SAND2013-10463P

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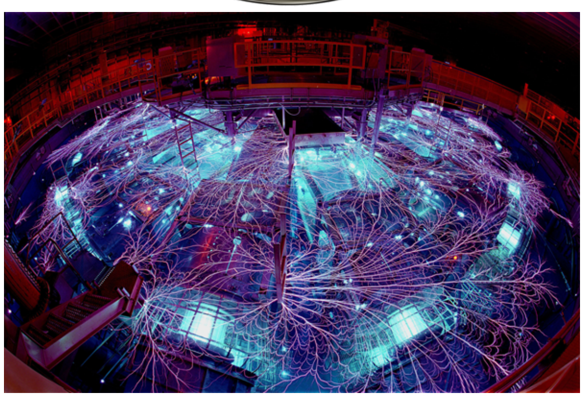
Pulsed Power Sciences
Harnessing Extreme Environments

Magnetic fields and tail-ion depletion in inertial confinement fusion (U)



Paul F. Schmit

Kinetic Processes in Extreme States of Matter 2013

December 18th, 2013

Los Alamos National Laboratory

(This presentation is Unclassified)

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U.S. DEPARTMENT OF ENERGY — NNSA National Nuclear Security Administration — LDRD Laboratory Directed Research & Development

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- Charles Nakhleh (LANL)
- Evan Dodd (LANL)
- Brian Albright (LANL)
- Adam Sefkow (SNL)
- Patrick Knapp (SNL)
- Dan Sinars (SNL)
- Ryan McBride (SNL)
- Steve Slutz (SNL)

Overview of Knudsen loss mechanism

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- First work on tail-ion depletion & Knudsen layers in ICF by Petschek and Henderson:

VOLUME 33, NUMBER 19 PHYSICAL REVIEW LETTERS 4 NOVEMBER 1974

Burn Characteristics of Marginal Deuterium-Tritium Microspheres

Dale B. Henderson
Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544
(Received 5 August 1974)

Long mean free paths for ions in the tail of the distribution may allow escape, quenching the burn of marginal ($\rho R < 10^{-2}$ g/cm^2) deuterium-tritium microspheres, possibly explaining the lack of success in experiments to date.

INFLUENCE OF HIGH-ENERGY ION LOSS ON DT REACTION RATE IN LASER FUSION PELLETS

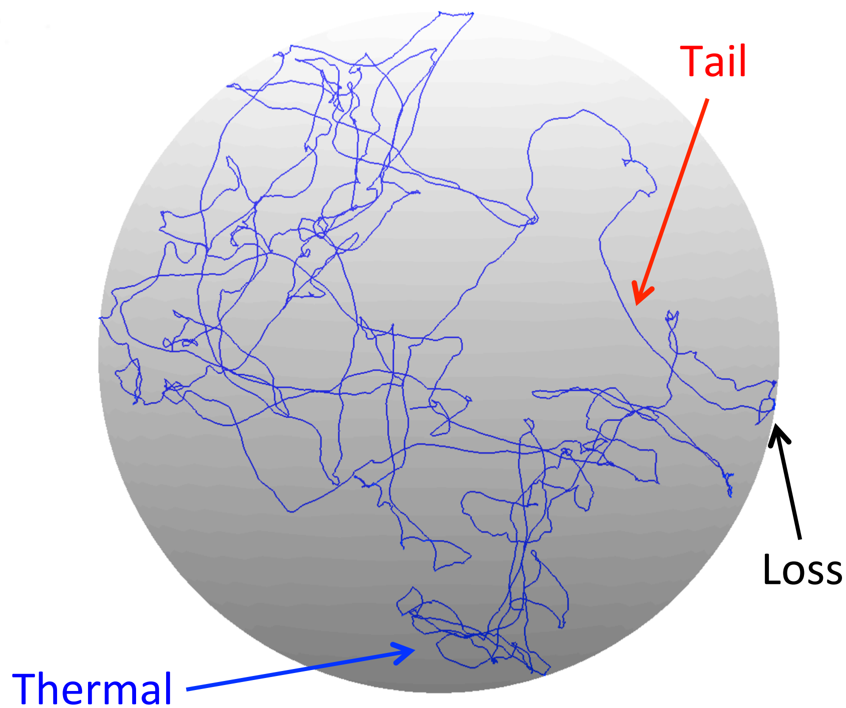
A.G. PETSCHEK*, D.B. HENDERSON (Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico, United States of America)

ABSTRACT. Because of the longer mean free path of high-energy ions, they will be preferentially lost from small pellets containing thermonuclear reactants. This effect has been calculated and, in the most extreme case calculated, a factor-of-about-four reduction of the reaction rate in DT from the Maxwell average rate at the same mean ion kinetic energy is found.

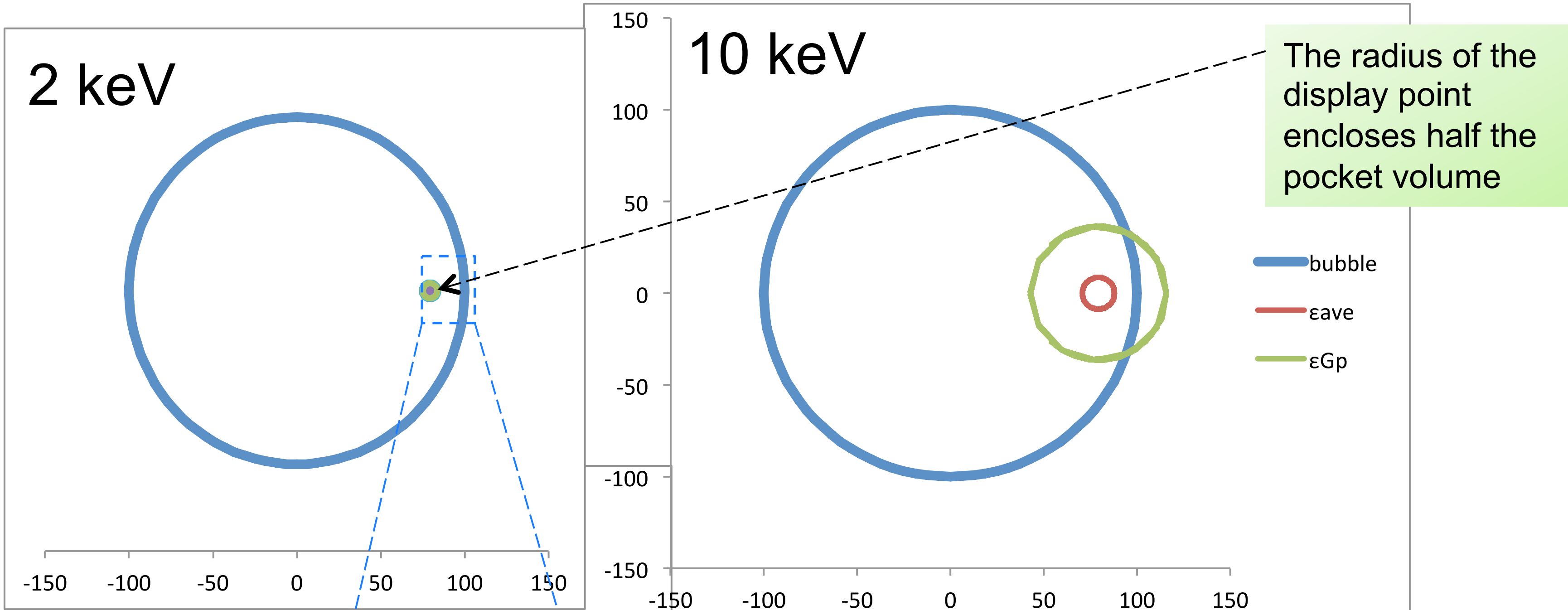
NUCLEAR FUSION, Vol.19, No.12 (1979)

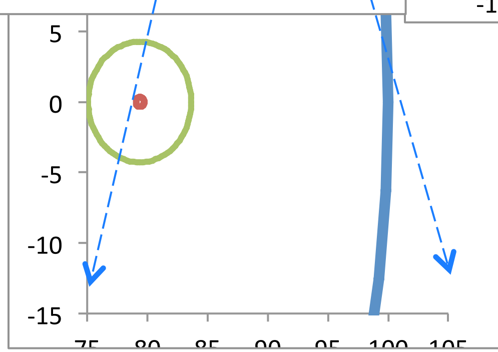
$$\text{Hydro}: \quad \frac{\lambda_{\rm mfp}}{L} \ll 1$$

$$\text{Fuel ions}: \quad \lambda_{\rm mfp} \sim v^4$$



Raising Ion Temperature from 2 to 10 keV causes Gamow peak energy ions to hit the wall in a 100 μm pocket of burning DT plasma at 5 g/cm³





Yes, Ion mean free paths are really that long!

		T (keV)	
		2.0	10.0
ε_{ave}	(1.5 T)	3.0	15.0
ε_{90}	(1.7 T)	6.3	31.3
ε_{90}	(2.38 T)	11.3	56.6
ε_{Gp}	$(\varepsilon_G/4T)^{1/3}$ T	10.6	30.9
ε_G	1183 keV	for D(T,n)^{4}He	

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(From Molvig *et al.*, LA-UR-12-25890)

Kinetic local loss model illustrates enhanced tail-ion losses at high energies

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Review Molvig *et al.* local loss model for tail-ions colliding with background Maxwellian*:

Planar tail-ion kinetic equation: $$\frac{\partial f_i}{\partial t} + v\mu\frac{\partial f_i}{\partial z} = \frac{1}{2}\nu_{ii}^{\mu}\frac{v_{Ti}^3}{v^3}\frac{\partial}{\partial \mu}\left(1-\mu^2\right)\frac{\partial f_i}{\partial \mu} + \mathcal{C}_{ii}^{E}\left(f_i, f_i\right)$$

$^*\mu = \cos\theta$ (wall-directed pitch-angle)

Advection + pitch-angle scattering act diffusively (spatially) on short timescales

$$\approx \mathcal{D}\frac{\partial^2 f_i}{\partial z^2} \approx \frac{\mathcal{D}}{L^2} f_i$$ (local loss approximation)

$$\mathcal{D} \approx \frac{\Delta z^2}{\Delta t} \approx \lambda_i^2 \nu_{ii}^{\mu} \approx \frac{v_{Ti}^2}{3\nu_{ii}^{\mu}}\varepsilon_k^{5/2}$$ (strong energy scaling)

$^*\varepsilon_k = m_i v^2/2T_i$

Local loss model:

$$\frac{\partial f_i}{\partial t} = \frac{1}{\varepsilon_k^{1/2}}\frac{\partial}{\partial \varepsilon_k}\left[f_i + \frac{\partial}{\partial \varepsilon_k} f_i\right] - N_K^2 \varepsilon_k^{5/2} f_i$$

$$N_K^2 = \frac{1}{3}\frac{v_{Ti}^2}{L^2}\frac{1}{\nu_{ii}^{\mu}\nu_{ii}^{E}} \approx \frac{\lambda_i^2}{L^2}$$ (Knudsen number)

Steady-state asymptotic solution:

$$f_K \approx \frac{2}{\sqrt{\pi + N_K \varepsilon_k^{3/2}}}\exp\left(-\varepsilon_k - \frac{2}{5}N_K \varepsilon_k^{5/2}\right)$$

Enhanced tail-ion depletion! Negative impact on reactivity

Maxwellian at low energies, consistent w/ assumptions

* K. Molvig *et al.*, Phys. Rev. Lett. 109, 095001 (2012).

Kinetic local loss model illustrates enhanced tail-ion losses at high energies

- Fuel performance determined primarily by two parameters, N_K and T_i:
  - N_K determines tail shape
  - T_i determines location of Gamow peak energy
- Analytic solution [1] indicates significant reduction of fuel reactivity even for modest (>0.01) N_K .
- Calculations by Wilks *et al*. [2] as well as our own calculations indicate that original model may have *overestimated* the reactivity reduction factor.

"Equation (8) is a simplified version of a more accurate Padé approximant, used here for its physical clarity and to avoid ***underestimating*** the ion-loss effect." [1]

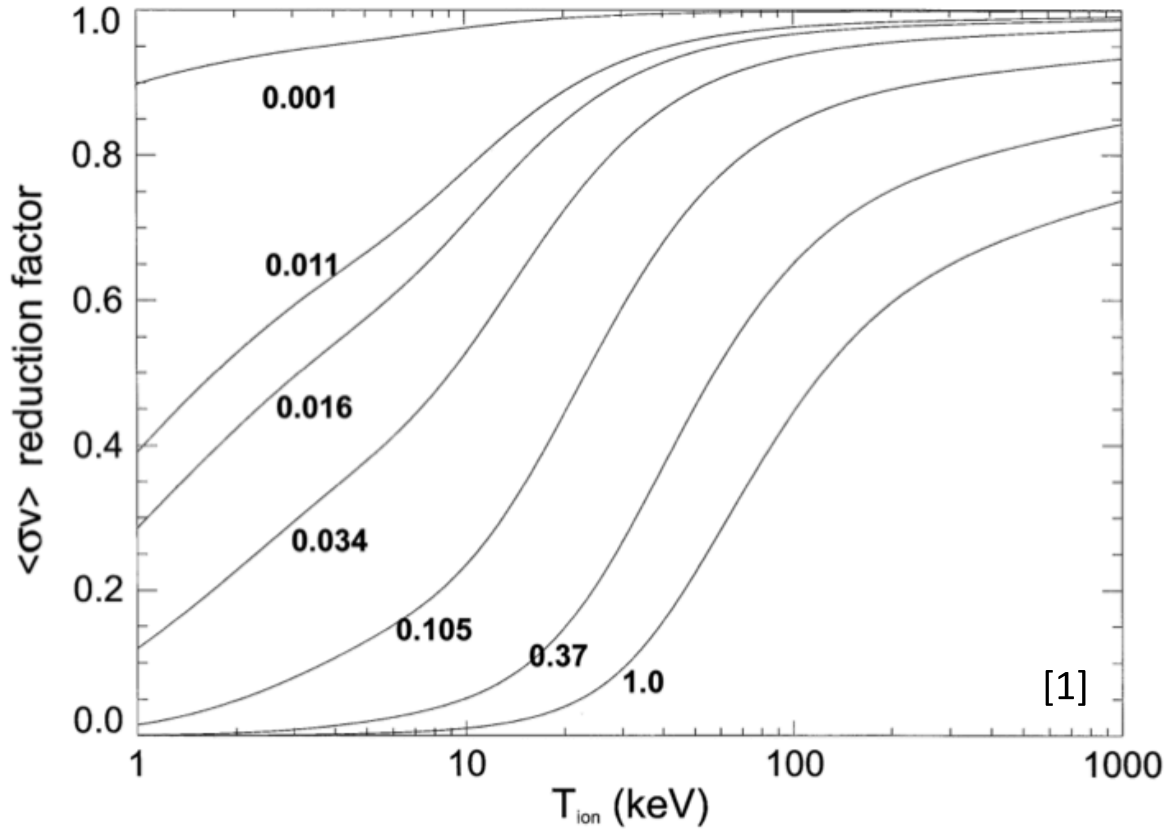


FIG. 1. Reduction ratio vs $T_{\rm ion}$ for values of N_K as shown.

$$N_K^2 = \frac{1}{3}\frac{v_{Ti}^2}{L^2}\frac{1}{\nu_{ii}^{\mu}\nu_{ii}^{E}} \approx \frac{\lambda_i^2}{L^2}$$

[1] K. Molvig *et al.*, PRL 109, 095001 (2012). [2] S. Wilks *et al.*, *Kinetic Effects in ICF Hot Spot Ignition*, LLNL-PRES-594912 (2012)

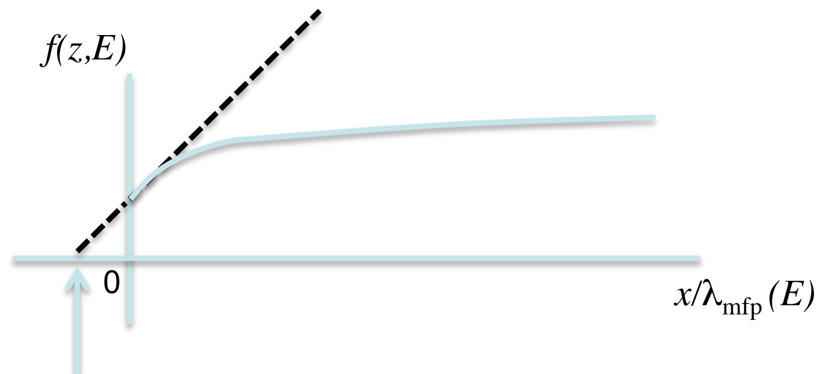
Substantial improvements to Molvig's Knudsen local loss model have been made[1]

- More accurate Padé solution to kinetic equation
- Full diffusion solution in place of local loss model
- Actual D and T ions used, not representative mass 2.5 ions
- Asymptotic matching to free-streaming half-space problem ("Coulomb-Milne" boundary condition) at edge of cavity

Spherical cavity solution

$$f_{K,\mathrm{sph}}^{CM}(x,\epsilon) \approx f_{\max}(\epsilon_{\mathrm{ref}}) \sum_{n=1}^{\infty} 2(-1)^{n+1} \frac{\sin\left\{n\pi x \Big/ \left[(N_K^{\mathrm{cav}})^{-1} + (q\epsilon^2/\alpha_i)\right]\right\}}{\left\{n\pi x \Big/ \left[(N_K^{\mathrm{cav}})^{-1} + (q\epsilon^2/\alpha_i)\right]\right\}} \frac{h_n(\epsilon)}{h_n(\epsilon_{\mathrm{ref}})},$$

$$h_n(\epsilon) \approx \frac{2\pi^{-1/2}}{\sqrt{1+\kappa_n \epsilon^{3/2}}} \exp\left[-\left(\frac{\epsilon + \frac{4}{5}\kappa_n \epsilon^{5/2} + \frac{8}{25}\kappa_n^2 \epsilon^4}{1+\frac{4}{5}\kappa_n \epsilon^{3/2}}\right)\right]$$



q = -0.615 ("extrapolation length")

[1] B. J. Albright, Kim Molvig, C.-K. Huang, A. N. Simakov, E. S. Dodd, N. M. Hoffman, G. Kagan, and P. F. Schmit, "Revised Knudsen-layer reduction of fusion reactivity," Phys. Plasmas (in press).

Including these effects lessens the reactivity reduction

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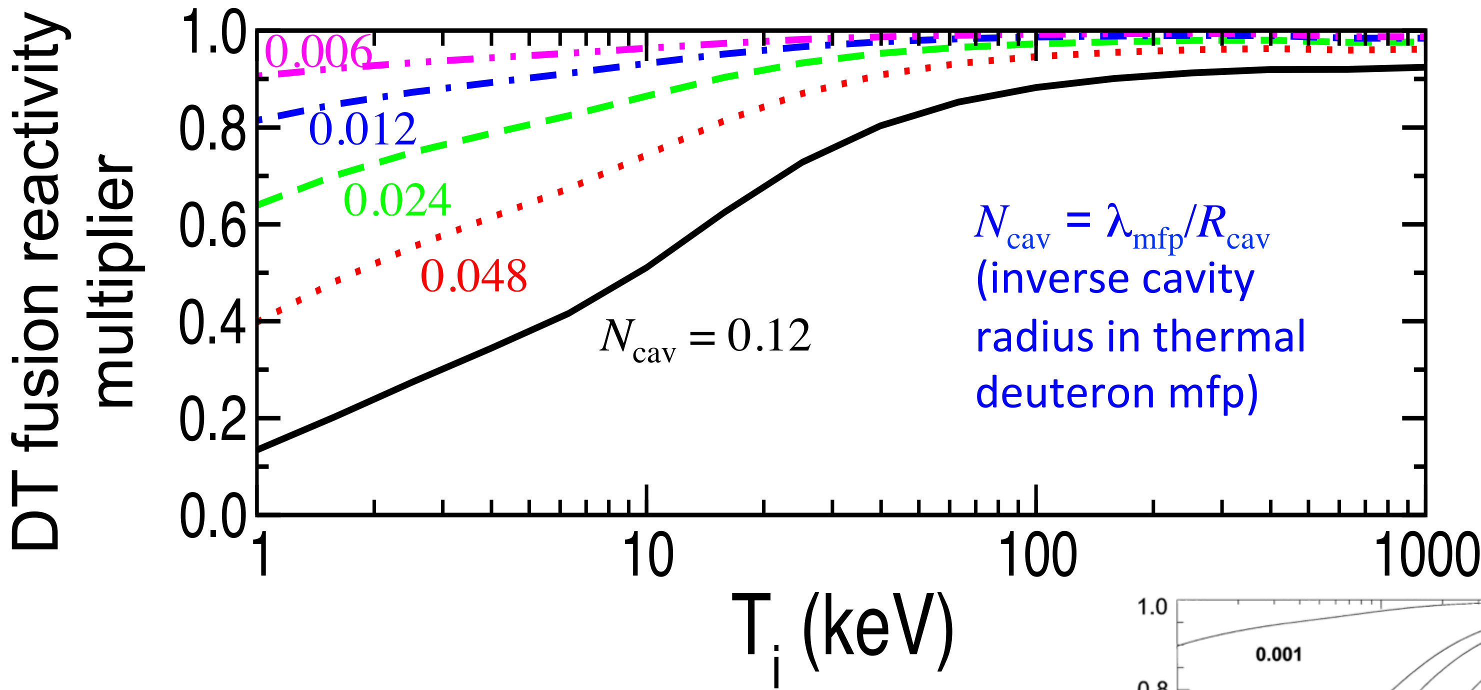

Original local loss WKB model*

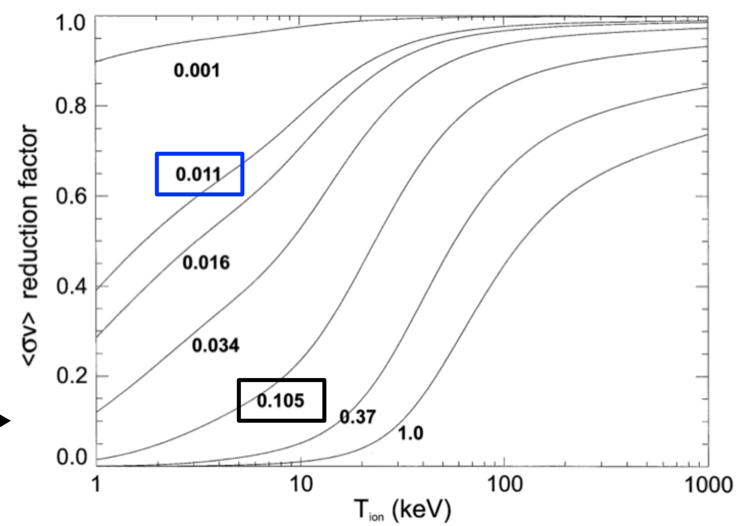

FIG. 1. Reduction ratio vs T_{ion} for values of N_K as shown.

* K. Molvig *et al.*, Phys. Rev. Lett. 109, 095001 (2012).

SNL is working toward the evaluation of a new Magnetized Liner Inertial Fusion (MagLIF)* concept

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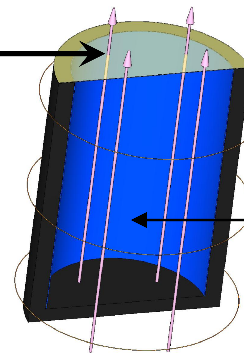
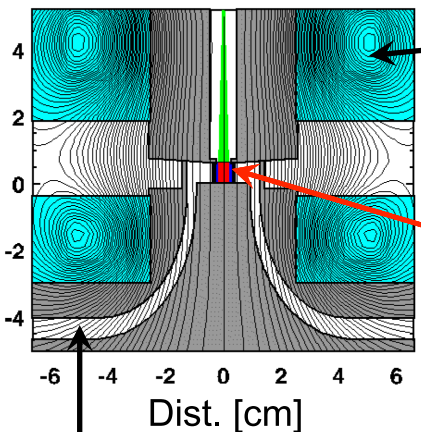
1. A 10–50 T axial magnetic field (B_z) is applied to inhibit thermal conduction losses and to enhance alpha particle deposition



Liner (Li, Be, or Al)

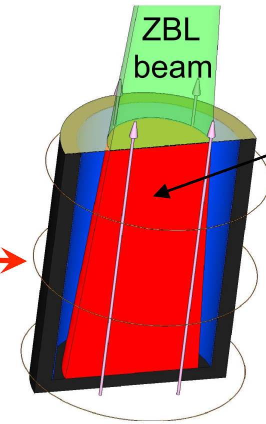
Cold DD or DT gas (fuel)



B_z coils

Z power flow (A-K gap)

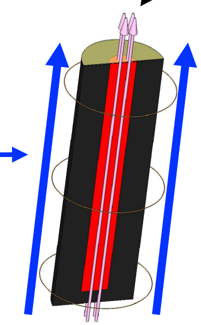
2. ZBL preheats the fuel to ~100–250 eV to reduce the required compression to CR≈20–30



ZBL preheated fuel

Compressed B_z field

3. Z drive current and B_θ field implode liner (via z-pinch) at 50–100 km/s, compressing the fuel and B_z field by factors of nearly 10^3 in ~100 ns

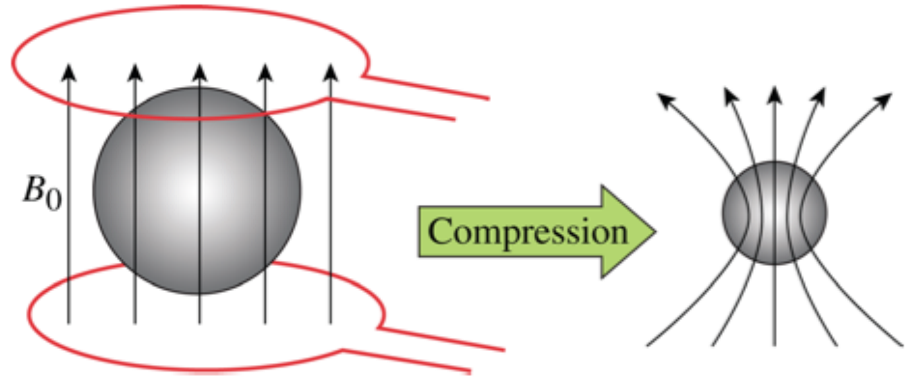


With DT fuel, simulations indicate scientific breakeven may be possible on Z (fusion energy out = energy deposited in fusion fuel)

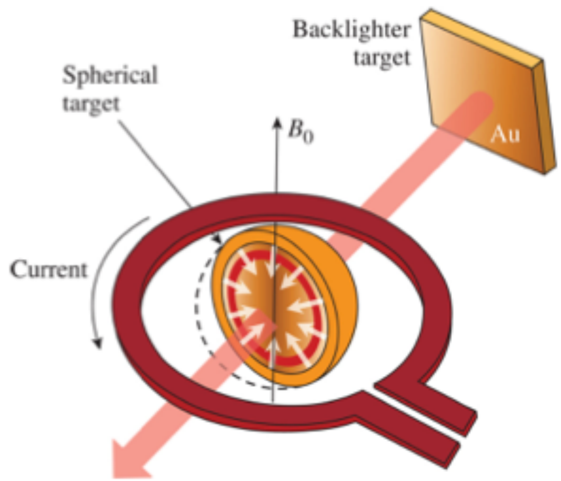
* S. A. Slutz *et al.*, PoP 17, 056303 (2010). S. A. Slutz and R. A. Vesey, PRL 108, 025003 (2012).

Recent experiments on OMEGA already have shown benefit of applied B-fields on temps/yields*

Spherical target geometry not optimum for realizing maximum performance gain with a solenoidal B-field, due to field line intersections with cold pusher.



Nevertheless, modest gains in measured ion temperature (15%) and yield (30%) are reported for magnetized (80 kG seed field) direct-drive DD shots. Compressed B-fields near 40 MG.



MagLIF's cylindrical geometry, higher predicted stagnation B-fields and lower hot spot densities suggest greater relative performance enhancement with ***B***.

Additional benefits probable for the tail-ion kinetics.

* P. Y. Chang *et al.*, PRL 107, 035006 (2011). M. Hohenberger *et al.*, Phys. Plasmas 19, 056306 (2012)

Heuristic model indicates B should mitigate Knudsen

Planar tail-ion kinetic equation: $$\frac{\partial f_i}{\partial t} + v\mu \frac{\partial f_i}{\partial z} = \underbrace{\frac{1}{2}\nu_{ii}^{\mu} \frac{v_{Ti}^3}{v^3} \frac{\partial}{\partial \mu}\left(1-\mu^2\right)\frac{\partial f_i}{\partial \mu}}_{\approx \mathcal{D}\frac{\partial^2 f_i}{\partial z^2} \approx \frac{\mathcal{D}}{L^2} f_i} + \mathcal{C}_{ii}^{E}\left(f_i, f_i\right)$$

$^*\mu = \cos\theta$ (wall-directed pitch-angle)

(diffusion → local loss)

"Local loss" model:

$$\mathcal{D} \approx \lambda_i^2 \nu_{ii}^{\mu} \propto \varepsilon_k^{5/2}$$

$$\frac{\partial f_i}{\partial t} = \frac{1}{\varepsilon_k^{1/2}} \frac{\partial}{\partial \varepsilon_k}\left[f_i + \frac{\partial}{\partial \varepsilon_k} f_i\right] - N_K^2 \varepsilon_k^{5/2} f_i$$

$^*N_K \approx \lambda_i / L$

$$f_K \approx \frac{2}{\sqrt{\pi + N_K \varepsilon_k^{3/2}}} \exp\left(-\varepsilon_k - \frac{2}{5} N_K \varepsilon_k^{5/2}\right)$$

Enhanced tail depletion

"Magnetized local loss" model:

$$\mathcal{D} \approx \rho_{\mathrm{L}}^2 \nu_{ii}^{\mu} \propto \varepsilon_k^{-1/2}$$

$$\frac{\partial f_i}{\partial t} = \frac{1}{\varepsilon_k^{1/2}} \frac{\partial}{\partial \varepsilon_k}\left[f_i + \frac{\partial}{\partial \varepsilon_k} f_i\right] - \frac{N_B^2}{\varepsilon_k^{1/2}} f_i$$

$^*N_B \approx \rho_{\mathrm{L}} / L$

$$f_B \propto \exp\left[-(1+\delta)\varepsilon_k\right]$$

No enhanced tail depletion

Suggests that *preferential* loss of high energy ions suppressed by magnetic field, mitigating Knudsen mechanism perpendicular to ***B***. **But what *B* do we need?**

Estimate of threshold to mitigate Knudsen depletion*

Objective: want *most reactive* ions to undergo magnetized (classical) diffusion to suppress preferential high energy losses. This requires:

- Fusing ions execute Larmor orbits between scattering events (*magnetization*)
- Larmor orbits smaller than fuel dimensions (*confinement*)

By balancing Maxwellian particle density ($f_1 \sim \exp[-\epsilon/T]$) and Gamow tunneling factor ($f_2 \sim \exp[-(\epsilon_{\mathrm{G}}/\epsilon)^{1/2}]$), peak reactivity occurs at Gamow peak energy, $\epsilon_{\mathrm{Gp}} = \xi T$:

- $\epsilon_{\mathrm{G}} = (\pi \alpha_f Z_1 Z_2)^2 2 m_r c^2 = 986.1 Z_1^2 Z_2^2 A_r \ \mathrm{keV}$ (Gamow energy)
- $\xi = 6.2696\,(Z_1 Z_2)^{2/3} \left(\dfrac{A_1 A_2}{A_1 + A_2}\right)^{1/3} [T\ (\mathrm{keV})]^{-1/3}$ (Gamow factor)

$\xi \approx 3.1\ (8\ \mathrm{keV\ DD})$

$\xi \approx 4.0\ (4\ \mathrm{keV\ DD})$

Threshold conditions to mitigate Knudsen-depleted reactivities should constrain the motion of ions near the Gamow peak energy.

Transform between Gamow peak and thermal Knudsen numbers:

$$N_{Kp} = \xi^2 N_K \qquad \left(N_K^2 = \frac{1}{3}\frac{v_{Ti}^2}{L^2}\frac{1}{\nu_{ii}^{\mu}\nu_{ii}^{E}} \approx \frac{\lambda_i^2}{L^2}\right)$$

$$N_{Bp} = \xi^{1/2} N_B \qquad \left(N_B^2 = \frac{v_{Ti}^2}{\omega_{ci}^2}\frac{\nu_{ii}^{\mu}}{\nu_{ii}^{E}}\frac{1}{L^2} \approx \frac{\rho_{\mathrm{L}}^2}{L^2}\right)$$

* P. F. Schmit, Kim Molving, and C. W. Nakhleh, *Physics of Plasmas* **20**, 112705 (2013).

Estimate of threshold to mitigate Knudsen depletion*

Threshold criteria (expressed in dimensionless parameters related to bulk plasma):

1) **Magnetization** of ions at Gamow peak energy:

$$\left(\frac{\nu_{ii}^{\mu}}{\omega_{ci}}\right)_p \approx \frac{N_{Bp}}{N_{Kp}} \lesssim 1 \quad \Leftrightarrow \quad \boxed{\frac{N_B}{N_K} \lesssim \xi^{3/2}}$$

2) **Confinement** of ions at Gamow peak energy:

$$\left(\frac{\rho_{\mathrm{L}}}{L}\right)_p \approx N_{Bp} \ll 1 \quad \Leftrightarrow \quad \boxed{N_B \ll \frac{1}{\xi^{1/2}}}$$

$$N_K \approx \frac{\lambda_i}{L}$$

$$N_B \approx \frac{\rho_{\mathrm{L}}}{L}$$

$$\xi \sim \mathcal{O}(1)$$

Condition (1) better satisfied at large N_K, which normally would produce significant depletion $(N_K \sim \mathcal{O}(1))$.

Important: we expect Maxwellian reactivities to be restored with magnetic fields too weak to magnetize thermal ions $(N_B/N_K < 1)$.

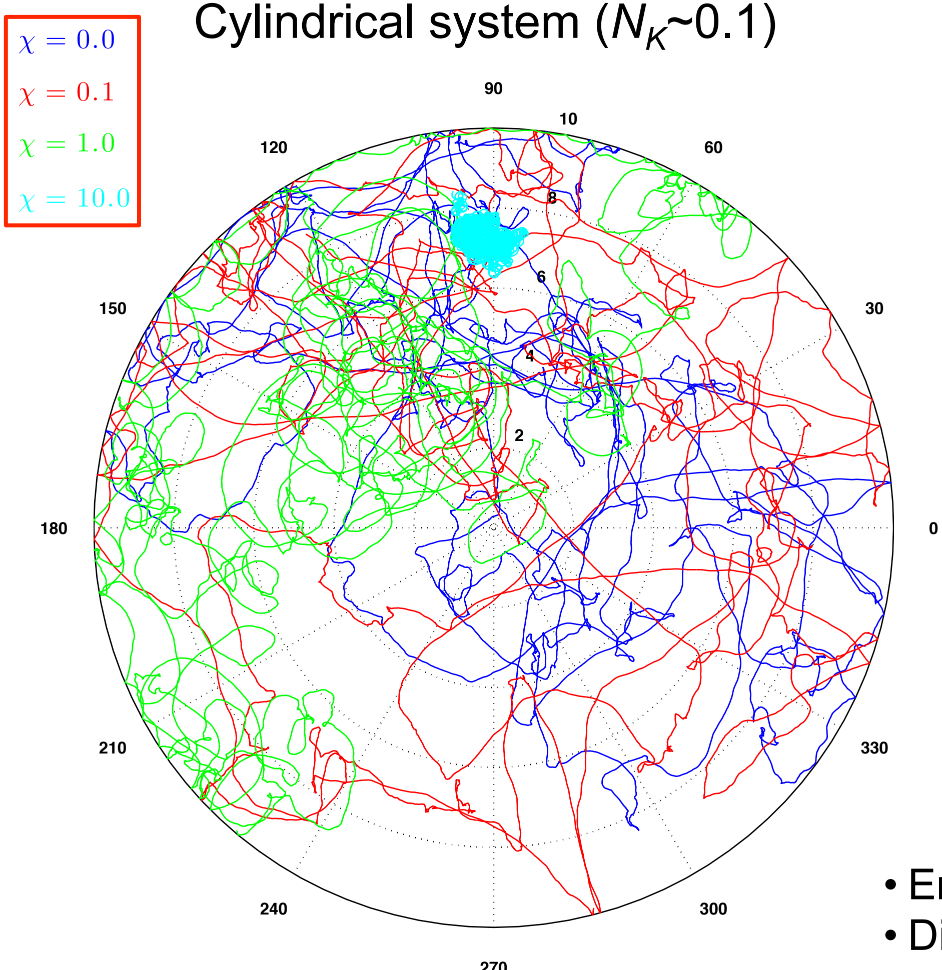
Even relatively modest magnetic fields could offer performance gains by suppressing Knudsen layer formation.

* P. F. Schmit, Kim Molving, and C. W. Nakhleh, *Physics of Plasmas* **20**, 112705 (2013).

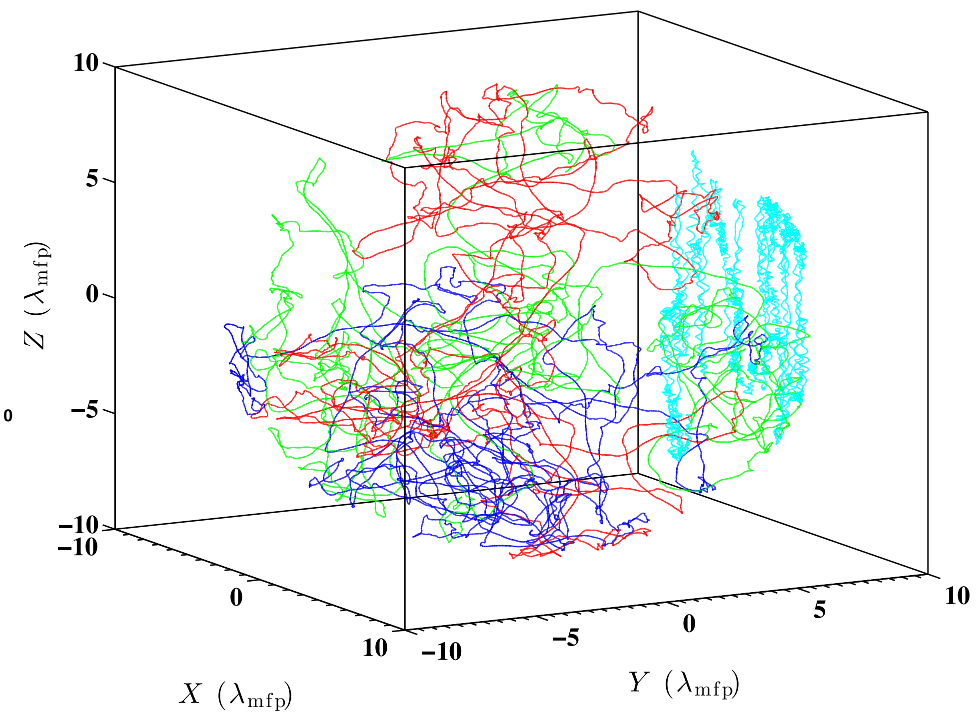
Simulations of tail-ion transport with B-fields*

New code solves complete tail-ion kinetic equation via equivalent set of stochastic differential orbit equations for ensemble of test particles.

Cylindrical system (N_K~0.1)



Spherical system (N_K~0.1)



- Ergodicity evident, especially with weaker ***B***
- Direct signature of weak magnetization difficult to detect from trajectories, but appears clearly in ensemble statistics

*χ defined alternatively as : $\chi \equiv \frac{\omega_{ca}}{\nu_{\mu a}} \approx \frac{N_K}{N_B}$

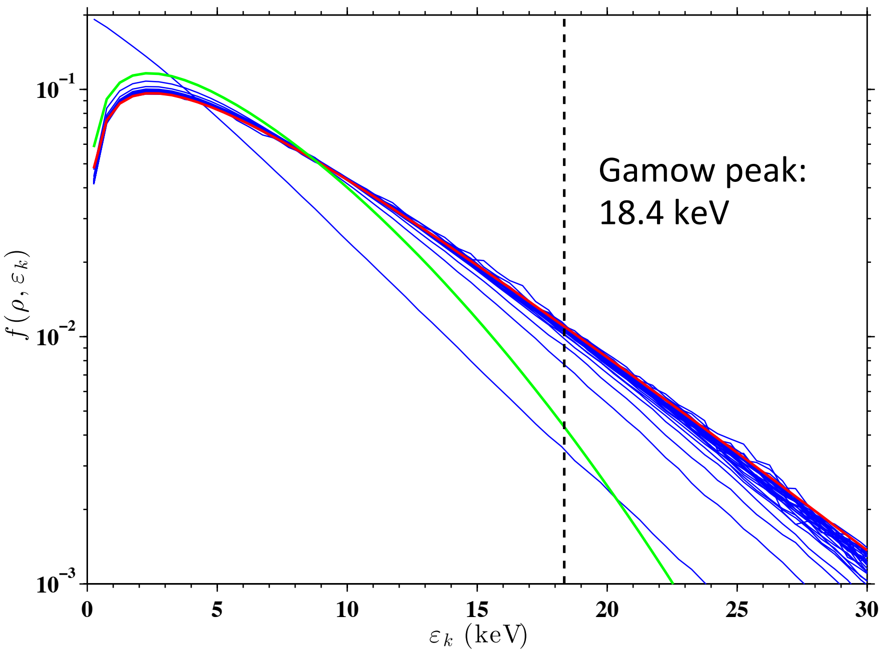
* P. F. Schmit, Kim Molving, and C. W. Nakhleh, *Physics of Plasmas* **20**, 112705 (2013).

Simulations of tail-ion transport with B-fields

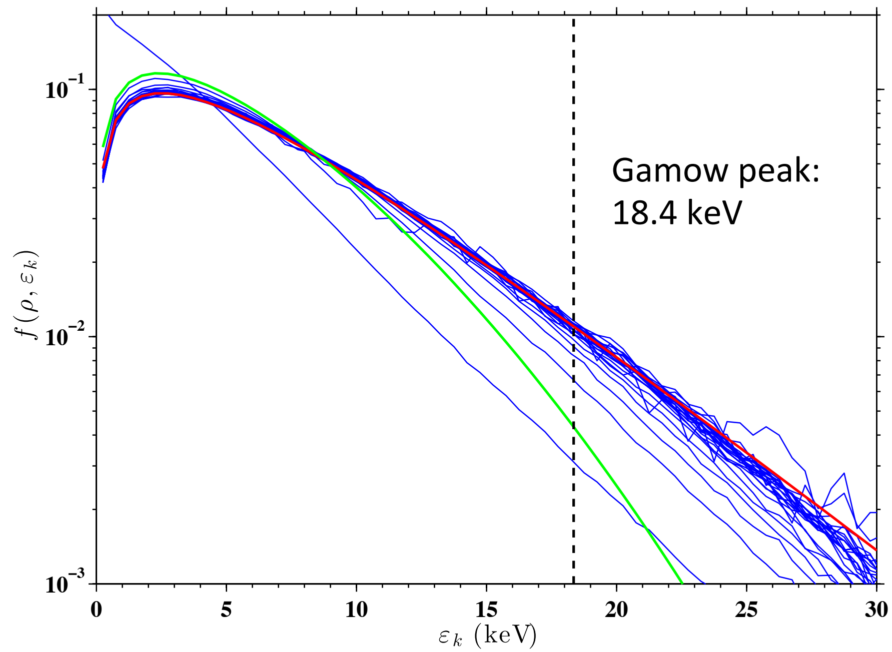
Distribution function at several radial position: 5 keV, 1 g/cc, DD plasma, N_K ~ 0.1, 10 eV wall

$\chi = 0$

Cylindrical system



Spherical system



Red line: Maxwellian distribution

Green line: Analytical 1D solution (unmagnetized) [Molvig *et al.*, PRL 109, 095001 (2012)]

- Knudsen depletion stronger in spherical geometry vs. cylindrical geometry
- Analytical model seems to overestimate depletion scaling significantly, particularly for core plasma. Consistent with observations by Wilks *et al.* using particle-in-cell code, Lsp*.
- Ion transport **model assumptions validated**, even very close to cold wall

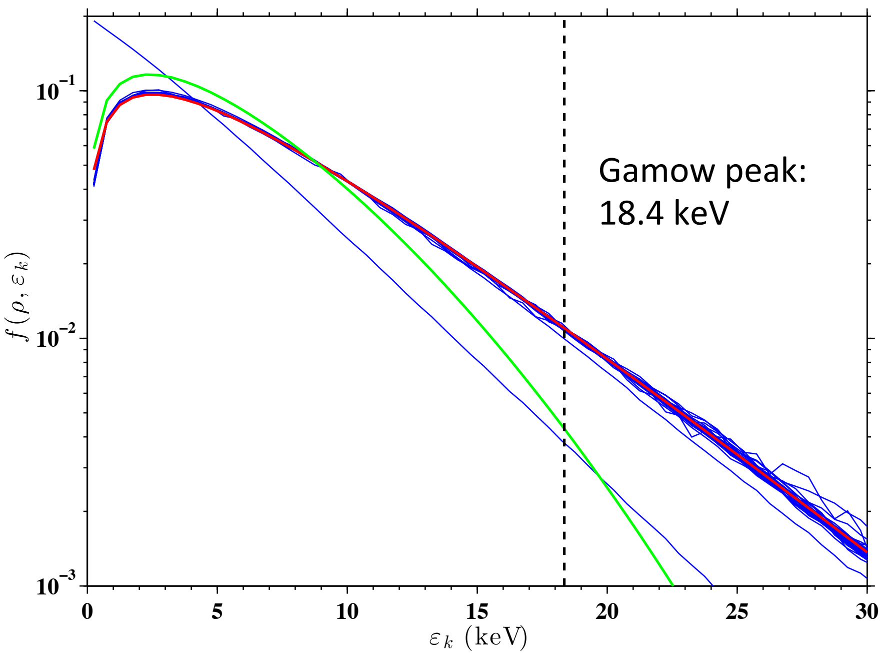
* S. Wilks *et al.*, *Kinetic Effects in ICF Hot Spot Ignition*, LLNL-PRES-594912 (2012).

Simulations of tail-ion transport with B-fields

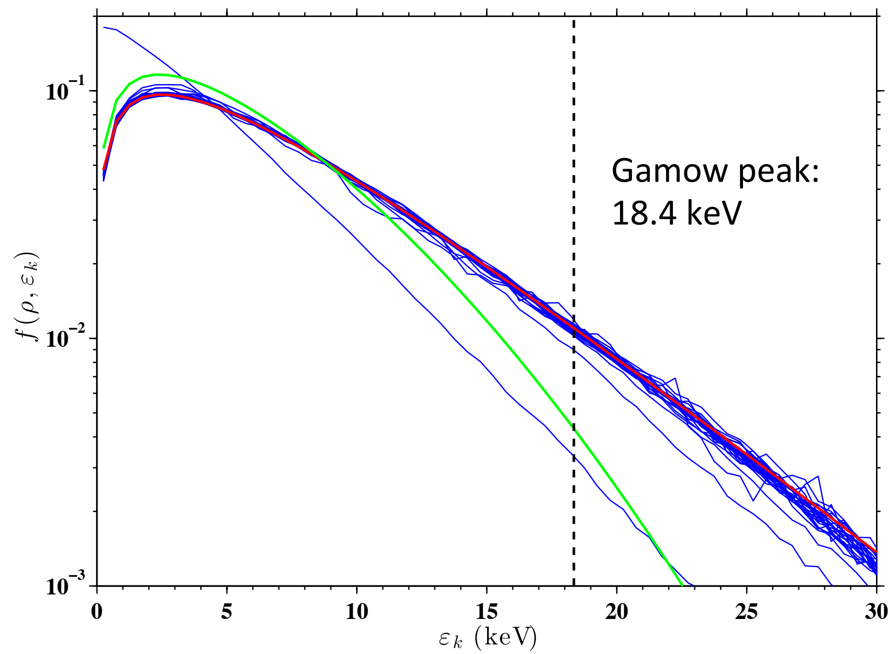
Distribution function at several radial position: 5 keV, 1 g/cc, DD plasma, N_K ~ 0.1, 10 eV wall

$\chi = 5$

Cylindrical system



Spherical system



Red line: Maxwellian distribution

Green line: Analytical 1D solution (unmagnetized) [Molvig *et al.*, PRL 109, 095001 (2012)]

- Magnetization mitigates Knudsen depletion substantially in cylindrical system
- Tail depletion weaker in spherical system, but **not to the same extent** as cylindrical system.
- Similar qualitative differences observed for alpha energy deposition*

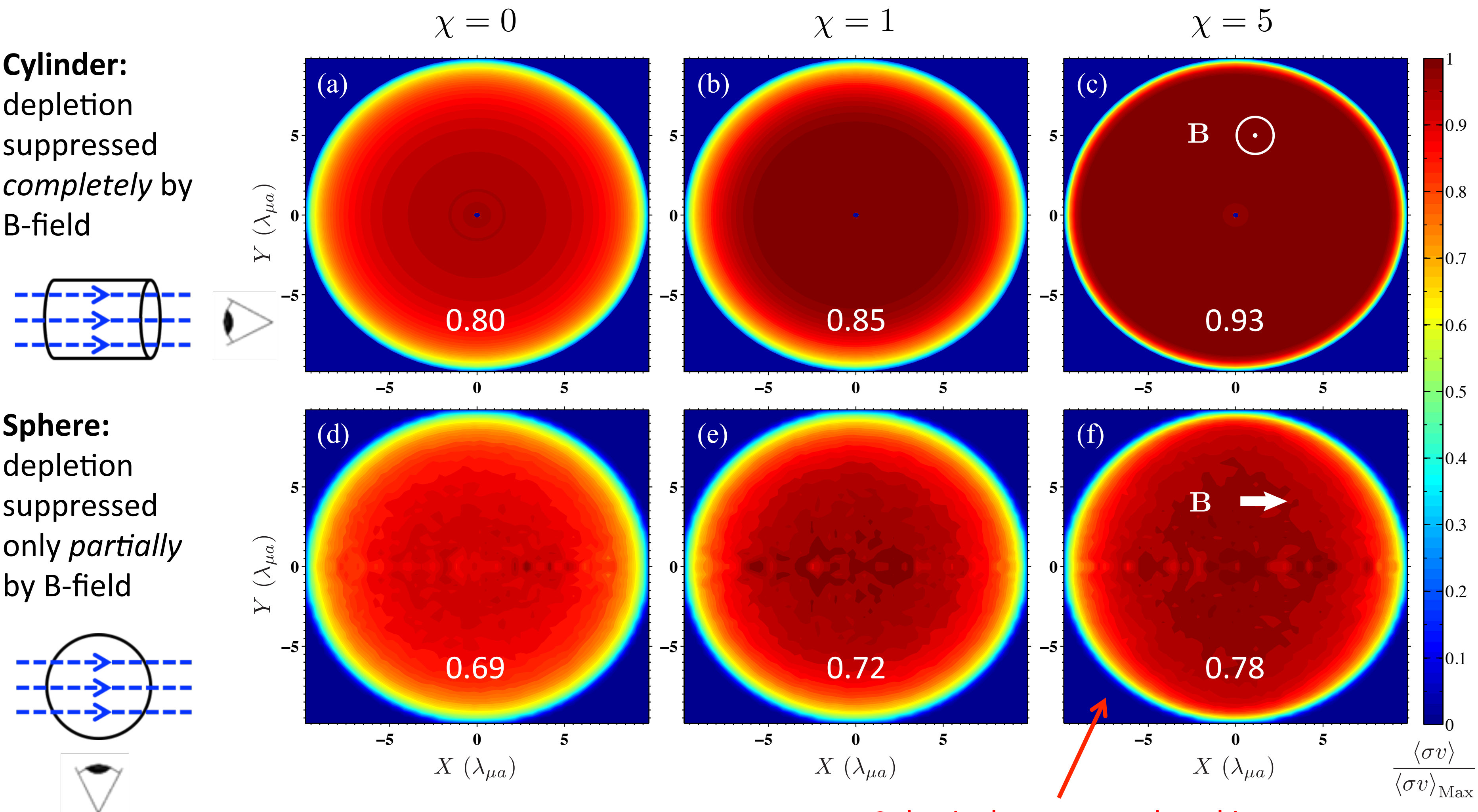
* M. M. Basko *et al*., Nucl. Fusion 40, 59 (2000). S. Yu. Gus'kov *et al*., Sov. J. Quantum Electron. 14, 1062 (1984)

Simulations of tail-ion transport with B-fields

Cylindrical and spherical systems: 5 keV, 1 g/cc, DD plasma, $N_K = 0.1$

Cylinder: depletion suppressed *completely* by B-field

Sphere: depletion suppressed only *partially* by B-field



Spherical symmetry-breaking of reactivity contours (3D→1D)

* P. F. Schmit, Kim Molving, and C. W. Nakhleh, *Physics of Plasmas* **20**, 112705 (2013).

Exploring the dimensionless parameter landscape

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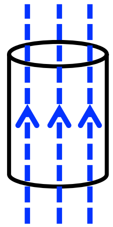
Cylindrical system: 8 keV, 1 g/cc, DD plasma: volume-averaged reactivity reduction

Relevant MagLIF timescales:

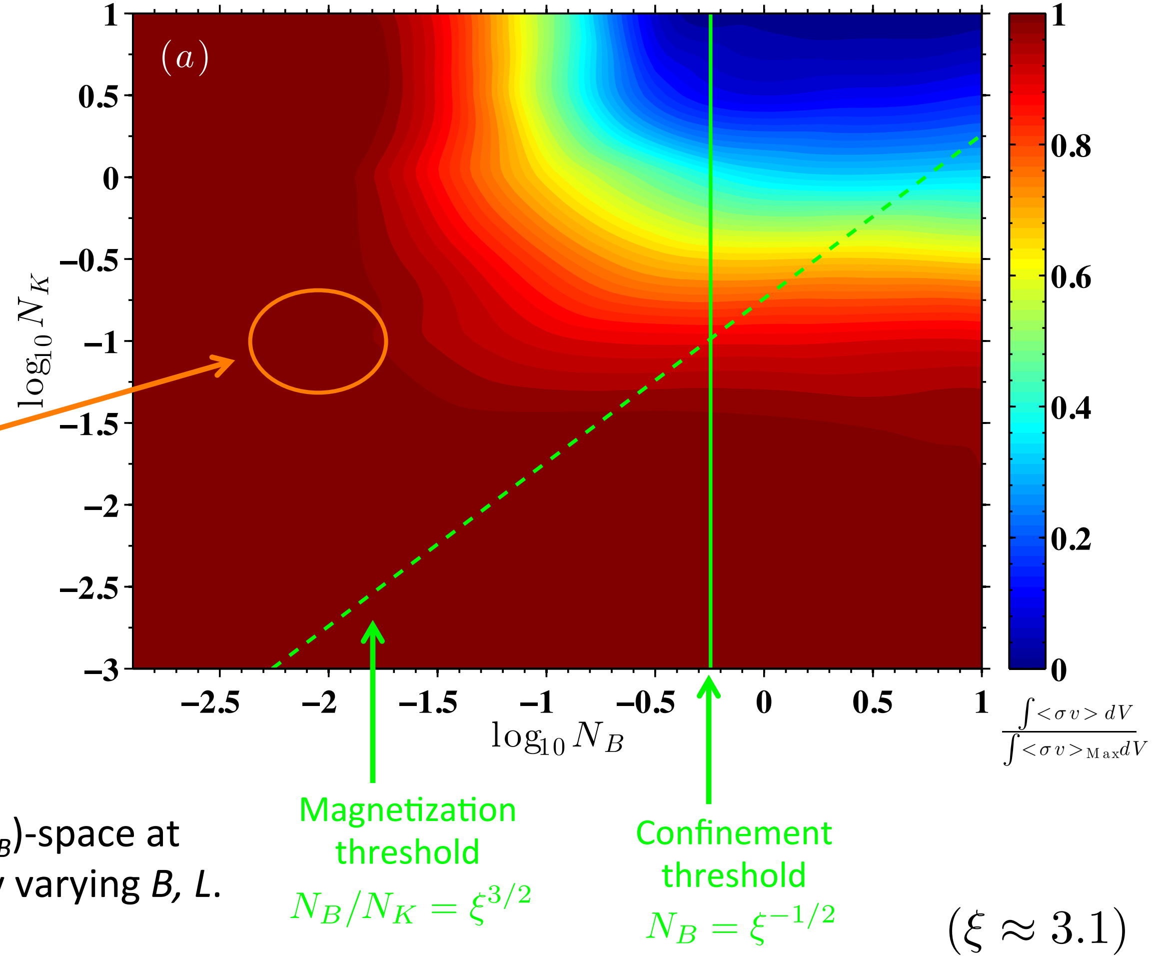
$$\tau_{\rm dd}^{\mu} \sim \mathcal{O}(10 \text{ ps})$$

$$\tau_{\rm eq} \sim \mathcal{O}(100 \text{ ps})$$

$$\tau_{\rm burn} \sim \mathcal{O}(1 \text{ ns})$$



(Knudsen numbers for thermal D ions)



$(\xi \approx 3.1)$

MagLIF point design is well within the plateau regime for fully restored Maxwellian reactivities.

$$N_K \sim \frac{\lambda_{\rm mfp}}{L} \sim \frac{T^2}{nL}$$

$$N_B \sim \frac{\rho_{\rm L}}{L} \sim \frac{T^{1/2}}{BL}$$

$\Rightarrow$ Scan (N_K, N_B)-space at fixed T, n by varying B, L.

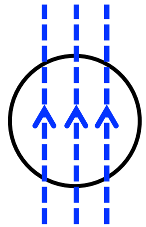
*Hot spot parameters: 8 keV, 0.5 g/cc, 100 MG B-field, 100 micron radius

* S. A. Slutz *et al.*, PoP 17, 056303 (2010). A. B. Sefkow *et al.*, SAND 2012-0876C (2012).

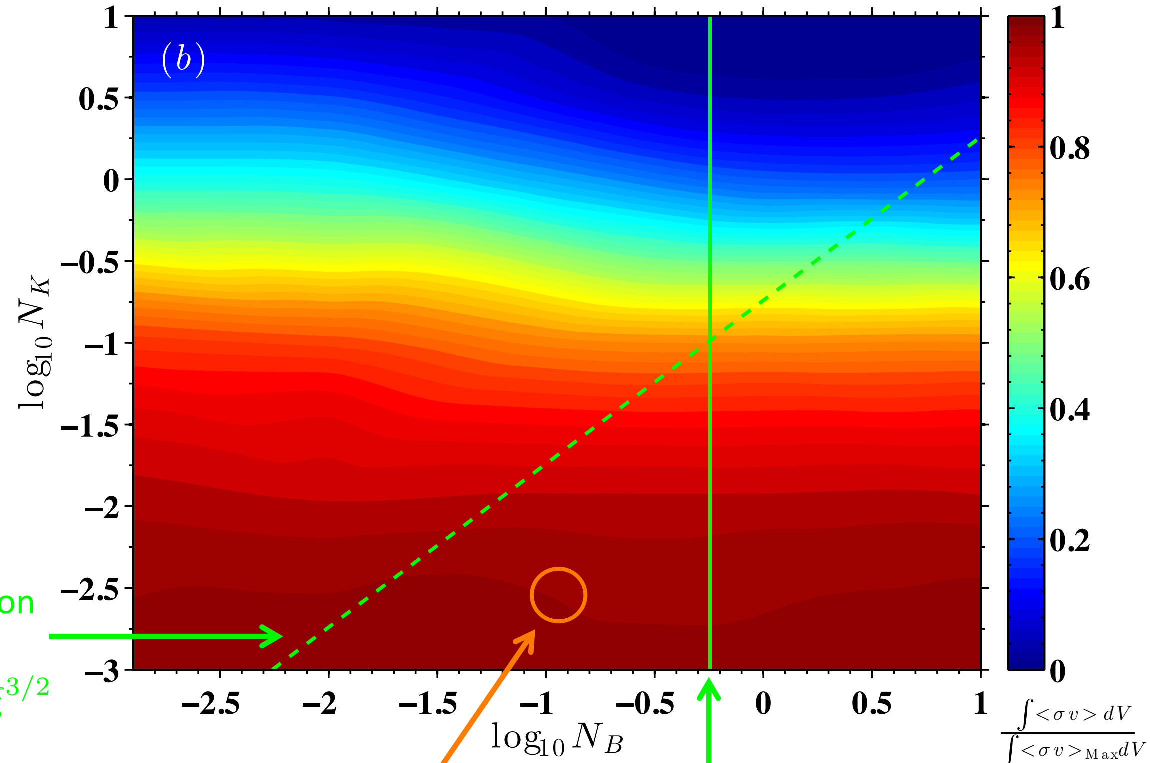
Exploring the dimensionless parameter landscape

Spherical system: 8 keV, 1 g/cc, DD plasma: volume-averaged reactivity reduction

Clearly only a limited benefit provided by magnetic field in spherical geometry. Essentially a transition from 3D to 1D depletion



(Knudsen numbers for thermal D ions)



Relevant OMEGA timescales:

$$\tau_{\mathrm{dd}}^{\mu} \sim \mathcal{O}(0.1\ \mathrm{ps})$$

$$\tau_{\mathrm{eq}} \sim \mathcal{O}(1\ \mathrm{ps})$$

$$\tau_{\mathrm{burn}} \sim \mathcal{O}(100\ \mathrm{ps})$$

Magnetized OMEGA experiments not in enhanced Knudsen regime

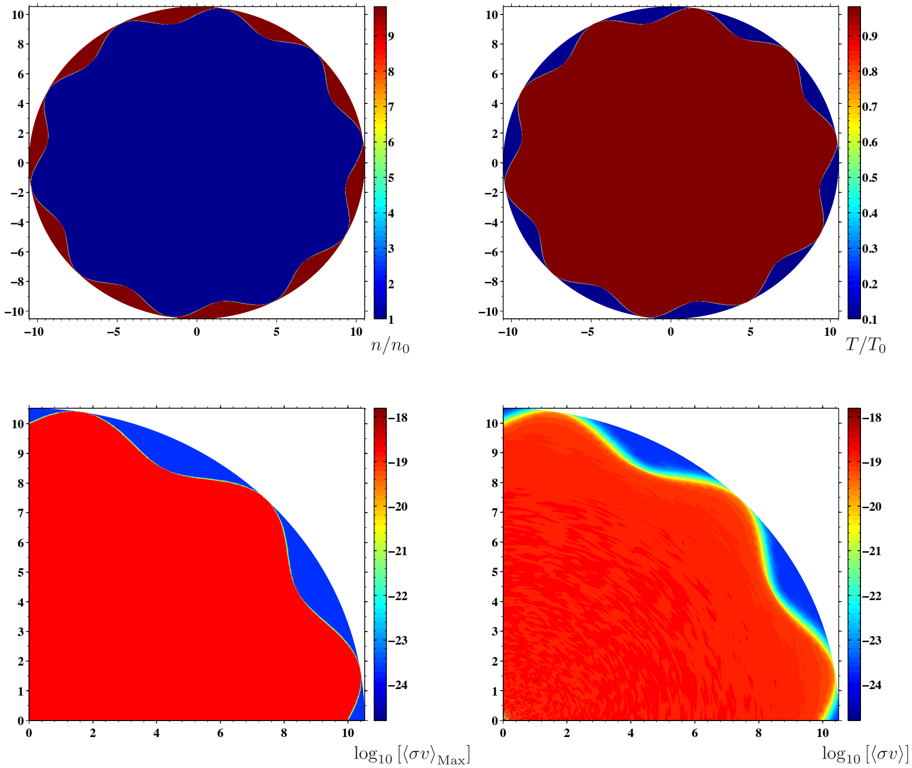
*Hot spot parameters: 3 keV, 30 g/cc, 44 MG B-field, 15 micron radius

* P. Y. Chang *et al.*, PRL 107, 035006 (2011). M. Hohenberger *et al.*, Phys. Plasmas 19, 056306 (2012)

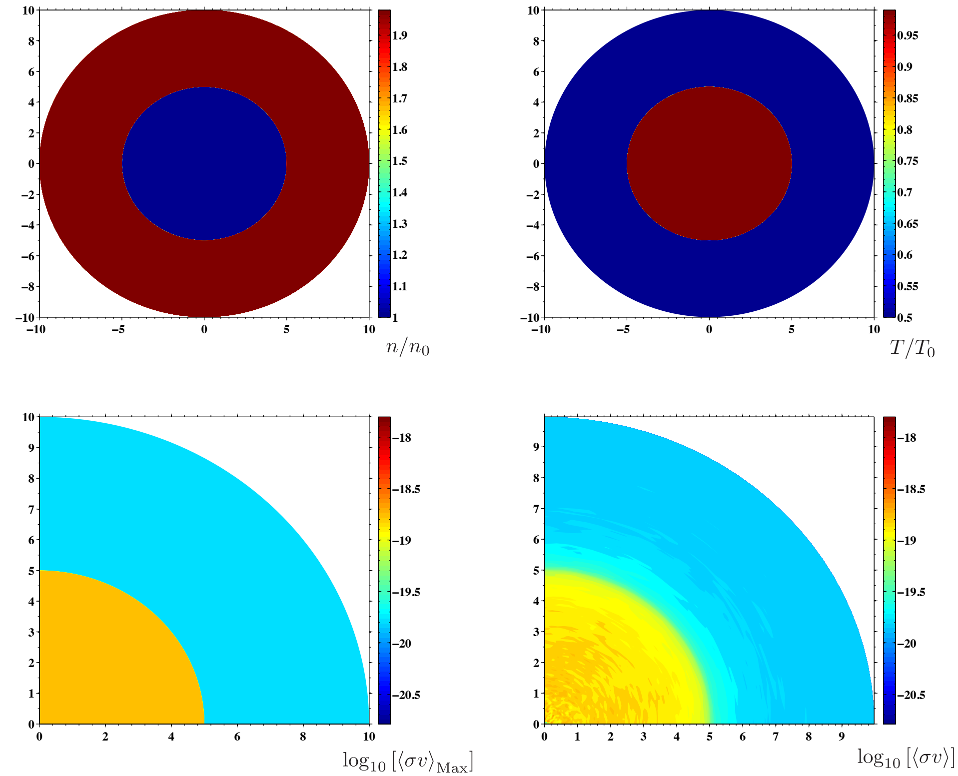
Currently testing new capability to simulate arbitrary multidimensional environments

Some initial tests:

Isobaric cylinder with perturbed "cryo" layer



Isobaric cylinder with smaller gradients



Rad-hydro codes cannot resolve nonthermal features in fuel ion distributions

Conclusions

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- Heuristic local loss model suggests ***B*** suppresses preferential losses of high energy ions.
- Tail-ion kinetic equation derived in hybrid cylindrical-spherical coordinates, determines tail-ion transport in arbitrary inhomogeneous dense plasma
- Numerical code developed to solve kinetic tail-ion equations in both cylindrical and spherical ICF configurations.
- Analytical Knudsen depletion model (Molvig *et al.*, PRL 2012) overestimates the extent and scaling of tail depletion, especially in core plasma.
- Uniform magnetization restores Maxwell-averaged reactivities throughout fuel volume by slowing down tail-ion diffusion rate at high ion energies.
- Strong, uniform ***B*** totally restores reactivity in cylindrical cavity. For spherical cavities, reactivity restoration is finite, but limited.

Conclusions

- Simulations confirm validity of threshold conditions for restoration of depleted fusion reactivity by magnetic fields.

- MagLIF should have significant margin to avoid tail-ion losses due to strong fuel magnetization at stagnation

Backup slides…

Tail-ion kinetic equations and model assumptions*

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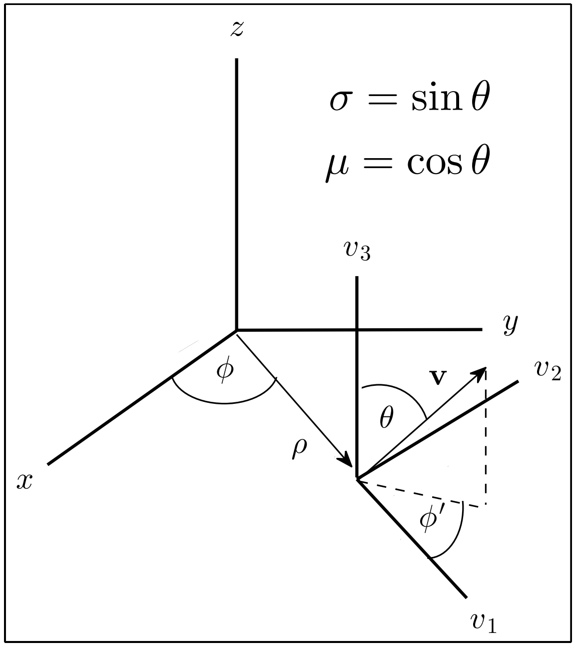
Ion Boltzmann equation: $\frac{\partial f_a}{\partial t} + \mathbf{v}\cdot\nabla f_a + \frac{eZ_a}{m_a}\mathbf{E}\cdot\frac{\partial f_a}{\partial \mathbf{v}} = \mathcal{C}_a(f_a) - \omega_{ca}\left(\mathbf{v}\times\hat{\mathbf{b}}\right)\cdot\frac{\partial f_a}{\partial \mathbf{v}}$

Model assumptions:

- Tail-ions dynamics don't feed back on each other or bulk, so can use linearized test-particle collision operator and prescribe steady-state bulk density, temperature, etc.
- Uniform applied magnetic field: $\mathbf{B} = B\hat{\mathbf{z}}$
- Cylindrically radial ambipolar electric fields: $\mathbf{E} = E(\rho)\hat{\boldsymbol{\rho}}$
- Hybrid cylindrical/spherical (spatial/velocity) coordinates
- **Solving for steady-state tail solutions in stationary bulk plasma state.**

Hybrid coordinate system:



In hybrid coordinates, Fokker-Planck form:

$$\frac{\partial f_a}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\sigma v \rho\cos\phi' f_a\right) - \frac{\partial}{\partial \phi'}\left(\sigma v\frac{\sin\phi'}{\rho} f_a\right)$$
$$+ \frac{1}{v^2}\frac{\partial}{\partial v}\left(\frac{eZ_aE}{m_a}\sigma v^2\cos\phi' f_a\right) - \frac{1}{v}\frac{\partial}{\partial \mu}\left(\frac{eZ_aE}{m_a}\sigma\mu\cos\phi' f_a\right) - \frac{\partial}{\partial \phi'}\left[\left(\frac{eZ_aE}{m_a}\frac{\sin\phi'}{\sigma v} + \omega_{ca}\right) f_a\right]$$
$$= \nu_a^E v_{Ta}^3\frac{1}{v^2}\frac{\partial}{\partial v}\left[D(v)\left(f_a + \frac{T_a}{m_a}\frac{1}{v}\frac{\partial f_a}{\partial v}\right)\right] + \nu_a^\mu\frac{v_{Ta}^3}{v^3}F(v)\left[\frac{1}{2}\left(\frac{\partial}{\partial \mu}\left(1-\mu^2\right)\frac{\partial f_a}{\partial \mu} + \frac{1}{1-\mu^2}\frac{\partial^2 f_a}{\partial \phi'^2}\right)\right]$$

* P. F. Schmit, Kim Molving, and C. W. Nakhleh, *Physics of Plasmas* **20**, 112705 (2013).

Tail-ion kinetic equations and model assumptions*

Further manipulations:

- Convert to dimensionless length, time, velocity, and potential units based on 1 keV, 1 g/cc *reference plasma*.

$$\nu_{\mu 0} \equiv \frac{Z_a^2}{\sqrt{A_a}} \frac{4\pi \rho_{m0} e^4 \langle Z_b^2 \ln \Lambda_{ab} \rangle_0}{m_p^3 v_{T0}^3 \langle A_b \rangle_0} \qquad \text{(1/time)}$$

$$v_{T0} \equiv \sqrt{\frac{2T_0}{m_a}} \qquad \text{(velocity)}$$

$$\lambda_{\mu 0} \equiv \frac{v_{T0}}{\nu_{\mu 0}} \qquad \text{(length)}$$

$$\Phi_0 \equiv \frac{T_0}{e} \qquad \text{(potential)}$$

- Transform velocity magnitude to energy variable: $\varepsilon_k = u^2 \equiv \frac{v^2}{v_{T0}^2}$

- Define new dependent variable, $F_a \equiv (1/2)\rho \varepsilon_k^{1/2} f_a$, such that number of particles in each differential volume element is given by: $dN = d\rho \, d\varepsilon_k \, d\mu \, d\phi' \, F_a$ ⟶ $^*||\mathcal{J}|| = (1/2)\rho\varepsilon_k^{1/2}$

- Cast into canonical Fokker-Planck form with clear *drag* and *diffusion* contributions for each variable

- Yielding…

* P. F. Schmit, Kim Molving, and C. W. Nakhleh, *Physics of Plasmas* **20**, 112705 (2013).

Tail-ion kinetic equations and model assumptions

Sandia National Laboratories

Tail-ion kinetic equation*:

$$\frac{\partial F_a}{\partial t} = -\frac{\partial}{\partial \rho}\mathcal{F}_\rho F_a - \frac{\partial}{\partial \phi'}\mathcal{F}_\phi F_a - \frac{\partial}{\partial \mu}\mathcal{F}_\mu F_a - \frac{\partial}{\partial \varepsilon_k}\mathcal{F}_\varepsilon F_a + \frac{1}{2}\frac{\partial^2}{\partial \phi'^2}\mathcal{D}_{\phi\phi}F_a + \frac{1}{2}\frac{\partial^2}{\partial \mu^2}\mathcal{D}_{\mu\mu}F_a + \frac{1}{2}\frac{\partial^2}{\partial \varepsilon_k^2}\mathcal{D}_{\varepsilon\varepsilon}F_a$$

Drag terms

$$\mathcal{F}_\rho = \sigma\varepsilon_k^{1/2}\cos\phi'$$

$$\mathcal{F}_\phi = \frac{Z_a}{2}\frac{\partial \Phi}{\partial \rho}\frac{\sin\phi'}{\sigma\varepsilon_k^{1/2}} - \sigma\varepsilon_k^{1/2}\frac{\sin\phi'}{\rho} + \chi_a$$

$$\mathcal{F}_\mu = \frac{Z_a}{2}\frac{\partial \Phi}{\partial \rho}\frac{\sigma\mu}{\varepsilon_k^{1/2}}\cos\phi' - \frac{\rho_m \Pi_a}{\varepsilon_k^{3/2}}\mu F(\varepsilon_k)$$

$$\mathcal{F}_\varepsilon = -\left(\frac{2\rho_m\Pi_a}{\varepsilon_k^{1/2}}A_a\left\langle\frac{1}{A_b}\right\rangle\left[D(\varepsilon_k) - T_a D'(\varepsilon_k)\right] + Z_a\frac{\partial \Phi}{\partial \rho}\sigma\varepsilon_k^{1/2}\cos\phi'\right)$$

$$^*\chi_a \equiv \frac{\omega_{ca}}{\nu_{\mu 0}}$$

(magnetic field **only** shows up in gyrophase drag term)

$$^*\Pi_a \equiv \frac{\langle Z_b^2 \ln \Lambda_{ab}\rangle}{\langle Z_b^2 \ln \Lambda_{ab}\rangle_0}$$

Diffusion terms

$$\mathcal{D}_{\phi\phi} = \frac{\rho_m \Pi_a}{\varepsilon_k^{3/2}}F(\varepsilon_k)\frac{1}{1-\mu^2}$$

$$\mathcal{D}_{\mu\mu} = \frac{\rho_m \Pi_a}{\varepsilon_k^{3/2}}F(\varepsilon_k)\left(1-\mu^2\right)$$

$$\mathcal{D}_{\varepsilon\varepsilon} = 4\rho_m\Pi_a T_a A_a\left\langle\frac{1}{A_b}\right\rangle\frac{D(\varepsilon_k)}{\varepsilon_k^{1/2}}$$

$$\frac{d\rho}{dt} = \mathcal{F}_\rho$$

$$\frac{d\phi'}{dt} = \mathcal{F}_\phi + \mathcal{D}_{\phi\phi}^{1/2}\Gamma_1(t)$$

$$\frac{d\mu}{dt} = \mathcal{F}_\mu + \mathcal{D}_{\mu\mu}^{1/2}\Gamma_2(t)$$

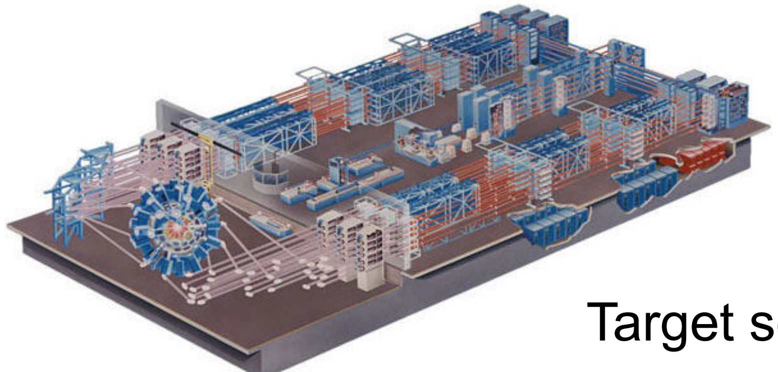
$$\frac{d\varepsilon_k}{dt} = \mathcal{F}_\varepsilon + \mathcal{D}_{\varepsilon\varepsilon}^{1/2}\Gamma_3(t)$$

The formal solution to this equation can be found by solving an equivalent set of single-particle stochastic differential orbital equations for an ensemble of test particles

* Similar advection-only terms appear for the other two spatial coordinates, ϕ and z

Fusion with long mean free path ions (OMEGA)

OMEGA Laser Facility



OMEGA drives DT gas fuel to burning (non-ignited) condition. At peak burn parameters are:

$$T_i \cong 9\ keV$$

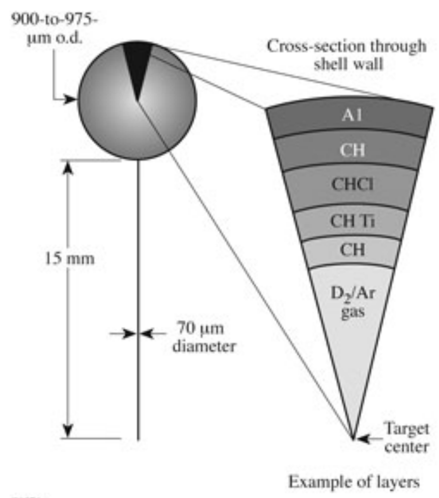
$$\rho \cong 6\ gm / cm^3$$

$$R \cong 25\ \mu m$$

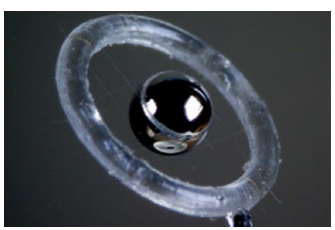
$$N_K = \frac{\lambda_i}{L} \cong 0.083$$

$$(L = R / 2.5)$$

Target schematic



Saturn mounted target



The small Knudsen number expansion that gives hydrodynamics is in question – plasma particularly

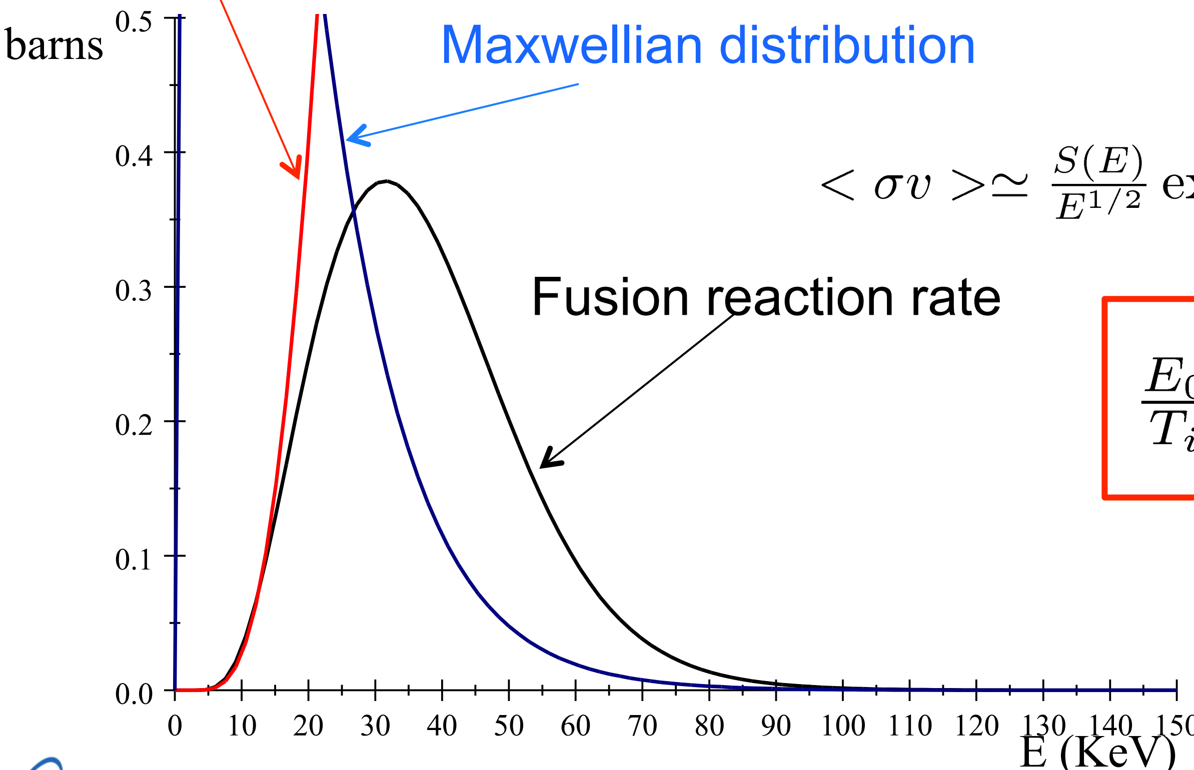
(From Molvig *et al.*, LA-UR-12-25890)

The Gamow Peak in Fusion Reactivity

Fusion cross section

$$\sigma_{fus} = \frac{S(E)}{E} \exp\left(-(E_G/E)^{1/2}\right)$$

$$E_G = 1182\ KeV$$



$$< \sigma v > \simeq \frac{S(E)}{E^{1/2}} \exp\left(-(E_G/E)^{1/2} - E/T_i\right)$$

$$\frac{E_0}{T_i} = \left(\frac{E_G}{4T_i}\right)^{1/3}$$

Gamow peak @ $\frac{E_0}{T_i} \approx 4$

Not the 10 – 20 familiar in astrophysics

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(From Molvig *et al.*, LA-UR-12-25890)

NNSA