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COOPR

Modeling and Optimization for the Electric Grid

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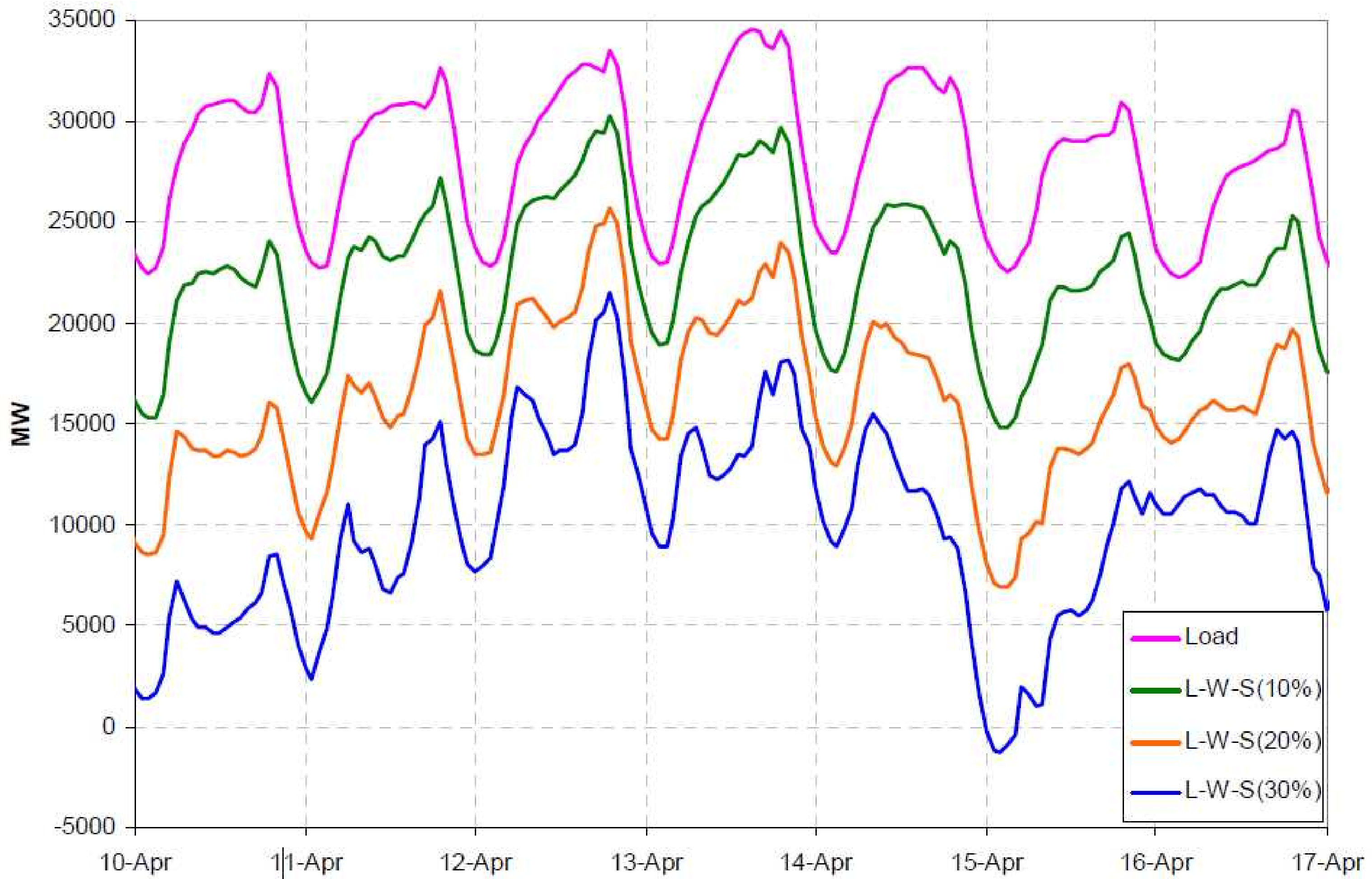
Integrating renewable energy generation

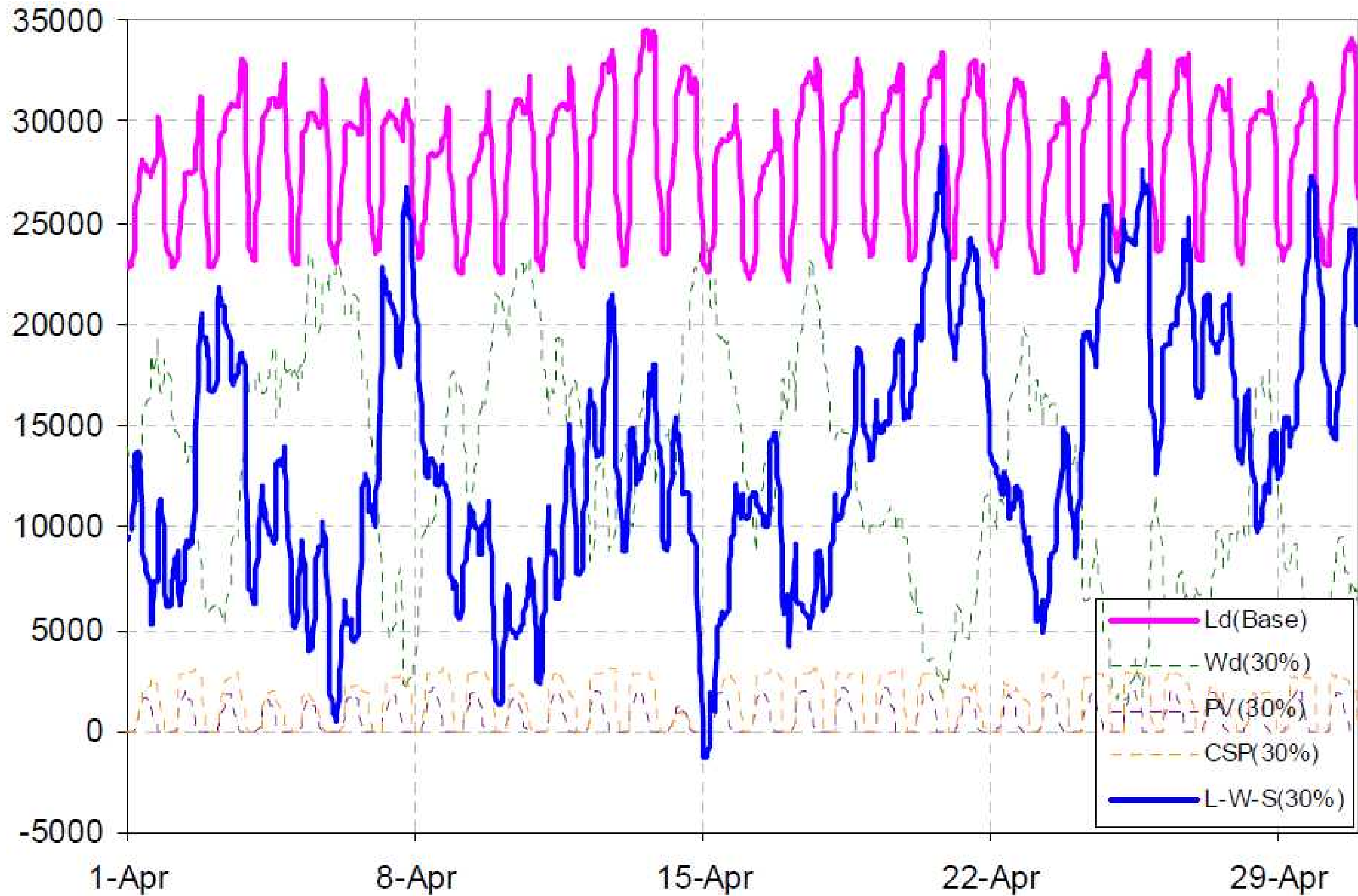
- The grid is managed by
  - Independent System Operators (ISO)
  - Regional Transmission Organizations (RTO)
  - Balancing Authorities
- Operator must balance load and generation at all times
  - Supply demand at lowest possible cost
  - Little to no storage in the grid
  - Unit-specific production ramp limits, startup and shutdown times
  - Disturbances absorbed by (spinning) reserve requirements
- Key challenges:
  - Load variability / forecast errors
  - Variability in *non-dispatchable* (renewable) generation

Dispatch must match *net load*



Plot reproduced from NREL "***Western Wind and Solar Integration Study***"
http://www.nrel.gov/electricity/transmission/western_wind.html

“Loss” of weekly periodicity



Plot reproduced from NREL “***Western Wind and Solar Integration Study***”
http://www.nrel.gov/electricity/transmission/western_wind.html

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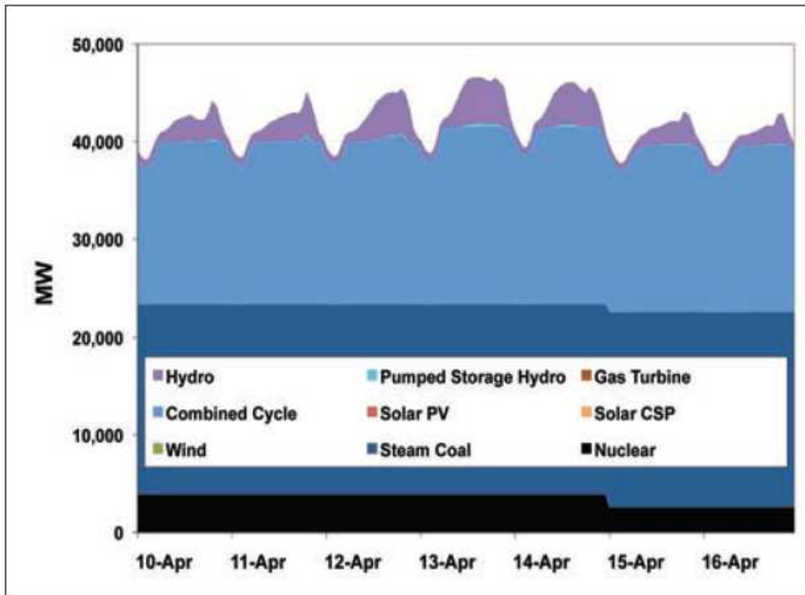
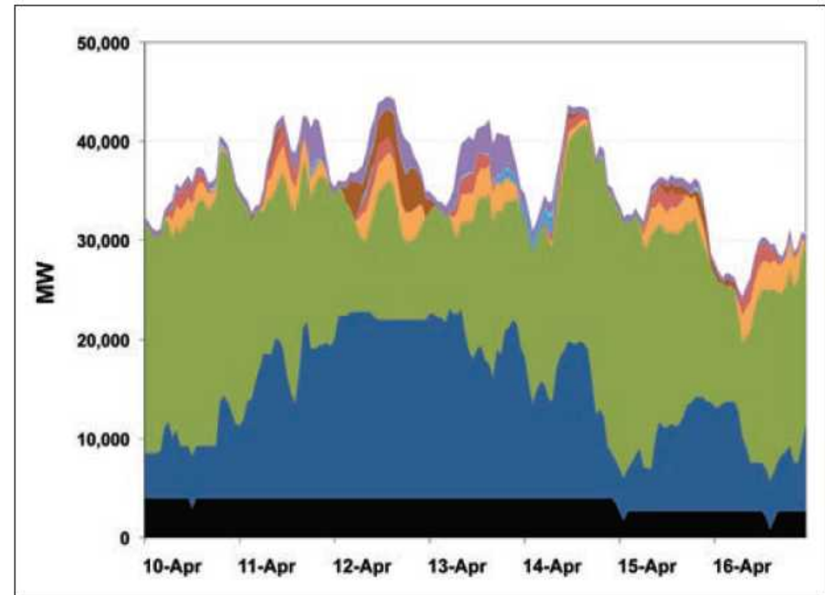
Significant impact on "base load" generators

0% Renewable Penetration



30% Renewable Penetration



Plot reproduced from NREL "***Western Wind and Solar Integration Study***"
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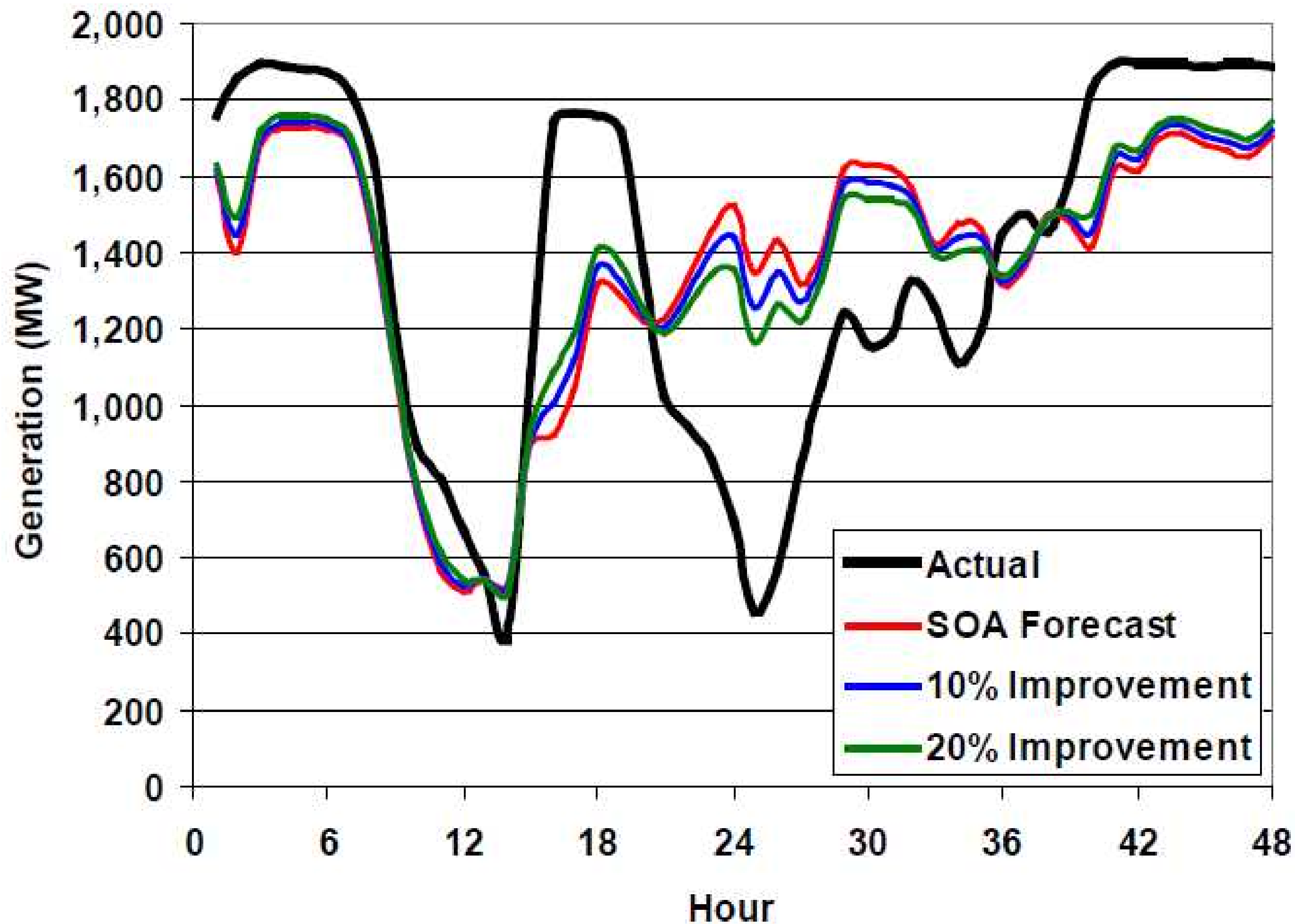
Significant gaps in renewable forecasts



Plot reproduced from NREL "***Value of Wind Power Forecasting***"
http://www.nrel.gov/electricity/transmission/western_wind.html

A word about the example problem…

- (In the US) Sequential markets (run by ISO/RTO):
  - “Unit commitment” (UC) / “Day-ahead Market” (DAM)
    - MIP run ~10 hours before the start of a day
    - Sets on/off state for all generator units hourly for 24 hours
  - “Reliability Unit Commitment” (RUC)
    - MIP run ~8 hours before the start of the day
    - Commits additional generators to meet spinning reserve and reliability (N-1 robustness) requirements
  - “Economic Dispatch” (ED) / “Security-constrained ED” (SCED)
    - “Real-time” markets: LP run hourly / every 5 minutes
    - Set generation levels, prices to meet realized demand

- Problem scale
  - 100’s – 1000’s of buses; 2-3x lines

The Challenge: MP is dense and subtle

$$\text{Minimize}: \sum_t \sum_g \left(c_g P_{g0t} + c_g^{SU} v_{gt} + c_g^{SD} w_{gt}\right) \quad (1)$$

$$\text{S.t.} \quad \theta^{\min} \le \theta_{nct} \le \theta^{\max}, \quad \forall\, n, c, t \quad (2)$$

$$\sum_{\forall\, k(n,.)} P_{kct} - \sum_{\forall\, k(.,n)} P_{kct} + \sum_{\forall\, g(n)} P_{g0t} = d_{nt},$$
$$\forall\, n, \quad c = 0, \text{ transmission contingency states } c, t \quad (3a)$$

$$\sum_{\forall\, k(n,.)} P_{kct} - \sum_{\forall\, k(.,n)} P_{kct} + \sum_{\forall\, g(n)} P_{gct} = d_{nt},$$
$$\forall\, n, \text{ generator contingency states } c, t \quad (3b)$$

$$P_{kc}^{\min} N1_{kc} z_{kt} \le P_{kct} \le P_{kc}^{\max} N1_{kc} z_{kt}, \quad \forall\, k, c, t \quad (4)$$

$$B_k(\theta_{nct} - \theta_{mct}) - P_{kct} + (2 - z_{kt} - N1_{kc}) M_k \ge 0, \quad \forall\, k, c, t \quad (5a)$$

$$B_k(\theta_{nct} - \theta_{mct}) - P_{kct} - (2 - z_{kt} - N1_{kc}) M_k \le 0, \quad \forall\, k, c, t \quad (5b)$$

$$P_g^{\min} N1_{gc} u_{gt} \le P_{gct} \le P_g^{\max} N1_{gc} u_{gt}, \quad \forall\, g, c, t \quad (6)$$

$$v_{g,t} - w_{g,t} = u_{g,t} - u_{g,t-1}, \quad \forall\, g, t \quad (7)$$

$$\sum_{q=t-UT_g+1}^{t} v_{g,q} \le u_{g,t}, \quad \forall\, g, t \in \{UT_g, \ldots, T\} \quad (8)$$

$$\sum_{q=t-DT_g+1}^{t} w_{g,q} \le 1 - u_{g,t}, \forall\, g, t \in \{DT_g, \ldots, T\} \quad (9)$$

$$P_{g0t} - P_{g0,t-1} \le R_g^{+} u_{g,t-1} + R_g^{SU} v_{g,t}, \quad \forall\, g, t \quad (10)$$

$$P_{g0,t-1} - P_{g0,t} \le R_g^{-} u_{g,t} + R_g^{SD} w_{g,t}, \quad \forall\, g, t \quad (11)$$

$$P_{gct} - P_{g0,t} \le R_g^{+}, \quad \forall\, g, c, t \quad (12)$$

$$P_{g0,t} N1_{gc} - P_{gct} \le R_g^{-}, \quad \forall\, g, c, t \quad (13)$$

$$0 \le v_{g,t} \le 1, \quad \forall\, g, t \quad (14)$$

$$0 \le w_{g,t} \le 1, \quad \forall\, g, t \quad (15)$$

$$u_{g,t} \in \{0, 1\}, \quad \forall\, g, t \quad (16)$$

Hedman, et al., "Co-Optimization of Generation Unit Commitment and Transmission Switching With N-1 Reliability," *IEEE Trans Power Systems*, 25(2), pp.1052-1063, 2010

The Challenge: MP is dense and subtle

Minimize : $\sum_t \sum_g (c_g P_{g0t} + c_g^{SU} v_{gt} + c_g^{SD} w_{gt})$

S.t. $\theta^{\min} \leq \theta_{nct} \leq \theta^{\max}, \quad \forall\, n, c, t$

$\sum_{\forall\, k(n,.)} P_{kct} - \sum_{\forall\, k(.,n)} P_{kct} + \sum_{\forall\, g(n)} P_{g0t} = d_{nt},$

$\forall\, n, \quad c = 0,$ transmission contingency states c, t

$\sum_{\forall\, k(n,.)} P_{kct} - \sum_{\forall\, k(.,n)} P_{kct} + \sum_{\forall\, g(n)} P_{gct} = d_{nt},$

$\forall\, n,$ generator contingency states c, t

$P_{kc}^{\min} N1_{kc} z_{kt} \leq P_{kct} \leq P_{kc}^{\max} N1_{kc} z_{kt}, \quad \forall\, k, c, t$

$B_k(\theta_{nct} - \theta_{mct}) - P_{kct} + (2 - z_{kt} - N1_{kc}) M_k \geq 0, \quad \forall\, k, c, t$

$B_k(\theta_{nct} - \theta_{mct}) - P_{kct} - (2 - z_{kt} - N1_{kc}) M_k \leq 0, \quad \forall\, k, c, t$

$P_g^{\min} N1_{gc} u_{gt} \leq P_{gct} \leq P_g^{\max} N1_{gc} u_{gt}, \quad \forall\, g, c, t$

$v_{g,t} - w_{g,t} = u_{g,t} - u_{g,t-1}, \quad \forall\, g, t$

$\sum_{q=t-UT_g+1}^{t} v_{g,q} \leq u_{g,t}, \quad \forall\, g, t \in \{UT_g, \ldots, T\}$

$\sum_{q=t-DT_g+1}^{t} w_{g,q} \leq 1 - u_{g,t}, \forall\, g, t \in \{DT_g, \ldots, T\}$

$P_{g0t} - P_{g0,t-1} \leq R_g^{+} u_{g,t-1} + R_g^{SU} v_{g,t}, \quad \forall\, g, t$

$P_{g0,t-1} - P_{g0,t} \leq R_g^{-} u_{g,t} + R_g^{SD} w_{g,t}, \quad \forall\, g, t$

$P_{gct} - P_{g0,t} \leq R_g^{+}, \quad \forall\, g, c, t$

$P_{g0,t} N1_{gc} - P_{gct} \leq R_g^{-}, \quad \forall\, g, c, t$

$0 \leq v_{g,t} \leq 1, \quad \forall\, g, t$

$0 \leq w_{g,t} \leq 1, \quad \forall\, g, t$

$u_{g,t} \in \{0, 1\}, \quad \forall\, g, t$

To a first approximation:

- DCOPF
- Economic dispatch
- Unit commitment
- Transmission switching
- N-1 contingency

Hedman, et al., "Co-Optimization of Generation Unit Commitment and Transmission Switching With N-1 Reliability," *IEEE Trans Power Systems*, 25(2), pp.1052-1063, 2010

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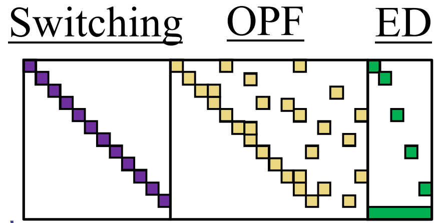
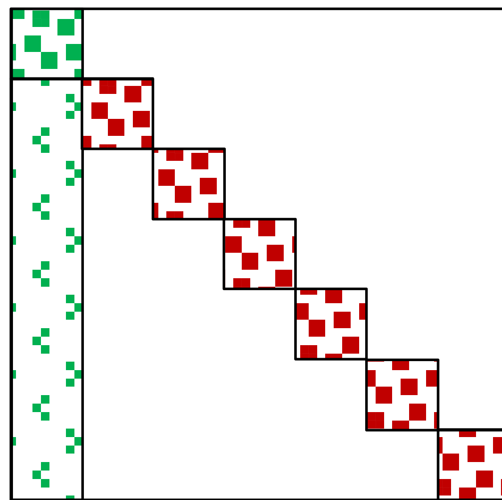
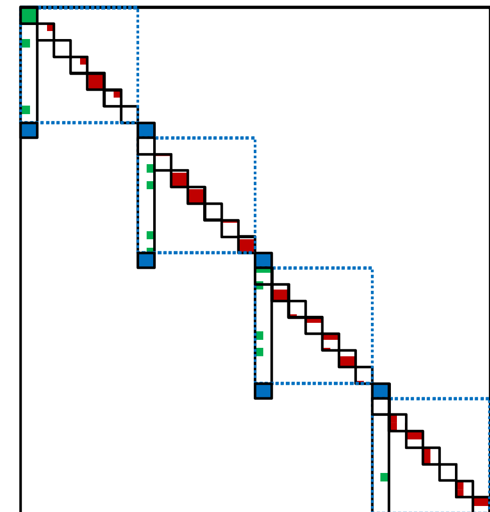
(Nonobvious) Inherent structure



Key feature:
Layered (nested) model complexity





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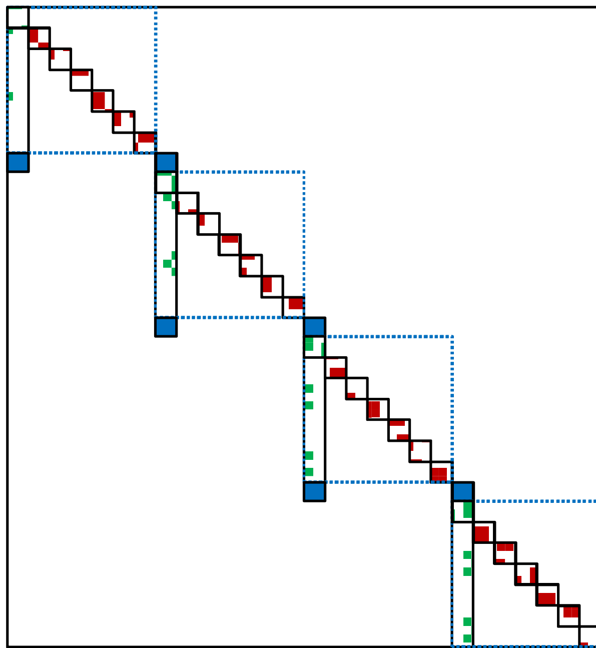
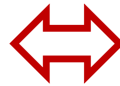
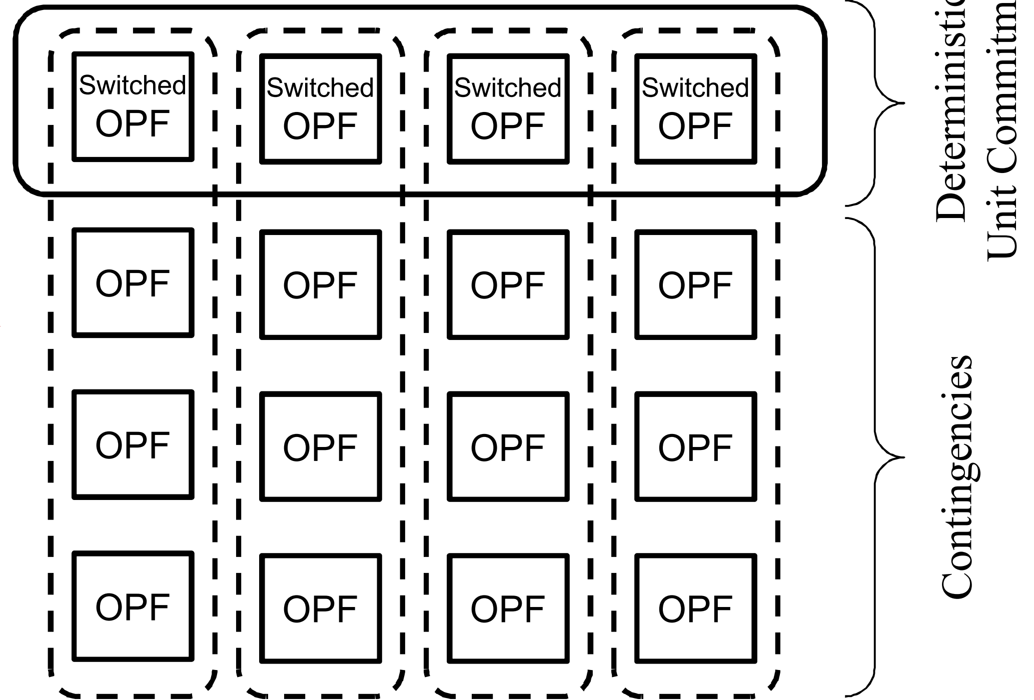
This still doesn't *quite* tell the whole story







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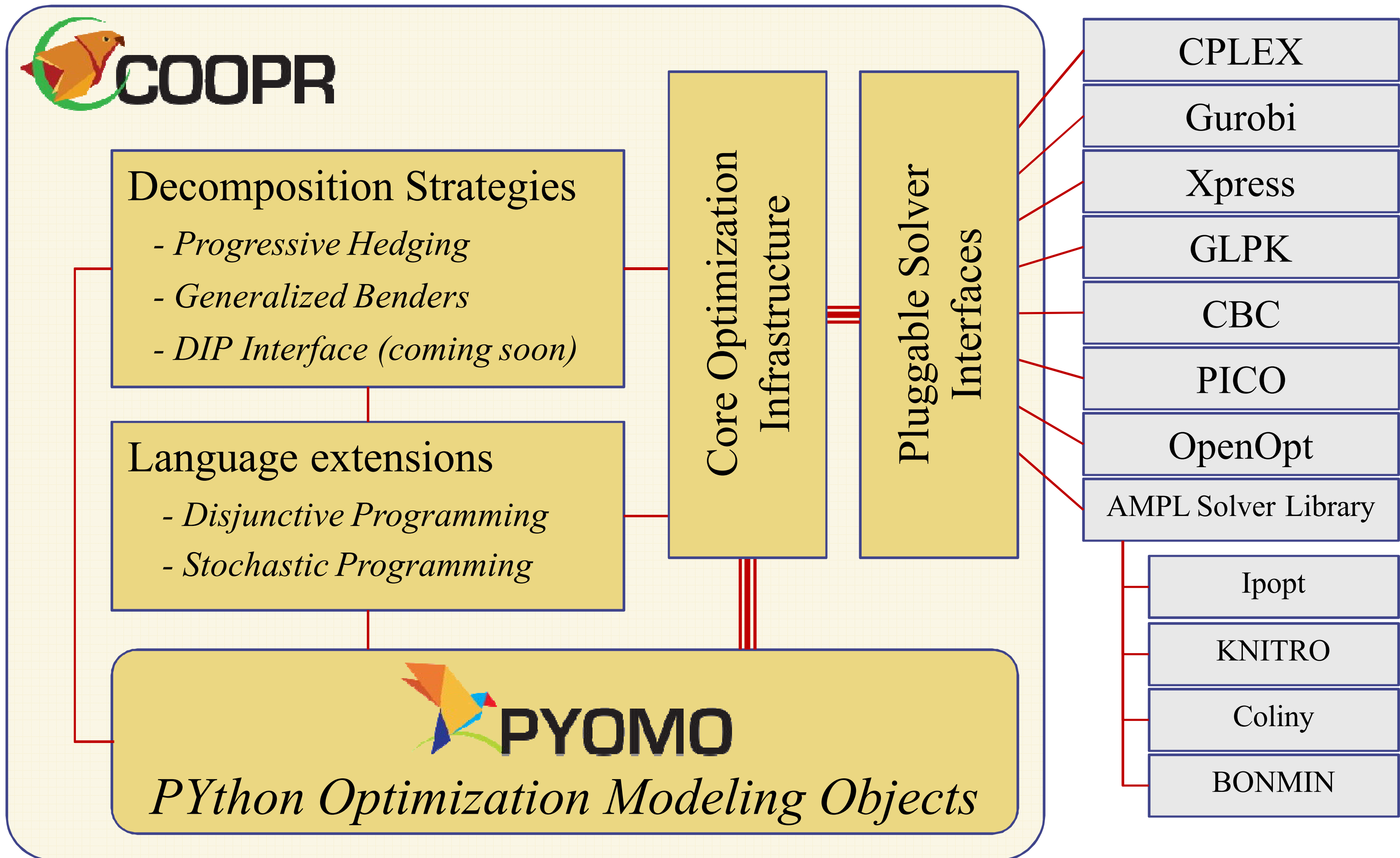
Block-oriented modeling

- "Blocks"
  - Collections of model components
    - Var, Param, Set, Constraint, etc.
  - Blocks may be arbitrarily nested
- Why blocks?
  - Support reusable modeling components
  - Express distinctly modeled concepts as distinct objects
  - Manipulate modeled components as distinct entities
  - Explicitly expose model structure (e.g., for decomposition)
- Prior art
  - Ubiquitous in the simulation community
  - Rare in Math Programming environments
    - *Notable exceptions:* ASCEND, JModelica.org
    - This is more than just suffixes!

Coopr: *a COmmon Optimization Python Repository*



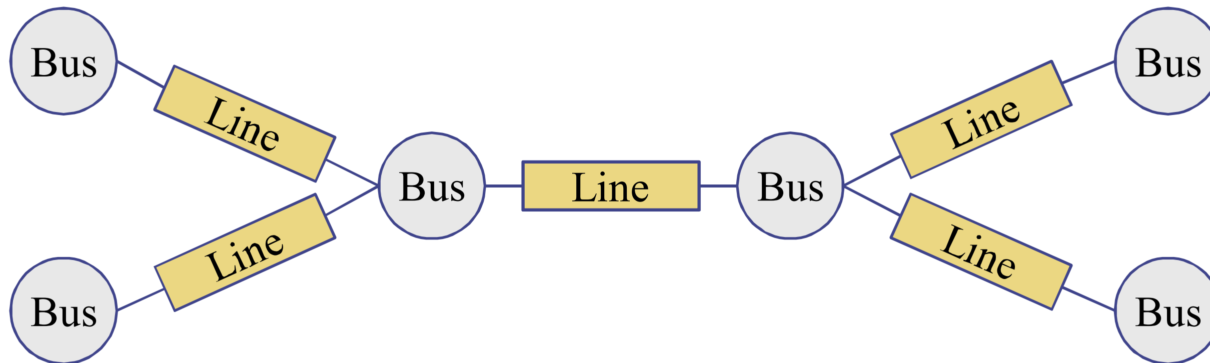
Pyomo overview

- Formulating optimization models natively within Python
  - Provide a natural syntax to describe mathematical models
  - Formulate large models with a concise syntax
  - Separate modeling and data declarations
  - Enable data import and export in commonly used formats
- Highlights:
  - Clean syntax
  - Python scripts provide a flexible context for exploring the structure of Pyomo models
  - Leverage high-quality third-party Python libraries, e.g., SciPy, NumPy, MatPlotLib

```
from coopr.pyomo import *

m = ConcreteModel()

m.x1 = Var()
m.x2 = Var(bounds=(-1,1))
m.x3 = Var(bounds=(1,2))

m.obj = Objective(
   sense = minimize,
   expr = m.x1**2 + (m.x2*m.x3)**4 +
          m.x1*m.x3 + m.x2 +
          m.x2*sin(m.x1+m.x3) )
model = m
```

Rethinking RUC: *a "Tinkertoy" approach*

- Capture connected block structure, e.g., *network flow*



  - Embed physical component models within separate blocks
  - Connect blocks using conceptual interfaces:
    - *Connectors*: groups of named numeric values
      - Constant, Parameter, Variable, Expression
    - "Connect" connectors with simple constraints

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Simple input-output blocks

```
def dc_line_rule(line, id):
    line.B          = Param()
    line.Limit      = Param()

    line.Angle_in   = Var()
    line.Angle_out  = Var()
    line.Power      = Var( bounds= ( -line.Limit, line.Limit ) )

    line.power_flow = Constraint( expr=
        line.Power == line.B*(line.Angle_in - line.Angle_out) )

    line.IN  = Connector( initialize=
        { “Power”: -line.Power, “Angle”: line.Angle_in } )

    line.OUT = Connector( initialize=
        { “Power”:  line.Power, “Angle”: line.Angle_out } )
```


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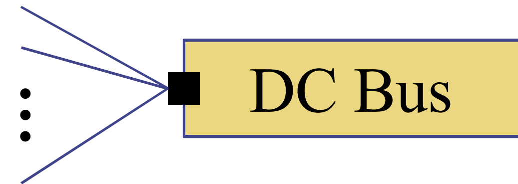
Arbitrary inputs: conservation blocks

```
def dc_bus_rule(bus, id):
    bus.D       = Param()

    bus.Angle   = Var()
    bus.Power   = VarList()

    def _power_balance(bus, P):
        return summation(P) == bus.D

    bus.BUS = Connector( initialize={ "Angle": bus.Angle })
    bus.BUS.add( bus.Power, "Power", aggregate=_power_balance )
```



- The *VarList* provides a unique local variable for every connection.
- The *aggregation* rule is called after expanding the connections

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General power flow model

```
from power_flow import \
        dc_line_rule as line_rule, \
        dc_bus_rule as bus_rule, \
        dc_generator_rule as generator_rule

model.BUSES      = Set()
model.LINES      = Set()
model.GENERATORS = Set()

model.links      = Param( model.LINES, ['IN', 'OUT'] )
model.bus        = Block( model.BUSES,      rule=bus_rule )
model.line       = Block( model.LINES,      rule=line_rule )
model.generator  = Block( model.GENERATORS, rule=generator_rule )

def _network(model, l):
   yield model.line[l].IN  == model.bus[ value(model.links[l, 'IN' ]) ].BUS
   yield model.line[l].OUT == model.bus[ value(model.links[l, 'OUT']) ].BUS
   yield ConstraintList.End
model.network = ConstraintList( model.LINES, rule=_network )

def _generator_placement(model, g):
   return model.generator[g].OUT == model.bus[ value(model.generator[g].bus) ].BUS
model.generator_placement = Constraint( model.GENERATORS, rule=_generator_placement )
```

Only domain-specific component

(Note: we have only shown the line rule and not the bus or generator rules)

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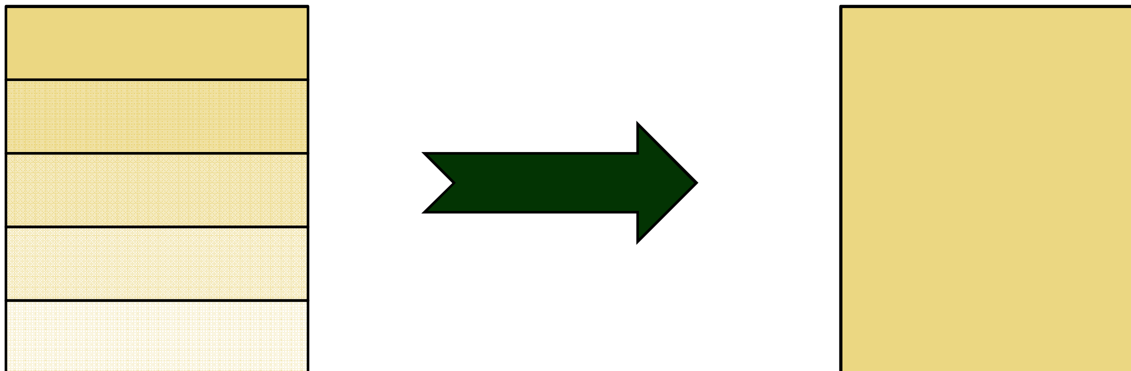
So, what’s really happening?

1) Construct hierarchical model
   - Generate blocks (Variables + Internal constraints)
   - “Connect” blocks by forming constraints over block connectors
2) An automatic *model transformation* “flattens” the model
   - Replicates connector constraints for each variable in connector
   - Generates aggregating constraints
   - (Eliminates redundant variables)



Leveraging components: AC power flow

```
from power_flow import \
        ac_line_rule as line_rule, \
        ac_bus_rule as bus_rule, \
        ac_generator_rule as generator_rule
model.BUSES      = Set()
model.LINES      = Set()
model.GENERATORS = Set()

model.links      = Param( model.LINES, [‘IN’, ‘OUT’] )
model.bus        = Block( model.BUSES,      rule=bus_rule )
model.line       = Block( model.LINES,      rule=line_rule )
model.generator  = Block( model.GENERATORS, rule=generator_rule )

def _network(model, l):
   yield model.line[l].IN  == model.bus[ value(model.links[l, ‘IN’ ]) ].BUS
   yield model.line[l].OUT == model.bus[ value(model.links[l, ‘OUT’]) ].BUS
   yield ConstraintList.End
model.network = ConstraintList( model.LINES, rule=_network )

def _generator_placement(model, g):
   return model.generator[g].OUT == model.bus[ value(model.generator[g].bus) ].BUS
model.generator_placement = Constraint( model.GENERATORS, rule=_generator_placement )
```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Manipulating model blocks

- Generalized Disjunctive Programming (GDP)
  - Switching entire blocks on/off through binary variables
- Introduce new Pyomo modeling components:
  - "Disjunct"
    - a new form of model block
  - "Disjunction"
    - a new constraint for enforcing logical XOR over disjunctive sets

$$
\begin{aligned}
\min \quad & \sum_k c_k + f(x) \\
s.t. \quad & g(x) \le 0 \\
& \bigvee_{i \in D_k} \begin{bmatrix} Y_{ik} \\ h_{ik}(x) \le o \\ c_k = \gamma_{ik} \end{bmatrix} \\
& \Omega(Y) = true \\
& Y_{ik} \in \{true, false\}
\end{aligned}
$$

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Creating a “switchable line”

- Sidebar: we need an “open line” model

```
def dc_line_rule(line, id):
    line.Limit     = Param()

    line.Angle_in  = Var()
    line.Angle_out = Var()
    line.Power     = Var( bounds= ( -line.Limit, line.Limit ) )

    line.power_flow = Constraint( expr= line.Power == 0 )

    line.IN  = Connector( initialize=
        { “Power”: -line.Power, “Angle”: line.Angle_in } )

    line.OUT = Connector( initialize=
        { “Power”:  line.Power, “Angle”: line.Angle_out } )
```

Creating a "switchable line"

```
def switchable_dc_line_rule(line):
   line.CLOSED = Disjunct(rule=closed_line_rule)
   line.OPENED = Disjunct(rule=opened_line_rule)
   line.switch = Disjunction(expr=[line.CLOSED, line.OPENED])

   line.FROM  = Connector()
   line.TO    = Connector()

   def connections_rule(line, id):
      yield line.FROM == line.CLOSED.FROM
      yield line.FROM == line.OPENED.FROM
      yield line.TO == line.CLOSED.TO
      yield line.TO == line.OPENED.TO
      yield ConstraintList.End
   line.connections = ConstraintList(rule=connections_rule)
```


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Creating a transmission switching model

```
from power_flow import  switchable_dc_line_rule as line_rule, \
                        dc_bus_rule as bus_rule, \
                        dc_generator_rule as generator_rule
```

```
model.bus       = Block( model.BUSES, rule=bus_rule )
model.line      = Block( model.LINES, rule=line_rule )
model.generator = Block( model.GENERATORS, rule=generator_rule )

def _network(model, l, end):
   if endpoint == 'IN':
      return model.line[l].IN == model.bus[ value(model.links[l, end]) ].PORT
   else:
      return model.line[l].OUT == model.bus[ value(model.links[l, end]) ].PORT
model.network = Constraint(model.LINES, model.ENDPOINTS, rule=_network)

def _generator_placement(model, g):
   return model.generator[g].OUT == model.bus[ value(model.generator[g].bus) ].PORT
model.generator_placement = Constraint(model.GENERATORS, rule=_generator_placement)
```


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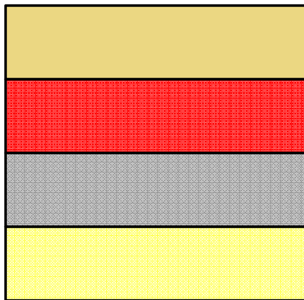
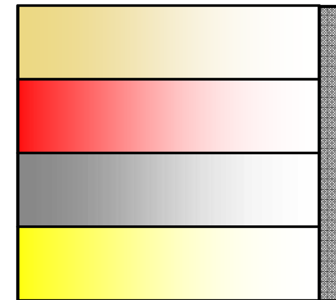
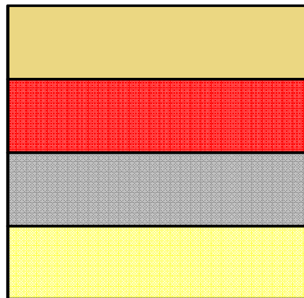
Solving GDP models

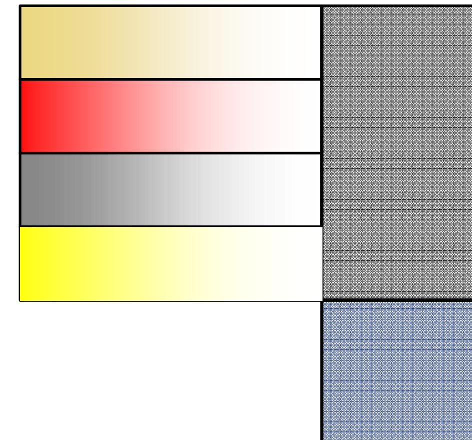
- Automated transformations generate "flat" MI(N)LPs
  - Big-M relaxation





  - Convex hull relaxation





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Putting it all together: UC + switching + N-1

Network Model

Switchable Transmission Line

Y_i

Transmission Line Power Flow Model

V

$\neg Y_i$

Current Balance (KCL)

Bus model

Y_i^{SU}

Start-Up Model

V

Y_i^{G}

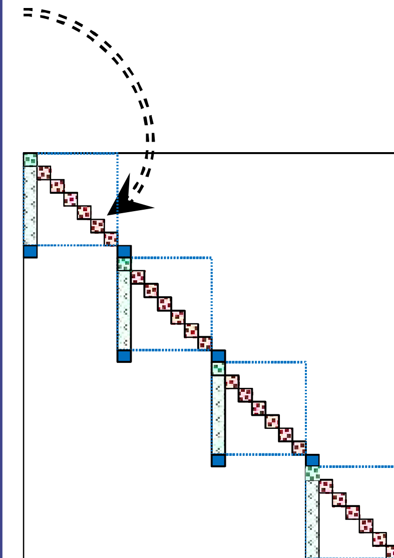
Generation Model

V

$\neg(Y_i^{SU} \mid Y_i^{G})$

Ramp Limits (Y_i)

Switchable Generator



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The cost of flexibility: transformation time

Instance	Flat	Blocked	Blocked+GDP (line only)	Blocked+GDP (line + generator)
5-bus (24-hour)				
Instantiation	0.88	4.75	5.56	12.2
Connector expansion	–	4.71	5.45	6.1
GDP transformation	–	–	3.11	8.2
Total	2.14	14.1	19.3	34.3
RTS-96 (4-hour)				
Instantiation	39.4	205	231	427
Connector expansion	–	206	224	184
GDP transformation	–	–	139	362
Total	67.6	574	788	1280
RTS-96 (8-hour)				
Instantiation	73.8	386	459	908
Connector expansion	–	389	449	272
GDP transformation	–	–	292	733
Total	148	1130	1610	2600
RTS-96 (12-hour)				
Instantiation	110	571	687	*
Connector expansion	–	657	679	*
GDP transformation	–	–	434	*
Total	219	1740	2470	*

Expanded constraints: *presolve required*

Model	Rows	Columns	Nonzeros	Binaries
Raw generated model				
Flat	20923	6697	53591	312
Blocked	46979	34961	121583	6727
Blocked+GDP (L)	64833	50943	157269	8599
Blocked+GDP (L+G)	88689	54142	211853	15806
After CPLEX presolve				
Flat	10640	4356	34344	285
Blocked	10327	4377	33100	292
Blocked+GDP (L)	10392	4377	33474	292
Blocked+GDP (L+G)	10643	4950	28582	1302

[5-bus, 24-hour test case]

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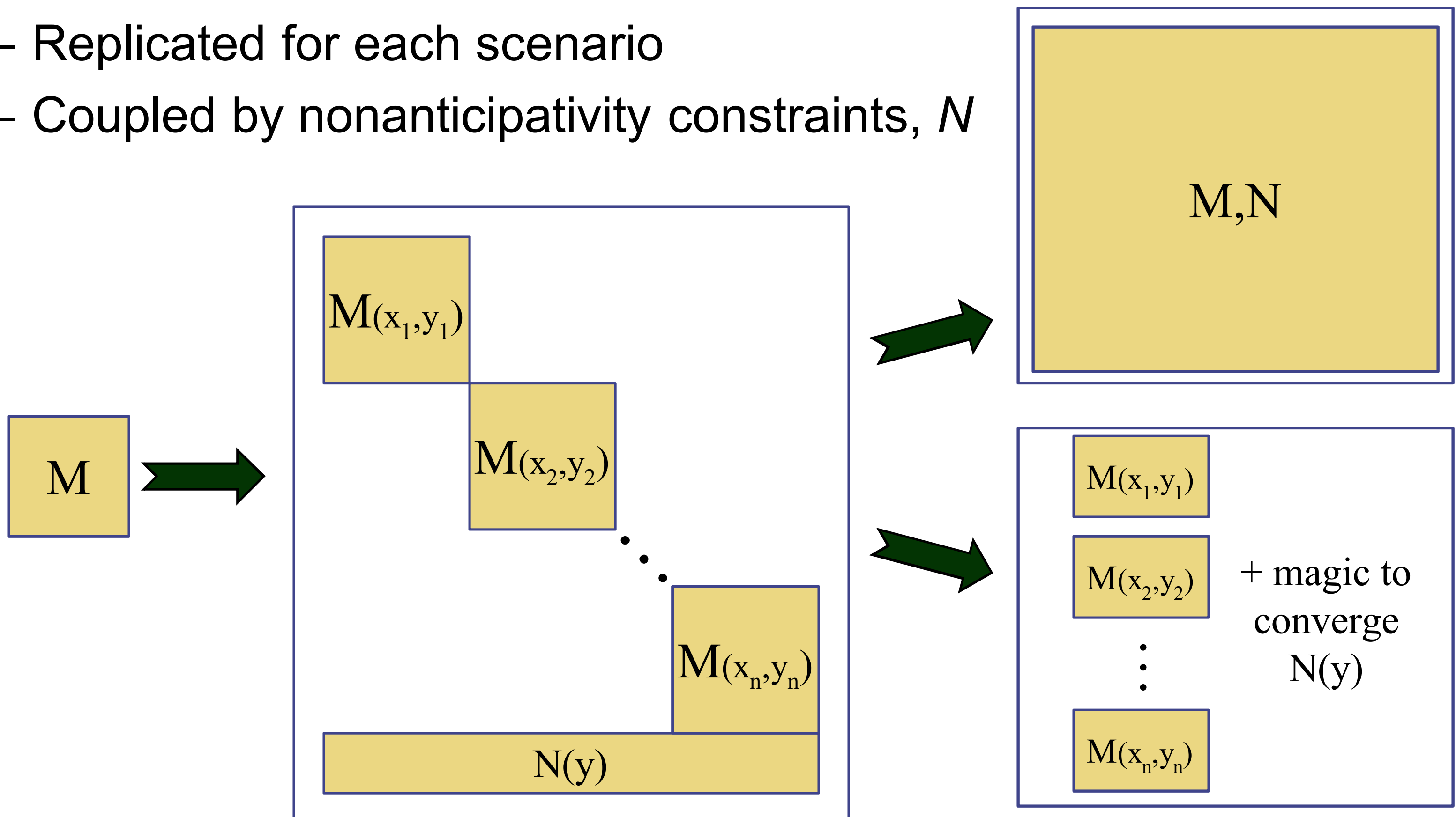
The key challenge is *managing uncertainty*

- Historically, absorbed by (spinning) reserves
  - Nominally, 5-10% base demand
  - Approximates the “true” constraint: reliability requirements
  - Absorbing non-dispatchable generation requires significantly higher reserves due to poor forecasts
- Alternative: directly model reliability requirement
  - Robust optimization (e.g., N-1)
  - Stochastic programming + “appropriate” expectation
    - Optimize expectation over a sufficiently large set of scenarios
- Challenges:
  - Multiple stages
  - Integer variables at *any* stage
  - Enormous scenario trees

Forming stochastic programs

- Exploit "block diagonal" structure
  - Deterministic model, *M*
  - Replicated for each scenario
  - Coupled by nonanticipativity constraints, *N*



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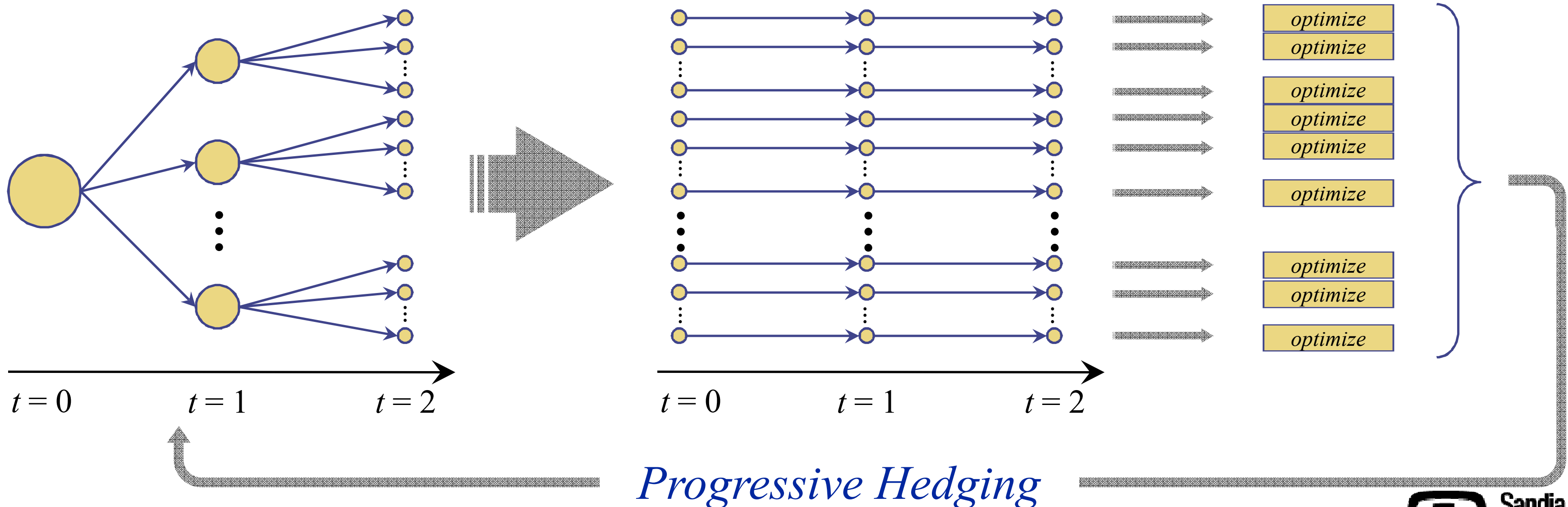
What about when the extensive form is too big / hard?

- Progressive Hedging (PH) [Rockafellar & Wets]
  - Solve scenarios independently
  - Iteratively converge nonanticipativity constraints
  - PySP: generic implementation of PH
    - Automatic problem construction
    - Numerous tricks / heuristics for handling integer decisions
    - Parallelization on large clusters



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Applying PH to the *N-1* problem

- CPLEX can solve the EF at the root node (for our test cases)
  - …using heuristics
  - …in 3 days [RTS-96 test case, with 217 contingencies]
- Scenario generation is slightly more complex
  - Choice of decomposition axis: contingency or time?
  - Bundles: nominal case + 1 contingency
- Using PH:
  - The good news:
    - Root nodes solve in < 1 minute
    - This parallelizes "trivially"
  - The bad news
    - Individual scenarios enter the B&B tree
    - … with a relatively large gap (>30%)
  - This is the focus of ongoing research; "so stay tuned"

“Blocks” fundamentally change modeling

- Explicit model blocks
  - Component reuse
  - Implicit transformations when generating model instances
- Generalized Disjunctive Programs
  - Explicit transformations to create standard forms
  - (Solver-specific decomposition)
- Block diagonal models
  - Implicit transformation to create standard forms
  - Solver-specific decompositions (e.g., progressive hedging)
- BUT… a parting shot:
  - The real problem is the ACOPF (nonconvex nonlinear)
  - Actually *solving* that problem is “nontrivial”

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- Los Alamos National Lab

COIN|OR — Computational Infrastructure for Operations Research

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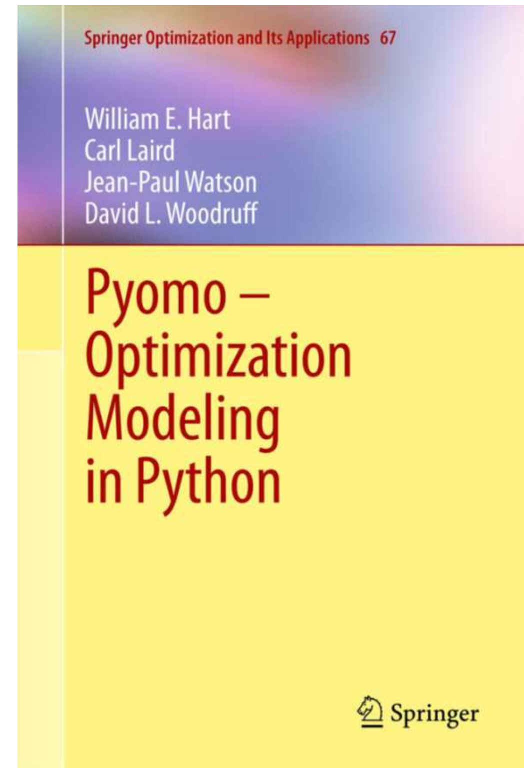
For more information…

- Project homepage
  - http://software.sandia.gov/coopr
- “The Book”



- Mathematical Programming Computation papers
  - Pyomo: Modeling and Solving Mathematical Programs in Python (Vol. 3, No. 3, 2011)
  - PySP: Modeling and Solving Stochastic Programs in Python (Vol. 4, No. 2, 2012)