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Modeling the Anisotropic Nonlinear Behavior of the Artery Wall

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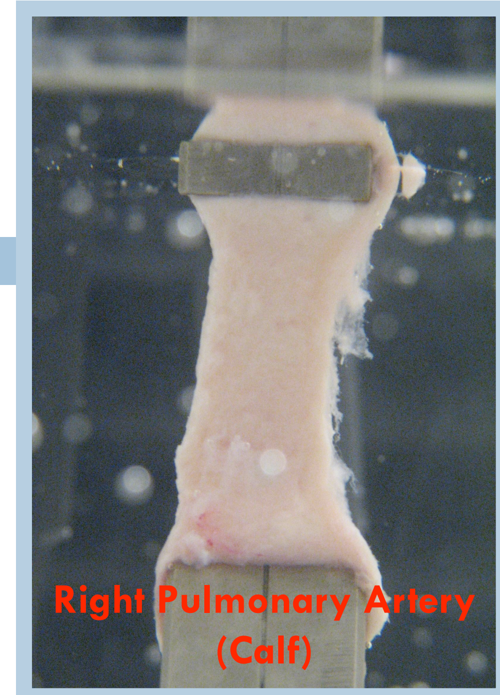
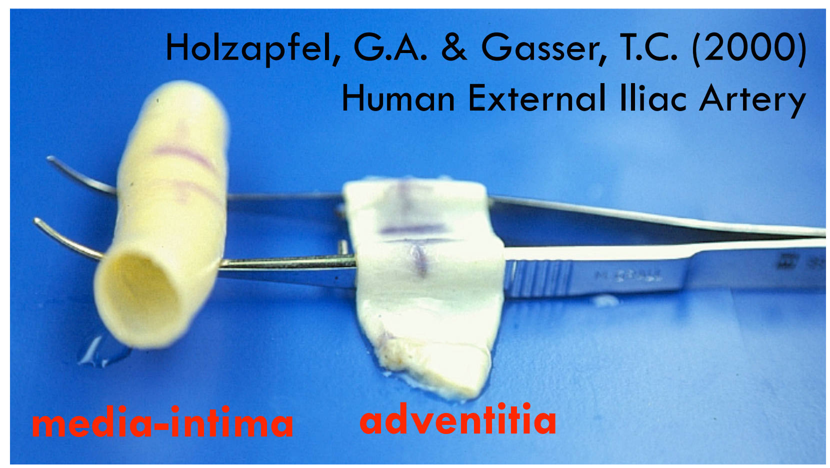
Overview

- Motivation
- Background
  - artery wall
  - material model
- Experimental Results
  - right pulmonary artery (calf)
- Data Fitting
  - approach & results
  - parameter comparison
- Future Work
  - experimental
  - modeling

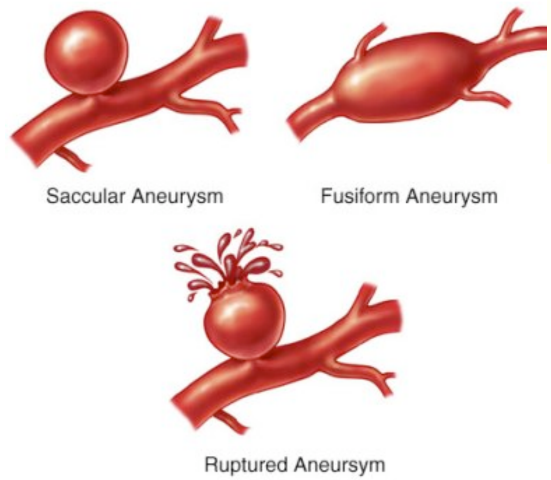


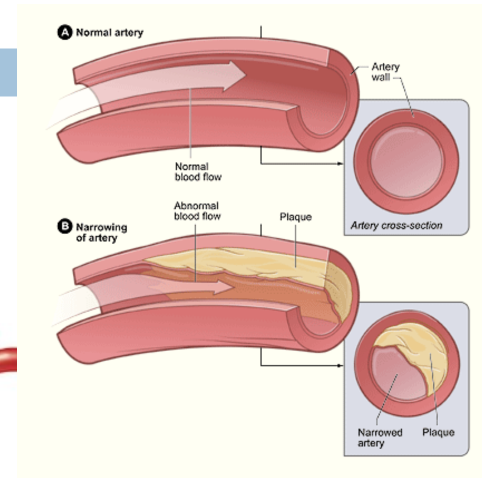


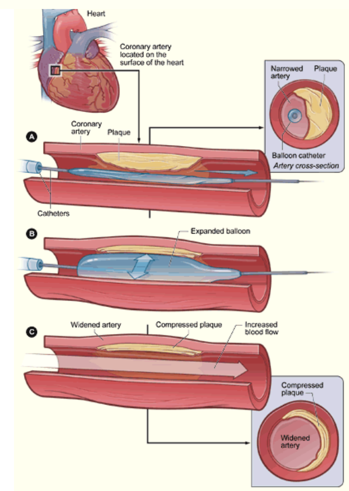
Motivation

- Clinical Aspects
  - prevention
  - diagnosis
  - treatment
- Diseases
  - aneurysms
  - atherosclerosis
    - balloon angioplasty
  - pulmonary hypertension
    - stiffening
    - change in relative amount of collagen and elastin

Severe Aortic Aneurysm







Balloon Angioplasty Procedure

Project Goal

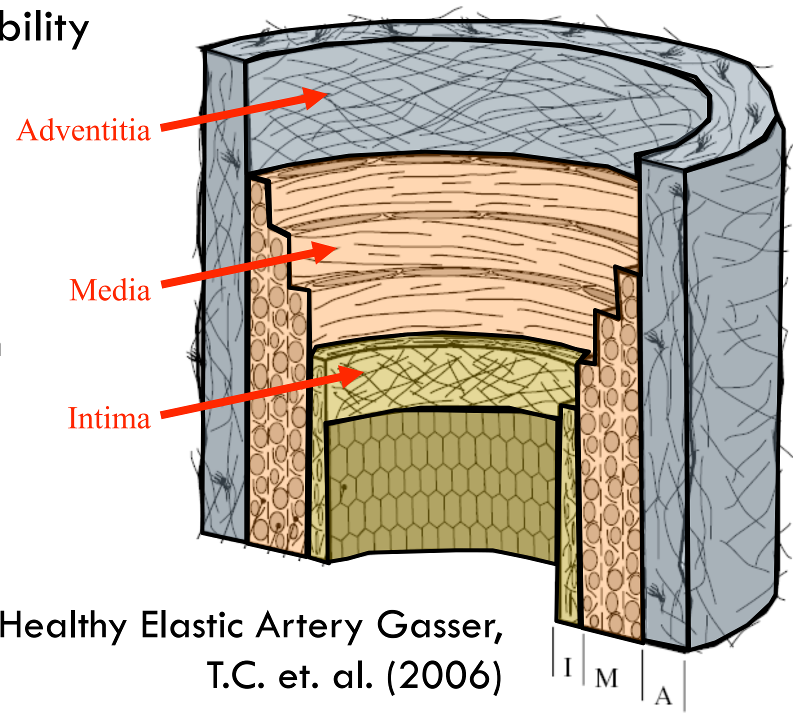
- Develop Physiologically Motivated Model
  - model parameters correlate to physical properties
    - e.g. strain stiffening effect → arterial limiting distension
  - parameter comparison between diseased and healthy tissue
    - degradation and/or evolution
      - structural alterations
      - individual components (elastin, collagen, SMC,…)

Artery Wall

Zoumi, et. al. (2004)
Rabbit Aortic Wall

Heterogenous, Fiber Reinforced Composite Structure

- Layer Specific Histological Features
  - Intima
    - endothelial cell monolayer
    - does not contribute to load-carrying capability of a healthy artery wall
  - Media
    - principal constituent – elastin
    - fenestrated elastic lamina
      - smooth muscle cells, elastin network, collagen fibers and proteoglycans
  - Adventitia
    - principal constituent – collagen
    - thick bundles of collagen fiber
      - helical arrangement
    - limits distention of artery walls



Healthy Elastic Artery Gasser, T.C. et. al. (2006)

Composite Material Model

Applied to Individual Medial & Adventitial Layer

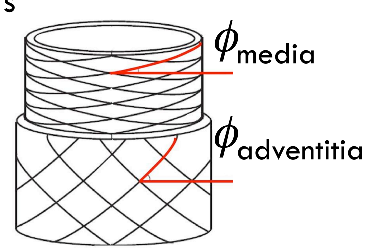
- Matrix Component: Neo-Hookean Model

$$\mathcal{W}_{\mathrm{M}} = \frac{\mu}{2}\left(\bar{I}_1 - 3\right) + \frac{\kappa}{4}\left(I_3 - \ln I_3 - 1\right)$$

- Fiber Component: Worm Like Chain Model

$$\mathcal{W}_{F_\beta} = \frac{\mu_\beta}{4}\left(2\frac{I_{\beta+3}}{\lambda_L^2} - \frac{\sqrt{I_{\beta+3}}}{\lambda_L} + \frac{1}{1-\sqrt{I_{\beta+3}/\lambda_L}}\right) + \frac{C}{\left(I_{\beta+3}\right)^{n/2}}$$

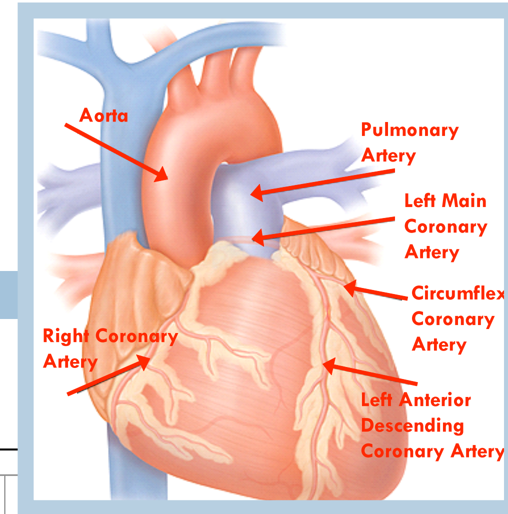
  - fiber stretch: $\lambda_\beta = \sqrt{I_{\beta+3}}$
  - use initial stress-free state to determine the constant → $C = \dfrac{\mu_\beta\left(6\lambda_L^2 - 9\lambda_L + 4\right)}{4n\lambda_L^2\left(\lambda_L - 1\right)^2}$
- Assumptions
  - quasi-incompressible formulation: $\kappa >> \mu$
  - repulsion exponent, n = 1
  - 2 fiber families per layer
    - equivalent model parameters
    - fiber orientation: $\pm\phi$



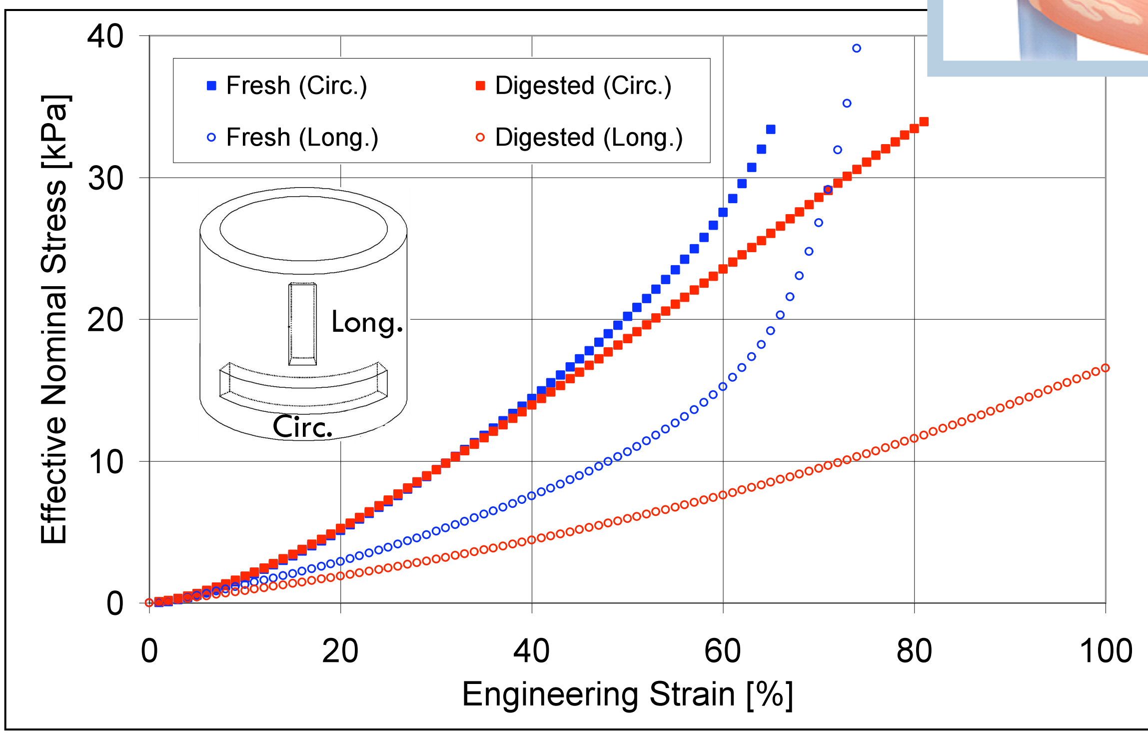
4 Model Parameters per Layer:
$\mu_M, \mu_F, \lambda_L, \phi$

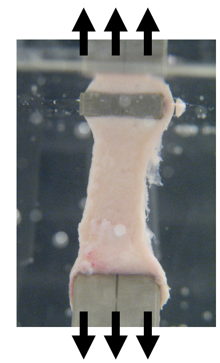
Experimental Results

Right Pulmonary Artery (Calf)



- Experimental Protocol
  - Test Fresh Sample
  - Perform Digestion Process
  - Test Digested Sample





Digestion Process Results In Elastin Network Only Samples

Approach for Parameter Determination

1. Fit Media Layer Parameters to Digested Sample

$$\left(\mu_{\mathrm{M}}^{\mathrm{media}}, \mu_{\mathrm{F}}^{\mathrm{media}}, \lambda_{\mathrm{L}}^{\mathrm{media}}, \phi^{\mathrm{media}}\right)$$

   - assume elastin-network is the primary load bearing constituent
   - determine model parameters via material point simulation

2. Fit Adventitia Layer Parameters to Fresh Sample

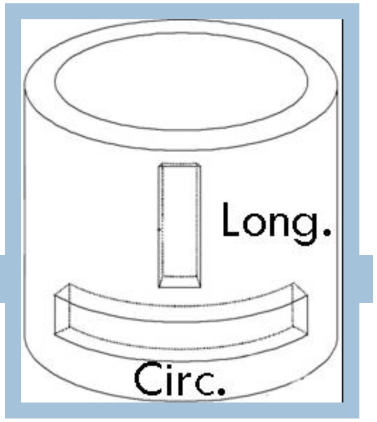
$$\left(\mu_{\mathrm{M}}^{\mathrm{adventitia}}, \mu_{\mathrm{F}}^{\mathrm{adventitia}}, \lambda_{\mathrm{L}}^{\mathrm{adventitia}}, \phi^{\mathrm{adventitia}}\right)$$

   - determine model parameters for the adventitial layer via bilayer finite-element simulation

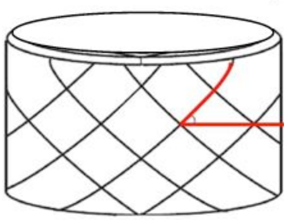
Curve Fitting Protocol for Each Layer

1. Simultaneously Fit Both the Circumferential and Longitudinal Data
2. Determine ϕ, μ_M and μ_F
   - fit to initial (0-15%) strain region
3. Determine λ_L
   - fit to entire strain region using ϕ, μ_M and μ_F
4. Minimize Error
   - root mean squared error $= \sqrt{\dfrac{\sum_{i=1}^{N}\left(x_{1,i} - x_{2,i}\right)^2}{N}}$
     - N = number of circ. and long. data points

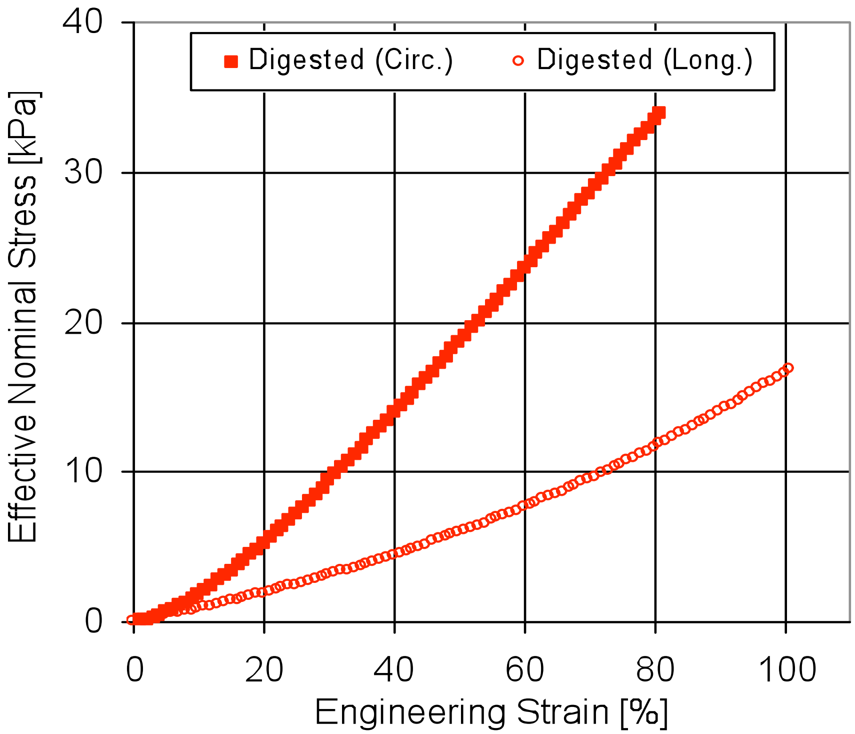
Digested Sample Data



- Inspection of Material Behavior
  - fiber orientation
    - 0° < ϕ < 45°

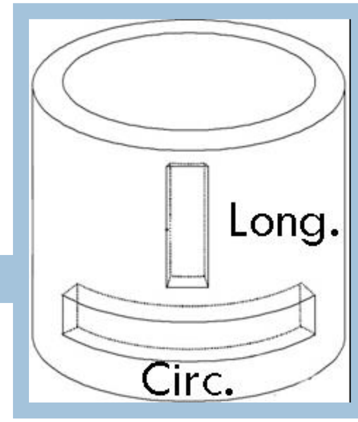


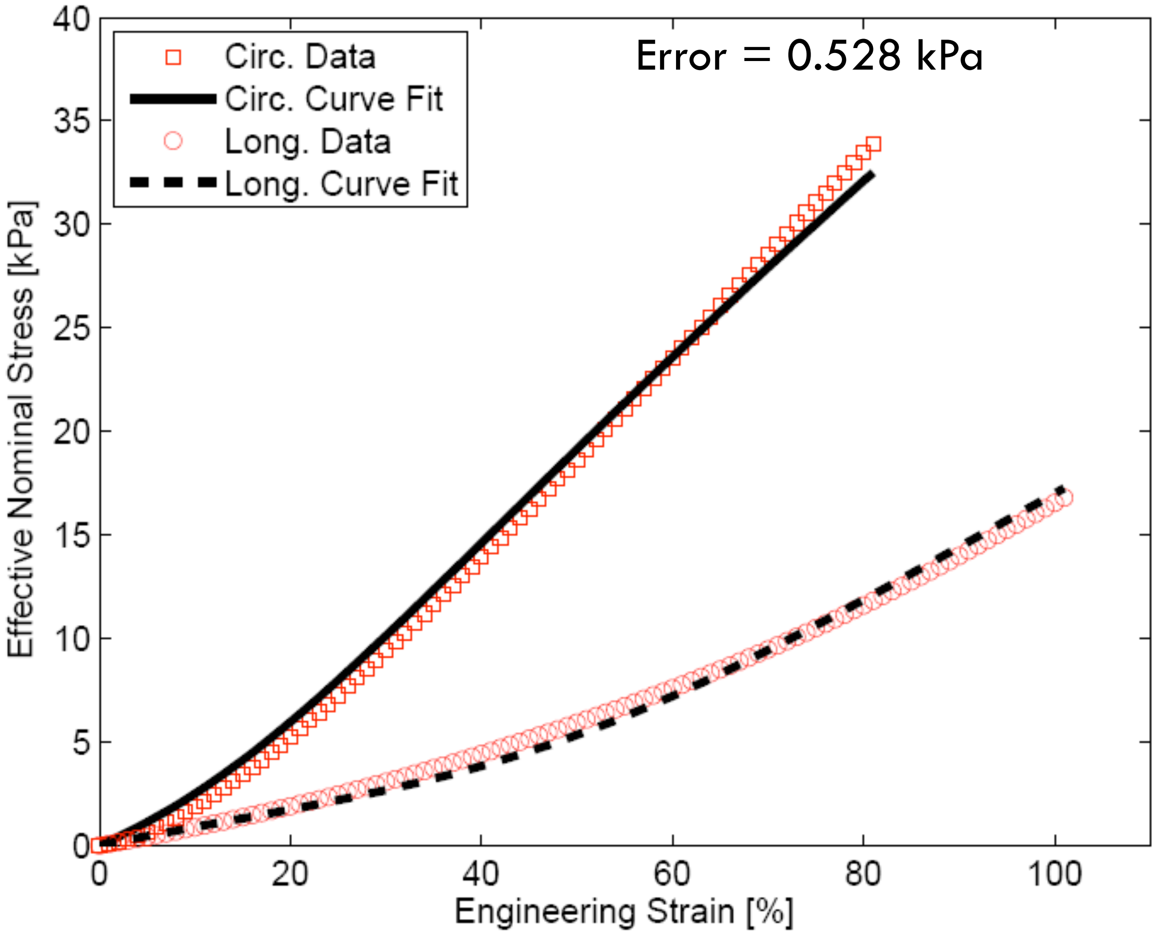
  - no appreciable strain stiffening
    - large locking stretch



Digested Sample Data

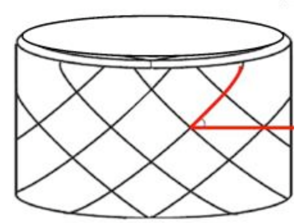
Material Point Simulation – Elastin Network Only





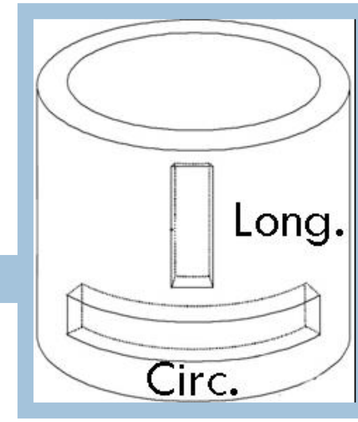
- Model Parameters

Media	
μ_M	2.5 kPa
μ_F	920 kPa
λ_L	10.35
ϕ	36.7°



Fresh Sample Data

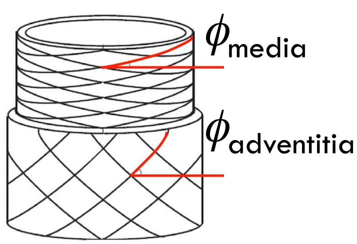
Finite Element Bilayer Discretization – Adventitia & Media

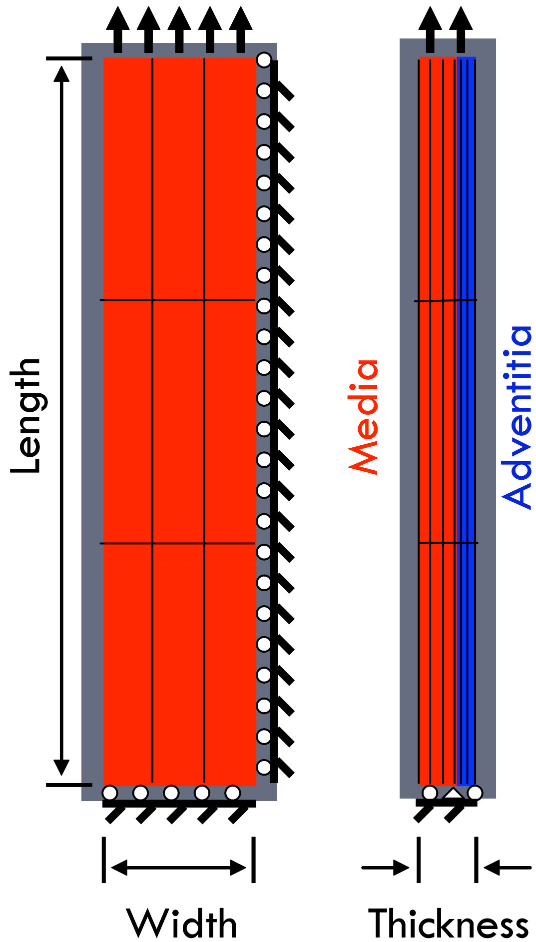


Dimensions (mm)

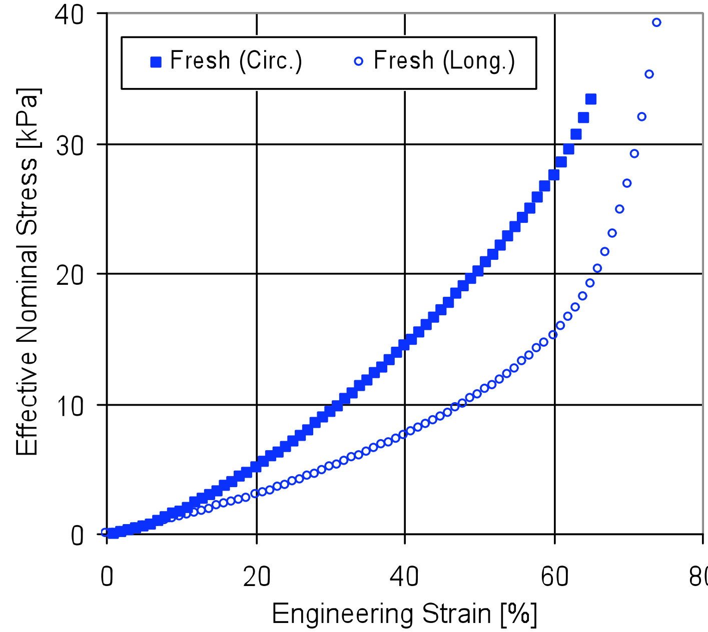
	Long.
Length	11.585
Width	2.9935
Thickness	1.53

	Circ.
Length	16
Width	3.345
Thickness	1.2



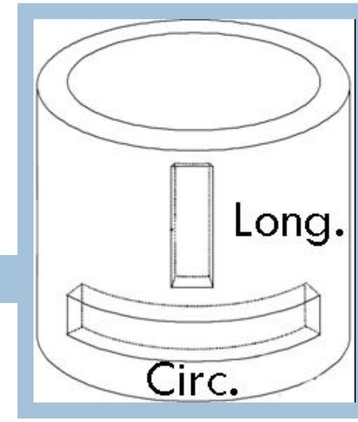


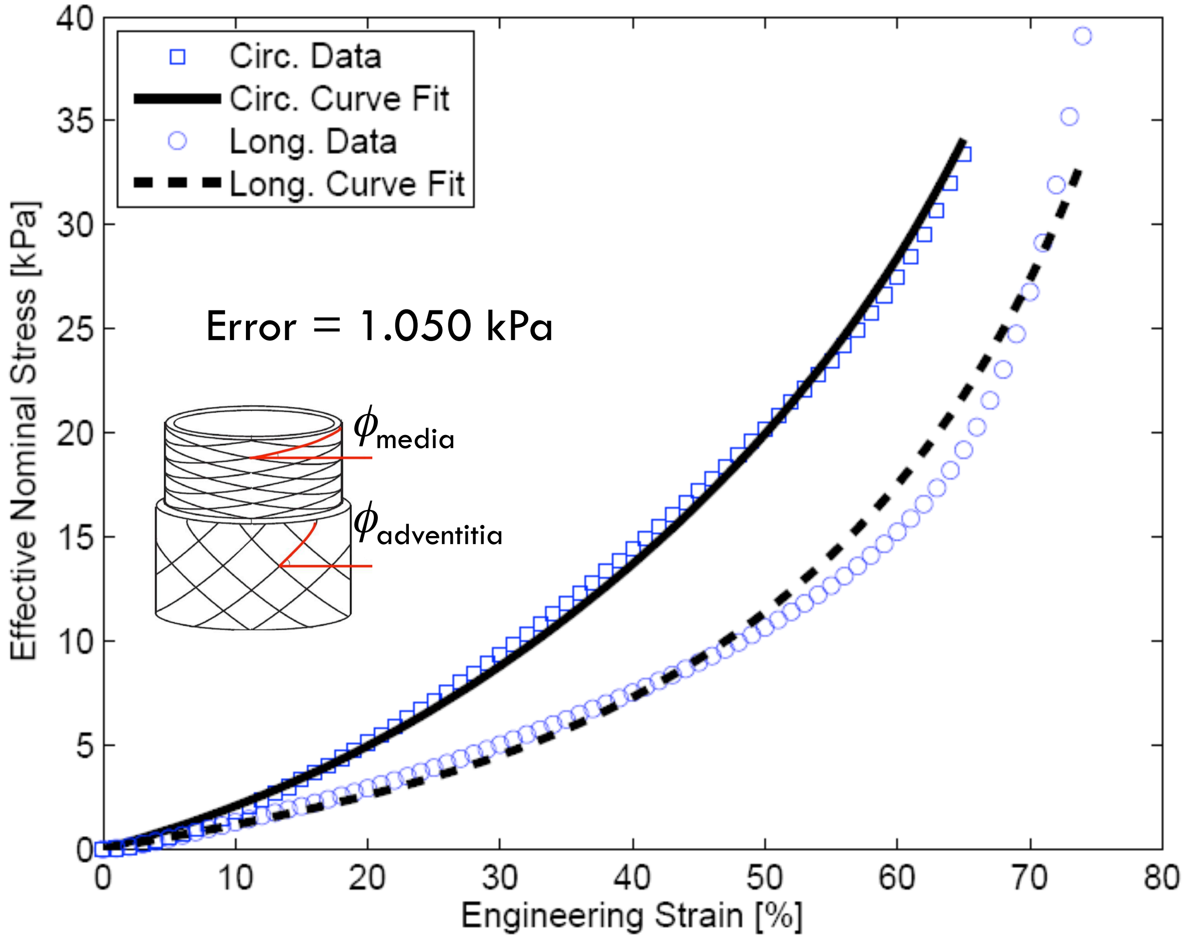
- Assumptions
  - medial layer thickness is 2/3 of total wall thickness



Fresh Sample Data

Finite Element Bilayer Simulation – Adventitia & Media





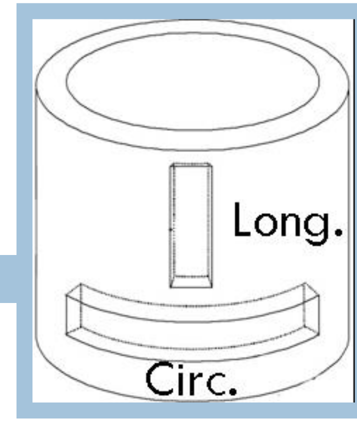
- Model Parameters

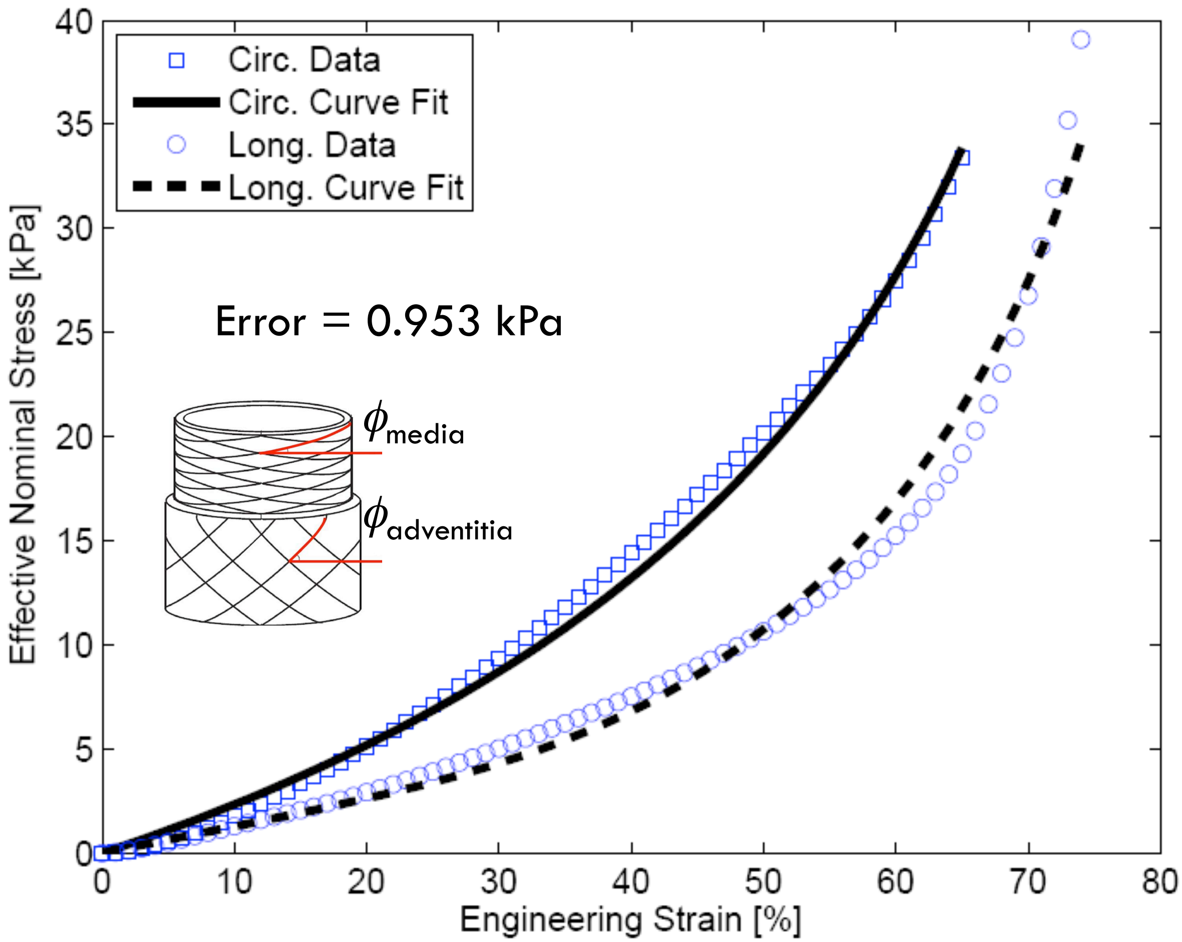
Media		Adventitia	
μ_M	2.5 kPa	μ_M	5 kPa
μ_F	920 kPa	μ_F	10 kPa
λ_L	10.35	λ_L	1.5
ϕ	36.7°	ϕ	44.5°

- Ratio of μ_M^{adv} and μ_M^{med} Not Consistent with Literature.

Fresh Sample Data

Finite Element Bilayer Simulation – Adventitia & Media





- Model Parameters

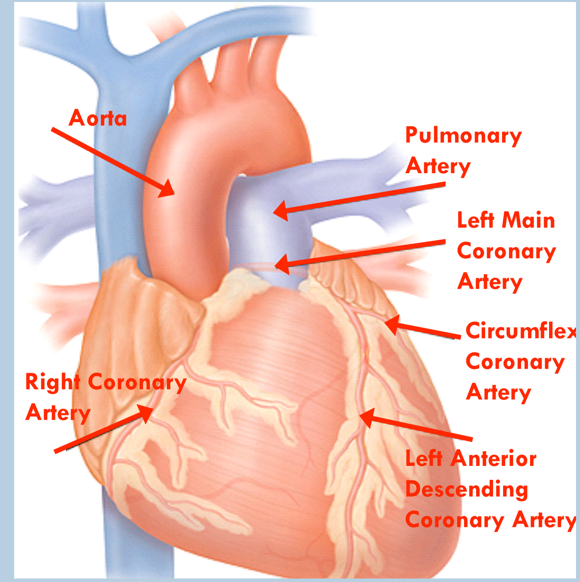
Media		Adventitia	
μ_M	2.5 kPa	μ_M	5 kPa
μ_F	920 kPa	μ_F	10 kPa
λ_L	10.35	λ_L	1.5
ϕ	36.7°	ϕ	44.5°

Media		Adventitia	
μ_M	6 kPa	μ_M	0.6 kPa
μ_F	920 kPa	μ_F	7 kPa
λ_L	10.35	λ_L	1.45
ϕ	36.7°	ϕ	44.5°

Parameter Comparison

- Initial Fiber Stiffness:

$$\mathbf{C}_F = \sum_\beta k_F \mathbf{M}_\beta \otimes \mathbf{M}_\beta \quad \text{where} \quad k_F = 4 \left. \frac{\partial^2 W_F}{\partial I_{\beta+3}^2} \right|_{I_{\beta+3}=1}$$

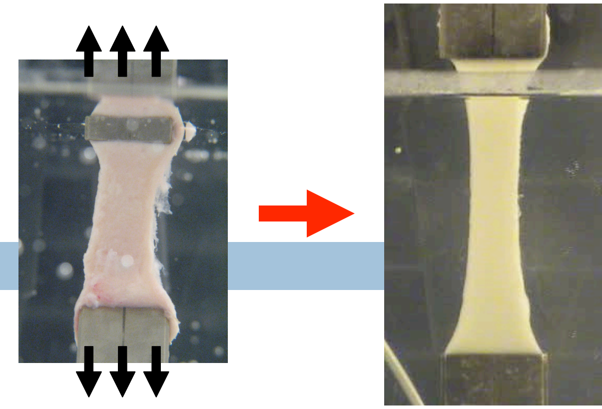


			2000	2004	2002
		Media			
Specimen		Calf RPA	Rabbit Carotid		Human LAD
Matrix Shear Modulus, (kPa)	μ_M	6	1.5	2.77	13.5
Initial Fiber Stiffness, (kPa)	k_F	41.6	9.45	8.61	2.56
Fiber Angle, (deg)	ϕ	36.7	29	26	10
		Adventitia			
Matrix Shear Modulus, (kPa)	μ_M	0.6	0.15	0.277	1.35
Initial Fiber Stiffness, (kPa)	k_F	88.33	2.25	21.53	20.4
Fiber Angle, (deg)	ϕ	44.5	62	45	40
$(\mu_M^{Media} / \mu_M^{Adventitia})$		10	10	10	10
$(k_F^{Media} / k_F^{Adventitia})$		0.47	4.20	0.40	0.13

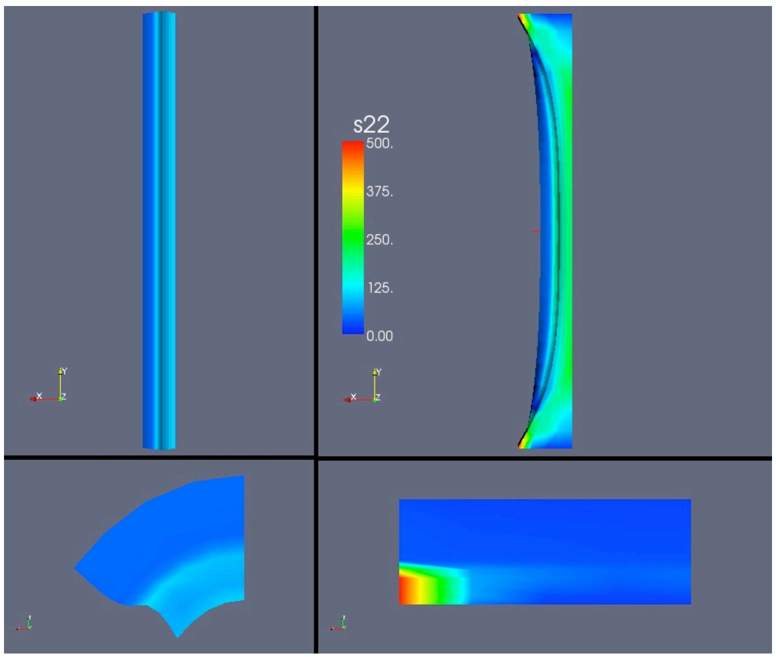
Holzapfel, G. A. et al.

Consider End Effects

Significant Variation in Response

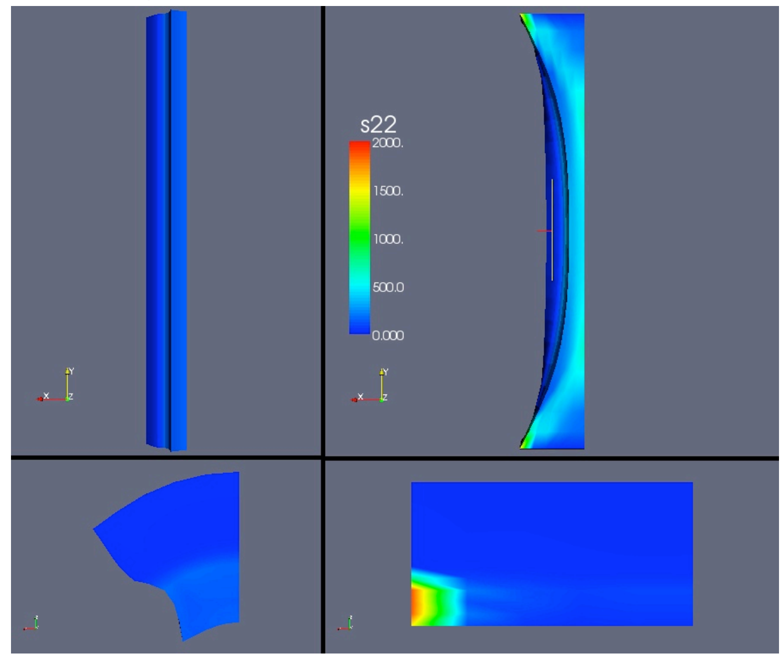


- Circumferential



Homogeneous Nonhomogeneous

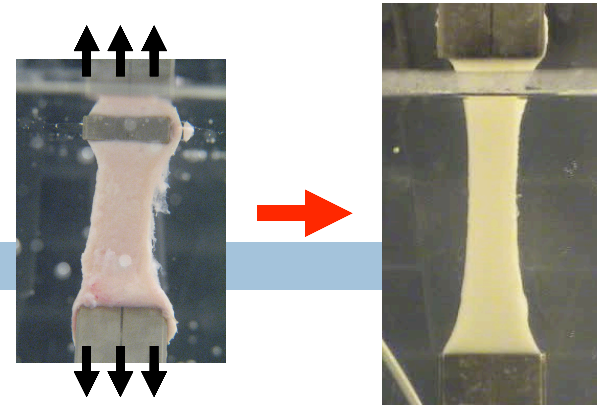
- Longitudinal

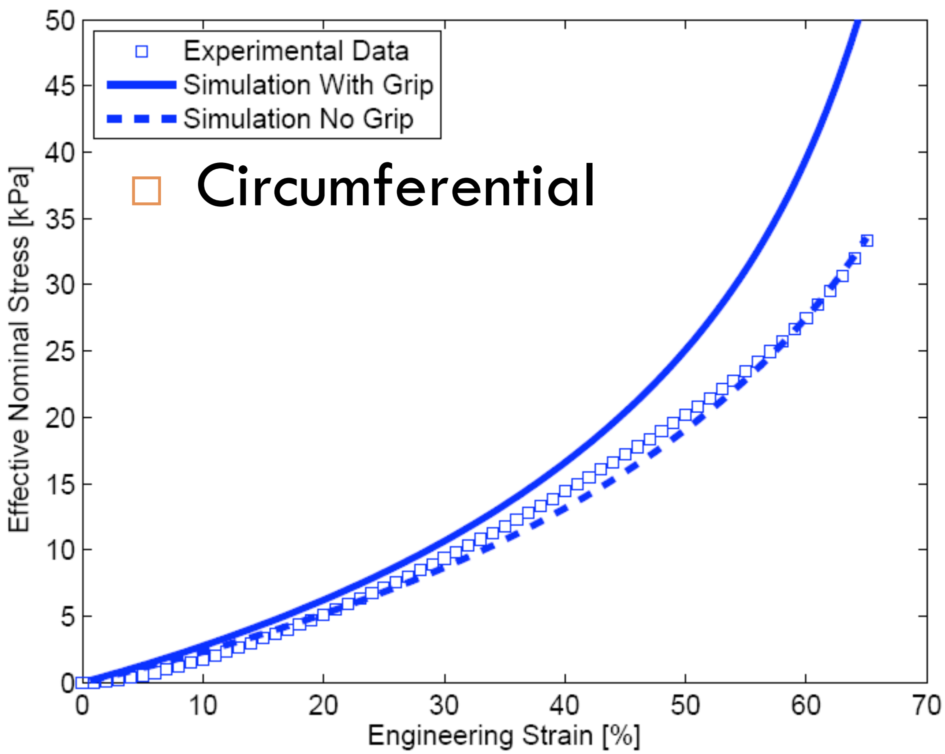


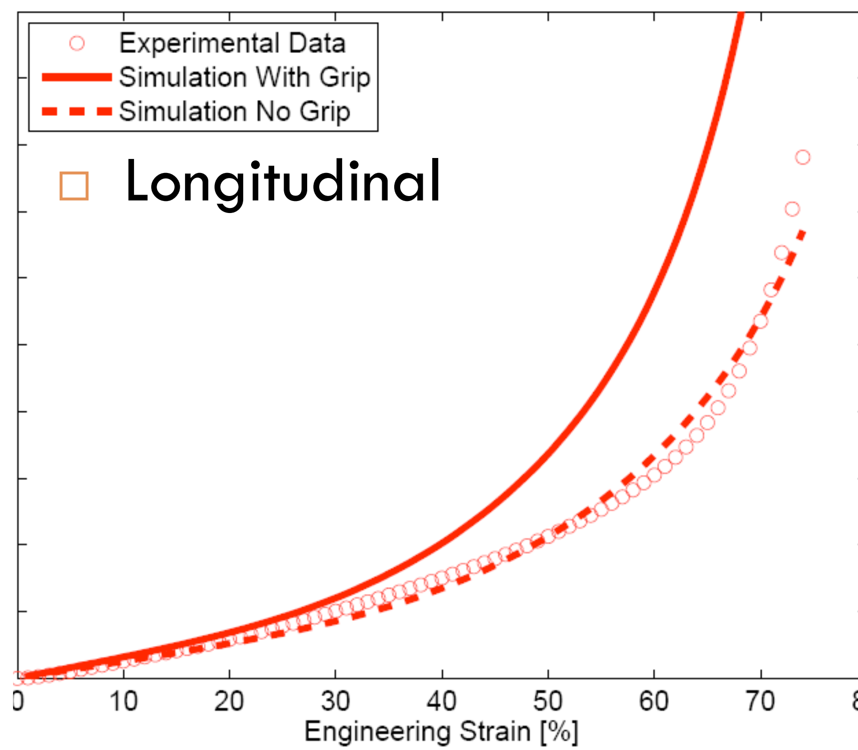
Homogeneous Nonhomogeneous

Consider End Effects

Finite Element Bilayer Simulation – Adventitia & Media

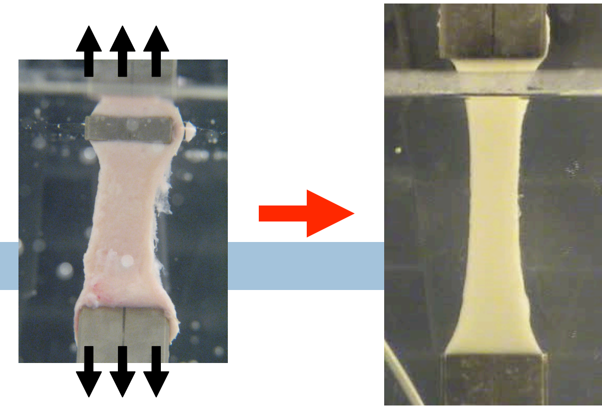






Neglecting End Effects Significantly Alter Response

Consider End Effects



- Clamp Effects
  - longitudinal response has a more pronounced difference
  - enhanced strain stiffening
    - response is also seen in the data
    - increase the locking stretch, λ_L
- Redo Data Fits to Include End Effects from the Grips

Conclusions

- Introduced Material Model for Artery Wall
  - included WLC model to capture strain stiffening behavior
- Good Modeling Assumption
  - assume elastin-network is the primary load bearing constituent in media layer under physiological loading
- Individual Layer Response
  - medial layer
    - does ***not*** display transversely isotropic behavior as literature suggests
  - adventitial layer
    - data fit displays near transversely isotropic behavior
  - must consider the comparison between different species & different artery segment

Conclusions

- Experiments
  - parameters are highly dependent on species and specific artery segment
  - inherently large amount of experimental error
  - end effects from the grips during simulations must be considered to predict material behavior appropriately

Future Work

- Material Model
  - develop a formal algorithm for determining the model parameters
  - include viscoelastic behavior
  - include smooth muscle cells
    - active & passive response
- Experimental
  - consider more experimental data to have a statistical sample
    - compare results from curve fit
  - perform rate-dependent experiments

Acknowledgements

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- Steve Lammers (Univ. of Colorado)

Digestion Protocol

Test (Fresh) → **Digestion Process** → Test (Elastin Network)

- Formic Acid-Cyanogen Bromide Solution Treatment
  - leaves only the elastic network
- Digestion Performed for a Variable Amount of Time
  - depending on sample thickness
- Samples are titrated back to pH=7.4 @ 37°C
  - using NaOH solution in HEPES buffer

Material Model

Cauchy Stress

- Matrix Component

$$\boldsymbol{\sigma}_{\mathrm{M}} = \frac{1}{\sqrt{I_3}}\left[\mu\left(\bar{\mathrm{b}} - \frac{1}{3}\bar{I}_1\mathbf{1}\right) + \frac{\kappa}{2}\left(I_3 - 1\right)\mathbf{1}\right]$$

- Fiber Component

$$\boldsymbol{\sigma}_{\mathrm{F}_\beta} = \frac{2}{\sqrt{I_3}}\sum_{\beta=1}^{2}\left[\frac{\mu_\beta\left(4I_{\beta+3} - 9\lambda_L\sqrt{I_{\beta+3}} + 6\lambda_L^2\right)}{8\lambda_L^2\left(\sqrt{I_{\beta+3}} - \lambda_L\right)^2} - \frac{Cn}{2\left(I_{\beta+3}\right)^{n/2+1}}\right] I_{\beta+3}\mathbf{m}_\beta$$

where

$$C = \frac{\mu_\beta\left(6\lambda_L^2 - 9\lambda_L + 4\right)}{4n\lambda_L^2\left(\lambda_L - 1\right)^2}$$

$$\lambda_\beta = \sqrt{I_{\beta+3}}$$

$$\lambda_L = L_c/r_0$$

Various Models from Literature

- Fung model
  - Anisotropic solid, 9 independent material constants

$$W = \tfrac{1}{2} c \left[\exp(Q) - 1 \right] - p(J - 1)$$

$$Q = \tfrac{1}{2} c_{abcd} E_{ab} E_{cd}$$

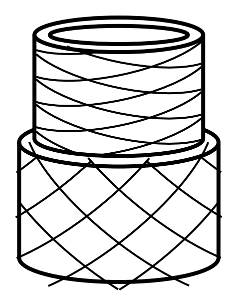
- Holzapfel, Gasser, Ogden, 2000
  - Dependent on invariants I_1, I_4, I_6 to achieve anisotropy
  - Two fiber families, with principal directions $\mathbf{a}_1$ and $\mathbf{a}_2$ and structure tensors $\mathbf{A}_1 = \mathbf{a}_1 \otimes \mathbf{a}_1$, $\mathbf{A}_2 = \mathbf{a}_2 \otimes \mathbf{a}_2$

$$W = \frac{c}{2}(I_1 - 3) + \frac{k_1}{2k_2} \sum_{i=4,6} \left\{ \exp\left[k_2 (I_i - 1)^2 \right] - 1 \right\}$$

$$I_1 = tr(\mathbf{C})$$

$$I_4 = \mathbf{C} : \mathbf{A}_1$$

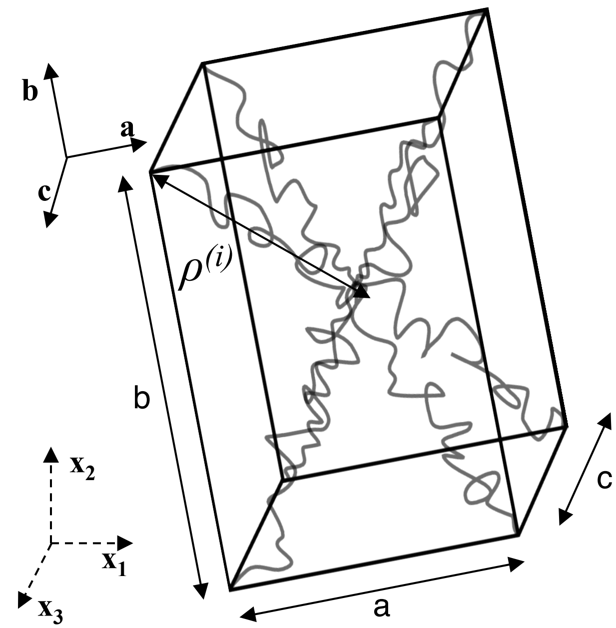
$$I_6 = \mathbf{C} : \mathbf{A}_2$$



Various Models from Literature

- Bischoff-Arruda, 2002
  - Orthotropic $a \neq b \neq c$

$$W(\mathbf{x}) = W_0 + \frac{\mu}{4}\left(N \sum_{i=1}^{4} \left[\frac{\rho^{(i)}}{N} \beta_\rho^{(i)} + \ln \frac{\beta_\rho^{(i)}}{\sinh \beta_\rho^{(i)}} \right] - \frac{\beta_\rho}{\sqrt{N}} \ln \left[\lambda_{\mathbf{a}}^{a^2} \lambda_{\mathbf{b}}^{b^2} \lambda_{\mathbf{c}}^{c^2} \right] \right) + \frac{B}{\alpha^2} \left\{ \cosh[\alpha(J-1)] - 1 \right\}$$



  - Transversely isotropic $a = b \neq c$
  - Isotropic $a = b = c$
    - Arruda-Boyce, 1993

$$W = nk\Theta N \left(\frac{r_{chain}}{Nl} \beta + \ln \frac{\beta}{\sinh \beta} \right) - \Theta c'$$

$$\beta = \mathcal{L}^{-1}\left[\frac{r_{chain}}{Nl} \right] \quad \mathcal{L}[\beta] = \coth \beta - \frac{1}{\beta}$$