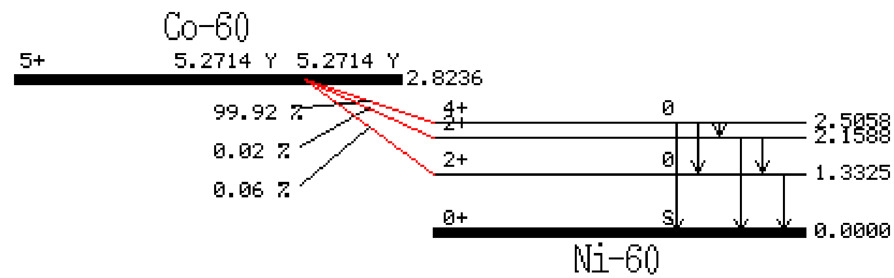
- Most externally observable radiation signatures result from gamma and neutron emissions
- Observables are usually differential over one or more independent variables (e.g., energy, position, time)
- Gamma spectrometry measures the distribution of photons versus energy
- Neutron multiplicity counting measures the distribution of neutrons versus number and counting time

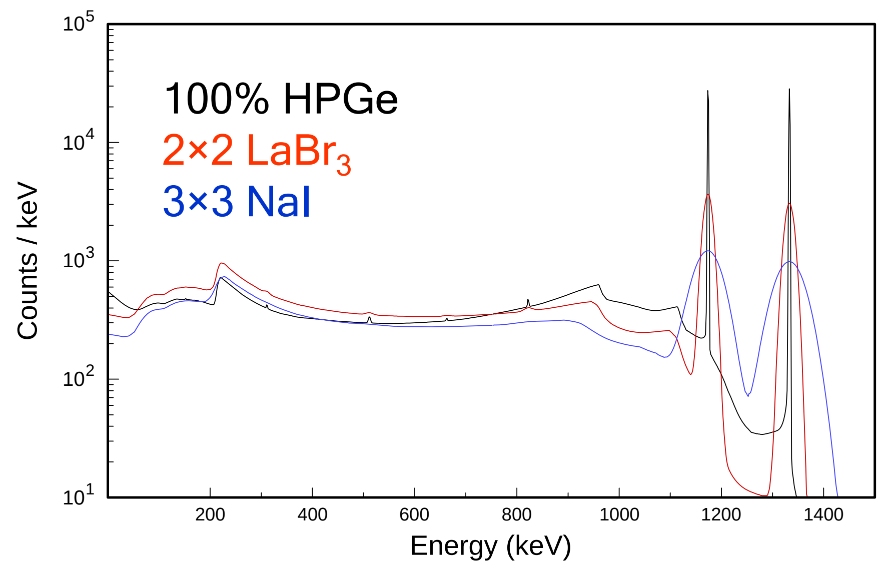
Gamma Spectrometers

Neutron Multiplicity Counters

Gamma Spectrometry

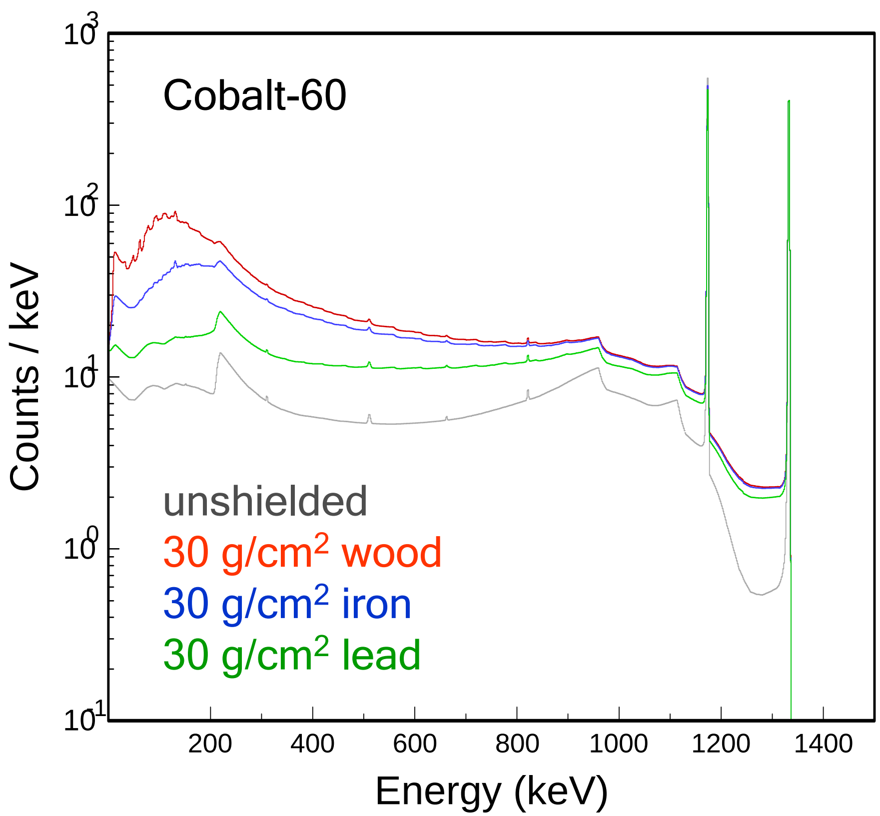
- Radionuclide decays from unstable ground state through series of discrete energy levels
- Decay between levels of a single daughter achieved via emission of discrete energy gammas
- Gammas characteristic of daughter level scheme
- Gamma spectrometers measure distribution of photon energies
- Spectrum can be used to identify radionuclide(s) **and** shielding

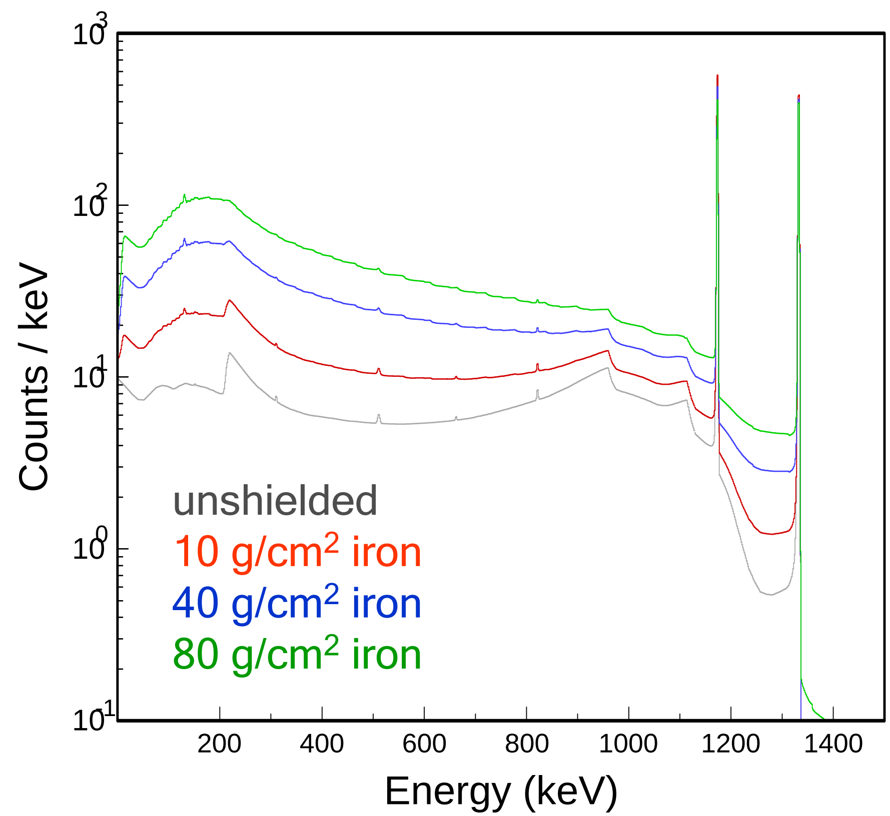




Photopeaks and Compton Continua

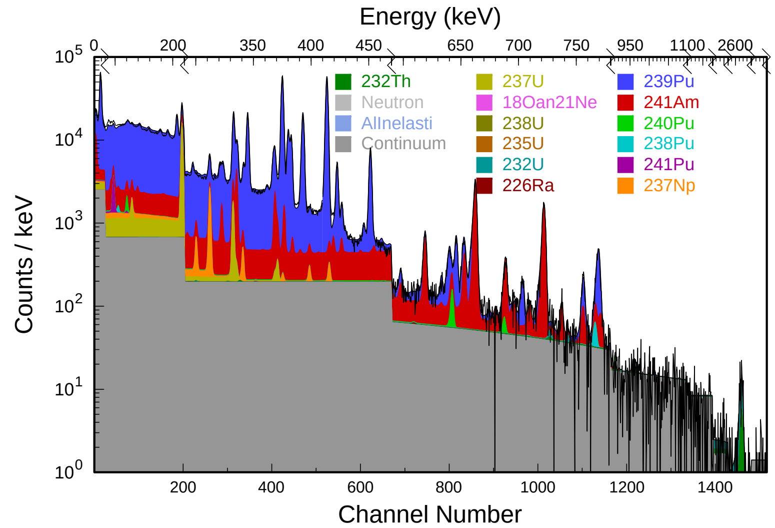
- Photopeak positions identify source
- Differential attenuation of photopeaks and Compton continua identify shielding

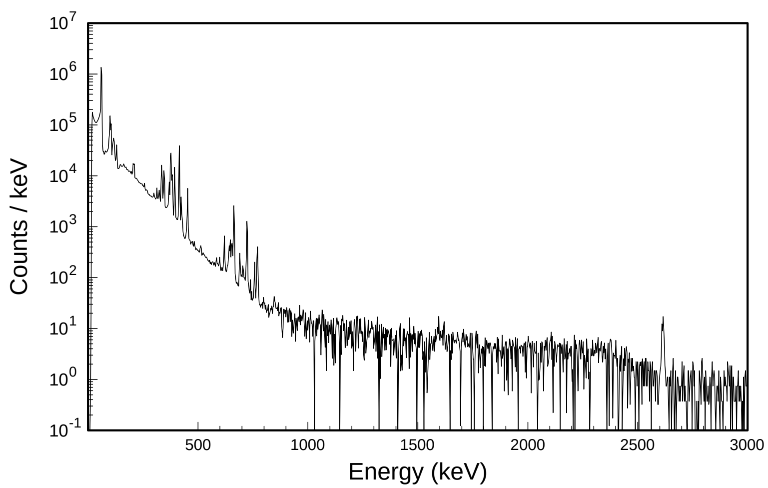




Example Problem

- Gamma spectrum (bottom-left) exhibits features consistent with plutonium
- Nonlinear regression (top-right) fits spectrum using variable isotopics, volume, shielding, and age
- Regression analysis (bottom-right) provides approximate model of source



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Plutonium wt.% (@ 33 +/-  1 years) __________ @ t=0 __________

Pu-236:   2.69E-12  +/-  1.19E-12    8.98E-09  +/-  4.75E-09
Pu-238:      0.014  +/-     0.008       0.019  +/-     0.010
Pu-239:     94.205  +/-     0.603      94.295  +/-     0.604
Pu-240:      5.279  +/-     1.157       5.298  +/-     1.161
Pu-241:      0.095  +/-     0.006       0.513  +/-     0.038
Pu-242:    [ 0.010]                   [ 0.010]
Am-241:      0.396  +/-     0.008
Np-237:      0.017  +/-     0.002
U-237:    2.94E-09  +/-  1.92E-10
U-232:    8.98E-09  +/-  3.97E-09

Confidence for measured 18O-an-21Ne gammas:        -1.6 sigma
Measured (a,n) relative to expected for oxide:     -3.0 sigma
Aluminum-inelastic peak:               1011.7 keV @  2.3 sigma
Chemical form:                                      METALLIC
External shielding:          AN = 30 +/- 1;  AD = 3.9 +/- 0.1
Outer radius if plutonium is spherical:             3.8 cm
Estimated void radius:                              3.3 cm
Estimated mass if delta-phase plutonium:            1.3 kg
___________________________________________________________

There is no evidence that uranium is present.
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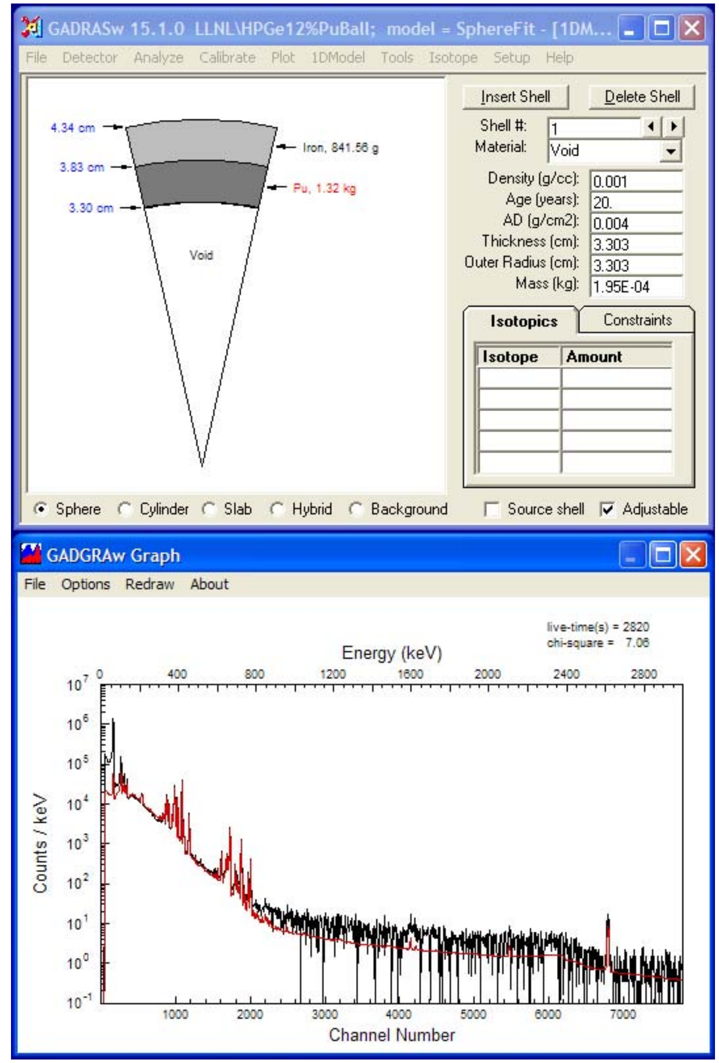
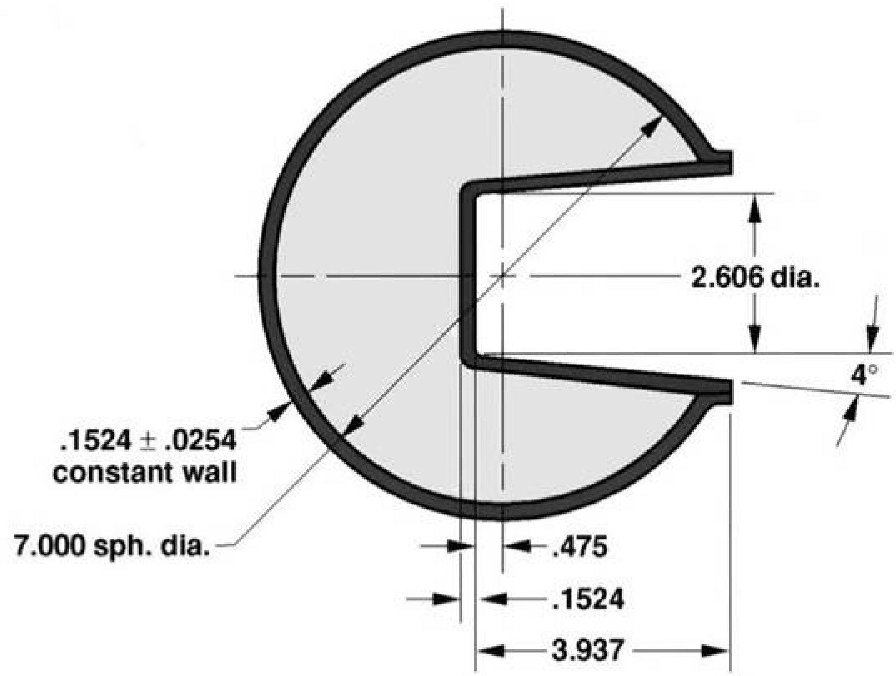
Example Problem

- One-dimensional transport model (top) generated from regression analysis
  - Model displayed as section of sphere with center at bottom and outer surface at top
  - Dimensions of model layers are treated as variable parameters
- Initial estimate of gamma spectrum (bottom) is generated from coupled neutron/electron/photon transport calculation
- Nonlinear optimization procedure finds model dimensions that minimize error in calculation

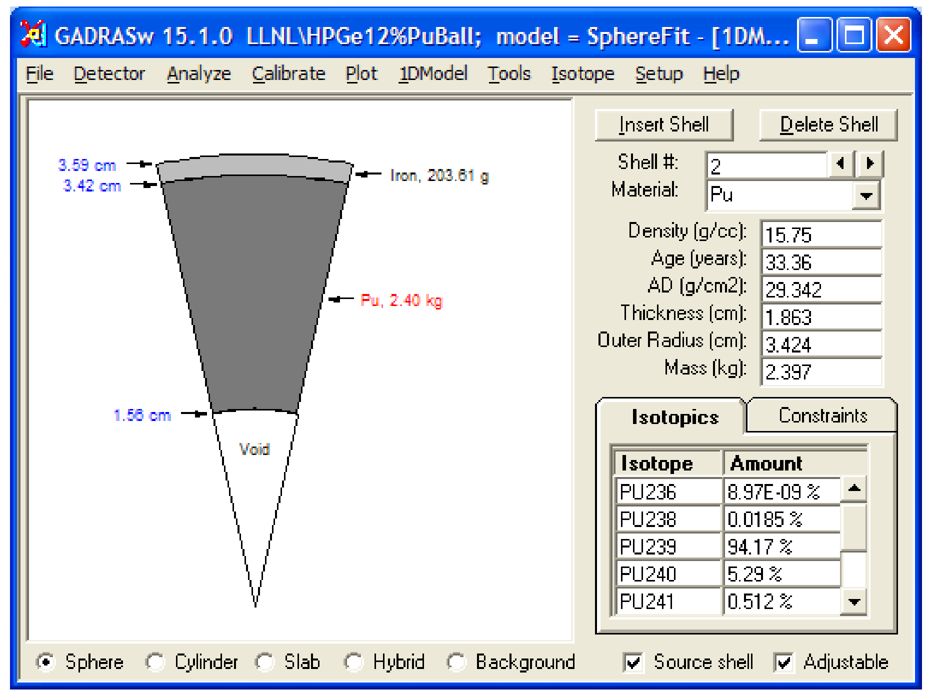


Actual Source

LLNL Plutonium Sphere

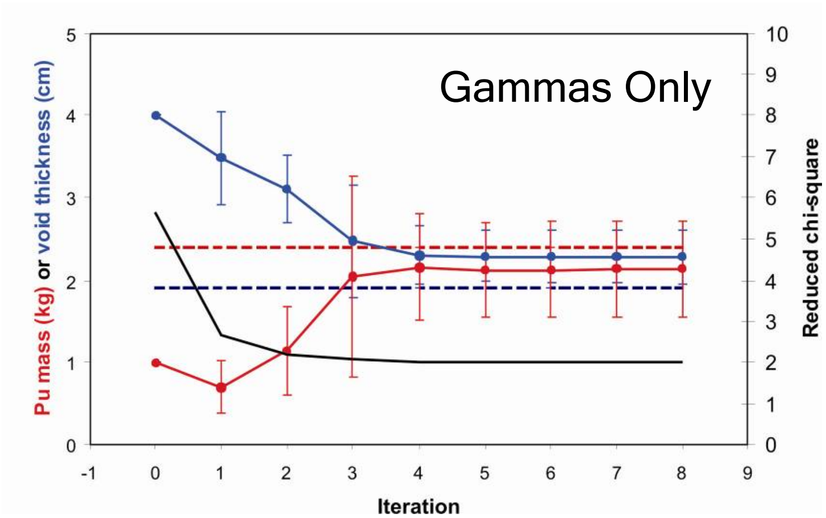


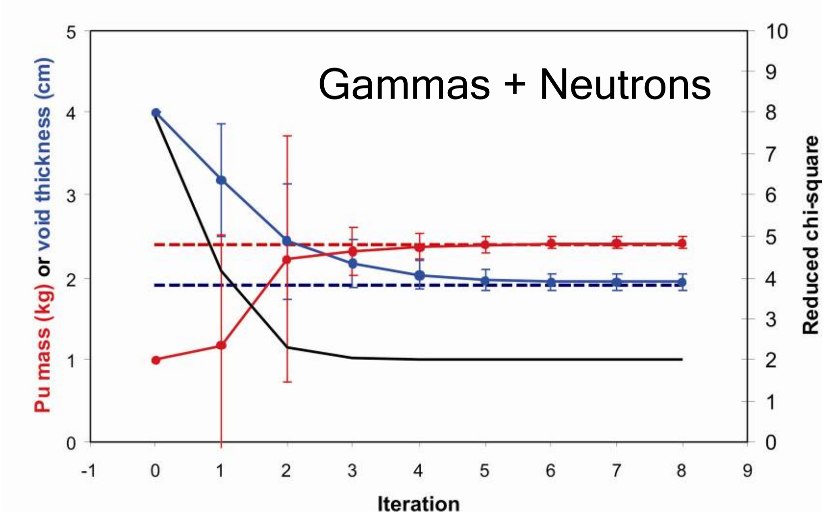
Optimized Transport Model



Solution using Gammas and Neutrons

- Gamma spectrum is primarily sensitive to outer surface of source
- Solution based on gamma spectrum alone is weakly constrained
- Neutron measurements (e.g., count rate) are more sensitive to entire volume of source
- Simultaneous solution based on analysis of gamma and neutron signatures is better constrained
- Neutron multiplicity counting provides a fairly rich signature of the neutron field



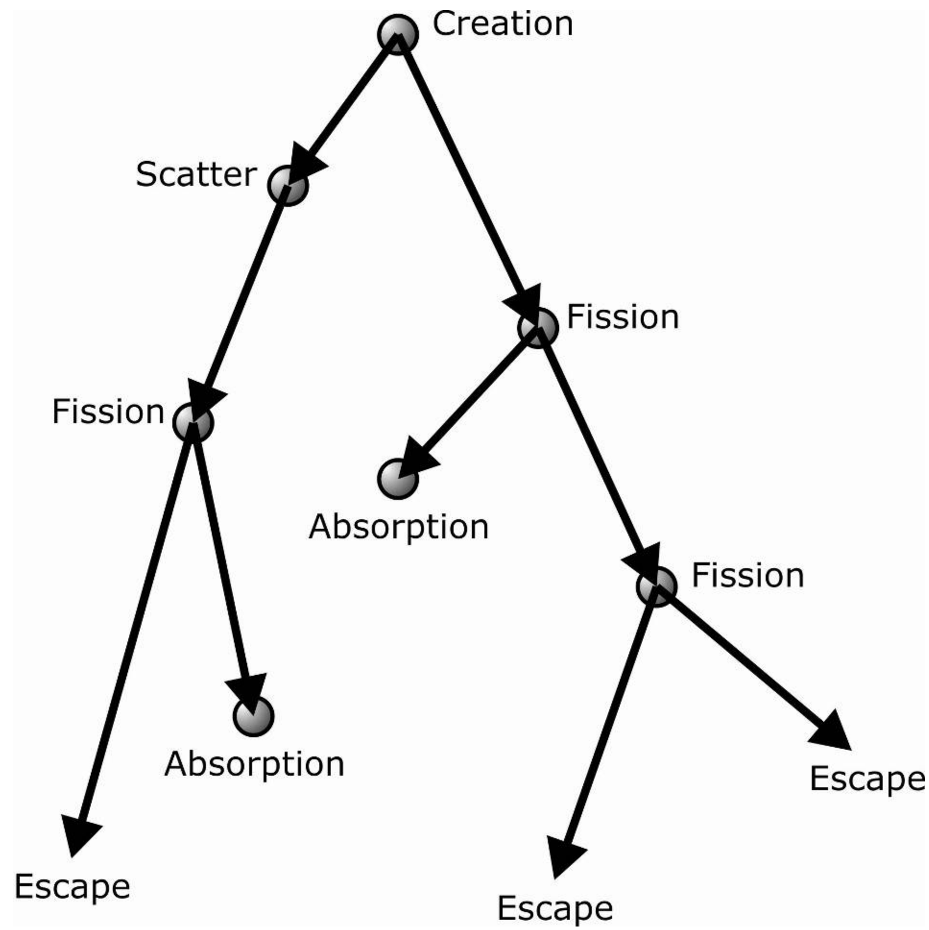


Fission Chain-Reactions

- Fission chain reactions multiply the number of neutrons in fissile transport medium

- Chain reaction characteristics:
  - Number of neutrons made during the chain reaction: neutron multiplication
  - Speed of chain reaction evolution: neutron generation time

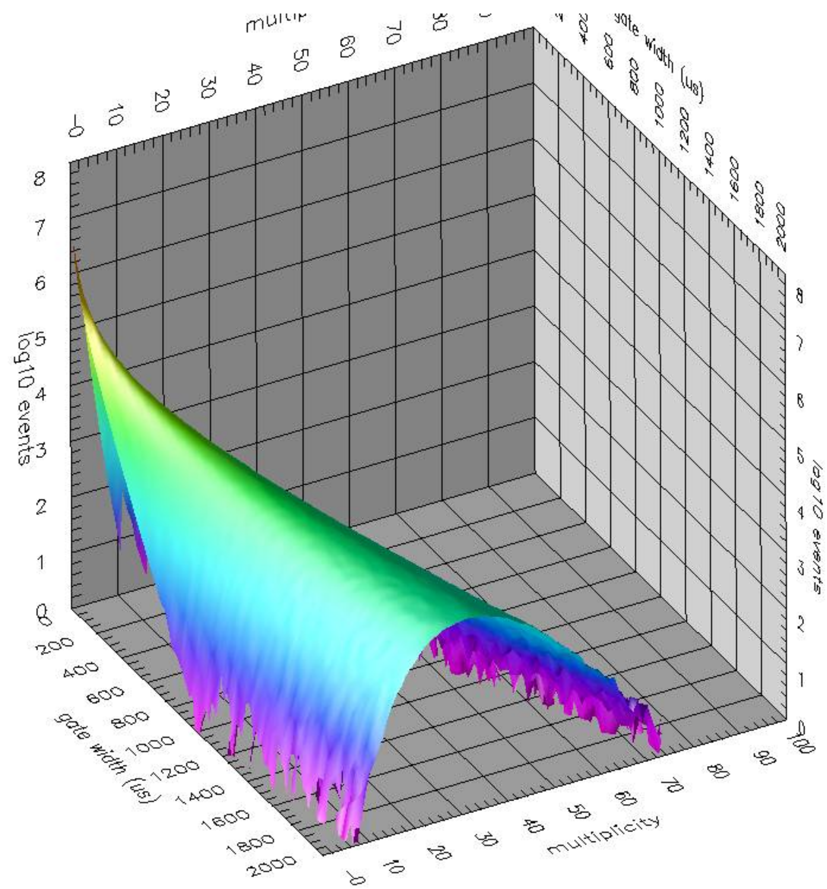
- Neutron multiplicity measurements are sensitive to both characteristics



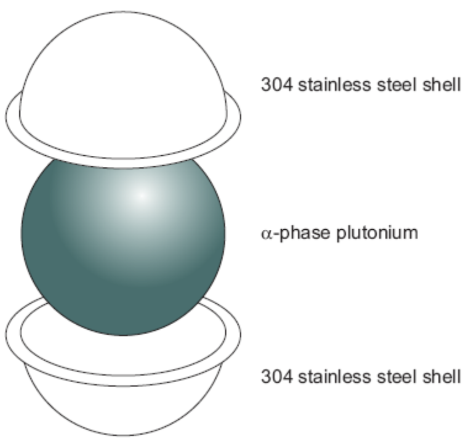
Neutron Multiplicity Counting

- Neutron multiplicity counting measures frequency of neutron detection versus:
  - Counting time, a.k.a., coincidence gate width, usually on order of microseconds
  - Number of coincident counts, a.k.a., multiplicity, usually between 10's and 100's of coincident neutrons

LANL BeRP Ball / 1.5" Poly Reflector

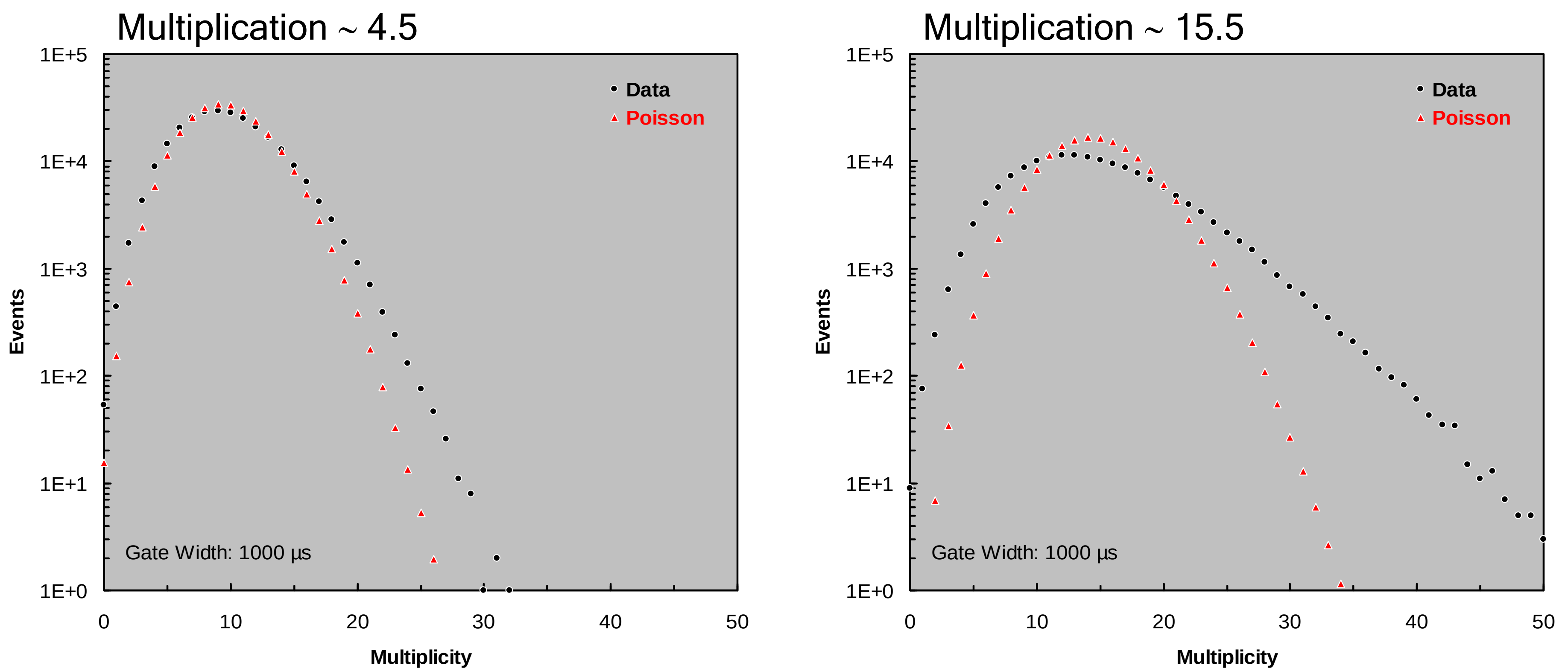


LANL BeRP Ball



Multiplication Produces Excess Variance

- Fission chain reactions are "bursty"
- Relative to a population of uncorrelated, Poisson-distributed counts, fission multiplicity distribution is wider



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Feynman-Y

- Feynman-Y measures variance in excess of Poisson distribution

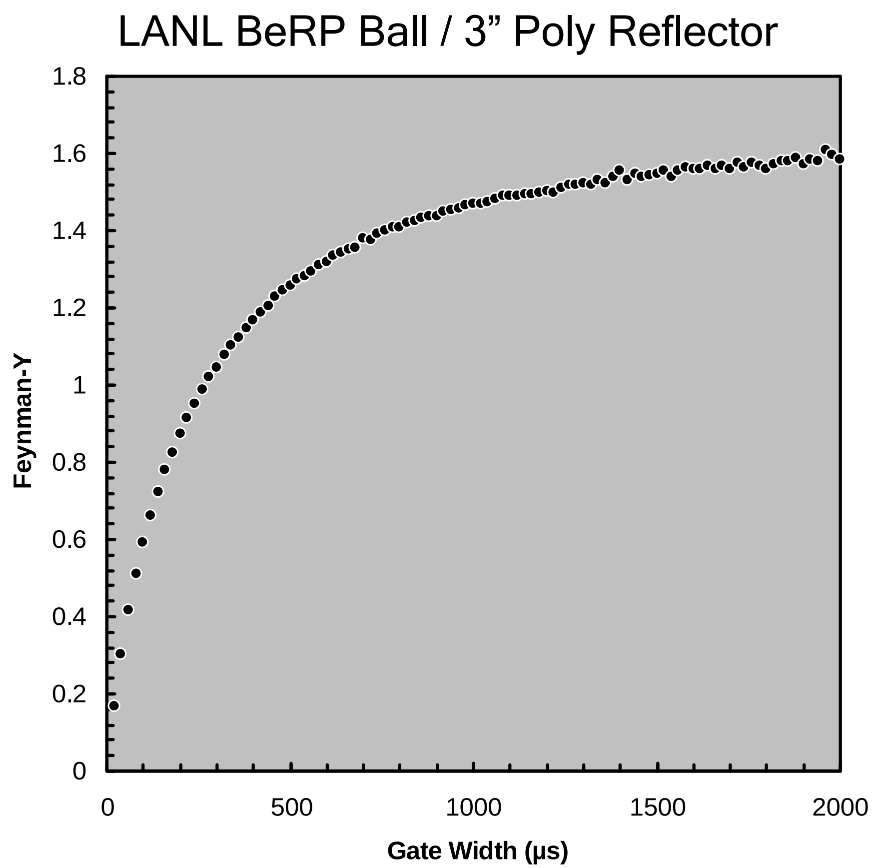
$$\frac{\sigma^2}{\mu} = 1 + Y$$

$$\sigma^2 : variance$$

$$\mu : mean$$

- Y vanishes if counting distribution is purely Poisson
- Y tends to increase with neutron multiplication

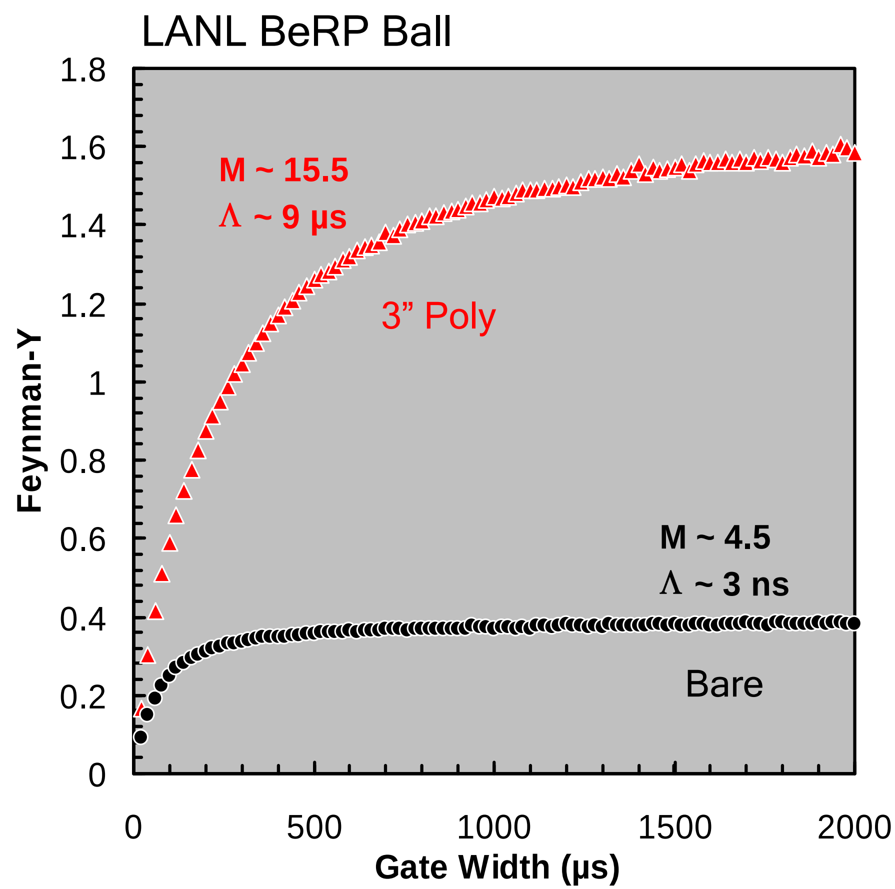
- Usually measured vs. coincidence gate width (counting time)



Effect of Multiplication and Generation Time

- Y is a measure of the second moment of the counting distribution
- Asymptotic value tends to increase with square of neutron multiplication

- Y is a measure of the system's dynamic response
- Shape vs. gate width tends to evolve more slowly with increasing neutron generation time



Neutron Multiplicity: Distribution vs. Moments

- Choice of observable dictated by solution requirements: accuracy, speed, and degrees of freedom

- Multiplicity distribution
  - Can be computed using Monte Carlo methods – accurate, but slow
  - Can be computed using recursive point models – faster, but still slow, and inaccurate for systems that violate point kinetics assumptions
  - Could be computed by deterministic solution of generalization to Boltzmann transport model – but no numerical solver exists
  - Degrees of freedom much lower than dimensionality – distribution dictated by product of source strength, multiplication, and leakage probability

- Multiplicity moments
  - Can be computed using analytical point models: fast, but inaccurate for non-point systems
  - Can be computed by deterministic solution of forward and adjoint Boltzmann transport models – fast and accurate
  - First few low-order moments convey most of the information contained in the distribution

Deterministic Computation of Feynman-Y

- Muñoz-Cobo, Perez, and Verdú developed method to compute neutron multiplicity moments via deterministic solution of Boltzmann transport equation (see NS&E #95)

- Method enables computation of first and higher moments (e.g., mean and variance) using existing deterministic transport solvers

- Three calculations required
  - Forward time-independent, fixed source
  - Adjoint time-independent, adjoint source is detector response
  - Forward time-dependent, instantaneous step in fixed source intensity

Deterministic Computation of Feynman-Y

- Feynman-Y exhibits two notional features
  - Asymptotic value
  - Shape dependent on coincidence gate width

- Asymptote
  - Computed from static forward and adjoint transport solution
  - Accounts for relative contribution of source and induced fission neutrons
  - Source term for adjoint problem is detection efficiency – adjoint flux "weighting function" represents importance to detection

- Shape
  - Computed from solution to dynamic step response problem
  - Forward source term is instantaneously stepped
  - Leakage current is folded with detector cross section & impulse response
  - Detector response is integrated over gate width

Feynman-Y Asymptote: Excess Variance

- Excess variance comes from source and induced fission

$$\frac{\sigma^2}{\mu} = 1 + Y \qquad \sigma^2 = \mu + {}_2S_0 + {}_2S$$

- Variance of source neutron production Q

$${}_2S_0 = \int d^3r \int dE \, \frac{\overline{\nu_0(\nu_0 - 1)}}{\overline{\nu_0}} Q(\vec{r}, E) \, I_0^2(\vec{r}) \qquad I_0(\vec{r}) = \int dE' \, \frac{\chi_0(\vec{r}, E')}{4\pi} \varphi^\dagger(\vec{r}, E')$$

- Variance of fission neutron production $\nu\Sigma_f\varphi$

$${}_2S = \int d^3r \int dE \, \overline{\nu(\nu - 1)} \Sigma_f(\vec{r}, E) \, \varphi(\vec{r}, E) \, I^2(\vec{r}) \qquad I(\vec{r}) = \int dE' \, \frac{\chi(\vec{r}, E')}{4\pi} \varphi^\dagger(\vec{r}, E)$$

- Importances I_0 and I weighted by adjoint flux $\varphi^\dagger$

Forward vs. Adjoint Flux

- Forward transport: rate of change = source + production – losses

$$\frac{1}{v}\frac{\partial}{\partial t}\varphi = Q + \mathbf{P}\varphi - \mathbf{L}\varphi$$

- Relationship between forward and adjoint

$$\left\langle \varphi^{\dagger}\, Q \right\rangle = \left\langle Q^{\dagger}\, \varphi \right\rangle$$

- Adjoint transport reverses time, direction, and change in energy

$$-\frac{1}{v}\frac{\partial}{\partial t}\varphi^{\dagger} = Q^{\dagger} + \mathbf{P}^{\dagger}\varphi^{\dagger} - \mathbf{L}^{\dagger}\varphi^{\dagger}$$

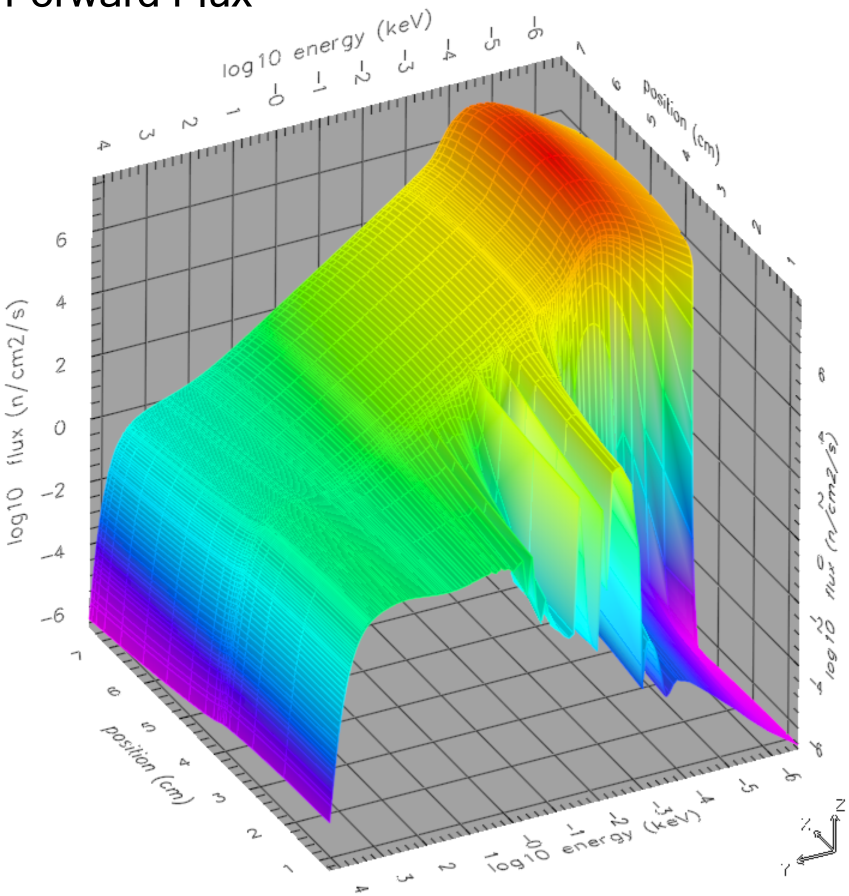
- Adjoint flux represents importance of source neutrons to interaction embodied in adjoint source

$$Q^{\dagger} = \Sigma_x \Rightarrow R_x = \left\langle \Sigma_x\, \varphi \right\rangle = \left\langle \varphi^{\dagger}\, Q \right\rangle$$

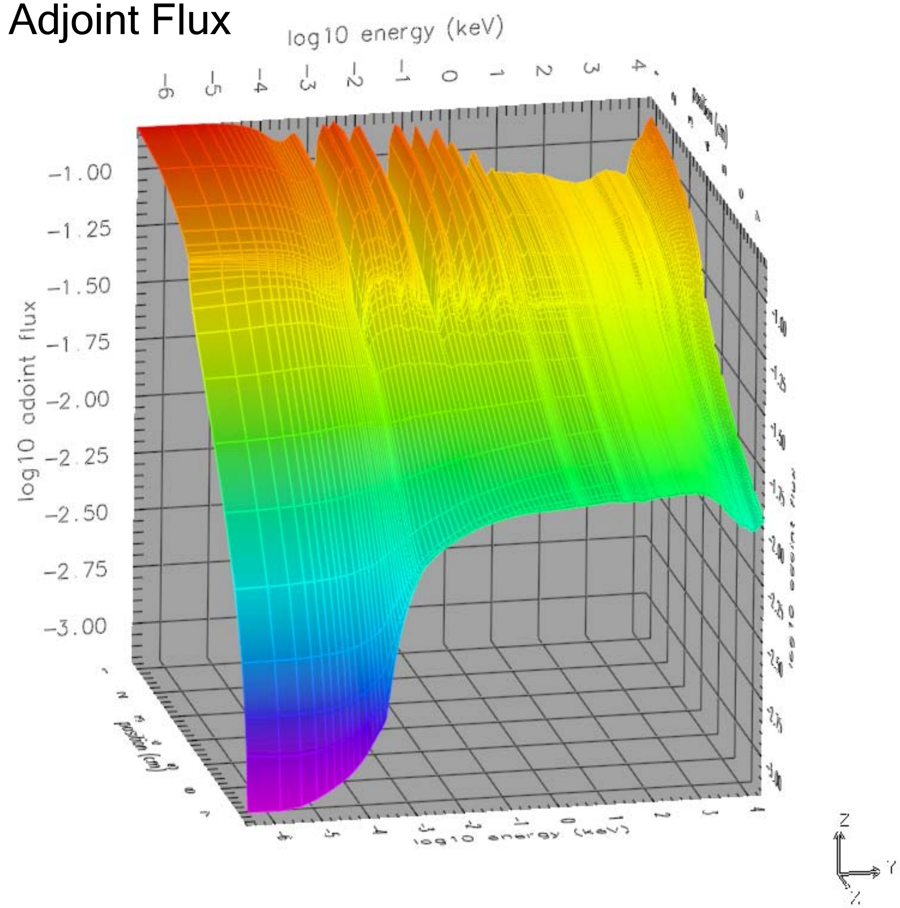
Static Forward and Adjoint Solutions

LANL BeRP Ball / 1.5" Poly Reflector

Forward Flux



Adjoint Flux



Feynman-Y Shape: Dynamic Response

- Feynman-Y shape computed from solution to forward dynamic step response problem

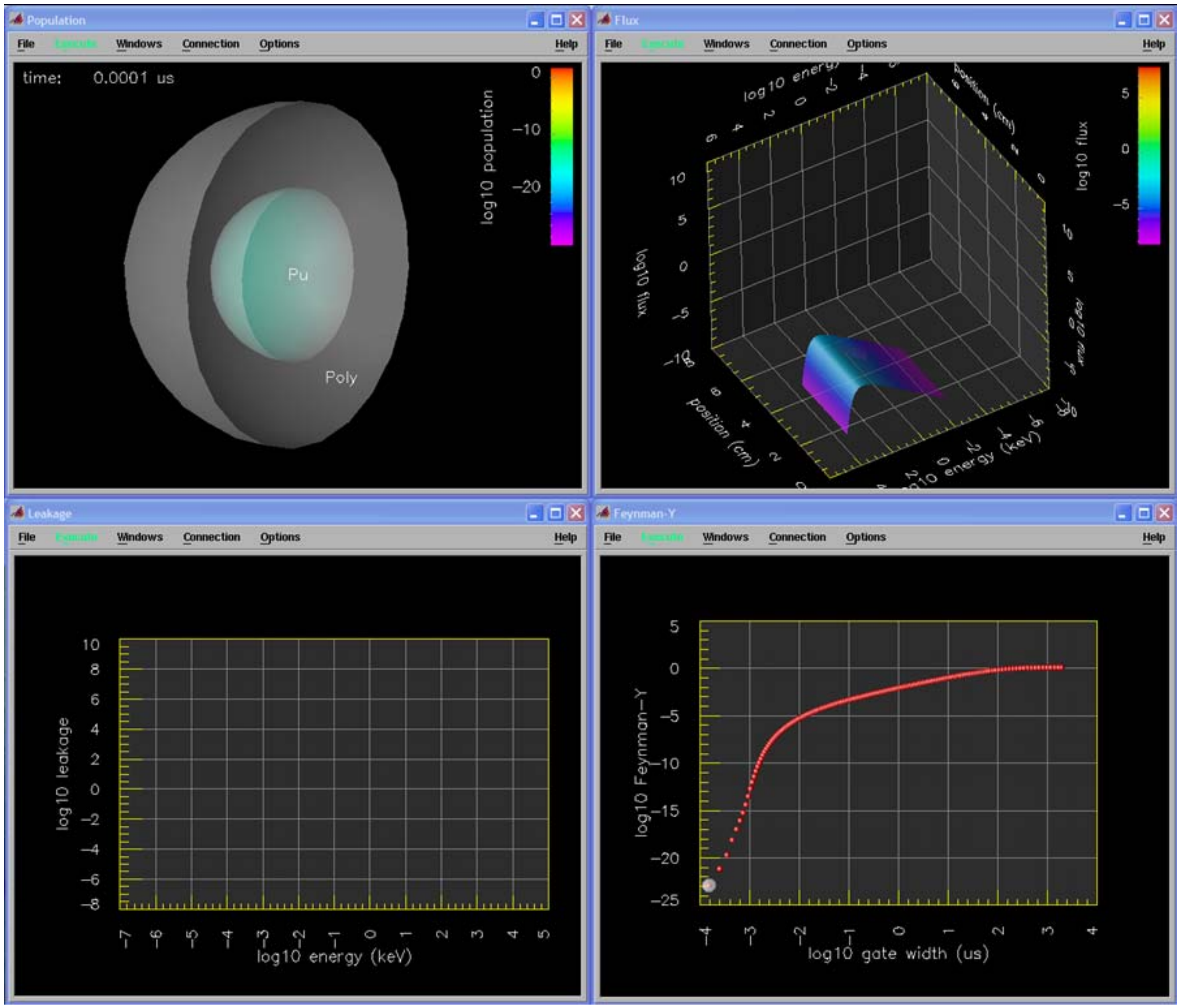
$$Y(T) \quad \propto \quad \frac{1}{T}\int_0^T dt \int_0^t dt' \, h(t-t')\Sigma_d(\vec{r},E)\,\varphi(\vec{r},E,t')$$

- Uses time-dependent transport solver to compute flux φ in response to instantaneous step in forward source term Q

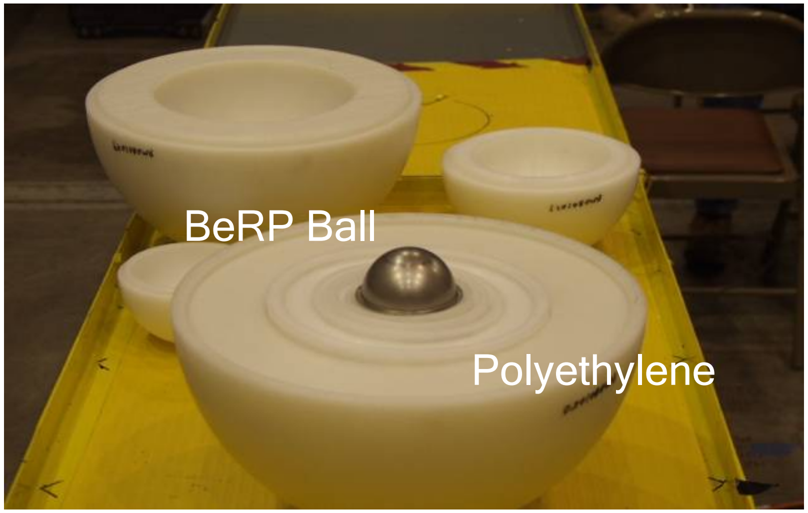
- Time-dependent flux folded with detector cross-section Σ_d and impulse response h

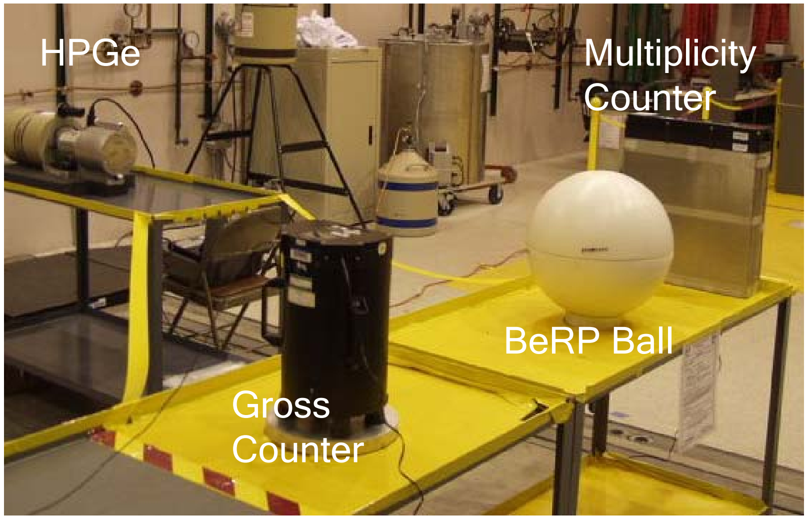
- Integrated over coincidence gate width T

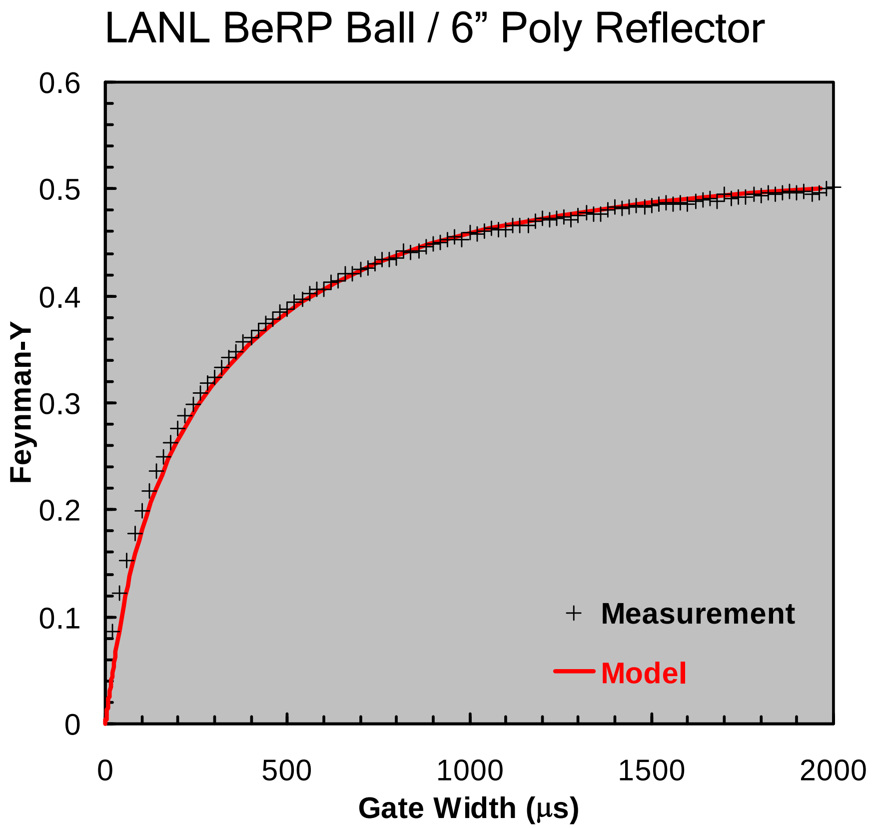
Dynamic Response Calculation



Initial Test Results







Summary

- Possible to infer configuration of an unknown radiation source from its radiation signatures

- Solutions based on multiple complementary signatures are better constrained

- Sandia is developing methods to solve for source configuration using gamma spectrometry and neutron multiplicity signatures

- Sandia developed fast method to accurately compute Feynman-Y
- Based on original work by Muñoz-Cobo, Perez, and Verdú
- Implementation uses LANL-developed, time-dependent transport solver PARTISN

- Initial test results confirm method's accuracy and potential speed

Ongoing and Future Work

- Just started 3-year project with University of Florida (UF) and University of Michigan (UM)

- Increase speed of calculations: UF, Prof. Glenn Sjoden, Dr. Ce Yi
  - Explore alternative cross-section generation schemes
  - Explore different solver options
  - Resolve slow convergence of adjoint solution

- Benchmark accuracy of calculations: UM, Prof. Sara Pozzi, Dr. Shaun Clarke
  - Augment MCNP-PoliMi tallies to accumulate neutron multiplicity distribution for proportional counters
  - Test MCNP-PoliMi against existing benchmark measurements
  - Generate synthetic test data for deterministic calculations