Final Report

FAST COMPUTATIONAL ALGORITHMS FOR PARTIAL DIFFERENTIAL EQUATIONS AND UNCERTAINTY QUANTIFICATION

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A summary of the accomplishments made during the term of this grant (July 2016 – June 2020) is given below. These accomplishments fall into three main categories: rank-reduced algorithms for parameter-dependent partial-differential equations (PDEs), discretization methods for parameter-dependent PDEs, and new and efficient solution algorithms for PDEs of various types. Publications cited in this section are those supported by the grant, listed immediately below.

Rank-reduced algorithms for parameter-dependent PDEs (Publications P2, P4, P5, P9, P10, P12, P13, P14). This represents the dominant effort of the project. Results include several studies (P5, P9) demonstrating the utility of multigrid methods for computing low-rank representations of parameter-dependent solutions to discrete PDEs, including the first proof establishing textbook convergence properties for low-rank methods. Related work (P4) showed the impact of spatial discretization on rank-compression strategies for constructing low-rank solutions and established the capability of low-rank methods for solving nonlinear PDEs (P10). Work in (P12) showed that low-rank versions of tensor methods can be used to significantly reduce the costs of solving the time-dependent parameter-dependent Navier-Stokes equations. Results in (P13) demonstrated that reduced-order models reduce the computational costs of performing Bayesian analysis of statistical inverse problems. Finally, results in (P2) demonstrated the effectiveness of empirical interpolation methods for the Navier-Stokes equations and introduced efficient preconditioning strategies for the associated discrete systems.

Impact of discretization strategies in parameter space (P6, P7). Results in (P6) demonstrated the effectiveness of so-called goal-oriented Petrov-Galerkin methods in enhancing the

utility of reduced-order models by designing strategies for model reduction to match properties of quantities of interest. Paper (P7) showed that ANOVA methods can be used to determine effective dimensions in parameter space for problems with inherent anisotropy such as models of convection-dominated flow.

Efficient algorithms for applications (P1, P3, P8, P11). Several studies (P3, P11, P16) were concerned with development of new efficient algorithms for models of multiphase flow; results in (P11) in particular established new robust nonlinear solvers for handling phase transitions. Publication (P1) developed a new preconditioning strategy for handling Schur complements that arise from the complex coupling between the Navier–Stokes equations and the Maxwell equations in models of magnetohydrodynamics. Results in (P8) used ideas from uncertainty quantification to construct new efficient algorithms for linear stability analysis of dynamical systems.

Support of doctoral students and postdoc (P13, P14, P15, P16, P18). The grant provided partial support for three students who completed their dissertations during the term of the grant. After graduating, Kookjin Lee (P14) obtained a postdoc at Sandia National Laboratories working with Kevin Carlberg. Quan Bui (P15) obtained a postdoctoral appointment at Lawrence Livermore National Laboratories under the direction of Daniel Osei-Kuffuor. Tengfei Su (P16) took a position in a financial group at Wells Fargo Bank. Akwum Onwunta (P13, P18) has been supported as a postdoc since July 2018.

Publications

P1. E. G. Phillips, J. N. Shadid, E. C. Cyr, H. C. Elman, and R. P. Pawlowski, Block preconditioners for stable mixed nodal and edge finite element representations of incompressible resistive MHD, *SIAM Journal on Scientific Computing* 38:B1009-B1031, 2016.

P2. H. C. Elman and V. Forstall, Numerical solution of the steady-state Navier-Stokes equations using empirical interpolation methods, *Computer Methods in Applied Mechanics and Engineering* 317:380-399, 2017.

P3. Q. Bui, H. C. Elman and D. J. Moulton, Algebraic multigrid preconditioners for multiphase flow in porous media, *SIAM Journal on Scientific Computing* 39:S662-S680, 2017.

P4. K. Lee and H. C. Elman, A preconditioned low-rank projection method with a rank-reduction scheme for stochastic partial differential equations, *SIAM Journal on Scientific Computing* 39:S828-S850, 2017.

P5. H. C. Elman and T. Su, A low-rank multigrid method for the stochastic steady-state diffusion problem *SIAM Journal on Matrix Analysis and Applications* 39:492-509, 2018.

P6. K. Lee, K. Carlberg and H. C. Elman, Stochastic least-squares Petrov-Galerkin method for parameterized linear systems, *SIAM/ASA Journal on Uncertainty Quantification* 6:374-396, 2018.

P7. H. Cho and H. C. Elman, An adaptive reduced basis collocation method based on PCM ANOVA decomposition for anisotropic stochastic PDEs, *International Journal for Uncertainty Quantification* 8:193-210, 2018.

P8. H. C. Elman and D. J. Silvester, Collocation methods for exploring perturbations in linear stability analysis, *SIAM Journal on Scientific Computing* 40:A2667-A2693, 2018.

P9. H. C. Elman and T. Su, Low-rank solution methods for stochastic eigenvalue problems *SIAM Journal on Scientific Computing* 41:A2657-S2680, 2019.

P10. K. Lee, H. C. Elman and B. Sousedík, A low-rank solver for the Navier-Stokes equations

with uncertain viscosity, *SIAM/ASA Journal on Uncertainty Quantification* 7:1275-1300, 2019.

P11. Q. Bui and H. C. Elman, Semi-smooth Newton methods for nonlinear complementarity formulation of compositional two-phase flow in porous media, arXiv:1805.05801v1, May 2018. To appear in *Journal of Computational Physics*.

P12. H. C. Elman and T. Su, A Low-Rank Solver for the Stochastic Unsteady Navier-Stokes Problem arXiv:1906.06785v1, June 2019. Submitted to *Computer Methods in Applied Mechanics and Engineering*.

P13. H. C. Elman and A. Onwunta, Reduced-Order Modeling for Nonlinear Bayesian Statistical Inverse Problems, arXiv:1909.02539v1, August 2019. Submitted to *Journal of Computational Physics*.

P14. K. Lee, "Fast and Optimal Solution Algorithms for Parameterized Partial Differential Equations," Doctoral Dissertation, University of Maryland, August 2017.

P15. Q. M. Bui, "Fast Solvers and Preconditioners for Multiphase Flow in Porous Media," Doctoral Dissertation, University of Maryland, May 2018.

P16. T. Su, "Low-Rank Solution Methods for Discrete Parametrized Partial Differential Equations," Doctoral Dissertation, University of Maryland, May 2019.

P17. Q. M. Bui, L. Wang and D. Osei-Kuffuor, Algebraic multigrid preconditioners for two-phase flow in porous media with phase transitions, *Advances in Water Resources* 114:19-28, 2018.

P18. P. Benner, A. Onwunta and M. Stoll, On the existence and uniqueness of the solution of a parabolic optimal control problem with uncertain inputs, arXiv:1809.10645v2, October 2018.

Presentations of Results

Mathematics Colloquium, Morgan State University, Oct. 2016.

Mathematics Colloquium, Colorado School of Mines, Dec. 2016.

GAMM Summer School, Berlin, Germany, Sept. 2017.

Center for Computational and Applied Mathematics Colloquium, Purdue University, Jan. 2017.

SIAM Conference on Computational Science and Engineering, Atlanta, GA, Feb. 2017.

Applied and Computational Mathematics Seminar, UC Irvine, May 2017.

Householder Symposium XX, Blacksburg, VA, June 2017.

Workshop on Quantification of Uncertainty: Improving Efficiency and Technology, Trieste, Italy, Jul. 2017.

Workshop on Key UQ Methodologies and Motivating Applications, Isaac Newton Institute for the Mathematical Sciences, University of Cambridge, England, Jan. 2018.

Conference on "Scientific Computing Around Louisiana," Louisiana State University, Baton Rouge, LA, Feb. 2018.

Copper Mountain Conference on Iterative Methods, Copper Mountain, CO, Apr. 2019. (Presentations by the PI and two supported students, Quan Bui and Tengfei Su.)

Chemnitz Finite Element Symposium, Chemnitz, Germany, Sept. 2018.

Tufts University, Applied Mathematics Seminar, Nov. 2018.

Applied Mathematics (CSCAMM) Seminar, University of Maryland, Nov. 2019.

SIAM Conference on Computational Science and Engineering, Spokane Washington, Feb. 2019. (Multiple presentations of supported work given by student advisee T. Su.)