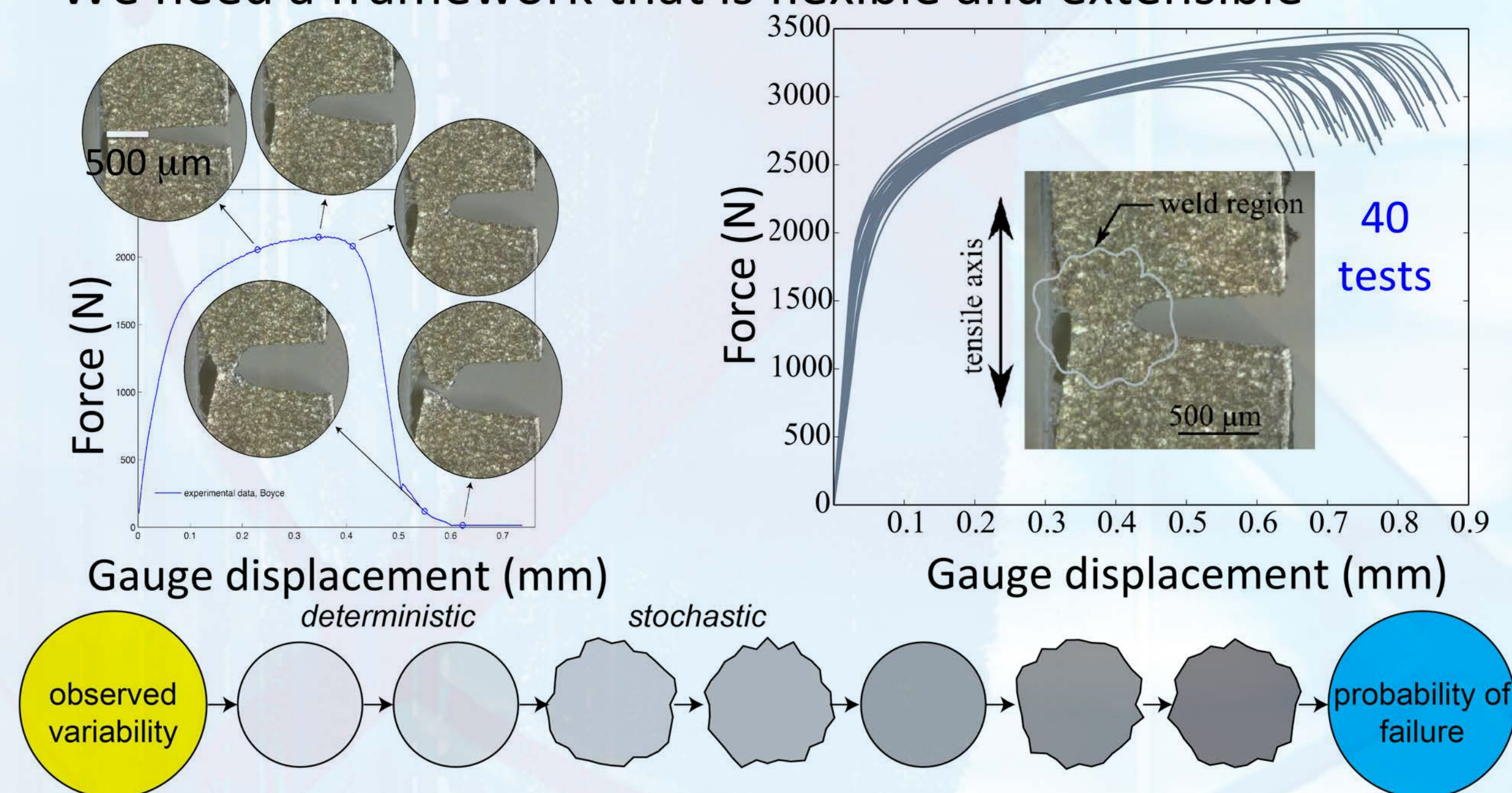


# A Framework for Predicting the Performance of A304L SS Laser Welds with Stochastic Reduced Order Models

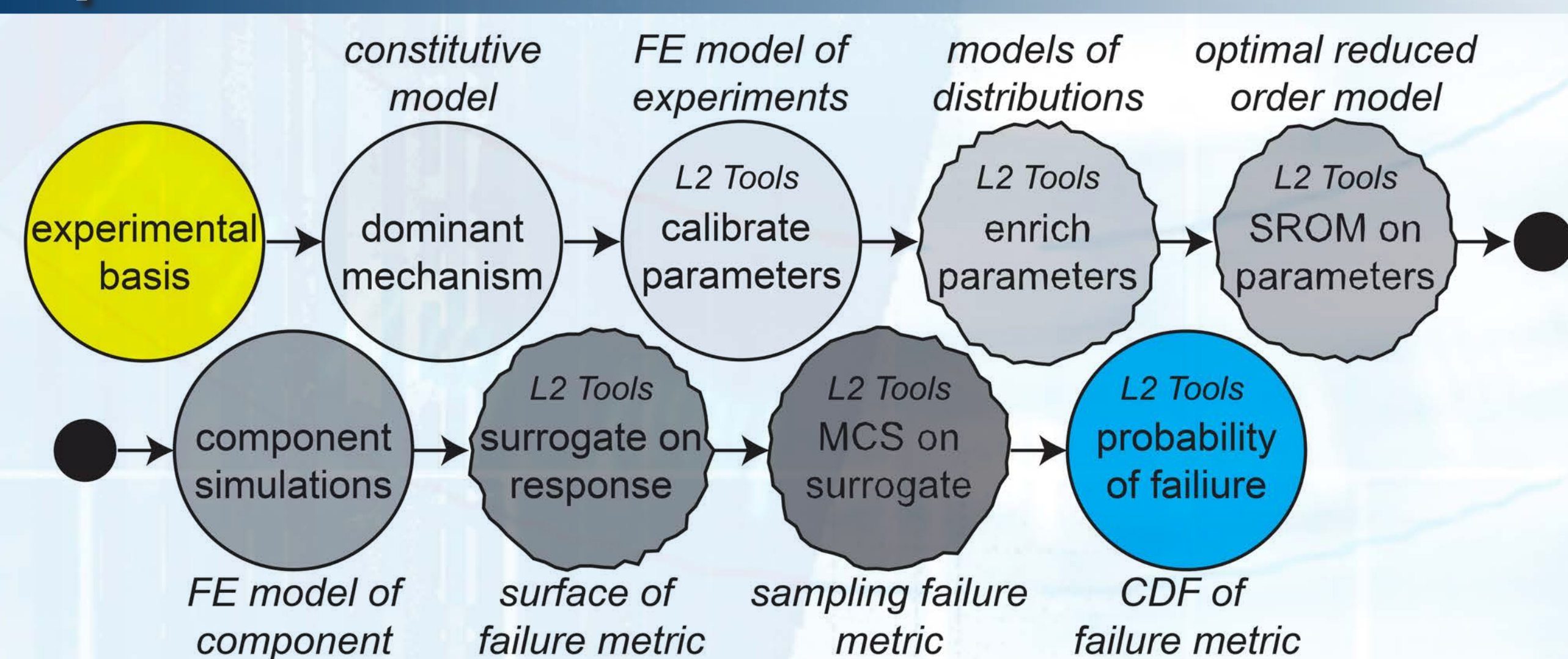
J. Foulk<sup>1</sup>, J. Emery<sup>1</sup>, R. Field<sup>1</sup>, K. Karlson<sup>1</sup>, M. Grigoriu<sup>2</sup>  
<sup>1</sup> Sandia National Laboratories; <sup>2</sup> Cornell University

## We Need a Framework for Prediction

- Nuclear safety is governed by the performance of 304L welds
- Observe geometric/material variability in laser welds
- Need to determine impact of variability on performance
- Capture tail of cumulative distribution function (CDF)
- We need a framework that is flexible and extensible



## Components of Stochastic Framework

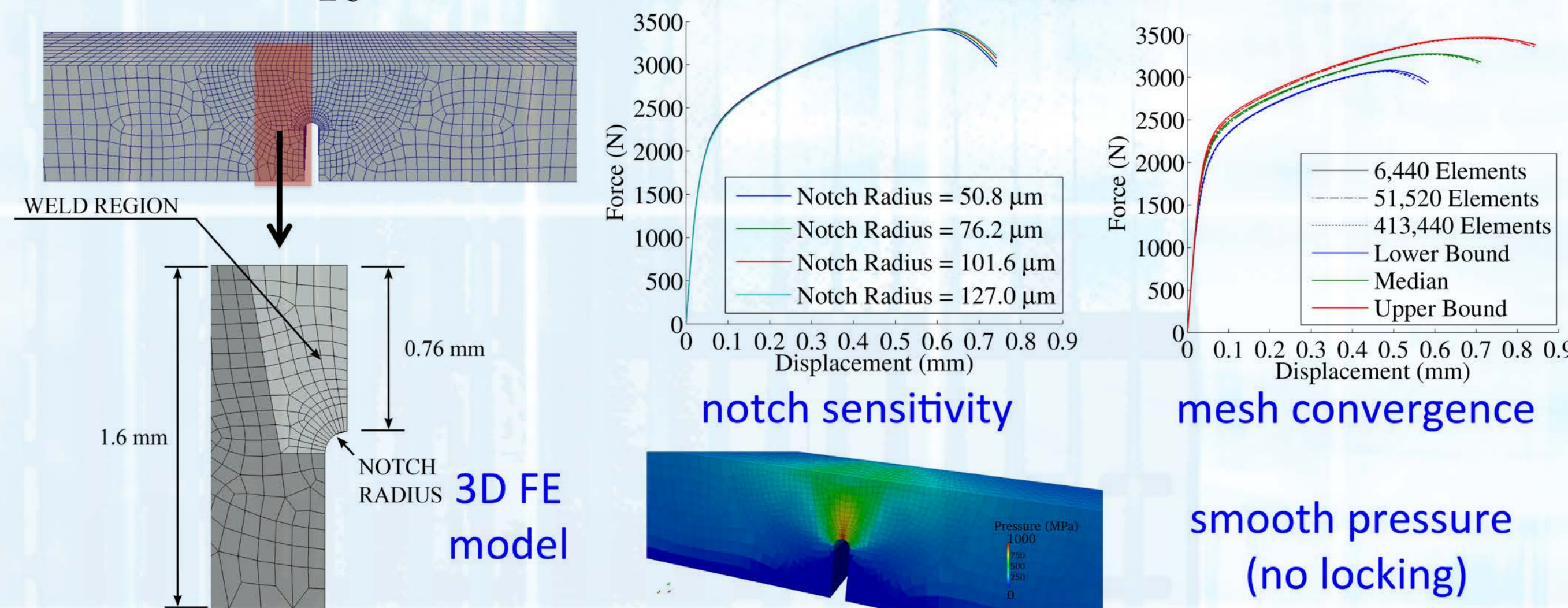


## Calibration of Stochastic Parameters

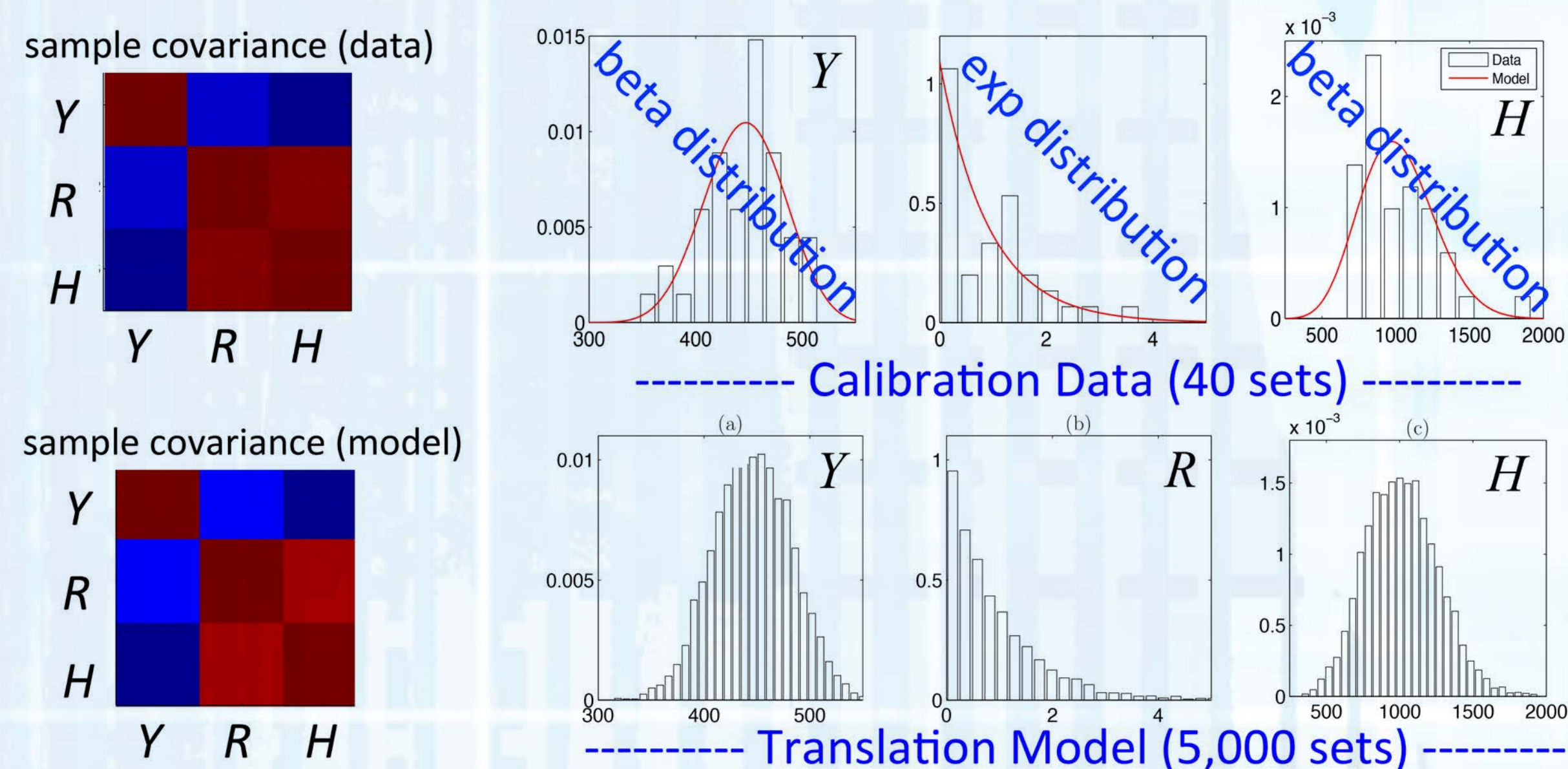
Model calibrated to the 40 available experiments using DAKOTA

$$\sigma_y = Y + \kappa \quad \dot{\kappa} = [H - R\kappa] \dot{\epsilon}_p \quad \Theta = \begin{bmatrix} Y \\ H \\ R \end{bmatrix}$$

initial yield stress  
hardening (linear)  
recovery coefficient



## Limited Data Requires Enrichment



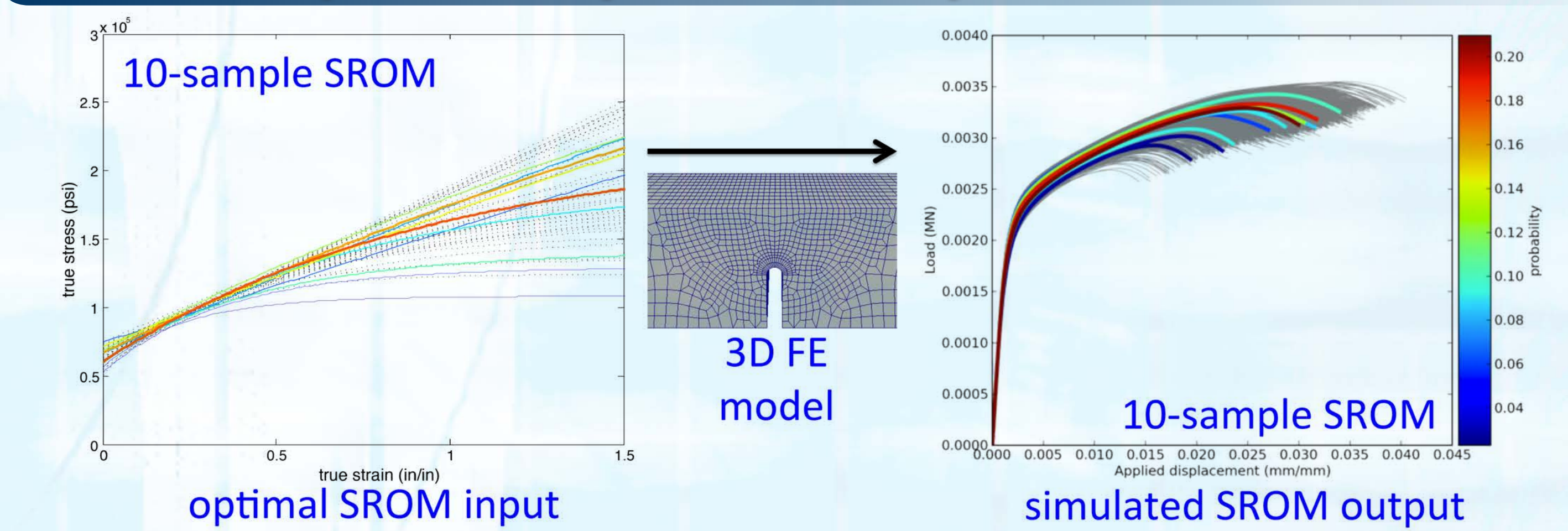
## Stochastic Reduced Order Model (SRM)

We choose to represent the uncertain input with a discrete random variable  $\tilde{\Theta}$ . The SRM is defined by the collection  $(\tilde{\theta}_k, \tilde{p}_k)$   $k = 1, \dots, m$  that minimizes an objective function of the form:

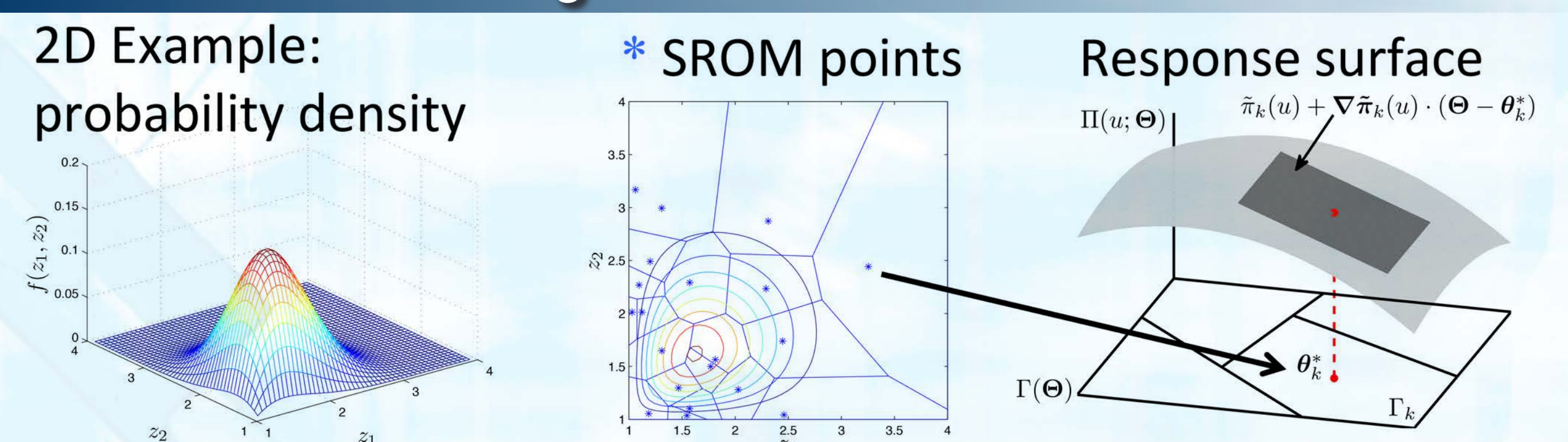
$$\max_{1 \leq r \leq \bar{r}} \max_{1 \leq s \leq d} \alpha_{s,r} |\tilde{\mu}_s(r) - \hat{\mu}_s(r)| + \max_x \max_{1 \leq s \leq d} \beta_s |\tilde{F}_s(x) - \hat{F}_s(x)| + \zeta_{s,t} \max_{s,t} |\tilde{c}(s,t) - \hat{c}(s,t)|$$

moments      cumulative distribution      correlation

## SRM Output, "Component" Response



## SRM-Based Surrogate Model

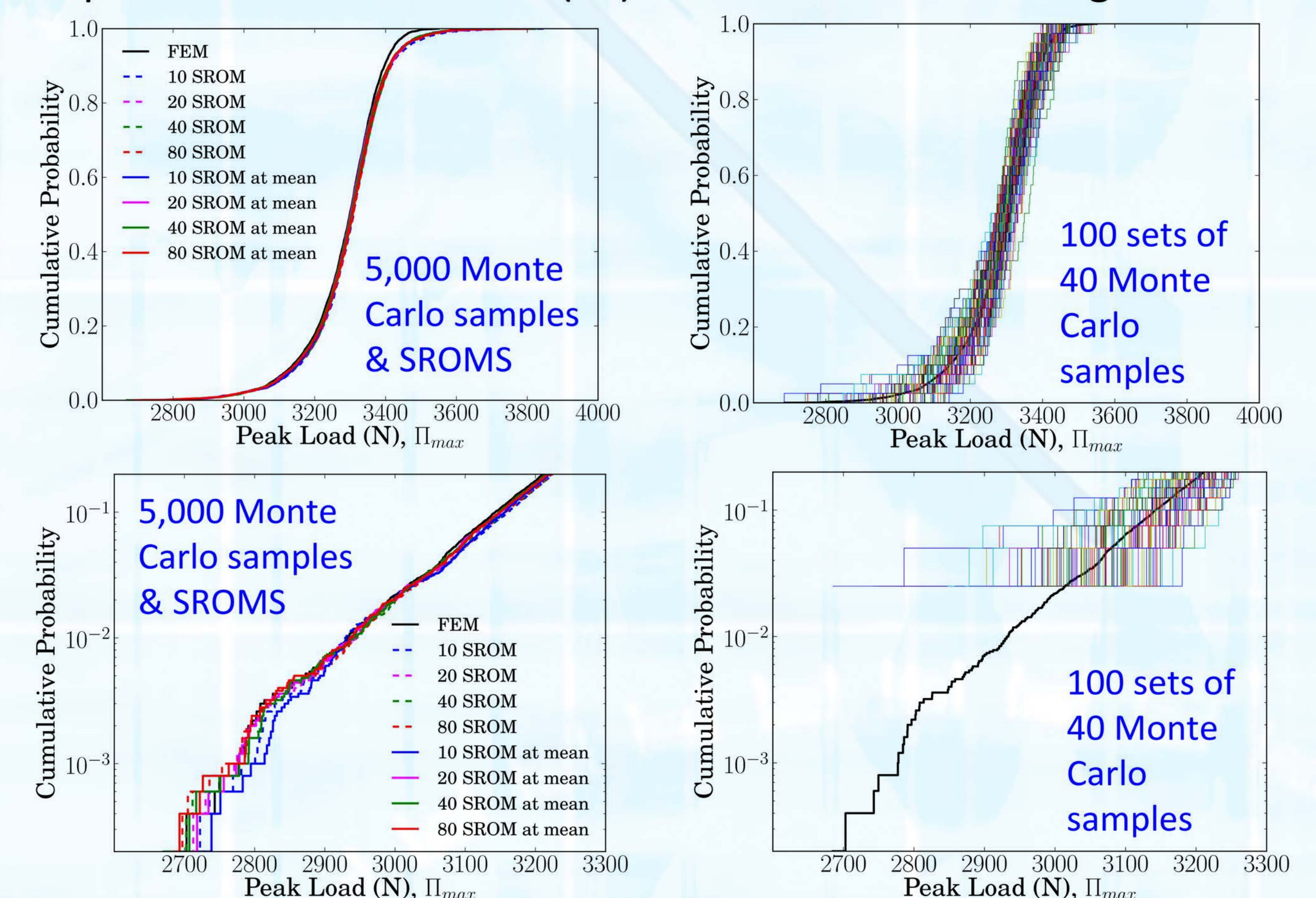


A response surface is constructed using a series of hyper-planes described with a first-order Taylor approximate:

$$\tilde{\Pi}_L(u; \Theta) = \sum_{k=1}^m 1(\Theta \in \Gamma_k) [\tilde{\pi}_k(u) + \nabla \tilde{\pi}_k(u) \cdot (\Theta - \theta_k^*)]$$

## MCS on SRM-based Surrogate Yields Tail in CDF

Compare "brute force" MCS (FE) with MCS on the surrogate model



For a linear response surface, 10-sample SRM = 40 calculations

## Summary of Findings

- We cannot afford "brute force" Monte Carlo Simulation (MCS)
- Developed framework for constructing and applying SRMs
- SRM-based surrogate accurately captures the tail of the CDF
- SRM-based surrogate is superior to "brute force" MCS

PREDICTIVE ENGINEERING SCIENCE PANEL