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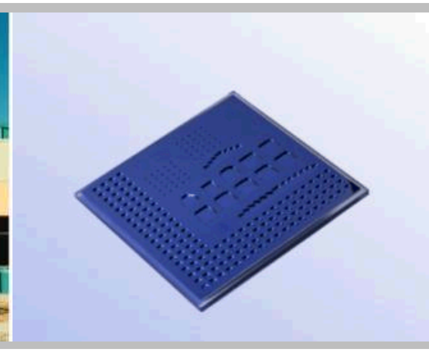

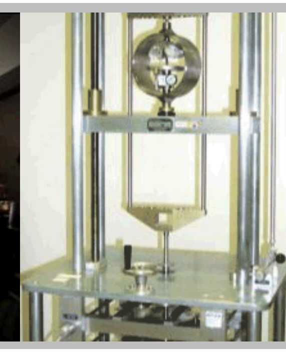

Dimensional Metrology NCSLI Tech Exchange Tutorial TE-14

Feb. 6, 2014

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Sandia National Laboratories

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Learning Outcomes

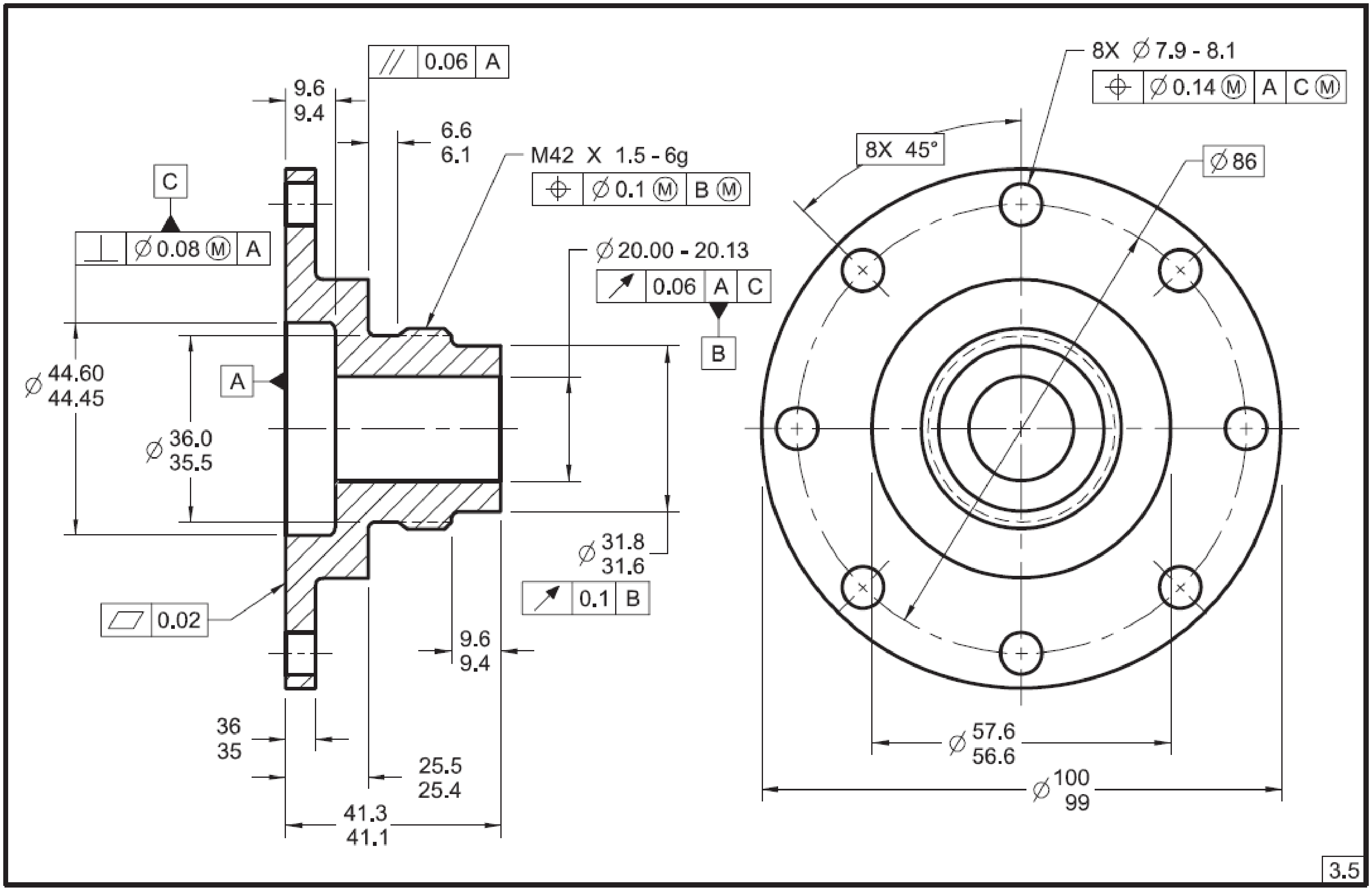
- Fundamentals of dimensional metrology
  - Geometric dimensioning and tolerancing; drawings
- Tools used in dimensional metrology
  - Brief survey
- Calibration of tools used in dimensional metrology
  - The SI
  - Performance characteristics of tools
  - How are they used in the field or lab
  - Typical calibration procedures
  - Uncertainties in calibration of tools
  - Some of the ASME standards
- Hands-on demos
- Where do you go for help?
  - ASME and ISO stds
  - NCSLI Dimensional Metrology committee
- Provide me with your contact info by end of this session, and I'll e-mail you copies of slides

The presenter gratefully acknowledges the courtesy of the provider of the materials used in the hands-on demos.

The big picture

- Dimensional characteristics of products include:
  - Size
  - Form
  - Orientation
  - Location
  - Texture
- Designers specify geometry as one of the requirements for functionality. For example:
  - (Typical drawing; assembly)
  - (Point out feature callouts—size, form, orientation, location, texture—point out how each has a functional requirement)
- Variation in manufacturing←→product tolerances
  - Metrology establishes conformance
  - Calibration assures metrology capability

from ASME Y14.5:2009, fig.3-29



Dimensional equipment

- Comparison
- Direct reading

- We calibrate:
  - The standards used, either as check stds for the instruments
  - The instruments themselves

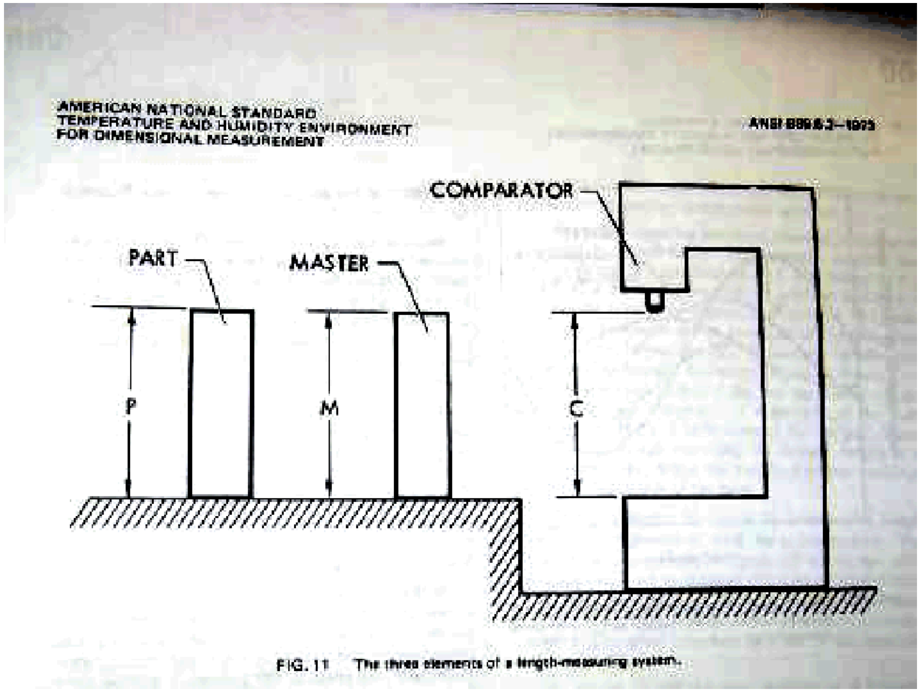
- Very important: All dimensional values are at 20°C (68°F)
  - All engineering materials have some coefficient of thermal expansion. Measuring and calibrating at temperatures other than 20°C will introduce some level of error; compensating for the error will introduce some level of uncertainty.

Thermal effects

- Historically, one of the largest contributor in dimensional metrology
- Dimensional measurements defined at 20°C (68°F)
- Corrections can be made, but good technical practice is to ***manufacture*** and ***inspect*** at 20°C
- Outside windows & exterior walls are both major sources of thermal disturbances
  - If you cannot relocate away from exterior window and wall, drapes & partition can attenuate disturbance

Static and dynamic disturbances

- Static: Constant disturbance (temperature is 69°F, not 68°F)
- Dynamic: Change (temperature changes from 69°F to 68°F)



ASME B89.6.2-1975

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Dynamic thermal effect

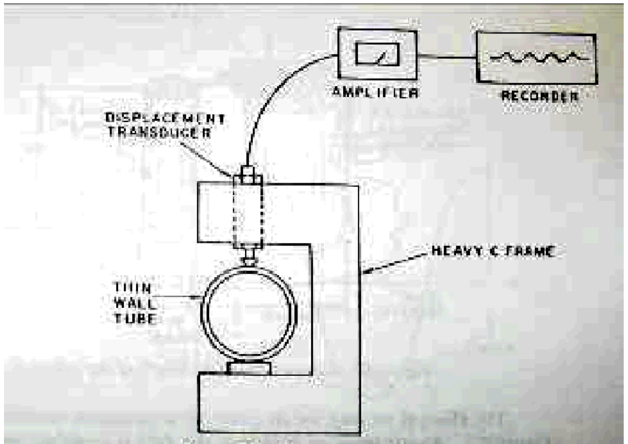


Figure 8.2. Comparator and part.

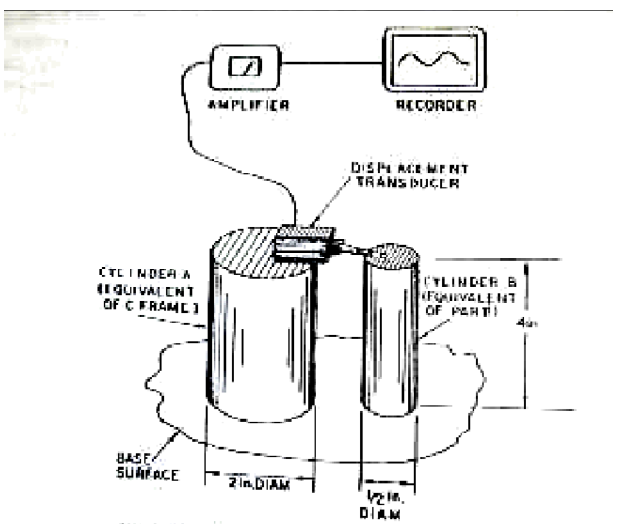


Figure 8.3. Comparison of the length of two cylinders.

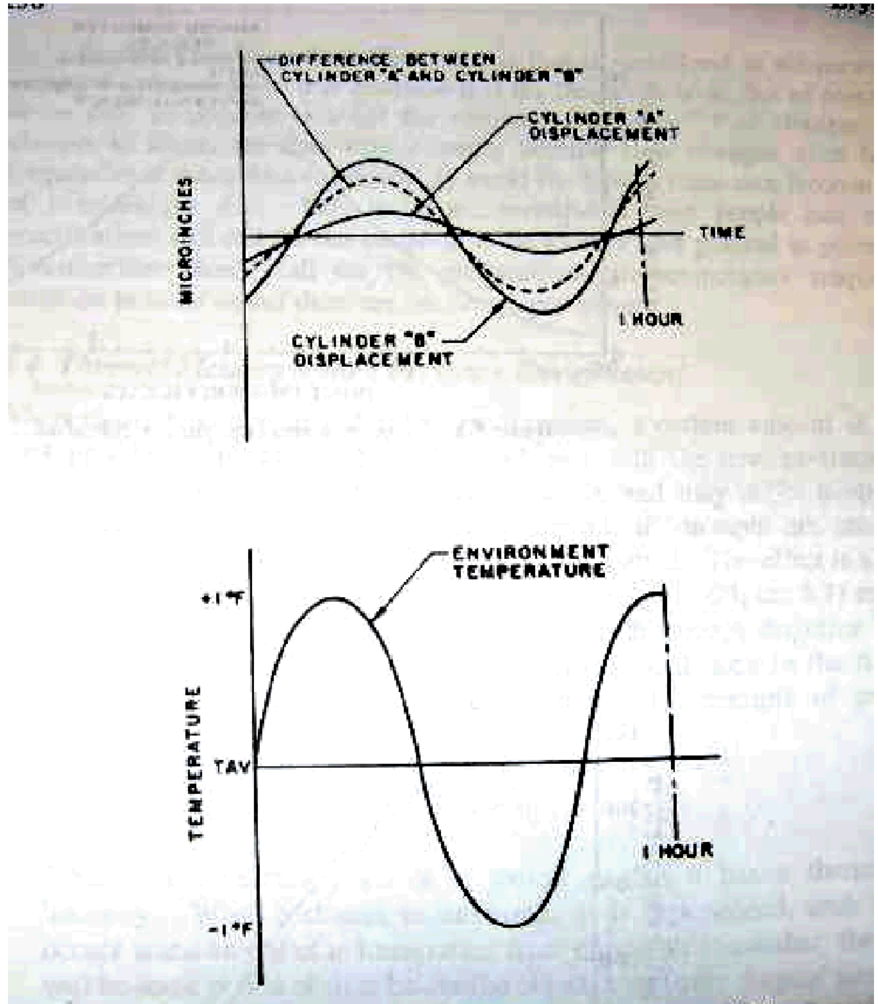
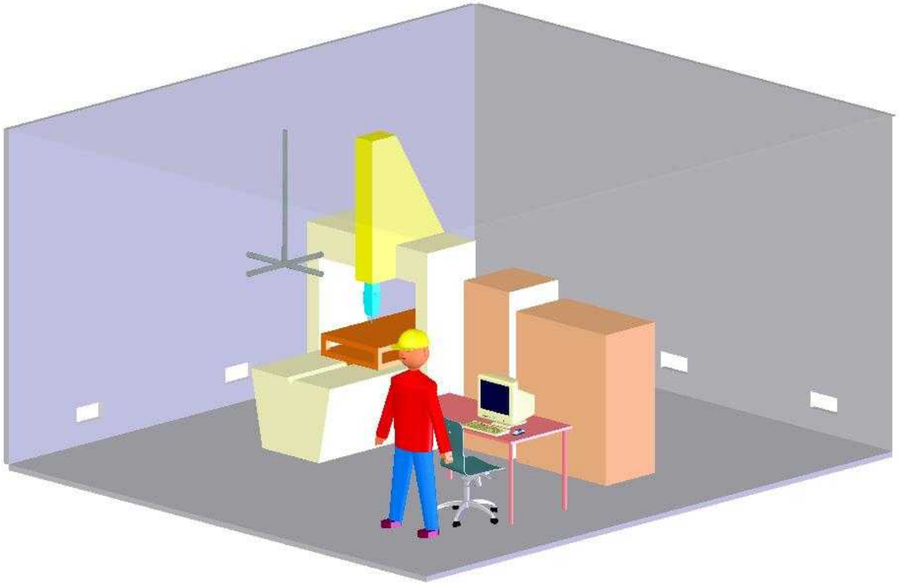


Figure 8.4. Plot of changes in length for the two cylinders.

Bryan, J. B., Brewer, W., McClure, E.R., and Pearson, J.W., “Thermal Effects in Dimensional Metrology,” ASME 65-PROD-13, 1965.

Sources of thermal disturbances

- Heat energy is transferred when there is a temperature difference
- Heat energy can be transferred by:
  - Conduction
  - Convection
  - Radiation
- Energy can also be generated (motors, electronics, friction of motion, etc.), which creates a temperature difference



Mitigation of thermal disturbances

- Attenuate sources (or sinks) of thermal energy
  - Human ~75-100 W; skin temperature ~90-100°F
  - Wear pants, lab coats (or long sleeves), & gloves
  - Minimize visitors, entry/egress
  - Circulate air to help keep parts & machines at thermal equilibrium (for nanotechnology, think about flooding with oil or water!)
  - Use indirect lighting; avoid incandescent lamps!!!!
  - Allow time to equilibrate after cleaning! (evaporative cooling…)
- Use engineering judgment for how much effort to take!
- NCSL RP-7: Laboratory design; RP-14: Selecting laboratory environment; RP-17: Verification of laboratory environments
  - Note that while neither atmospheric pressure nor humidity have (major) influence on dimensional characteristics, humidity is a factor for corrosion & durability of dimensional stds & equipment

In view of these facts, and the absence of any material normal standards of customary weights and measures, the Office of Weights and Measures, with the approval of the Secretary of the Treasury, will in the future regard the International Prototype Metre and Kilogramme as fundamental standards, and the customary units—the yard and the pound—will be derived therefrom in accordance with the Act of July 28, 1866. Indeed, this course has been practically forced upon this Office for several years, but it is considered desirable to make this formal announcement for the information of all interested in the science of metrology or in measurements of precision.

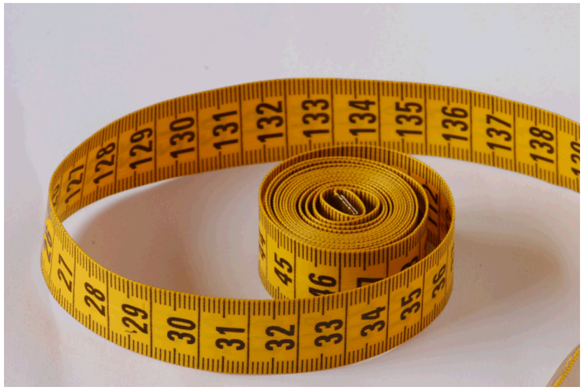
T. C. MENDENHALL,
Superintendent of Standard Weights and Measures.

Approved:
J. G. CARLISLE,
Secretary of the Treasury.
APRIL 5, 1893.

[United States Coast and Geodetic Survey.—Office of Standard Weights and Measures—T. C. Mendenhall, Superintendent.]

Dimensional M&TE

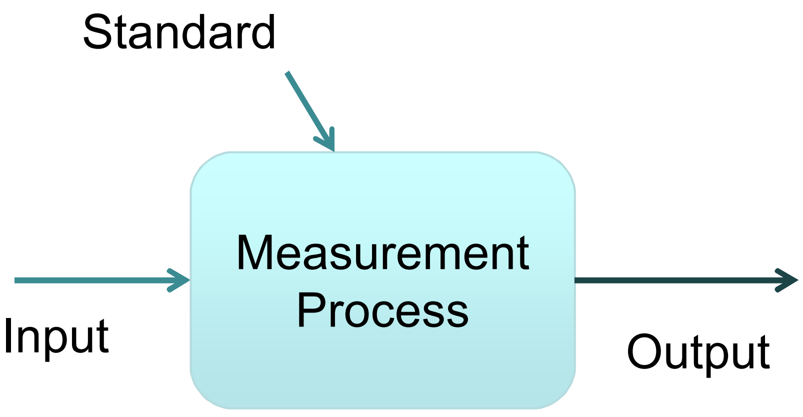
- All measurements are fundamentally comparisons
- Dimensional M&TE is operated either as a comparator to a reference, or as "direct reading"
- Example of direct reading:
  - Ruler
- Example of comparator:
  - Go/No Go gage

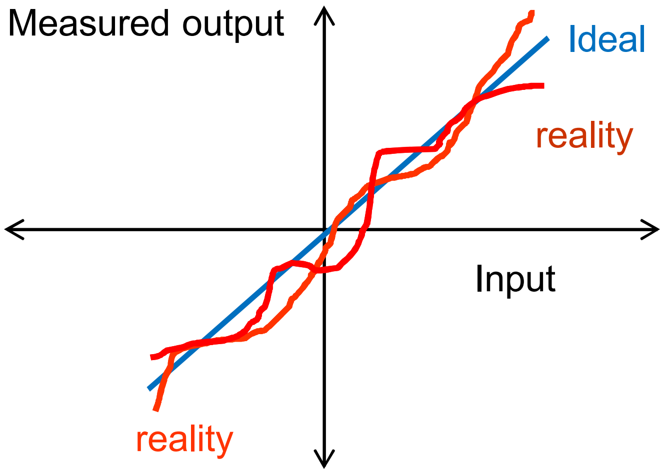


- A direct reading instrument can also be used in a comparison mode

Measurement model of M&TE

- When is it easier to measure to 10 nm than to measure to 10 mm?
- Resolving to 10 nm over a span of 1000 nm is easier than resolving to 10 mm over a span of 100 km!
- Calibration of dimensional M&TE is typically performed as a comparison to reference stds, to reduce dynamic range





Transducers, readouts, and instrument design

- Design principles with dimensional equipment
  - Knowing the design ideas helps with understanding the calibration methods

- Isolation
- Kinematic mounting
- Abbe principle and alignment
- Metrology frames
- Bearings
- Material selection
- Sensors & transducers
- Structural loop
- Kinematic drives and couplings
- Thermal effects
- Controls
- Error budgets
- Error mapping and compensation
- …

Machines are deterministic—not random. With sufficient effort and resources, a designer can achieve high repeatability and high accuracy.

Measurement of length

- Fundamentally, distance between two points
- Instruments/standards don't measure the distance between 2 points:
  - Distance between point and plane
  - Distance between 2 planes
  - etc
- Reference ***surface*** and measurand surface

Calibration, verification, validation

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2.39 (6.11)

calibration

operation that, under specified conditions, in a first step, establishes a relation between the **quantity values** with **measurement uncertainties** provided by **measurement standards** and corresponding **indications** with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a **measurement result** from an indication

NOTE 1 A calibration may be expressed by a statement, calibration function, **calibration diagram, calibration curve,** or calibration table. In some cases, it may consist of an additive or multiplicative **correction** of the indication with associated measurement uncertainty.

NOTE 2 Calibration should not be confused with **adjustment of a measuring system,** often mistakenly called "self-calibration". nor with **verification** of calibration.

NOTE 3 Often, the first step alone in the above definition is perceived as being calibration.

validation

verification, where the specified requirements are adequate for an intended use

EXAMPLE A **measurement procedure,** ordinarily used for the **measurement** of mass concentration of nitrogen in water, may be validated also for measurement in human serum.

2.44

verification

provision of objective evidence that a given item fulfils specified requirements

EXAMPLE 1 Confirmation that a given **reference material** as claimed is homogeneous for the **quantity value** and **measurement procedure** concerned, down to a measurement portion having a mass of 10 mg.

EXAMPLE 2 Confirmation that performance properties or legal requirements of a **measuring system** are achieved.

EXAMPLE 3 Confirmation that a **target measurement uncertainty** can be met.

NOTE 1 When applicable, **measurement uncertainty** should be taken into consideration.

NOTE 2 The item may be, e.g. a process, measurement procedure, material, compound, or measuring system.

NOTE 3 The specified requirements may be, e.g. that a manufacturer's specifications are met.

NOTE 4 Verification in legal metrology, as defined in VIML[53], and in conformity assessment in general, pertains to the examination and marking and/or issuing of a verification certificate for a measuring system.

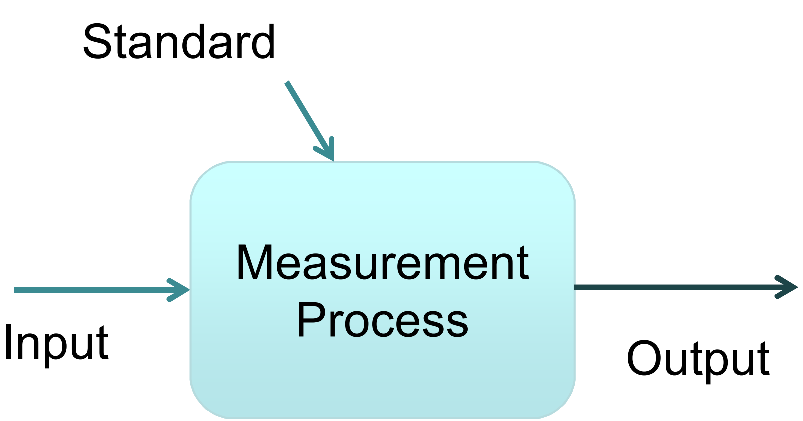
NOTE 5 Verification should not be confused with **calibration.** Not every verification is a **validation.**

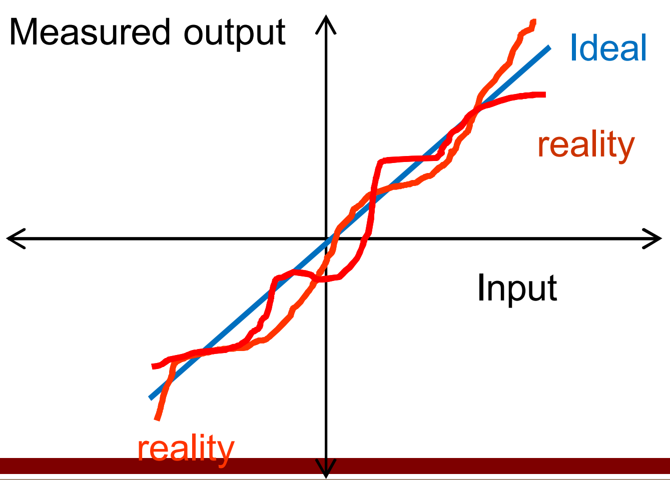
Calibration in more detail

- You could follow the same procedure—and the result could be viewed as a calibration, a verification, or a validation, depending on the application
- The documentary requirements for a calibration report (or calibration certificate) are generally more rigorous than for a verification
- Best practices with dimensional M&TE (or other M&TE) generally include using check standards, which would be viewed as a ***verification*** at (or near) time-of-use:
  - Auto-cal button on balances or oscilloscopes
  - Control chart of M&TE on check standards
  - etc
- What do you do with the verification data?
  - Procedures should include a “call engineering” or “call management” step if appropriate! (I have seen procedures that say “record the data and proceed”—even if the data say your equipment is bad)
  - Achieve a balance between using your equipment & verifying the equipment. (Equipment that’s known good, but always tied up in verification, is just as good as bad).

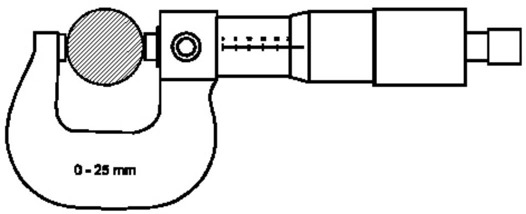
Where does measurement uncertainty come from?

- The measurand itself (intrinsic variability in the properties of the item being measured)
- The measurement process; instrument
- External factors (such as environment)
- Analyzing (and expressing) measurement uncertainty can aid engineering and decision-making



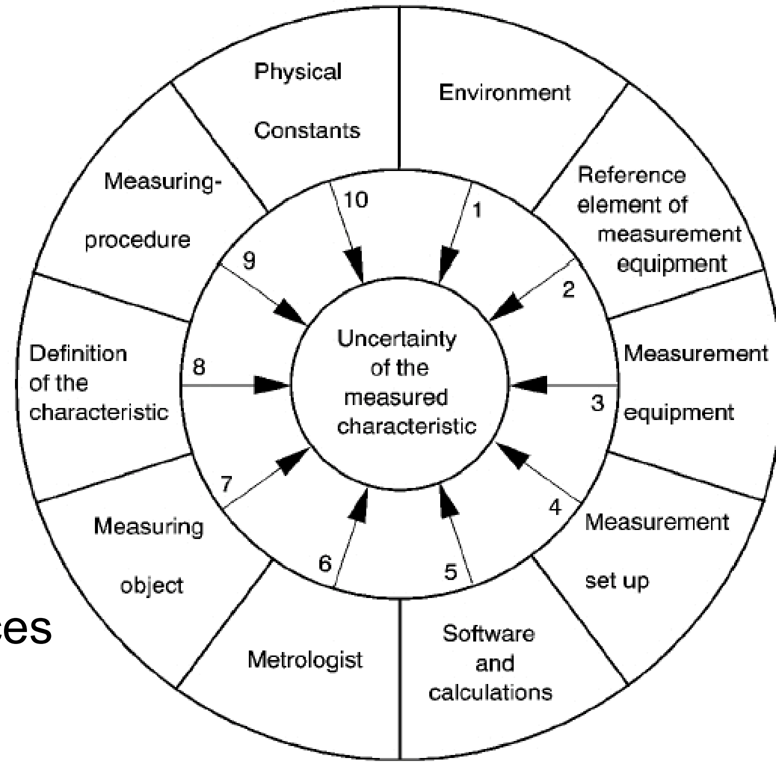


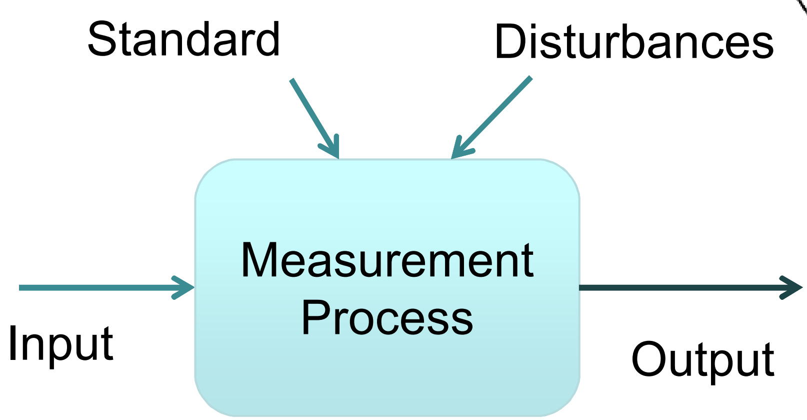
Measurement model, refined

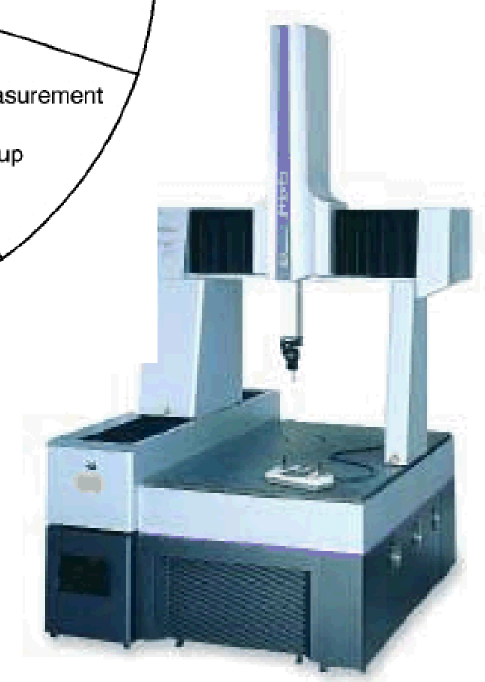


Sources of Uncertainty

(From ISO/TS 14253-2:1999)







Reporting a measurement

- x=0.010 254 m ± 0.000 012 m at a level of confidence of approximately 95%
- x=0.010 254 m ± 0.000 012 m, with a coverage factor k=2, where the combined standard uncertainty u_c = 0.000 0060 m
- Various other ways (including ways of expressing without the ±)

- Don't be excessively verbose or redundant
- Somewhere in the document, document how the expanded uncertainty was estimated

Calibration procedures & results

- All calibration results should result from a procedure. Procedures may be written procedures or data sheet procedures.
- All procedures (my opinion, not Sandia's, NCSLI, etc) should include:
  - Author
  - Date
  - Meaningful title (your company may require a name such as CP-12345-67-ASDF)
  - Safety considerations
  - Training requirements for personnel
  - Scope & limits: What are you calibrating? How well can you calibrate the item? What are the conditions you need?
  - Required equipment, materials
  - Actual procedure: How do you perform the calibration?
  - Data sheet if applicable
  - Uncertainty associated with calibration
  - How to report results

Walk through some hand tools

- ASME micrometer standard (2013)
- Gage block standard (reaffirmed 2007)
- Surface plate standard (2013)
- Caliper discussion
- Test indicator discussion

General process for estimating measurement uncertainty

- What are you measuring? (Is it really what you want to measure?)
- What are possible sources of errors? Those that cannot be reasonably eliminated are your sources of uncertainty
- Quantify effect of uncertainties on your measurement:
  - Statistical evaluations where appropriate—use std unc.
  - Other evaluations where appropriate—convert to std unc.
  - Influence factors (weighting factors)
- Document your model
- Combine the uncertainties
- Expand to get expanded uncertainty (coverage factor, level of confidence)
- Report result

Some probability & statistics

- Statistics is the study of data, especially relating to the analysis of data
- Statistics makes use of some probability theory, including probability distributions

- A measurement (and its associated uncertainty) is an ***inference***:
  - The diameter of a nickel lies between 21.19 mm and 21.23 mm
  - The degree of belief in the truth of this statement can be expressed as a number between 0 and 1 (0: I don't believe this statement at all. 1: I really believe that statement)

Probability of events & statistics

- Random events (we model our measurements as having some random events) can be modeled with a probability distribution

SECTION 7: DENSITY CALIBRATION DATA:

Nominal Value	Density	Uncertainty (k=2)	Standard Set	Balance
1 kg	8.0289 g/cm^3	0.0014 g/cm^3	D1	PR10003

UNCERTAINTY - The error in assignment of the density due to the measurement process. Uncertainty is calculated per NIST Technical Note 1297 using a coverage factor of k = 2 (k = 2 defines an interval having a level of confidence of approximately 95 percent).

- I believe that this weight has a density between 8.0275 and 8.0303 gram/cm^3.
  - How much do I believe it? To 95%
  - I believe that the weight's density is < 8.0275 gram/cm^3 to 2.5%
  - I believe that the weight's density is > 8.0303 gram/cm^3 to 2.5%
- I ***model*** the probability distribution as a Gaussian (aka normal)

coverage factor of k=2 (…interval having a level of confidence ~95%)

Probability distributions

- An event will (or will not) occur.
  - Probability that a coin will land heads is 0.4999999
  - Probability that a coin will land tails is 0.4999999
  - Probability that the coin lands on edge is 0.0000002
  - Sum of probabilities of all possible events is exactly 1 (100%)
- An event with a numerical value
  - A fair, six sided die lands on the "1"—1/6
  - The sum of 5 rolls of a six sided die is ≤ 10
  - The distance flown by a thrown coconut

A "virtual" experiment with dice

- We roll a fair die. Landing on edge or corner is unstable, so the die will always show a number.
- We expect that the probability of landing on "1" = prob of "2" = prob of "3", etc. = 1/6
- Let's use MS-Excel to roll the die
  - We roll 5 dice 60 times. (Doing it by hand would take a long time!)

Warning: MS-Excel is not proven to be a reliable tool for complex statistics/probability computations.

Some Excel demos

- Discrete random numbers
- Making a histogram
  - Excel's default histograms are ***<u>REALLY UGLY!</u>***
  - (but sometimes, that's all we have…)
- Other types of random numbers in Excel

Warning: MS-Excel is not proven to be a reliable tool for complex statistics/probability computations.

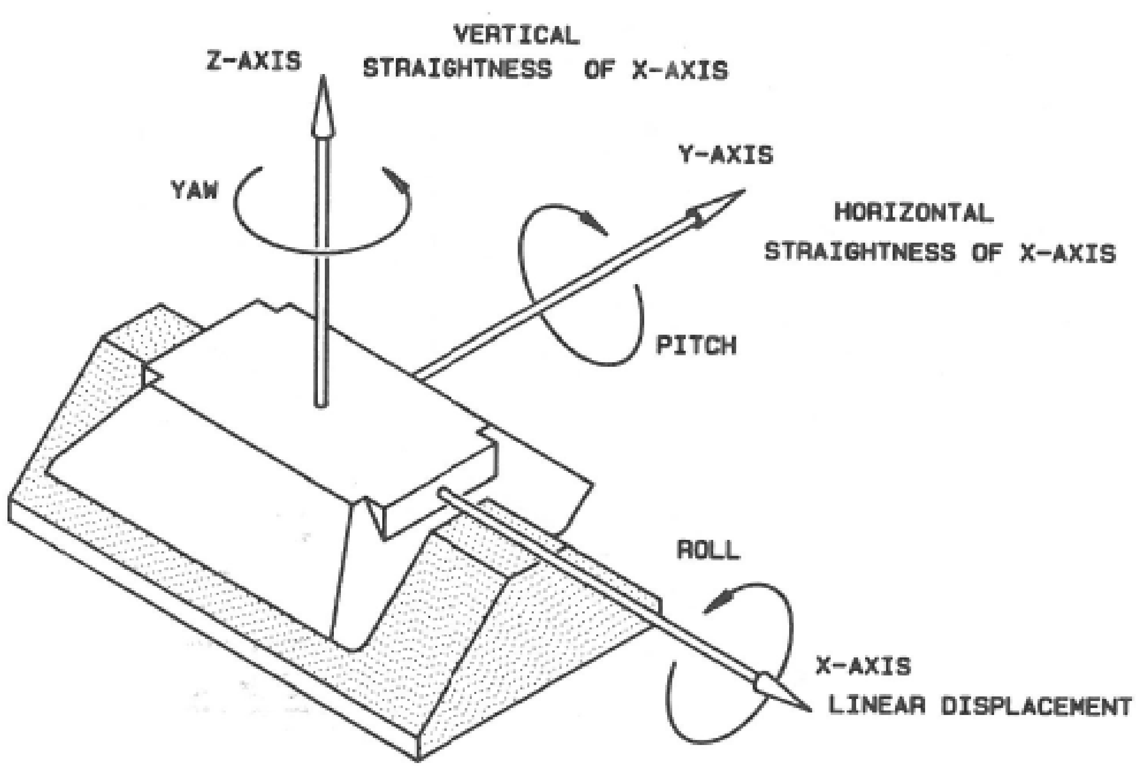
Measurement model and the GUM

- Measurement result is an ***estimate*** of the value. The statement of uncertainty makes the estimate more complete.
  - Include unpredictable variations, for example, in environment
  - Include effect of inadequate knowledge in the process, for example, correction factors
- “Type A” and “Type B” are for convenience in discussion
  - Not “random” vs “systematic”
- Type A: Use statistical methods to evaluate
- Type B: Not type A
- Eventually, model Type B as if it were statistical so you can do math

Repeatability

- If your instrument is not repeatable, you can't get a good measurement.
- Designers design (among other things) for good repeatability

Repeatable motion



- A rigid body has 6 degrees of freedom (6 numbers specify location and orientation of a body)
  - No such thing as a rigid body…
- Diagram shows Cartesian coordinates, but you have 6 DOFs regardless of whether Cartesian, Cylindrical, Spherical, …

Materials

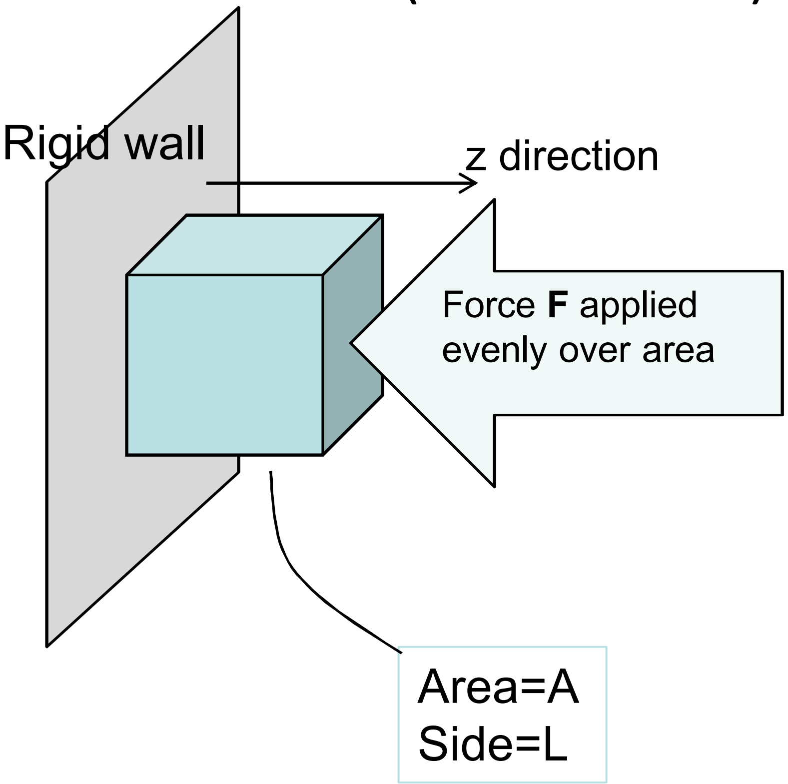
- Solids, liquids, and gases
  - (OK also, plasmas & Bose-Einstein condensates)
- All solids are elastic—when you apply a force, they deform. When you remove the force, the solid returns to the original shape.
- Resistance to deformation: Hooke's law
  - Materials behave like linear springs (unless you break them!)
  - Deformation is *proportional* to force (without considering geometry)
- Isotropic materials: Properties the same in all directions (piece of steel)
- Anisotropic: Properties are different in different directions (carbon fiber composite)
- At small scale (micro/nano), all materials are anisotropic!

Material properties

- Isotropic elastic materials have:
  - Stiffness, which can be described by E (Young's modulus or modulus of elasticity) and ν (Poisson's ratio)
  - Strength (how much can you deform before it's permanent)
  - Density
  - Thermal conductivity
  - Thermal expansion
  - Temporal stability
  - Electrical and magnetic properties, which we ignore for now…
- Stress: Force per unit area on a body or inside a body
- Strain: Relative deformation of a body (change in length divided by original length)

Stress, Strain etc.

- Let's look at a small cube of an isotropic engineering material (such as steel)



- Cube has stress $\sigma = F/A$
  - Units = Pa
- Shortens by $\varepsilon = \delta L/L$
  - Units = dimensionless
- $\varepsilon = \sigma/E$ (or $\sigma = E\varepsilon$)
- $\delta L = \varepsilon \cdot L$
  - Units = m

- Cube bulges sideways (x, y) by $\varepsilon_x = \nu \cdot \varepsilon_z$
  - Dimensionless

- E→Young's modulus or modulus of elasticity; units = Pa. Steel ~205 GPa
- ν→Poisson's ratio; dimensionless. Steel ~ 0.29

Hooke's law

- The extension of a spring is proportional to the force: $F = k \cdot x$ (k=spring constant)
- Stiff spring (k big): Large force, small displacement
- Compliant spring (k small): Small force, large displacement
  - Want stiff machines! (measure the UUT length, and not the machine change in length!)

$$\delta L = \varepsilon \times L = \left(\frac{F}{AE}\right) \cdot L$$

- If we want small δL, → small F, small L, large A, large E
- Instruments should be short and fat!

GB comparator

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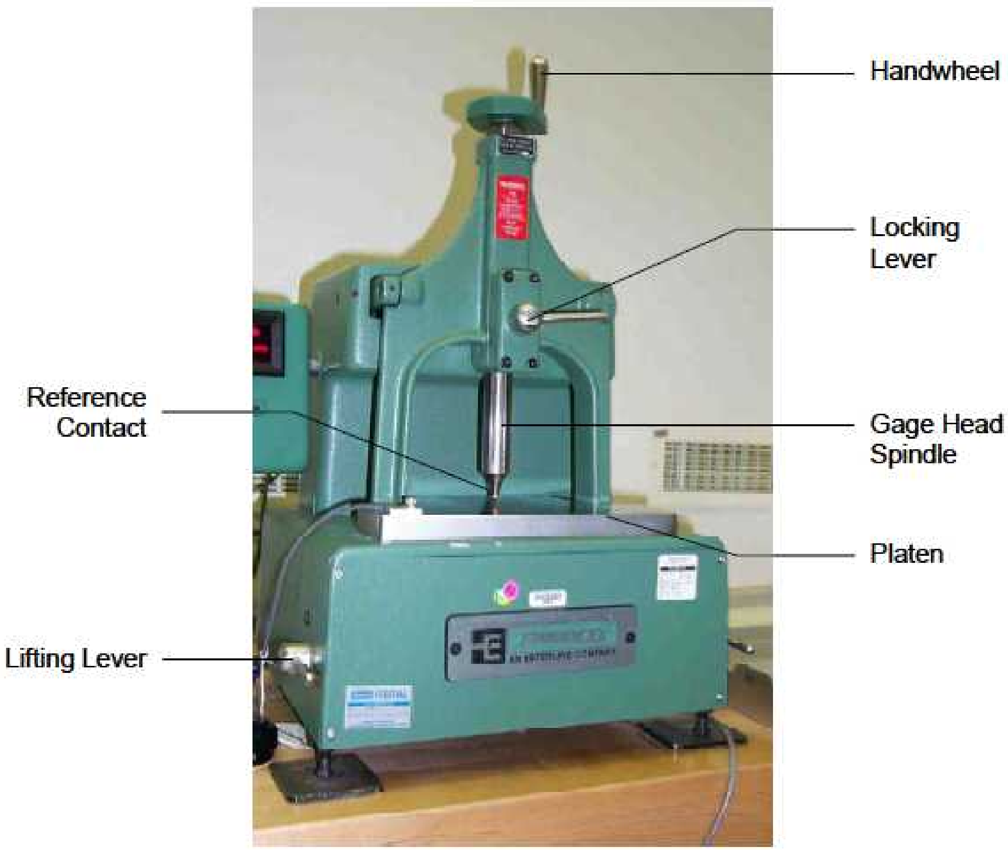
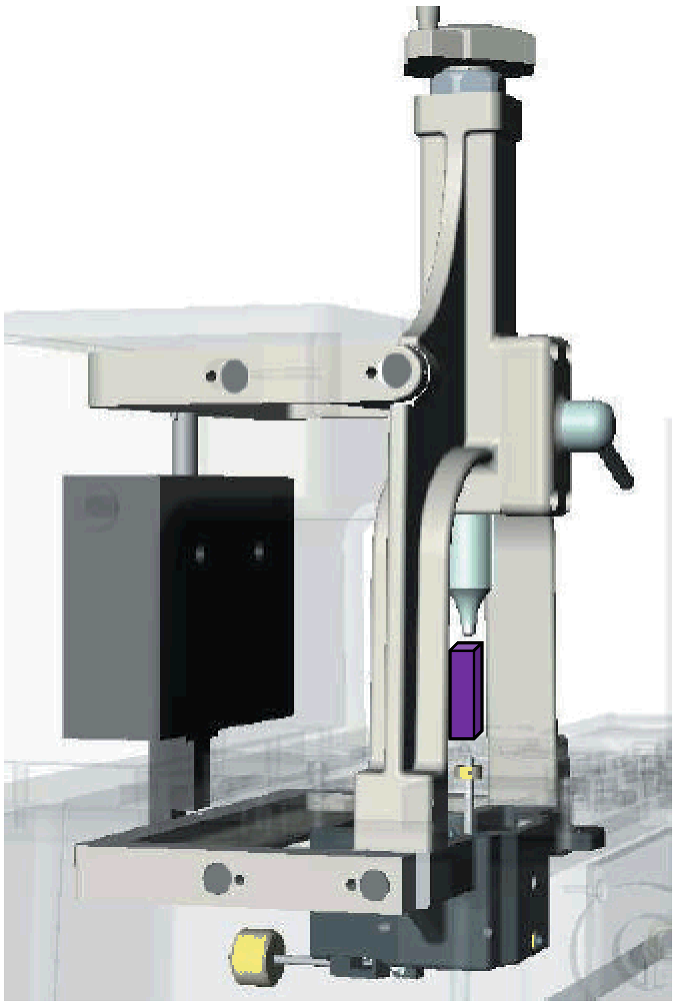
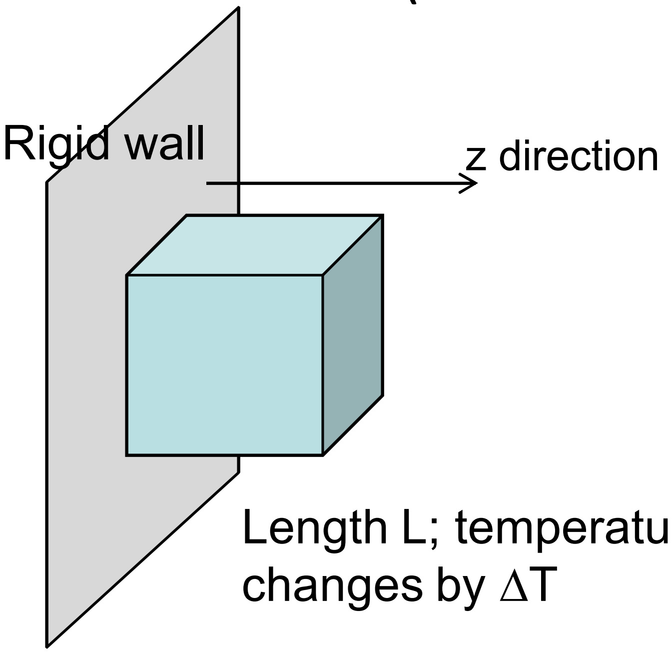




Temperature

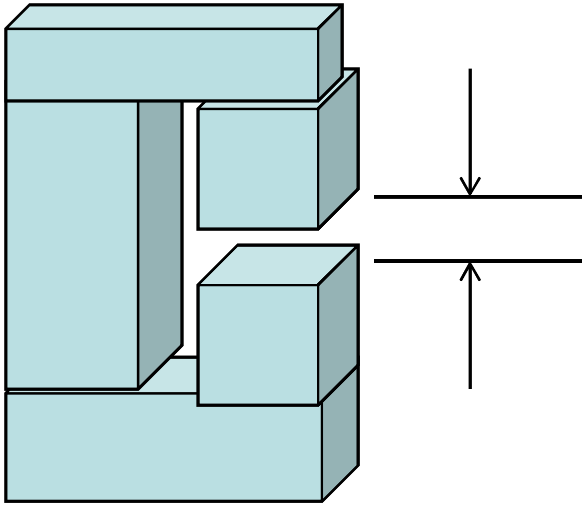
- Let's look at a small cube of an isotropic engineering material (such as steel)



Length L; temperature changes by ΔT

- Cube has CTE. For steel, 11.5 ppm/K
- $\Delta L = L \times CTE \times \Delta T$
- Thermal strain: $\Delta L/L$

- What if ?



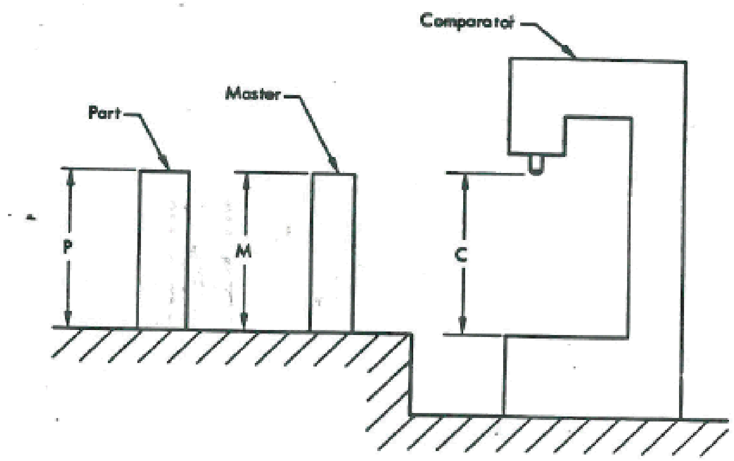
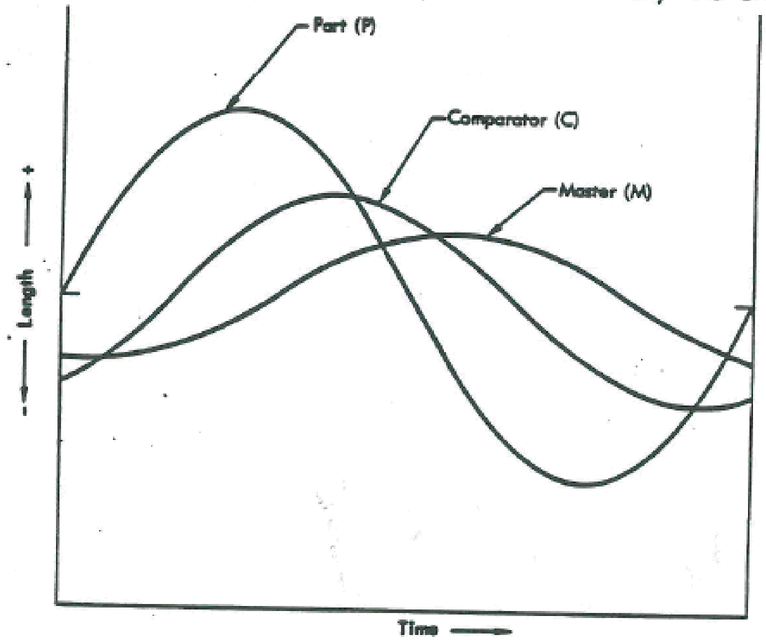


Fig. 6. The three elements of a length-measuring system.

From E. Ray McClure's Thesis, 1969



Materials selection

- Stiffness, strength for structural considerations
- Low density for dynamic (vibration) considerations
- Thermal expansion
- Thermal conductivity
- Heat capacity
- (Thermal conductivity ÷ (heat capacity×density) = thermal diffusivity)→high value = fast equilibrium
- Don't forget to consider the stability of the material over time! (Invar very sensitive to history of "abuse")
- Make all things from the same material if possible—comparing "like-to-like" makes differential CTE correction a smaller factor

Reserve slides after

Some probability & statistics

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- We roll a fair die. Landing on edge or corner is unstable, so the die will always show a number.
- We expect that the probability of landing on "1" = prob of "2" = prob of "3", etc. = 1/6
- Let's use MS-Excel to roll the die
  - We roll 5 dice 60 times. (Doing it by hand would take a long time!)

Warning: MS-Excel is not proven to be a reliable tool for complex statistics/probability computations.

Some Excel demos

Sandia National Laboratories

- Discrete random numbers
- Making a histogram
  - Excel's default histograms are ***<u>REALLY UGLY!</u>***
  - (but sometimes, that's all we have…)
- Other types of random numbers in Excel

Warning: MS-Excel is not proven to be a reliable tool for complex statistics/probability computations.

Measurement model and the GUM

- Measurement result is an ***estimate*** of the value. The statement of uncertainty makes the estimate more complete.
  - Include unpredictable variations, for example, in environment
  - Include effect of inadequate knowledge in the process, for example, correction factors
- "Type A" and "Type B" are for convenience in discussion
  - Not "random" vs "systematic"
- Type A: Use statistical methods to evaluate
- Type B: Not type A
- Eventually, model Type B as if it were statistical so you can do math

Density of a sphere

Sandia National Laboratories

- Measurement model:
  - Measure mass (including uncertainty u_m) (convention here: u is standard uncertainty; U is expanded uncertainty)
  - Measure radius (including uncertainty u_r)
  - Derive density ρ

$$\rho = \frac{M}{V} = \frac{M}{\left(\frac{4}{3}\pi R^3\right)} = \frac{3M}{4\pi R^3}$$

5.1.2 The combined standard uncertainty $u_c(y)$ is the positive square root of the combined variance $u_c^2(y)$, which is given by

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) \tag{10}$$

where f is the function given in Equation (1). Each $u(x_i)$ is a standard uncertainty evaluated as described in 4.2 (Type A evaluation) or as in 4.3 (Type B evaluation). The combined standard uncertainty $u_c(y)$ is an estimated standard deviation and characterizes the dispersion of the values that could reasonably be attributed to the measurand Y (see 2.2.3).

Math…

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Variances

5.1.2 The combined standard uncertainty $u_c(y)$ is the positive square root of the combined variance $u_c^2(y)$, which is given by

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \qquad (10)$$

where f is the function given in Equation (1). Each $u(x_i)$ is a standard uncertainty evaluated as described in 4.2 (Type A evaluation) or as in 4.3 (Type B evaluation). The combined standard uncertainty $u_c(y)$ is an estimated standard deviation and characterizes the dispersion of the values that could reasonably be attributed to the measurand Y (see 2.2.3).

Influence factors

Radius or diameter?

- Our instrument measures diameter
- Measurement equation becomes:

$$\rho = \frac{M}{V} = \frac{M}{\left(\frac{4}{3}\pi\left(\frac{D}{2}\right)^3\right)} = \frac{3M}{4\pi\left(\frac{D}{2}\right)^3} = \frac{6M}{\pi D^3}$$

$$\rho = f(M, D)$$

$$\frac{\partial\rho}{\partial M} = \left.\frac{6}{\pi D^3}\right|_{D_{measured}}$$

$$\frac{\partial\rho}{\partial D} = \left.\frac{-18M}{\pi D^4}\right|_{D,M_{measured}}$$

$$u_\rho = \sqrt{\left(\frac{\partial\rho}{\partial M}\right)^2 u_M^2 + \left(\frac{\partial\rho}{\partial D}\right)^2 u_D^2}$$

More math…

- Always do a dimensional analysis; always carry your units!

$$u_\rho = \sqrt{\left(\frac{\partial \rho}{\partial M}\right)^2 u_M^2 + \left(\frac{\partial \rho}{\partial D}\right)^2 u_D^2}$$

m^{-3} kg kg m^{-4} m

kg m^{-3} ← kg^2 m^{-6} kg^2 m^{-8} m^2

- Note that you can also express the uncertainties & weighting factors as *relative* uncertainties & weighting factors (% of weight, % of diameter, etc.)

If you don't want to remember calculus

- Simulation is a perfectly acceptable tool.
  - If you are more comfortable using computational tools,
  - or if your measurand is more complicated than "a couple" of inputs,
  - or if your inputs might be correlated
- We're going to simulate the measurement of density & the associated uncertainties

- We're using MS-Excel in the demo
- We're going to find the sensitivity coefficients first
  - Tab "Sims to get sens. Coefs."

Simpler approach to sensitivity coeffs

- Consider the Steinhart-Hart equation for a NTC thermistor:

$$T = \left(A + B \cdot \ln(R) + C \cdot (\ln(R))^3\right)^{-1}$$

- T is temp in °K; A,B,C are Steinhart-Hart coeffs, R is resistance of thermistor in Ω

- A typical thermistor:
- A=1.468×10^{-3} ; B=2.383×10^{-4} ; C=1.007×10^{-7}

$$T_{\text{Celsius}} = \left(A + B \cdot \ln(R) + C \cdot (\ln(R))^3\right)^{-1} - 273.15$$

Taylor series w/o calculus

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Given: $T = \left(A + B \cdot \ln(R) + C \cdot (\ln(R))^3 \right)^{-1}$

Find: $\dfrac{\partial T}{\partial A};\ \dfrac{\partial T}{\partial B};\ \dfrac{\partial T}{\partial C};\ \dfrac{\partial T}{\partial R}$

- Remember: For $\dfrac{\Delta A}{A} \ll 1,\ T(A+\Delta A) \approx T(A) + \dfrac{\partial T}{\partial A}\Delta A$
- We can just numerically evaluate T(A) and T(A+ΔA); then ge

$$\frac{\partial T}{\partial A} \approx \frac{T(A+\Delta A) - T(A)}{\Delta A}$$

- Excel— “Taylor series numerically” tab
- Similar process for other partial derivatives
- Note: Making a graph of your results is (usually) very useful!

Back to uncertainty of density

- With a number of measurements, we estimate the density
- If making measurements is expensive, but we have a good model:
- We can simulate the measurements—use “sims to get unc.” tab

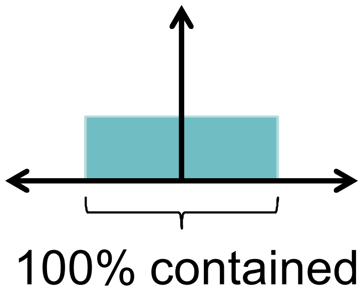
- Everyone report their measurements; we will enter data in “Density calcs” tab

More on simulations

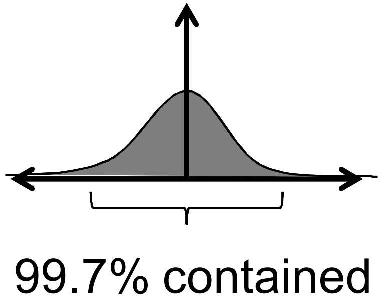
- Do as I say, not as I do: USE MANY MORE SIMULATIONS! (10^6 simulations are often expected to get 2 significant figures at 95% coverage!)
- Other tools are more appropriate: R, Octave, MATLAB, MINITAB…
  - Use Excel to get an "order-of-magnitude" type of number
- If your measurand is not analytic (maybe involves a BC in FEA and expensive to compute), use your judgment for # of simulations—but 50-100 at the minimum as a rule of thumb

Back to probability density functions

- The measurement model & the GUM method are based on an approximation of propagation of error sources through a Taylor series approximation
- For ease of analysis, various models of error sources are converted to the equivalent “Gaussian”:
  - Example: Rectangular distribution→



  - Approximated by normal (Gaussian)→
  - → $u \approx \frac{\text{half-width}}{\sqrt{3}}$



- With Gaussian distributions, variances add

Commonly used PDF's

- Gaussian, aka Normal
  - Standard deviation is standard deviation
- Rectangular (A/D converters, etc.)
  - Equiv. standard deviation = (half-width)/sqrt(3)
- Triangular (setting an instrument at a setpoint; machined part size w/o re-measuring)
  - Equiv. standard deviation = (half-width)/sqrt(6)
- U-shaped (Typical A/C system cycling)
  - Equiv. standard deviation = (half-width)/sqrt(2)

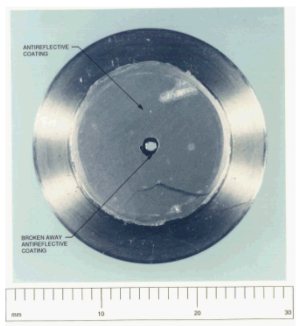
Remember: These are estimates

- Pay attention, but don't get obsessive
  - Your uncertainty estimate is given to two significant figures, but is (seldom) good to two significant figures
  - 95% coverage factor vs 95.4% coverage factor? (do I really care?)
- Your estimates are only as good as your model
- And will not cover "blunders"
  - You can do a really good analysis, with careful error budgets and uncertainty estimates, but these only cover what you know
  - Miscommunication:
  - Specific impulse in lb_f-s vs N-s → Mars Climate Orbiter crashes

But estimates could help judgment

- NASA Hubble Space Telescope
  - Refractive Null Corrector (RvNC) method not adequate to requirements
  - Reflective Null Corrector (RNC) potentially much better
  - Data taken using both methods—Discrepancies between both methods
  - Discrepancies not critically analyzed (NASA Technical Report NASA-TM-103443)

Figure D-1. RNC interferogram of the primary mirror, taken in February 1982.



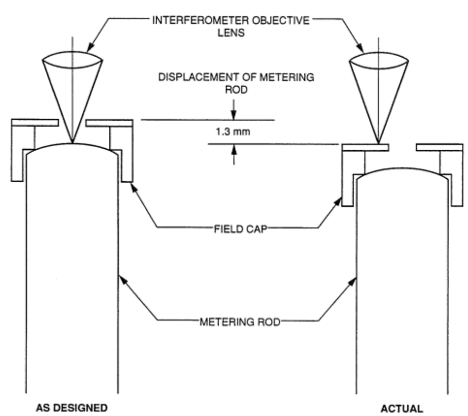


Figure 7-4. Displacement due to the interferometer focusing on the field cap instead of the metering rod.

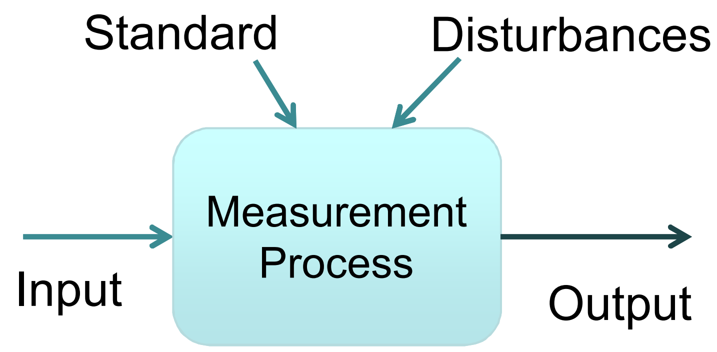
Figure D-2. RvNC interferogram of the primary mirror, taken in May 1981.

Break

Inference questions:

- Given a small(ish) sample:
  - What is a good estimate of the standard deviation of the entire population?
- Measurement model:
  - Disturbances have Gaussian behavior
- Small samples will generally *undersample* the variability of the population

- Use the t-distribution to get a better estimate of the population variance*



t-distribution

- Table G.2 in GUM, page 90/134 in PDF reader; labeled page 78
  - Use DOF# = "number of samples – 1"
  - Calculate std deviation, expand using t-distribution to ~95%
  - Or use Excel TINV() function (or T.INV.2T())→ TINV(0.05,4)=2.776445

Table G.2 — Value of $t_p(\nu)$ from the t-distribution for degrees of freedom ν that defines an interval $-t_p(\nu)$ to $+t_p(\nu)$ that encompasses the fraction p of the distribution

Degrees of freedom	Fraction p in percent					
ν	68,27[a)]	90	95	95,45[a)]	99	99,73[a)]
1	1,84	6,31	12,71	13,97	63,66	235,80
2	1,32	2,92	4,30	4,53	9,92	19,21
3	1,20	2,35	3,18	3,31	5,84	9,22
4	1,14	2,13	2,78	2,87	4,60	6,62
5	1,11	2,02	2,57	2,65	4,03	5,51
6	1,09	1,94	2,45	2,52	3,71	4,90
7	1,08	1,89	2,36	2,43	3,50	4,53
8	1,07	1,86	2,31	2,37	3,36	4,28
9	1,06	1,83	2,26	2,32	3,25	4,09
10	1,05	1,81	2,23	2,28	3,17	3,96

As # of DOFs → (∞-1), t distribution approaches Gaussian distribution

Inference questions:

- Given a small(ish) number of samples: Do I have confidence in what I measured?
  - Is the process in control? Is the variance of the measurement within expectation? How do I check that?
  - Is the measurand within expectation?
- Statistical tests for "in-control": Typically, the "f" test for equal variances. With *n* repeat measurements, is my standard deviation in line with my historical process standard deviation?
- Statistical test for measurand: Use a check standard as part of your measurement. Is the check standard value in line with its history? (presupposes a stable check standard; check standard need not be a calibrated item). Typically a "t" test for equal means
- If setting up a complex system: Intermediate checks to ensure you know what's going on.
- Back later, if time permits; if not—it's in your slides

Degrees of freedom

- "excess" values in fitting a model.
- 2 data points→straight line $y=mx+b$ No degrees of freedom
- 3 data points→straight line $y=mx+b$ 1 DOF
- In statistical evaluations, # of data points taken are usually used to calculate a mean (average)→
  - 1 data point, 0 DOF
  - 2 data points, 1 DOF
  - 3 data points, 2 DOF
  - By induction, # DOFs = number of data points – 1

t-distribution, DOFs, effective DOFs

- Appendix G (“Annex G”) in GUM
- t-distribution of (x) + t-distribution of (y) ≠ $(t_x^2 + t_y^2)^{1/2}$
- Better approximation: Calculate the *effective* degrees of freedom, using the Welch-Satterthwaite formula:

$$\nu_{eff} = \frac{\left(\sum \left(w_i u_i\right)^2\right)^2}{\sum \frac{\left(w_i u_i\right)^4}{\nu_i}}$$

- ν → DOFs; *w* → weighting factors; *u* → std unc.
- Truncate ν_{eff} to next smaller integer; then, expand using t-table

Working an example w/ Welch-Satterthwaite

- Numerator of W-S equation is the square of the combined weighted variance
- Denominator weights each individual variance by its DOFs
- Let’s do density again:
  - Measure diameter *n* times (number of students or number of students/2), with a different orientation of the sphere
  - Use micrometer—some groups use inch; others use metric micrometer
  - Weigh just once. Assume weighing process is “in control”, Type B for weight, ±0.1 gram, rectangular distribution

Following the process—sources of unc. and their influence

- Uncertainty of measurement instrument itself
  - Reproducibility of micrometer (counted with repeated measurement of sphere)
  - Accuracy of micrometer: ±0.0002” for English; ±0.01 mm for SI
- Classify above: Type A or B?
- Uncertainty in the operator and measurand: Repeated measurement of sphere
- Let’s go to Excel & calculate

Whew, another break

Traceability

- Measurements are not traceable to NIST (or NPL or PTB or…)—they are traceable to the SI

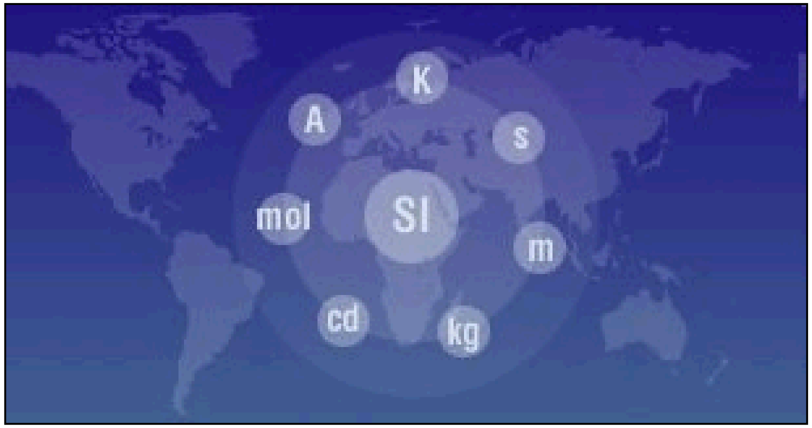
2.41 (6.10)
metrological traceability
property of a **measurement result** whereby the result can be related to a reference through a documented unbroken chain of **calibrations**, each contributing to the **measurement uncertainty**

The International System of Units

See the SI Brochure published by the BIPM.

Think Americans don't use the metric system? Secretly we have all been using it since 1893.

Mechanical Base Units



The **second** is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

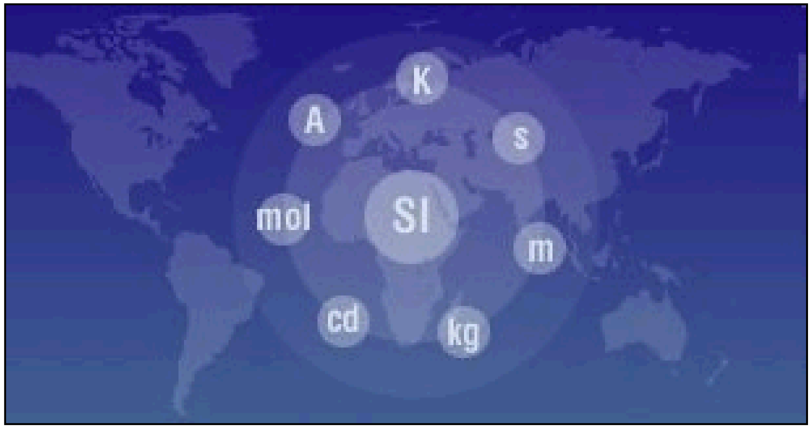
The **meter** is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.

The **kilogram** is equal to the mass of the international prototype of the kilogram.

Electrical and Thermodynamic Base Units

The **ampere** is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2 x 10^{-7} newton per meter of length.

The **kelvin** is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.



The Other Two Base Units

The **mole** is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.

The **candela** is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 x 10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

Derived Units

All other SI units can be formed by combinations of the base units.

For example:

Frequency, Hz = s^{-1}

Pressure, Pa = m^{-1} kg s^{-2}

Energy, J = m^{2} kg s^{-2}

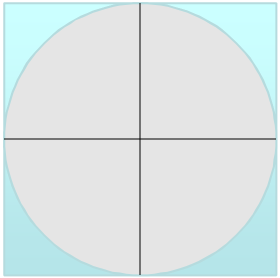
Voltage, V = m^{2} kg s^{-3} A^{-1}

Traceability, continued

- Is generally achieved not to the base SI unit, but to a generally accepted value
  - Laser wavelength of iodine-stabilized laser
- Through accredited services, via a mutual recognition agreement
- Rigor level is based on your needs

Digression into Monte Carlo simulations

- Monte Carlo method: called that because gambling casinos in Monte Carlo, Monaco
- Early discussions in the 19^{th} century; used by Enrico Fermi & colleagues during WW II
- Let's do a very crude way of calculating the value of π :
  - Consider a square target, side = 2r
  - Throw darts randomly at target
  - Of $(2r)^2$ darts thrown, πr^2 darts will fall in the circle



  - We can do this in a quadrant: Throw darts at upper right quadrant.
  - If dart has a distance $(x^2+y^2)^{1/2}$ <=1, it is in the circle. Count that as success.
  - Ratio "success" to "all throws" $\approx \pi/4$

Some Monte Carlo trials

```
#simulate pi
import random
import math

def simpi(num):
    ''' (int)-->float
    Runs num Monte Carlo simulations, returns pi

    Calls random.random() for x coord, random.random() for y coord
    if sqrt(x^2+yY2)<=1, accumulate
    pi = accumulate/num
    '''
    count=0.0
    for i in range(num):
        x=random.random()
        y=random.random()
        if math.sqrt(x**2+y**2)<=1.000:
            count+=1.0
    return 4.0*count/float(num)
```

```
>>> simpi(10)
2.8
>>> simpi(100)
3.0
>>> simpi(1000)
3.096
>>> simpi(10000)
3.1404
>>> simpi(10000)
3.1572
>>> simpi(10000)
3.1176
>>> simpi(1000)
3.144
>>> simpi(1000)
3.168
>>> simpi(1000)
3.216
>>>
```

- We can also set this up to try it in Excel
- Tab “calculate pi”

More Monte Carlo simulations

- Commercial programming codes (MATLAB, MINITAB, etc.) or open-source (R, Octave, Python, etc.) have robust random number generators; generally run fairly quickly
- Need lots of trials!
- Some commercial codes estimate uncertainty, e.g. CMM task-specific uncertainty (Metrosage; Calypso, Quindos add-ons)
- Shilling et al.; finding location of an edge on a CMM:

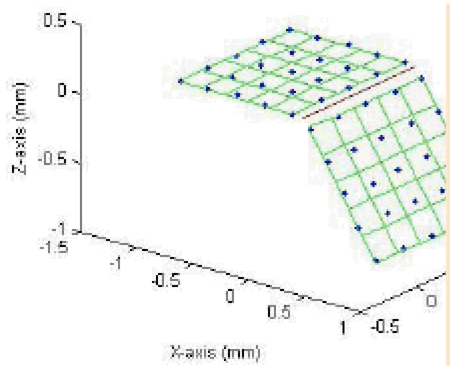


FIGURE 3. Example of Intersection Li
Planes

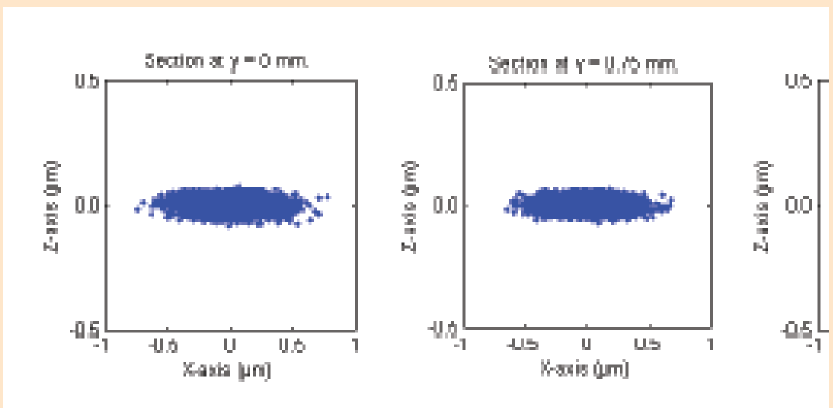


Figure 14. 5 000 resulting intersection line cross-sections at ends and center of l

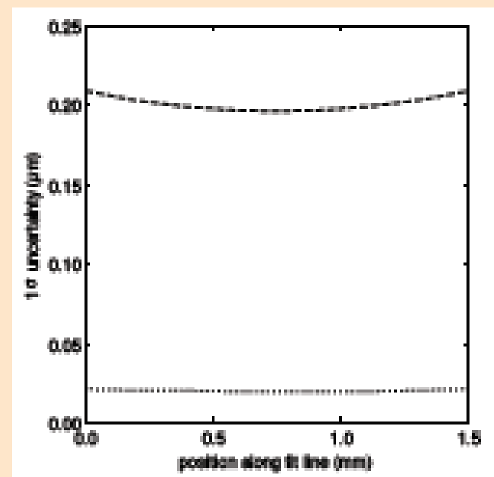


Figure 15. 1σ uncertainty along intersection line.