

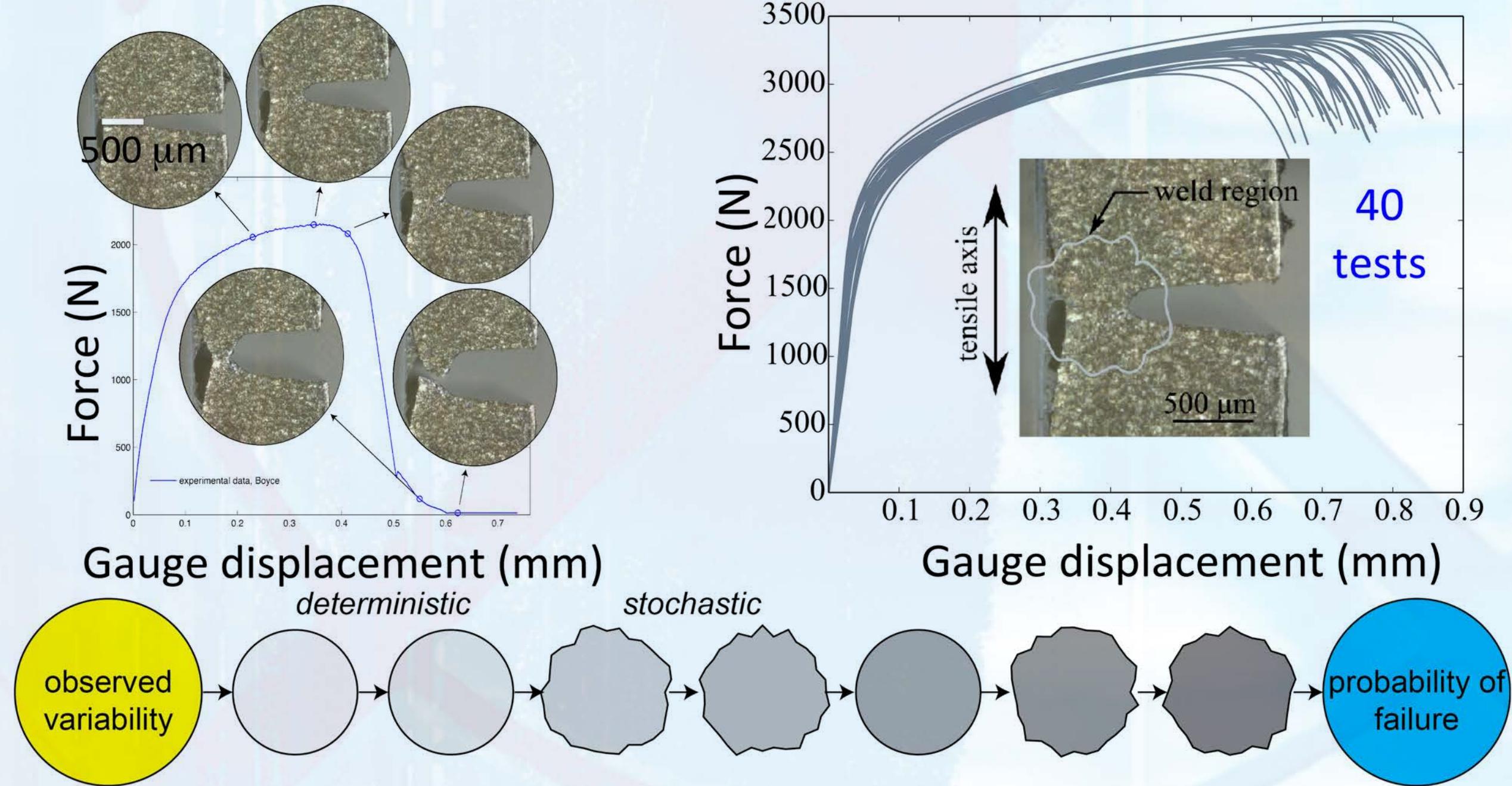
A Framework for Predicting the Performance of A304L SS Laser Welds with Stochastic Reduced Order Models

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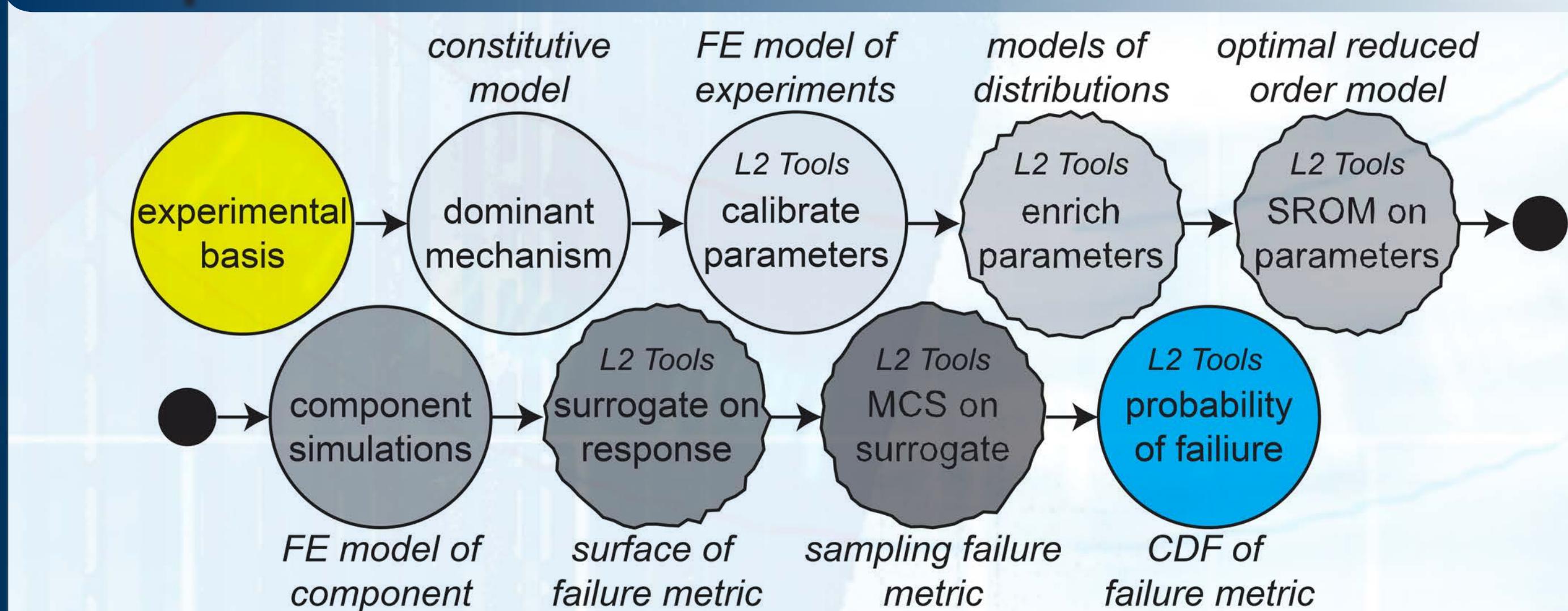
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We Need a Framework for Prediction

- Nuclear safety is governed by the performance of 304L welds
- Observe geometric/material variability in laser welds
- Need to determine impact of variability on performance
- Capture tail of cumulative distribution function (CDF)
- We need a framework that is flexible and extensible



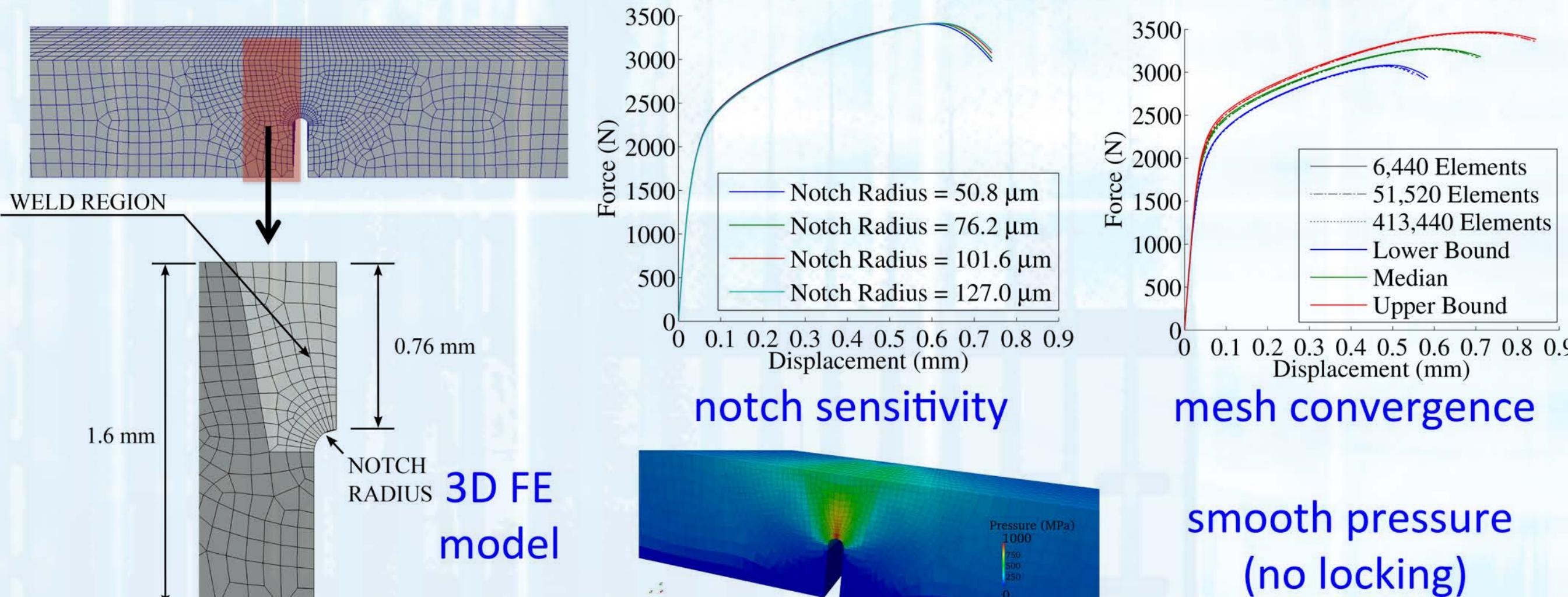
Components of Stochastic Framework



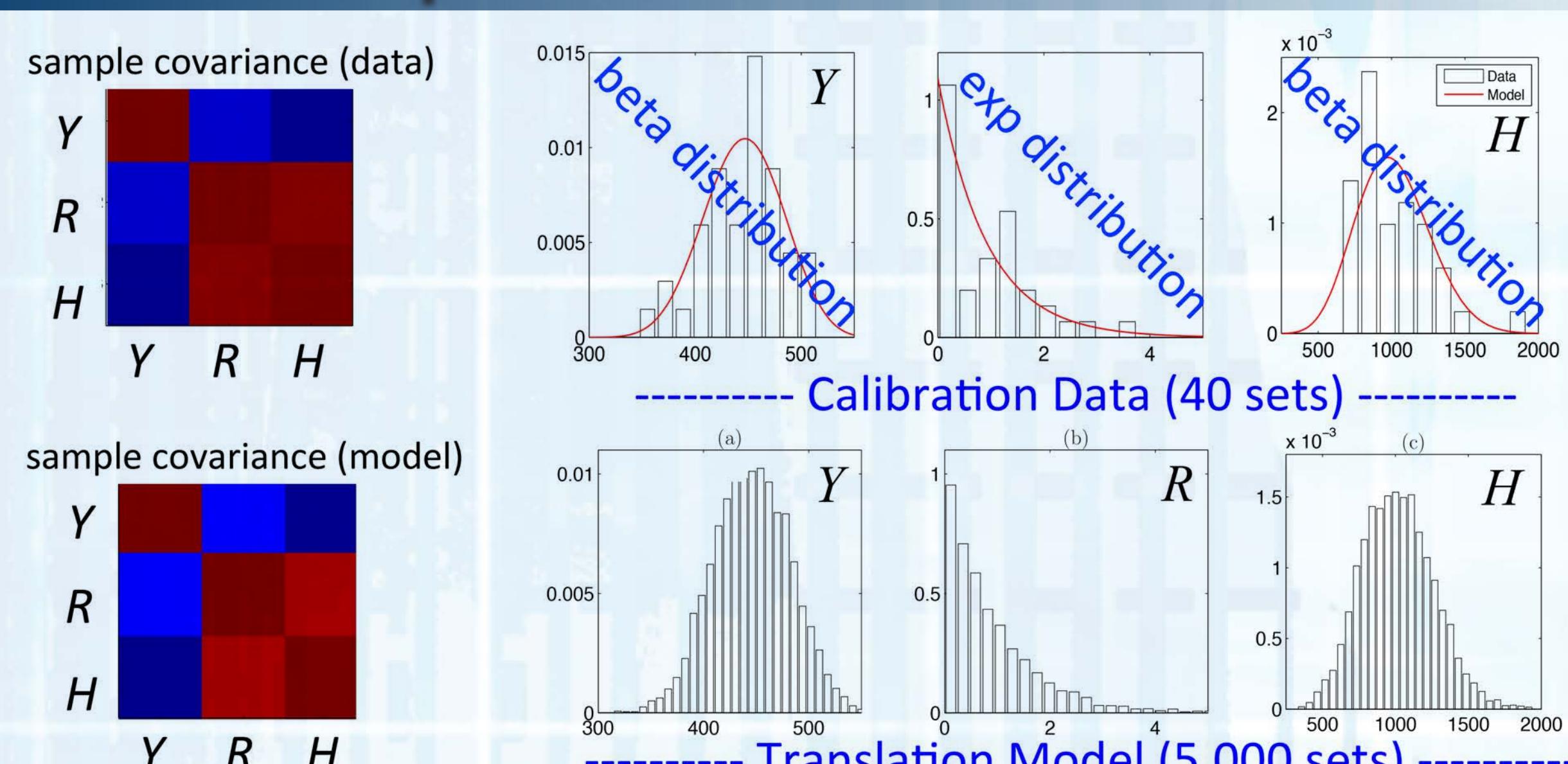
Calibration of Stochastic Parameters

Model calibrated to the 40 available experiments using DAKOTA

$$\sigma_y = Y + \kappa \quad \dot{\kappa} = [H - R\kappa] \dot{\epsilon}_p \quad \Theta = \begin{bmatrix} Y \\ H \\ R \end{bmatrix} \quad \begin{aligned} \kappa(\epsilon_p) &= \frac{H}{R} [1 - \exp(-R\epsilon_p)] \end{aligned}$$



Limited Data Requires Enrichment

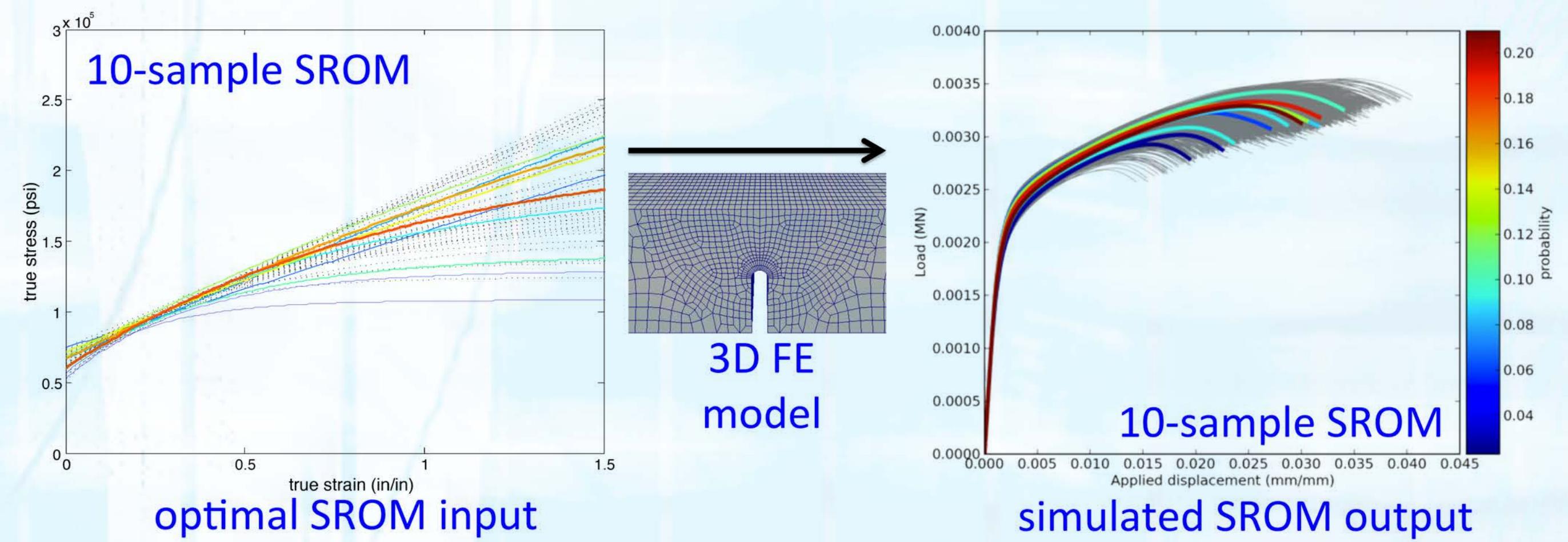


Stochastic Reduced Order Model (SROM)

We choose to represent the uncertain input with a discrete random variable $\tilde{\Theta}$. The SROM is defined by the collection $(\tilde{\theta}_k, \tilde{p}_k)$ $k = 1, \dots, m$ that minimizes an objective function of the form:

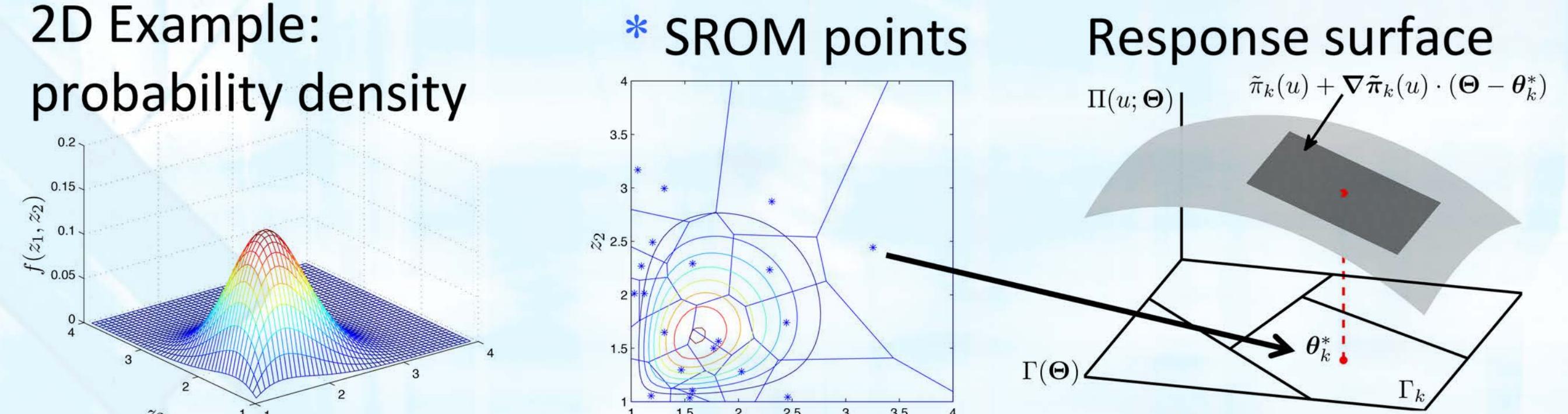
$$\max_{1 \leq r \leq \bar{r}} \max_{1 \leq s \leq d} \underbrace{\alpha_{s,r} |\tilde{\mu}_s(r) - \hat{\mu}_s(r)|}_{\text{moments}} + \max_x \max_{1 \leq s \leq d} \underbrace{\beta_s |F_s(x) - \hat{F}_s(x)|}_{\text{cumulative distribution}} + \underbrace{\zeta_{s,t} \max_{s,t} |\tilde{c}(s,t) - \hat{c}(s,t)|}_{\text{correlation}}$$

SROM Output, "Component" Response



SROM-Based Surrogate Model

2D Example:
probability density

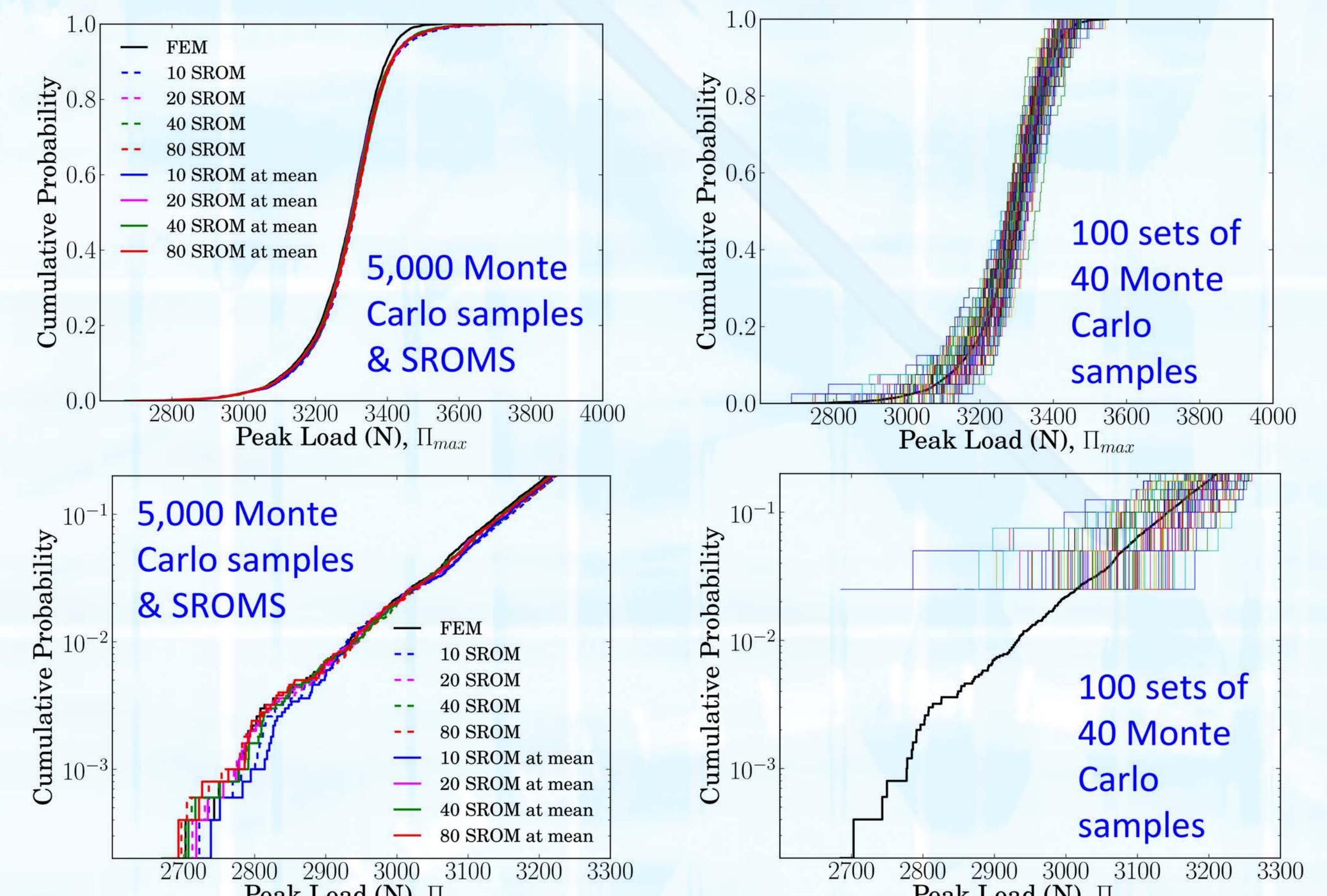


A response surface is constructed using a series of hyper-planes described with a first-order Taylor approximate:

$$\tilde{\Pi}_L(u; \Theta) = \sum_{k=1}^m \mathbb{1}(\Theta \in \Gamma_k) [\tilde{\pi}_k(u) + \nabla \tilde{\pi}_k(u) \cdot (\Theta - \theta_k^*)]$$

MCS on SROM-based Surrogate Yields Tail in CDF

Compare "brute force" MCS (FE) with MCS on the surrogate model



For a linear response surface, 10-sample SROM = 40 calculations

Summary of Findings

- We cannot afford "brute force" Monte Carlo Simulation (MCS)
- Developed framework for constructing and applying SROMs
- SROM-based surrogate accurately captures the tail of the CDF
- SROM-based surrogate is superior to "brute force" MCS

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