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Parameterized Reduced Order Models Constructed Using Hyper Dual Numbers

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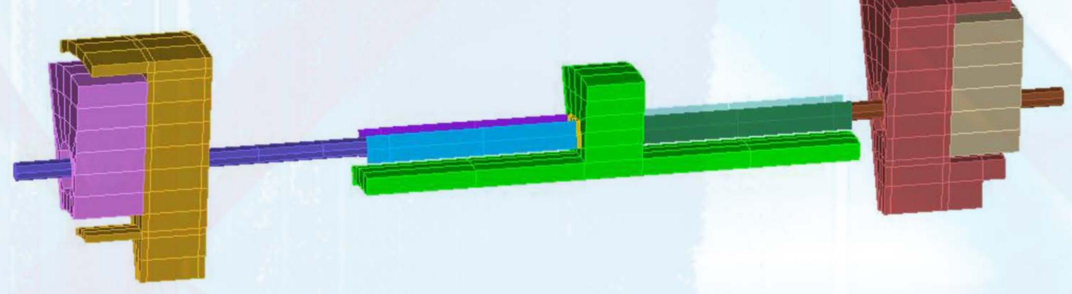
The Challenges of Design for High Consequence Applications

- Goal: Assessment of uncertainty in and design optimization of real systems
- Uncertainties omnipresent
  - Environmental specifications, manufacturing tolerances, defects, epistemic sources, etc.
- Consequence: robust design requirements
  - Can require thousands of perturbed models
- Rough estimate: 10 years of human effort, plus 3 years of a dedicated super computer using high fidelity FEA...



Derivatives Calculated by the Complex Step Expansion

Ordinary complex numbers ($E^2 = i^2 = -1$)

$$f(x+hi) = \underbrace{\left(f(x) - \frac{1}{2!}h^2 f''(x) + \cdots\right)}_{\text{Real}} + h\underbrace{\left(f'(x) - \frac{1}{3!}h^3E^3 f'''(x) + \cdots\right)}_{\text{Imaginary}} i$$

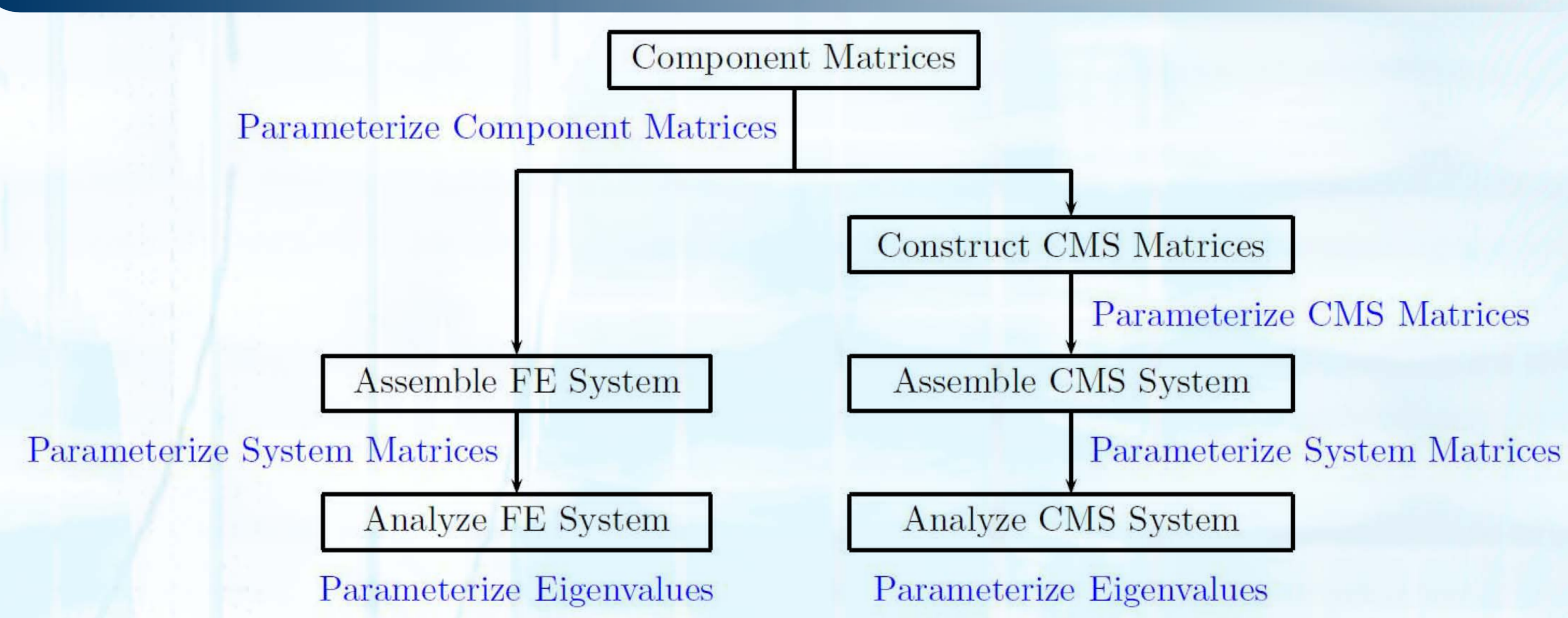
Double numbers ($E^2 = e^2 = 1$)

$$f(x+he) = \underbrace{\left(f(x) + \frac{1}{2!}h^2 f''(x) + \cdots\right)}_{\text{Real}} + h\underbrace{\left(f'(x) + \frac{1}{3!}h^3E^3 f'''(x) + \cdots\right)}_{\text{Non-Real}} e$$

Dual numbers ($E^2 = \varepsilon^2 = 0$)

$$f(x+h\varepsilon) = \underbrace{f(x)}_{\text{Real}} + \underbrace{hf'(x)\varepsilon}_{\text{Non-Real}}$$

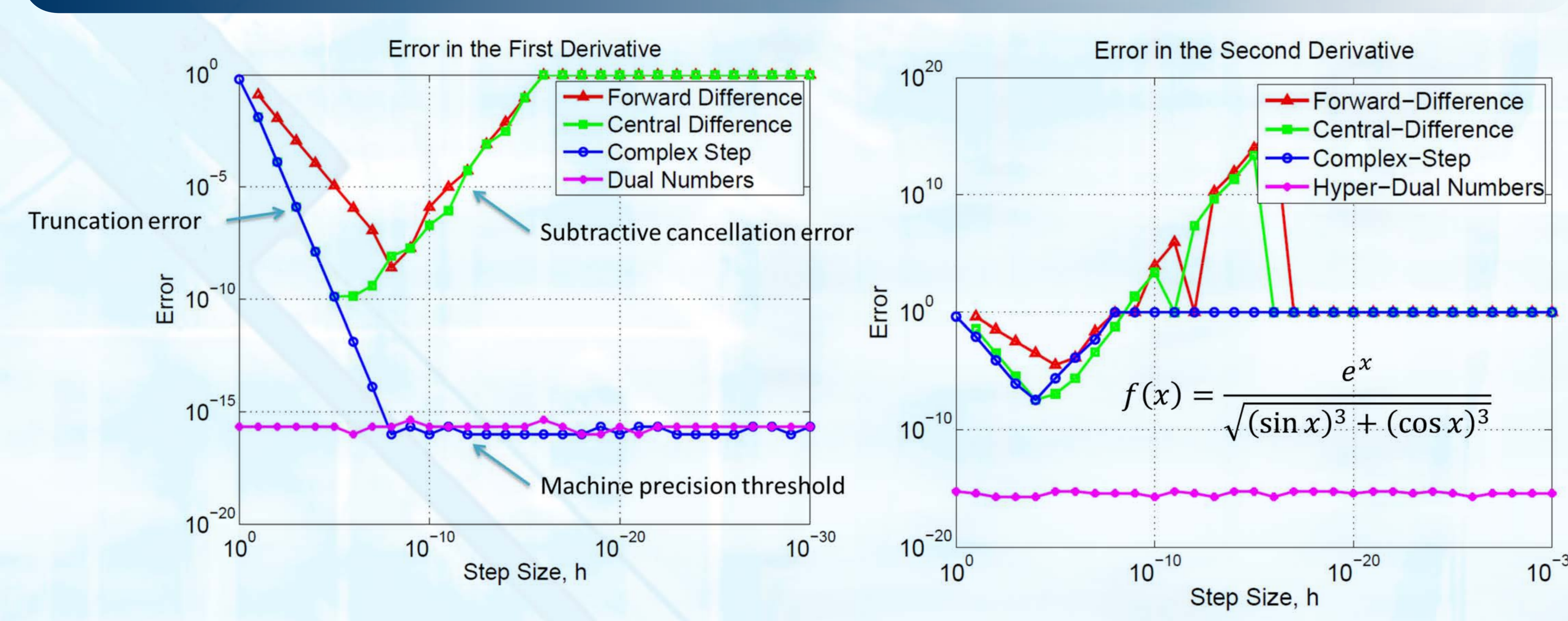
Development of a Parameterized Reduced Order Model



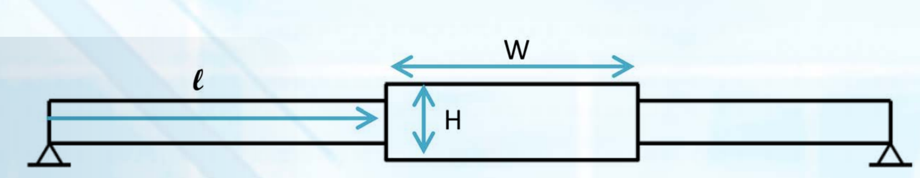
Models parameterized in terms of the variables of interest using

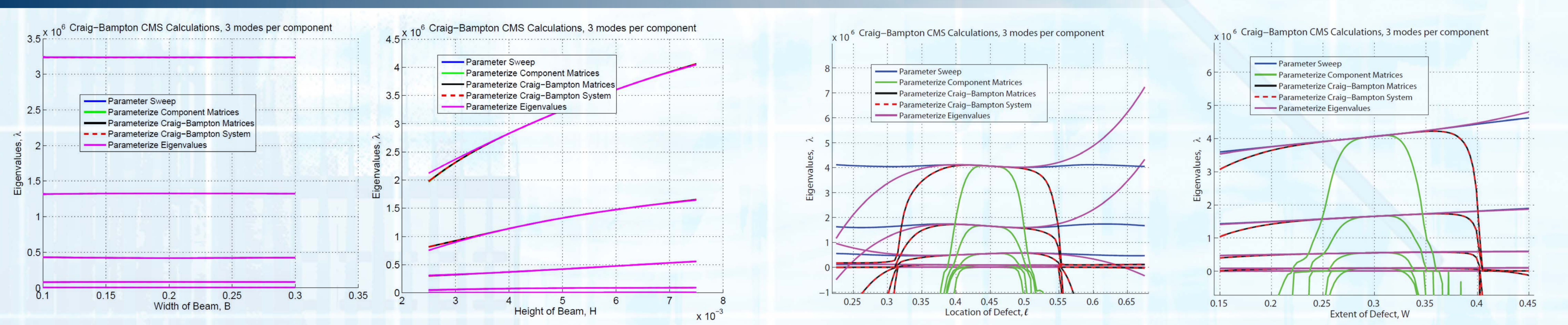
$$f(a+h) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$$

Accuracy of Derivative Calculations

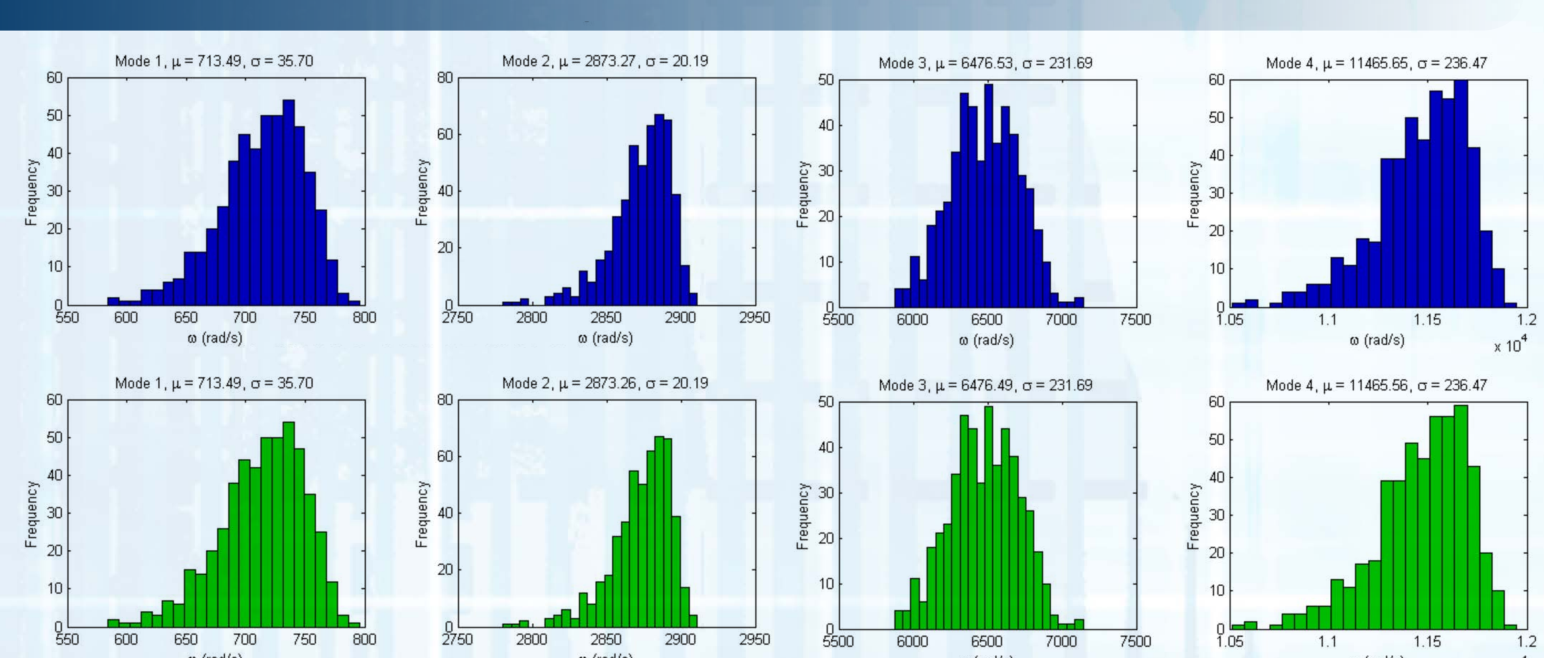


Results of PROMs Constructed With Hyper Dual Numbers





Multivariate Parameterization



Major Conclusions

- Hyper dual numbers are an extension of a branch of generalized complex number theory, dating back to 1900
- Building hyper dual numbers into our FEA code allows us to develop parameterized reduced order models (PROMs) with a single mesh
- Results match analytical solutions very well for PROMs constructed from Craig-Bampton models or Eigen representations

PREDICTIVE ENGINEERING SCIENCE PANEL

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