



Quantum Error Correction

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Overview and Motivation:

- Unlike classical computing that is either in '0' or '1' state (bit), quantum computing allows for being in multiple states at the same time (superposition)
- Quantum computing promises efficient solution to "difficult" problems that has no classically efficient counterpart today (number factorization)
- Classical bits (0,1) replaced by Quantum bits or "qubits". Single qubits are represented as 2-element column vector, for example:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Quantum computation are realized through "gate" operations. Similar to classical computing with such gates as AND, OR, NOT etc.
- The other types of quantum operation are the Preparation of qubits, and Measurement

Specific Technical Challenges:

- *No cloning principle*: It is impossible to make a copy of unknown quantum state (qubits)
- Observing a qubit by measurement "alters" the qubit; measurement is an irreversible process
- Qubits in superposition of states have associated probability (not certainty) of yielding certain measurement outcomes
- Quantum computing are intrinsically prone to errors. Unlike classical computing with only two bits, there are infinite number of realizable qubits
- Scheduling of Quantum Operations (slide 10) - Hardware constraints must be considered in scheduling quantum computation for real systems since scheduling has significant impact on the ability to be fault-tolerant and the computation time.

Goals:

- Improve on threshold search confidence that currently uses Monte-Carlo simulation
- Provide faster simulation with tight lower bounds on threshold values

Impact and Benefits:

- Cryptographic revolution
- Instantaneous computation
- Exponentially less communication between computers to solve certain problems
- Quantum Teleportation
- Become leaders in the development of a fast-growing research area: quantum information science
- Demonstrate the realms of feasible fault-tolerant quantum computation
- Guide engineers and scientists in the design and development stages of realizing fault-tolerant quantum circuits



Algorithms and Applications:

- Efficient factorization (Shor's Algorithm) which breaks prime number factorization-based encryption (e.g. RSA)
- Secure communication with Quantum Key Distribution which prevents eavesdroppers from intercepting private messages
- Polynomial time (efficient) solution to Pell's equation ($x^2-dy^2=1$) based on Hallgren's algorithm allows rational number representation of irrational square-roots

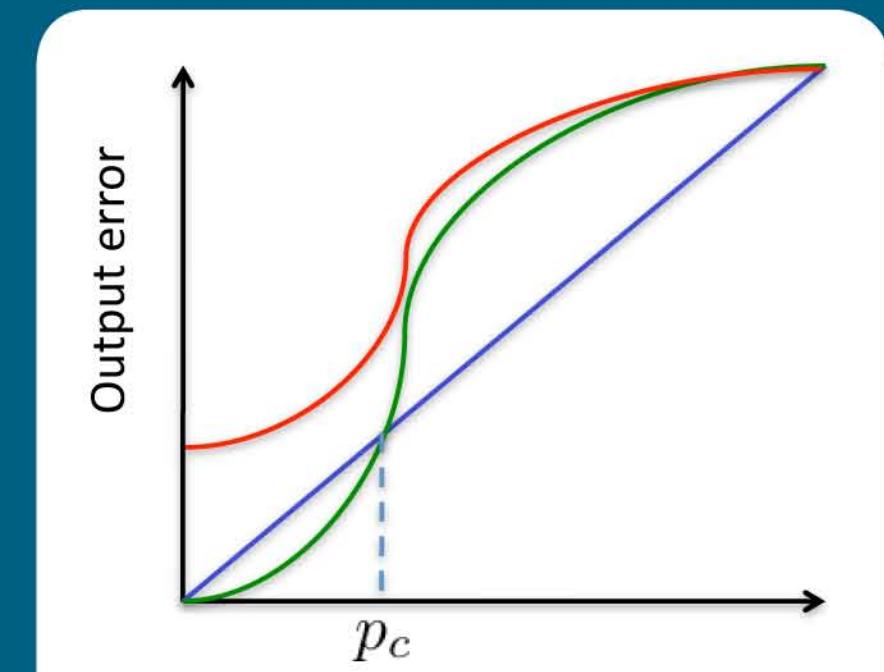
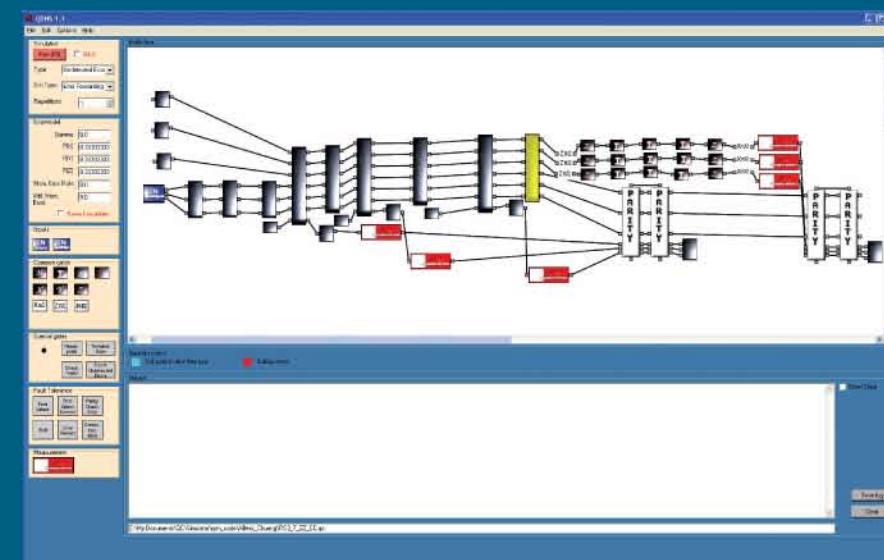
Example: set $d=2$, to obtain the fourth solution of $(x,y) = (577,408)$

$$\sqrt{2} = 1.4142135\dots \approx 577/408 = 1.4142156$$

No efficient classical solution is known to exist today

Simulation of Quantum Protocols:

Fault-tolerant protocols simulated for threshold search. Source code is written in MPI/C# and simulated efficiently (Gottesman-Knill theorem)



Curve of output error equals input error

Fault-tolerant scheme has no threshold under the noise model

Fault-tolerant scheme has a threshold (p_c) below which we can do arbitrarily long computation asymptotically error-free

Current Work:

- Verification of quantum gate schedules
- Exploring the implementation and underlying mathematical concepts of quantum error correction
- Comparing quantum accuracy thresholds using Monte Carlo and Counting Algorithms