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Viscous Added Mass of a Moving Solid Object in a Closed Liquid-Filled Container

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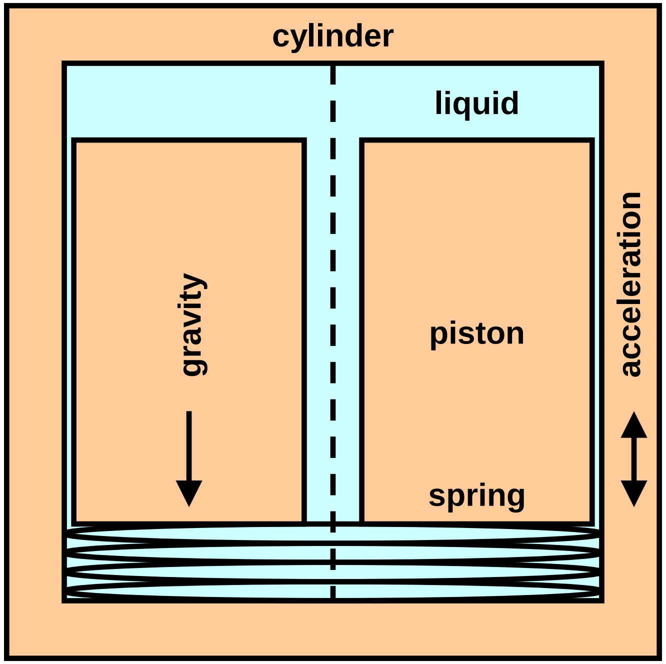
Piston in Liquid-Filled Cylinder



$$\frac{dZ_P}{dt} = U_P$$

$$M_P \frac{dU_P}{dt} = ???$$

Motion of piston in closed cylinder

- Piston fills most of cylinder
- Spring supports it against gravity
- Viscous liquid fills open volume
- Cylinder is transiently accelerated

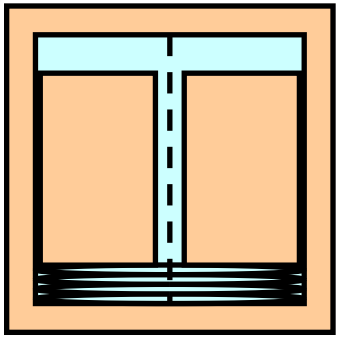
Develop ODE model for dynamics

- Consider cylinder reference frame
- Find liquid forces on piston
- Quasi-steady approximation

General approach is simplified here

- Axisymmetric geometry
- Motion only along axis

Rigorous Dynamics



$$\frac{dZ_P}{dt} = U_P, \quad M_P \frac{dU_P}{dt} = -K_S\left(Z_P - Z_S\right) - M_P\left(1 - \left(\rho_L/\rho_P\right)\right)\left(g + A_C\right) + F_L$$

spring gravity/acceleration/buoyancy liquid

Liquid force on piston in cylinder can be found rigorously

- Unsteady incompressible Navier-Stokes equations
- Cylinder reference frame: acceleration acts like gravity
- Subtract hydrostatic pressure: gravity/acceleration/buoyancy
- Piston position and velocity both vary in time

Liquid force is integral of stress tensor over piston surface

$$\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{u} = 0 \qquad \rho_L \left\{ \frac{\partial \mathbf{u}}{\partial t} + \left(\mathbf{u} \cdot \frac{\partial}{\partial \mathbf{x}} \right) \mathbf{u} \right\} = \frac{\partial}{\partial \mathbf{x}} \cdot \boldsymbol{\sigma} \qquad \boldsymbol{\sigma} = -p\mathbf{I} + \mu_L \left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}}{\partial \mathbf{x}}^{\mathrm{T}} \right)$$

$$\mathbf{u} = U_P \hat{\mathbf{e}}_z \text{ on piston} \qquad \mathbf{u} = \mathbf{0} \text{ on cylinder} \qquad F_L = \int_{S_P} \hat{\mathbf{e}}_z \cdot \boldsymbol{\sigma} \cdot \hat{\mathbf{n}} \, dS$$

Quasi-Steady Approximation

Steady: Drag

$$\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{u}_0 = 0 \qquad \rho_L \frac{\partial}{\partial \mathbf{x}} \cdot (\mathbf{u}_0 \mathbf{u}_0) = \frac{\partial}{\partial \mathbf{x}} \cdot \boldsymbol{\sigma}_0 \qquad \boldsymbol{\sigma}_0 = -p_0 \mathbf{I} + \mu_L \left(\frac{\partial \mathbf{u}_0}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}_0}{\partial \mathbf{x}}^{\mathrm{T}} \right)$$

$$\mathbf{u}_0 = \hat{\mathbf{e}}_z U_P \text{ on piston} \qquad \mathbf{u}_0 = \mathbf{0} \text{ on cylinder} \qquad F_{L0} = \int_{S_P} \hat{\mathbf{e}}_z \cdot \boldsymbol{\sigma}_0 \cdot \hat{\mathbf{n}} dS \equiv -\beta_D U_P$$

Unsteady: Added Mass

$$\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{u}_1 = 0 \qquad \rho_L \frac{\partial \mathbf{u}_0}{\partial U_P} \frac{dU_P}{dt} + \rho_L \frac{\partial}{\partial \mathbf{x}} \cdot (\mathbf{u}_0 \mathbf{u}_1 + \mathbf{u}_1 \mathbf{u}_0) = \frac{\partial}{\partial \mathbf{x}} \cdot \boldsymbol{\sigma}_1 \qquad \boldsymbol{\sigma}_1 = -p_1 \mathbf{I} + \mu_L \left(\frac{\partial \mathbf{u}_1}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}_1}{\partial \mathbf{x}}^{\mathrm{T}} \right)$$

$$\mathbf{u}_1 = \mathbf{0} \text{ on piston} \qquad \mathbf{u}_1 = \mathbf{0} \text{ on cylinder} \qquad F_{L1} = \int_{S_P} \hat{\mathbf{e}}_z \cdot \boldsymbol{\sigma}_1 \cdot \hat{\mathbf{n}} dS \equiv -M_A (dU_P/dt)$$

- **Find liquid-force expression for quasi-steady flow**
  - Closed container has time scale to achieve steady flow
  - Piston position & velocity change little over this time scale
- **Decompose flow field into sum of two contributions**
  - Steady part ("0", large) yields drag force
  - Unsteady part ("1", small) yields added-mass force

Quasi-Steady Stokes Limit

Stokes Limit

$$\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{u} = 0 \qquad \mathbf{0} = \frac{\partial}{\partial \mathbf{x}} \cdot \boldsymbol{\sigma} \qquad \boldsymbol{\sigma} = -p\mathbf{I} + 2\mu_L \mathbf{S}$$

$$\mathbf{u} = U_P \hat{\mathbf{e}}_z \text{ on piston} \qquad \mathbf{u} = \mathbf{0} \text{ on cylinder} \qquad \mathbf{S} = \frac{1}{2}\left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}}{\partial \mathbf{x}}^{\mathrm{T}}\right)$$

$$F_L = -\beta_D U_P - M_A \left(dU_P / dt\right) \qquad \frac{1}{2}\beta_D U_P^{\ 2} = \mu_L \int_{V_L} \mathbf{S} : \mathbf{S} dV \qquad \frac{1}{2} M_A U_P^{\ 2} = \frac{1}{2}\rho_L \int_{V_L} \mathbf{u} \cdot \mathbf{u} dV$$

Expressions simplify considerably in Stokes limit

- Navier-Stokes: 3 CFD solutions for each piston velocity
  - Drag coefficient & added mass depend on piston velocity
- Stokes: only 1 CFD solution for all piston velocities
  - Drag coefficient & added mass are constants in this limit

Added mass caused by change in liquid kinetic energy

- Acceleration changes both piston & liquid kinetic energy
- Liquid acts like flywheel storing kinetic energy

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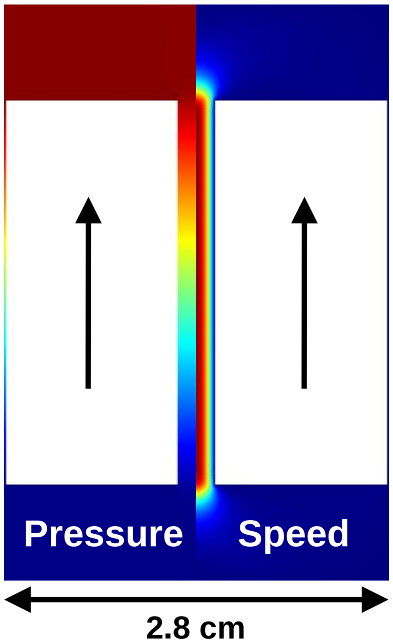
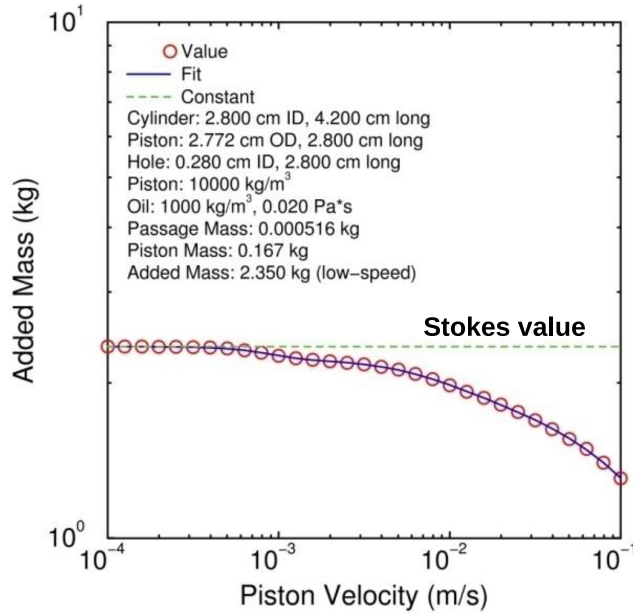
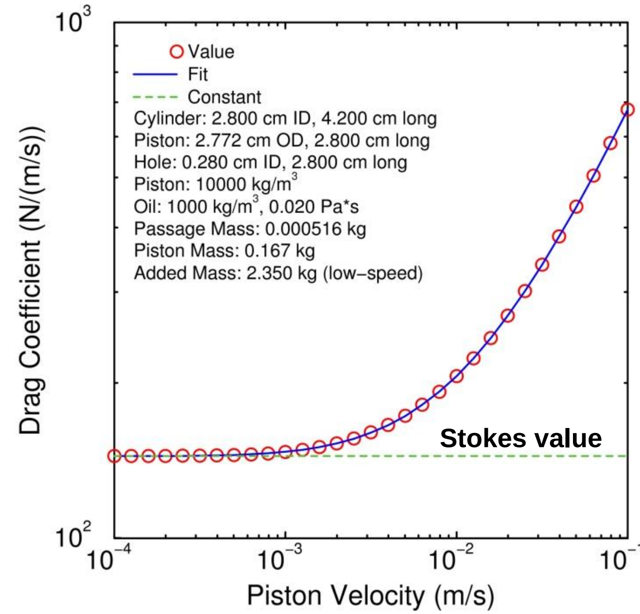
Piston-Cylinder Example





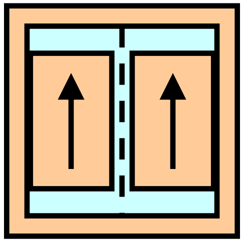


Added mass is very large (Stokes below)

- **Liquid mass: 0.000516 kg (in passages)**
- **Piston mass: 0.167 kg**
- **Added mass: 2.350 kg (~14 × piston, ~4500 × liquid)**

At large piston velocities, added mass & drag coefficient depart from corresponding Stokes values

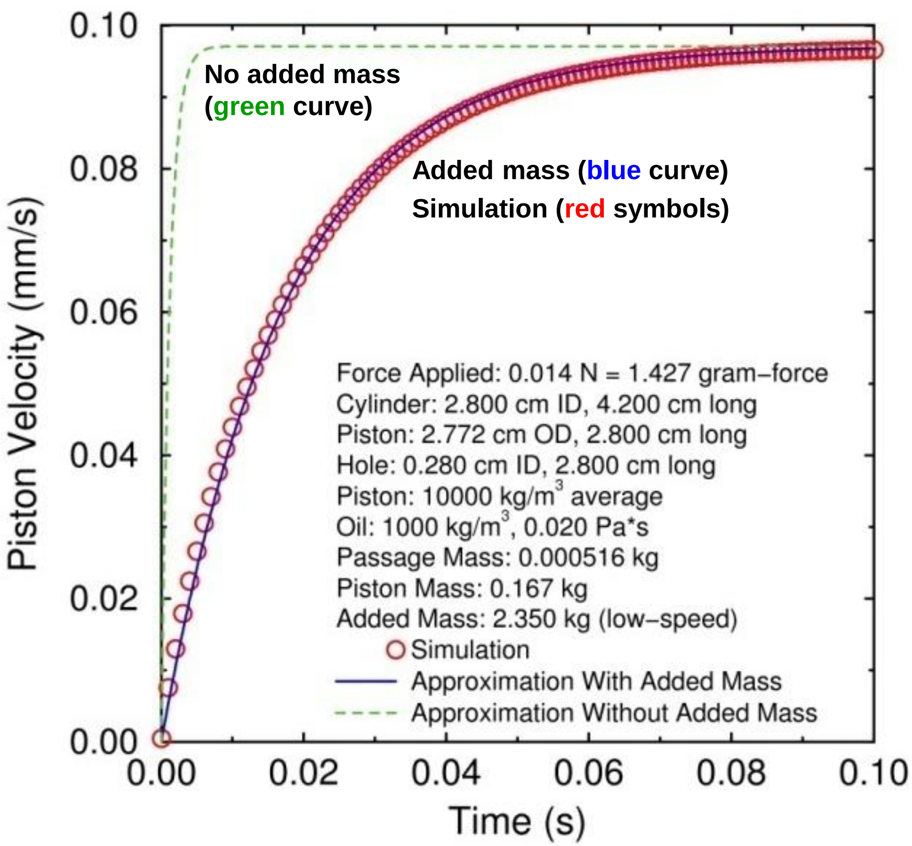
Response to Constant Force



applied force
but no spring

$$\frac{dZ_P}{dt} = U_P, \quad \left(M_P + M_A[U_P]\right)\frac{dU_P}{dt} = F_{\text{applied}} - \beta_D[U_P]U_P$$

added mass — applied force — drag force



Compare to rigorous Navier-Stokes simulation

- Apply constant force to piston (no spring)
- Piston approaches terminal velocity
- Added mass determines time scale of response

Added mass is essential for accurate dynamics

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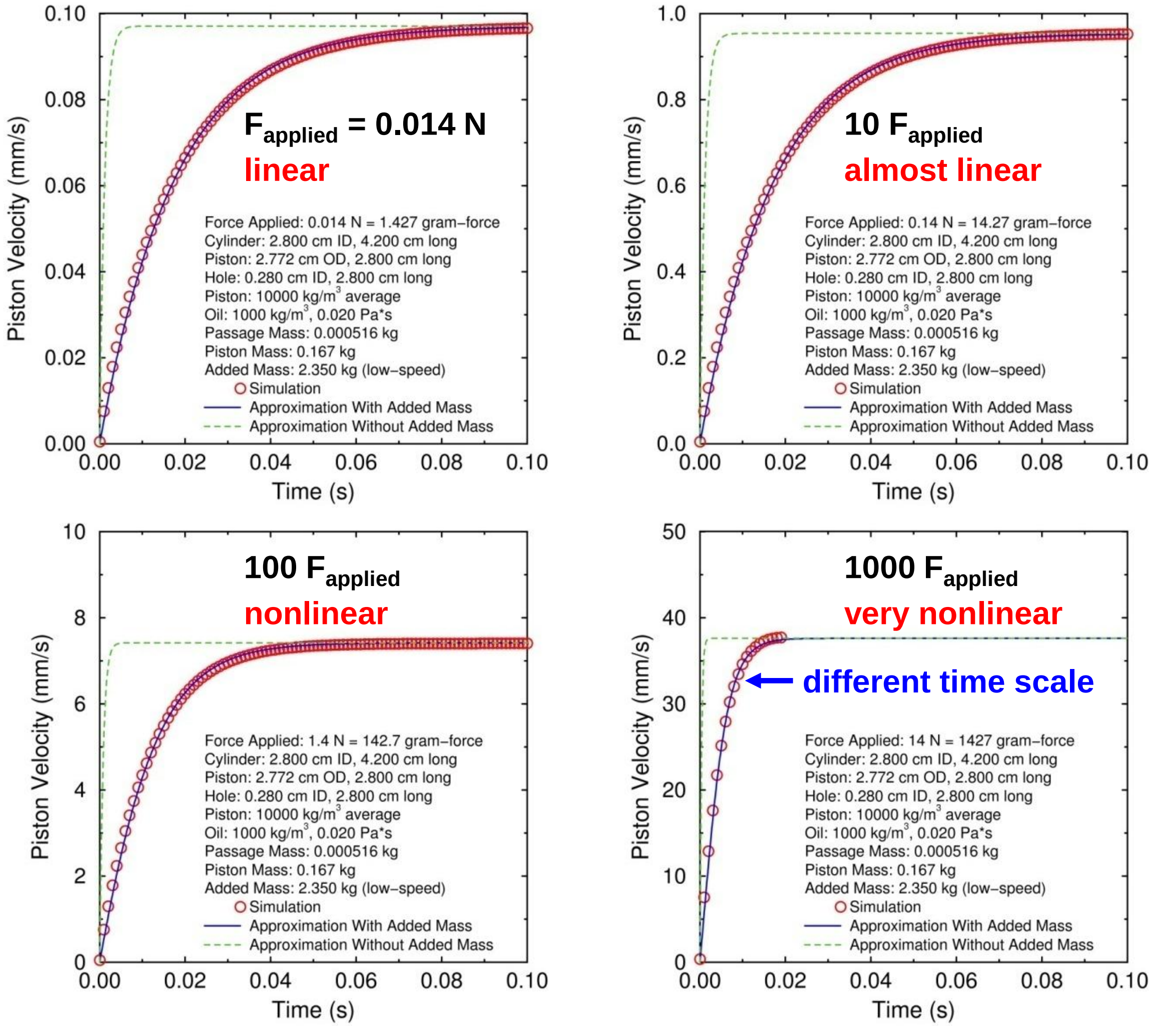
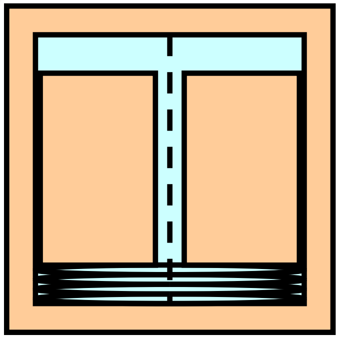
Linear to Nonlinear Conditions

Added mass produces accurate dynamics for all cases

– From linear (Stokes) to very nonlinear (Navier-Stokes)



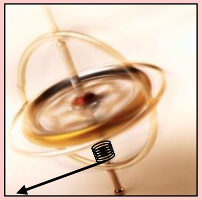
Viscous Added Mass



added mass — drag coefficient

$$\frac{dZ_P}{dt} = U_P, \quad \left(M_P + M_A\left[U_P\right]\right)\frac{dU_P}{dt} = -\beta_D\left[U_P\right]U_P$$
$$-M_P\left(1-\left(\rho_L/\rho_P\right)\right)\left(g + A_C\right) - K_S\left(Z_P - Z_S\right)$$

gravity/acceleration/buoyancy — spring force



Kinetic Energy
piston ~ cord
liquid ~ rotor

Observed when piston accelerates in liquid-filled cylinder
- In low-frequency limit, how liquid kinetic energy changes

Can be accurately computed for complicated shapes
- General approach based on standard CFD simulations

Large for pistons with thin passages in closed cylinders
- Relative to piston mass & liquid mass in piston passages

Dynamically significant during transient acceleration
- Must include it to agree with rigorous dynamics

Enables rich dynamic behavior during vibration
- Position dependence, interaction with gas pockets