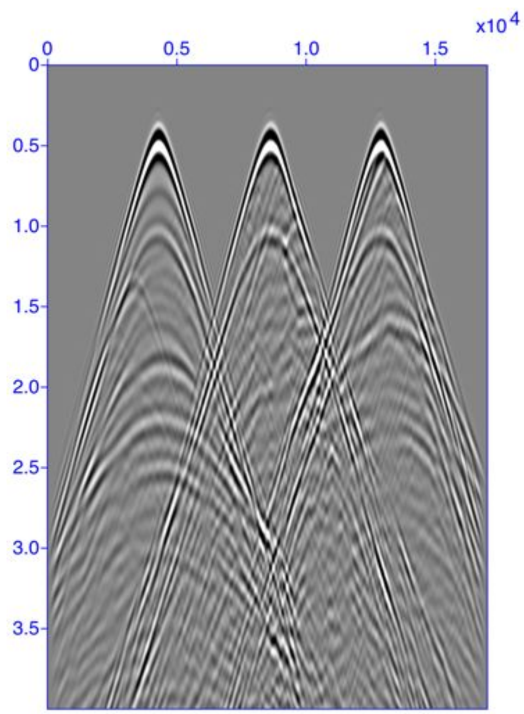
SAND2010-7032P

Discontinuous Galerkin for Wave Propagation and Inversion



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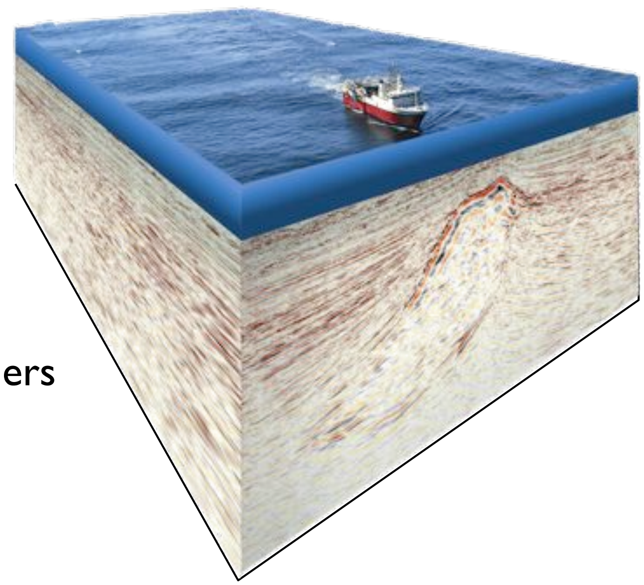
Hans Schwaiger, Bart van Bloemen Waanders
and Joe Young



Presented at:
Oak Ridge National Laboratory
October 11, 2010

Approved by sponsor for release

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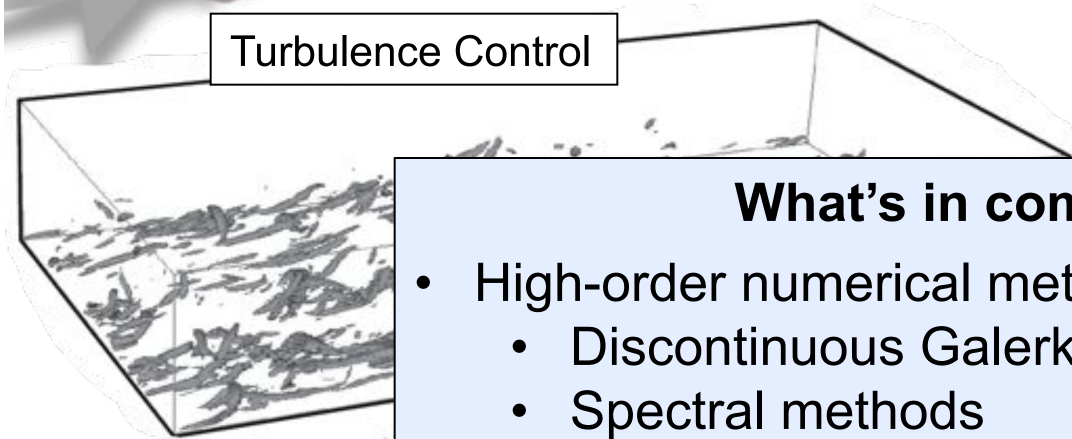
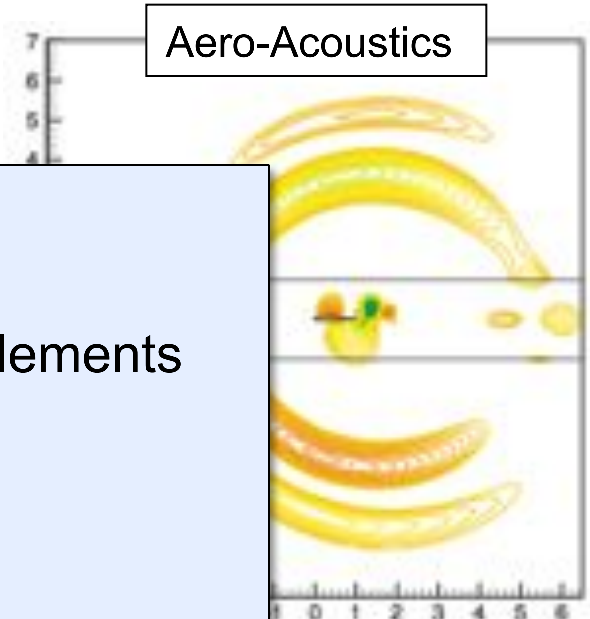
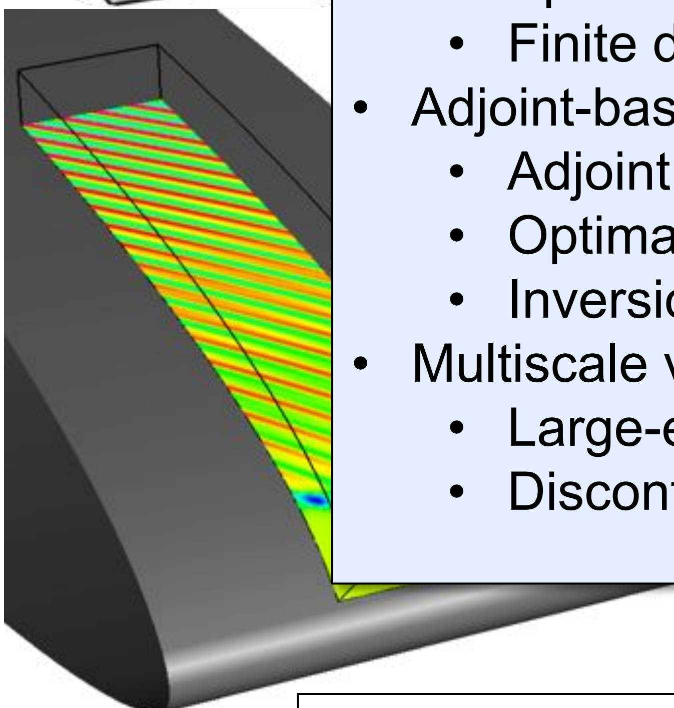
Other things I've done…

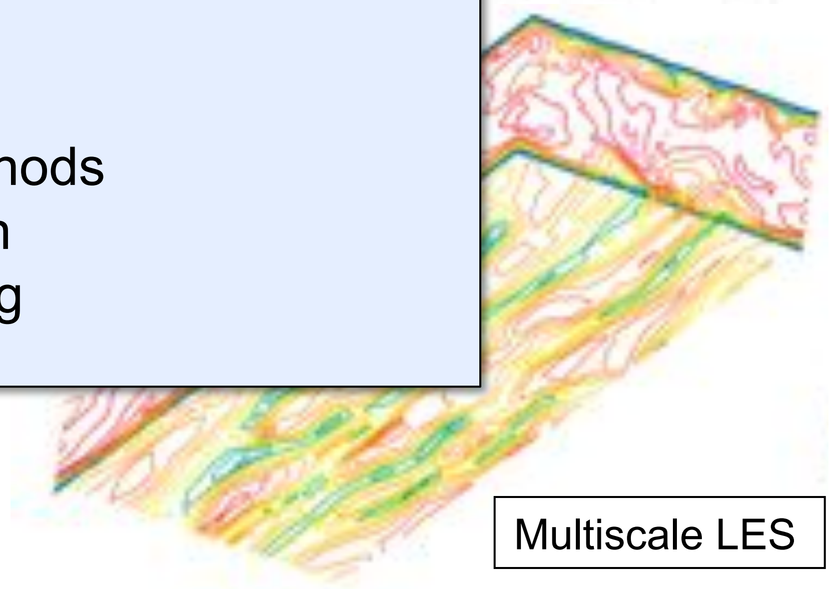




What's in common:

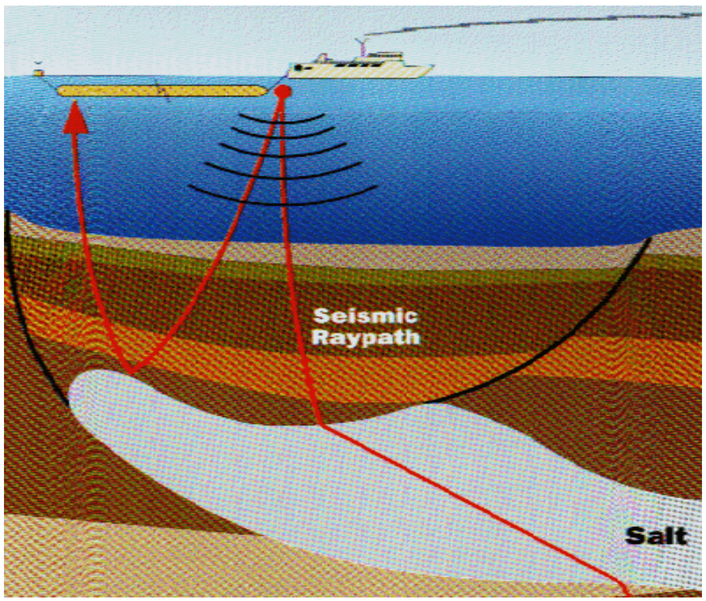
- High-order numerical methods
  - Discontinuous Galerkin finite elements
  - Spectral methods
  - Finite difference
- Adjoint-based methods
  - Adjoint sensitivities
  - Optimal control
  - Inversion
- Multiscale variational methods
  - Large-eddy simulation
  - Discontinuity capturing





Full Waveform Inversion (FWI)

Seismic Experiment ("shot")



- 100,000's shots per survey
- 1000's receivers per shot
- 12 seconds at 1msec sampling

= **4.8 TB of data per survey!**

- **Seismic Imaging (traditional)**
  - Acoustic wave propagation
  - "Manually" inverting for wave speed
  - Primarily utilizes travel time
  - Methods neglect waveform/amplitude
  - ~20 exaFLOP (FD elastic)
- **Seismic Inversion (FWI)**
  - Matching full waveform of the wavelets
  - Acoustic, Elastic, Attenuation, Anisotropic
  - Inverting for wave speeds and density $O(10^8)$ parameters
  - ~20,000 exaFLOP (FD elastic)
- **Interest to Sandia and DOE**
  - Nuclear non-proliferation monitoring
  - Underground structure identification
  - Site characterization (CO_2 sequestration and waste repositories)

Project Goals

- Investigate use of unstructured Discontinuous Galerkin (DG) for seismic modeling & inversion
- Leverage DG capabilities for seismic modeling
  - Unstructured meshes can accurately capture discontinuous material interfaces (faults, ocean bottom, salt structures).
  - Local polynomial refinement enables improved resolution for localized geological features.
- Demonstrate DG for seismic inversion
  - Utilizing above advantages
  - Use Simultaneous Source Inversion (SSI), see Krebs et al. 2009
- Utilize algorithms from Trilinos and Dakota toolkits

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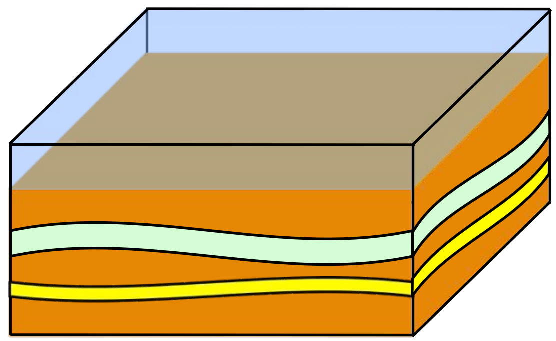
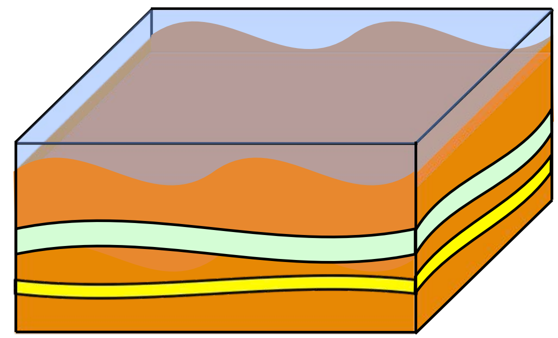
Why discontinuous Galerkin?

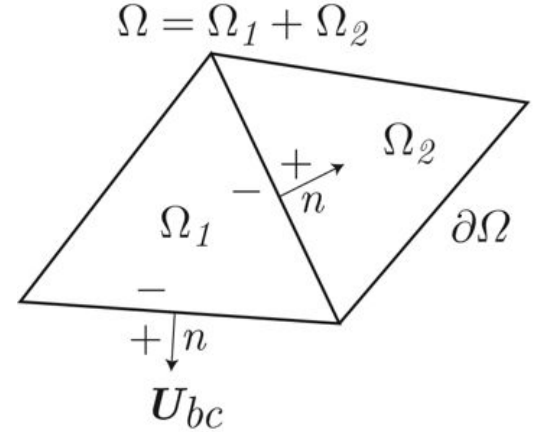


- **Finite Difference**
  - Uniform structured mesh
    - Hard to coarsen with depth
    - Hard to refine near targets
    - "Stair-step" interfaces
  - Difficulty aligning mesh with
    - Surface topologies
    - Material interfaces
  - Can high order be maintained?



- **Discontinuous Galerkin**
  - Unstructured mesh
    - Can coarsen with depth
    - Can refine near targets
    - Exact interfaces
  - Can match
    - Surface topologies
    - Material interfaces
  - Provably high order

Discontinuous Galerkin



Strong form:

$$\boldsymbol{U}_{,t} + \boldsymbol{F}_{i,i} = \mathbf{S}, \quad \text{in } \Omega$$

$$\boldsymbol{U}(\boldsymbol{x}, 0) = \boldsymbol{U}_0(\boldsymbol{x}), \quad \text{at } \; t = 0$$

and appropriate boundary conditions on $\partial\Omega$.

Partition Ω into N subdomains Ω_e.

$$\int\limits_{\Omega_e} \left(\boldsymbol{W}^T \boldsymbol{U}_{,t} - \boldsymbol{W}^T_{,i} \boldsymbol{F}_i \right) d\boldsymbol{x} + \int\limits_{\partial\Omega_e} \boldsymbol{W}^T \boldsymbol{F}_n \, d\boldsymbol{s} = \int\limits_{\Omega_e} \boldsymbol{W}^T \mathbf{S} \, d\boldsymbol{s}$$

Introduce numerical fluxes $\boldsymbol{F}_n(\boldsymbol{U}) \to \widehat{\boldsymbol{F}}_n(\boldsymbol{U}^-, \boldsymbol{U}^+)$ and sum over all elements

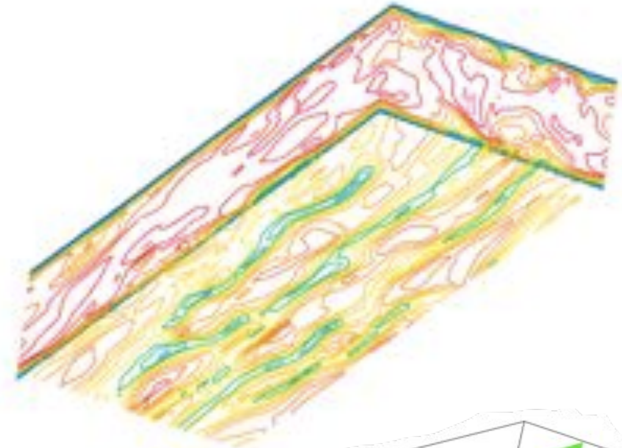
$$\sum_{e=1}^{N} \int\limits_{\Omega_e} \left(\boldsymbol{W}^T \boldsymbol{U}_{,t} - \boldsymbol{W}^T_{,i} \boldsymbol{F}_i - \boldsymbol{W}^T \mathbf{S} \right) d\boldsymbol{x} + \int\limits_{\partial\Omega_e} \boldsymbol{W}^T \widehat{\boldsymbol{F}}_n(\boldsymbol{U}^-, \boldsymbol{U}^+) \, d\boldsymbol{s} = 0$$

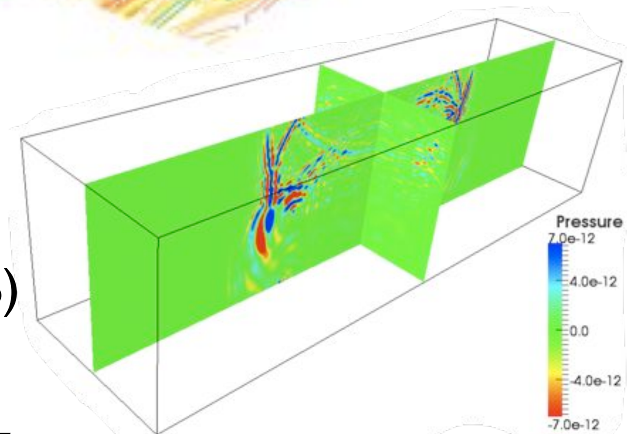
for all $\boldsymbol{W} \in \mathcal{V}$.

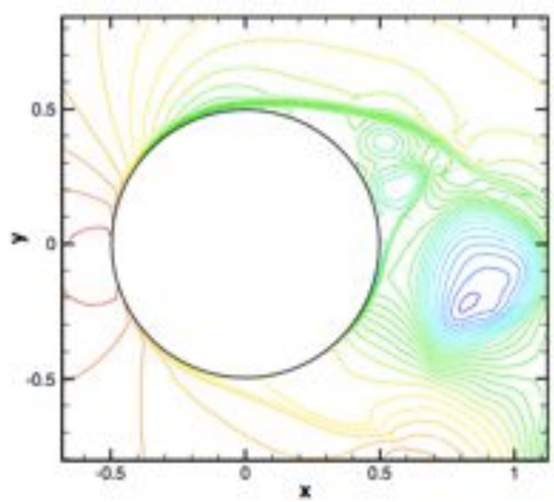
Benefits: High accuracy, unstructured, local hp-refinement, local conservation

DGM Toolkit

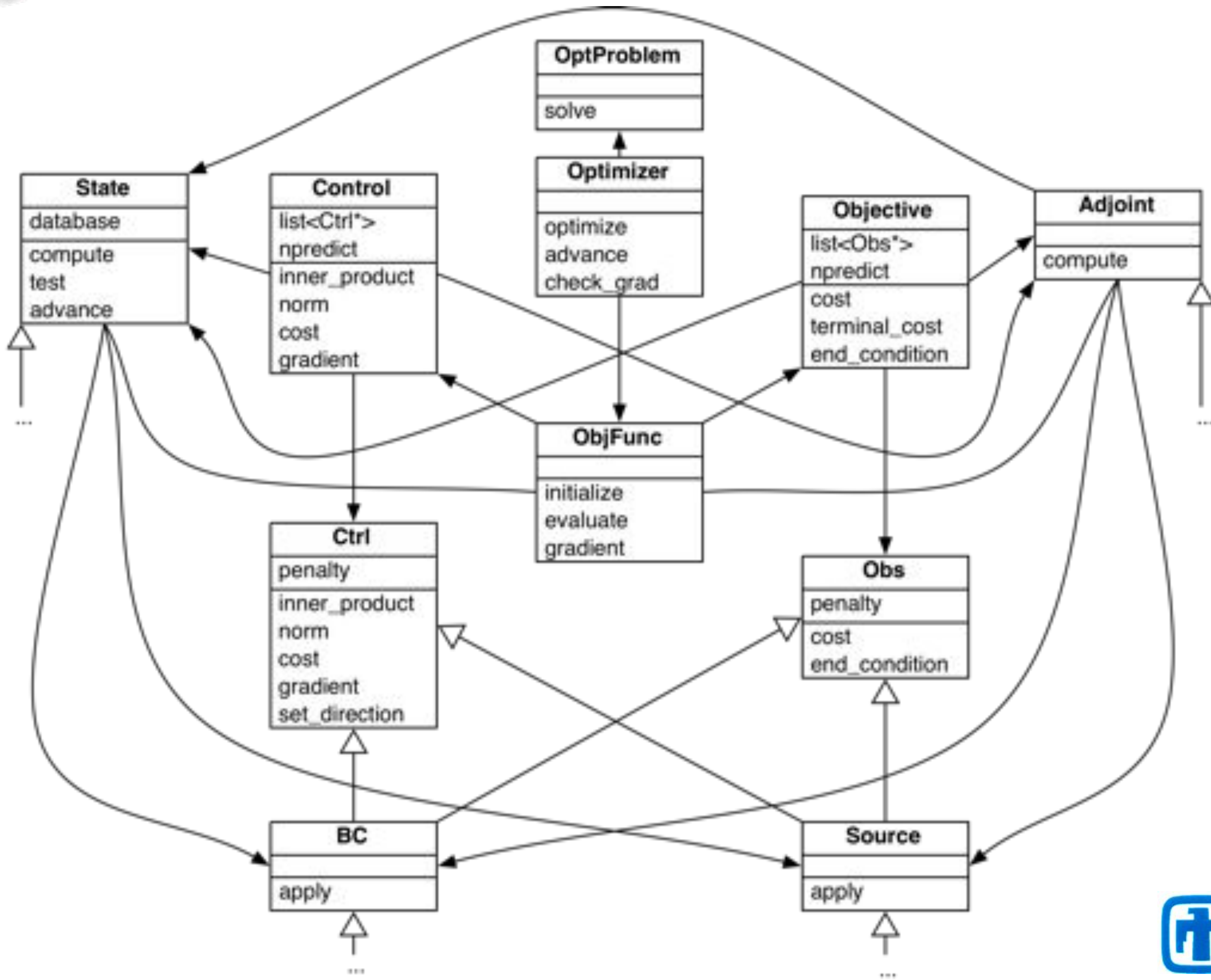
- High-order on unstructured meshes
  - Line, Quad, Tri, Hex elements
- Supports local, *p*-refinement
- Object-oriented software design
- Physics independent: examples for
  - Compressible Euler & Navier-Stokes (with LES)
  - Incompressible Euler & Navier-Stokes
  - Advection-diffusion, Burgers, Darcy, Helmholtz
- Designed for adjoint-based optimization
  - Steady-state and transient with checkpointing
- MPI with MPI-IO
- Version 0.0 released open-source (Rice)
- Version 1.0 on the way…







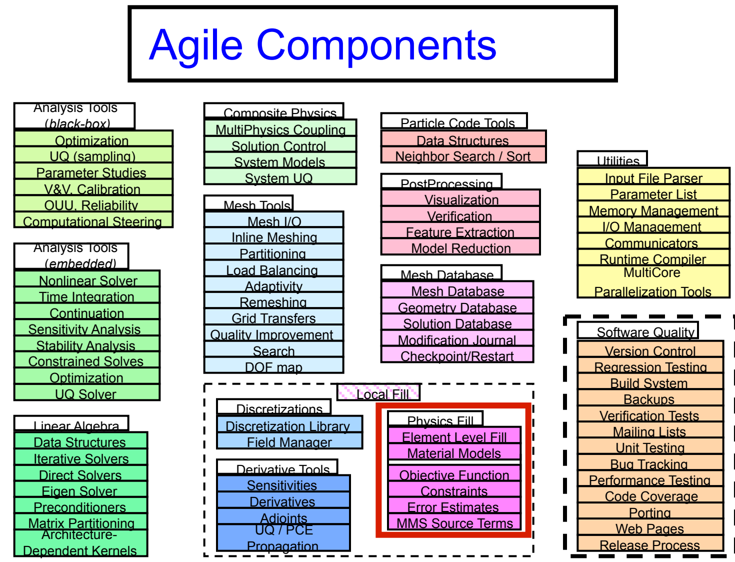
DGM Inversion Infrastructure

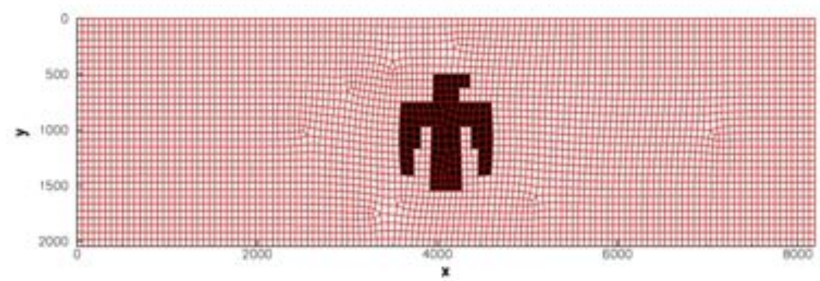


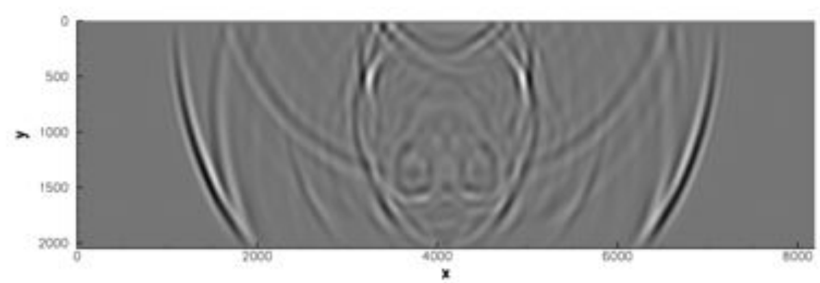
Ichos

(Greek for sound or Tune)

- **Discontinuous Galerkin**
  - Unstructured meshes
  - Variable-order polynomial representation
    - » For both the solution and the media
  - Local polynomial de-/refinement
  - Curved and non-simplicial elements
- **Component Technology**
  - Built on DGM Toolkit
    - » Component-based software design for DG
  - Agile Components (ModelEvaluator)
    - » Access to Trilinos (OptiPack) and Dakota
  - Multiple physics (acoustic, elastic and attenuation)
- **Optimization and Inversion**
  - Transient optimization
  - Adjoint-based optimization/inversion
  - Simultaneous Source Inversion

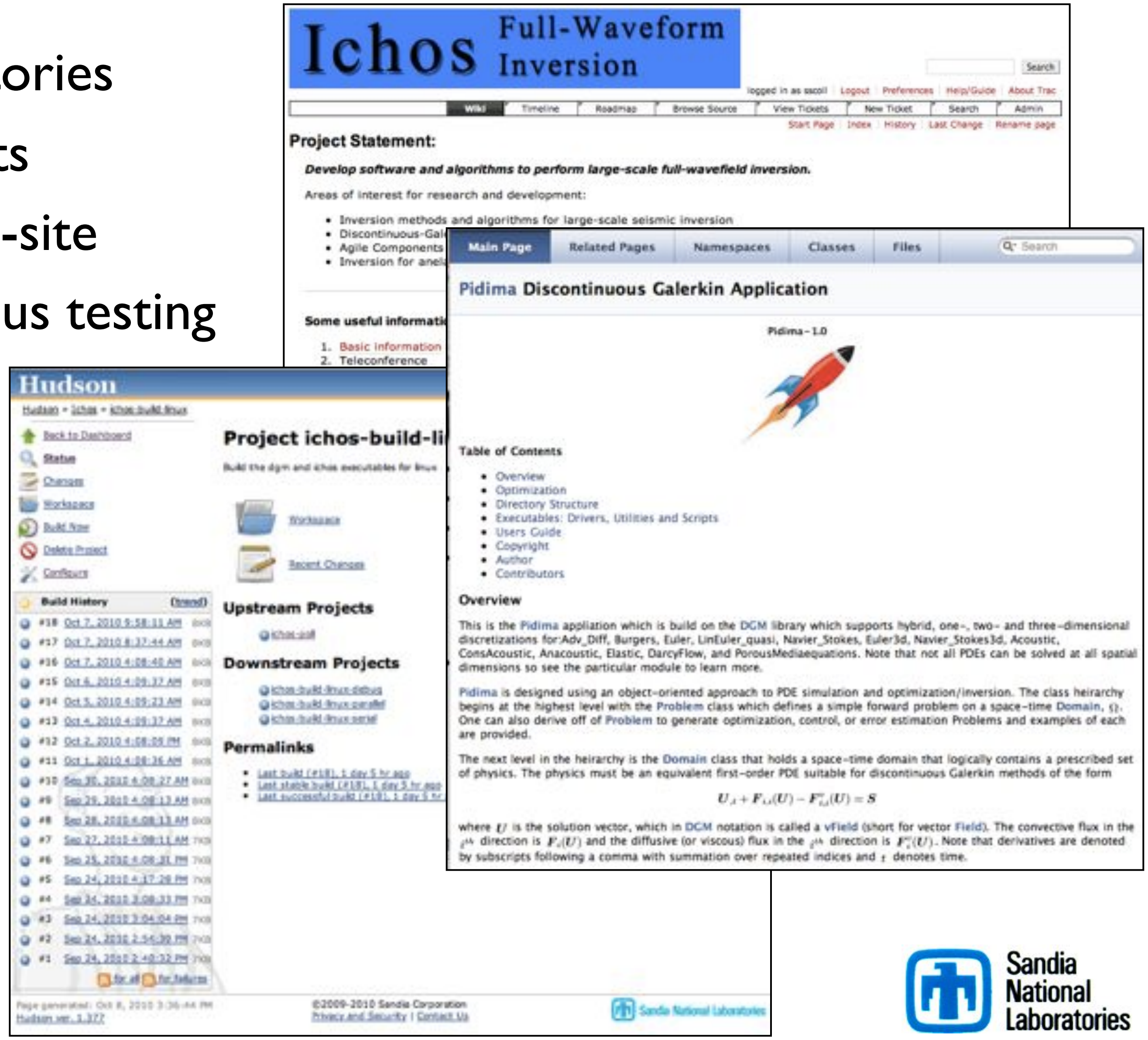






Project Management

- SVN/CVS repositories
- Mailman email lists
- Trac project web-site
- Hudson continuous testing
  - Linux
  - Mac
  - Clusters
- Full Doxygen documentation



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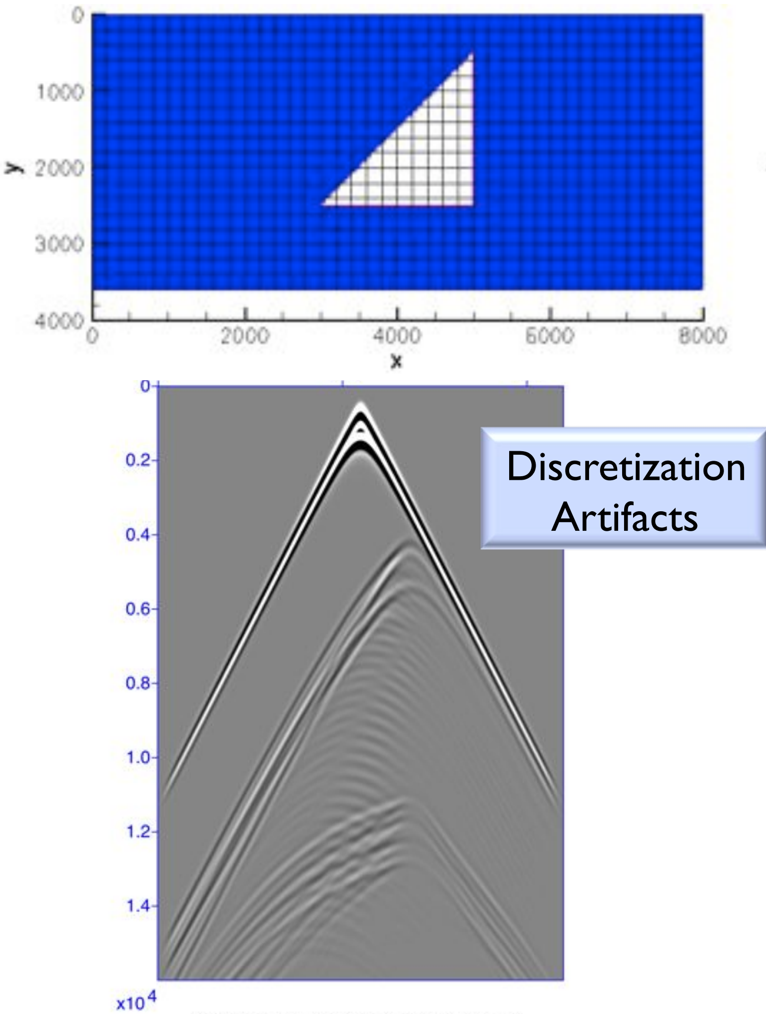
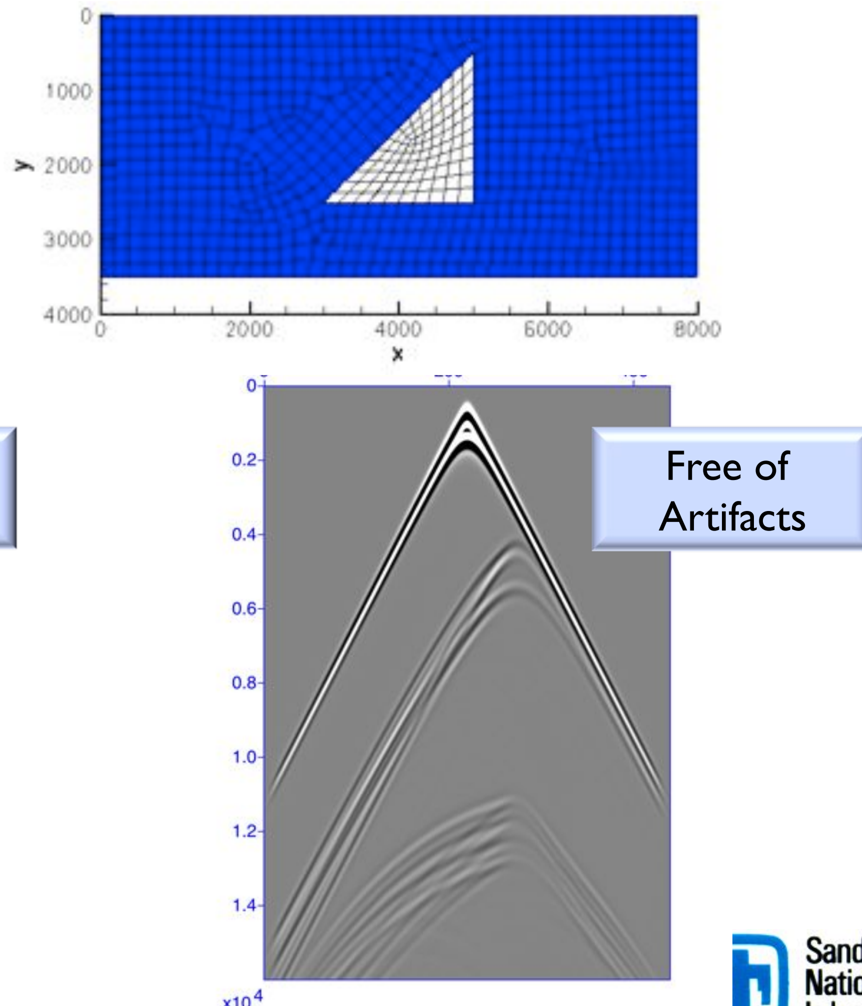
Forward Modeling

Discretization Challenges

Traditional structured meshes have difficulty capturing geological features accurately

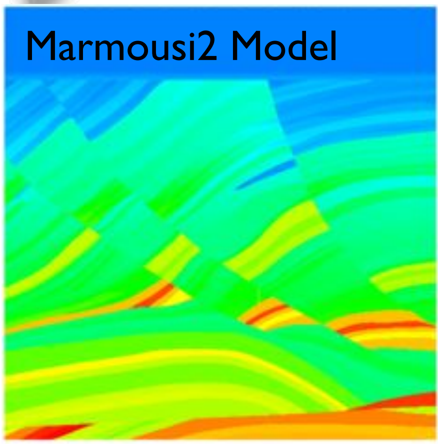


Triangle: p=10 uniform mesh

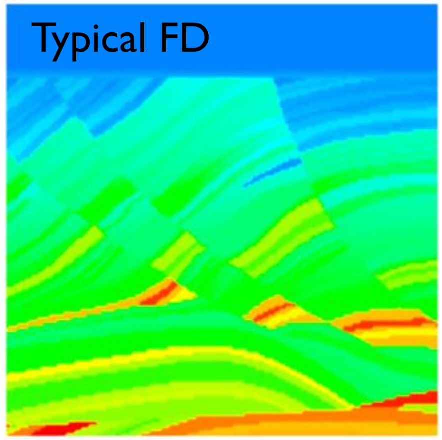


Triangle: p=10 unstructured

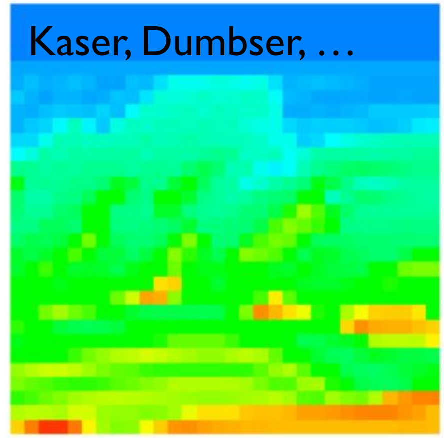
Better Media Representation



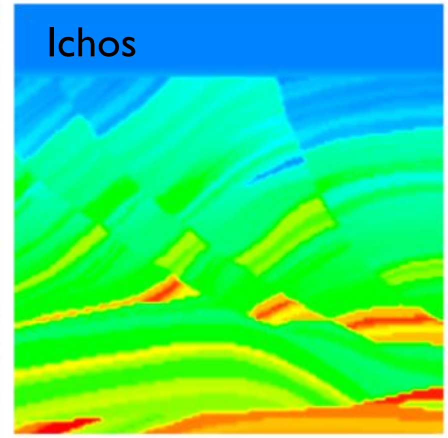
h = 1.25m



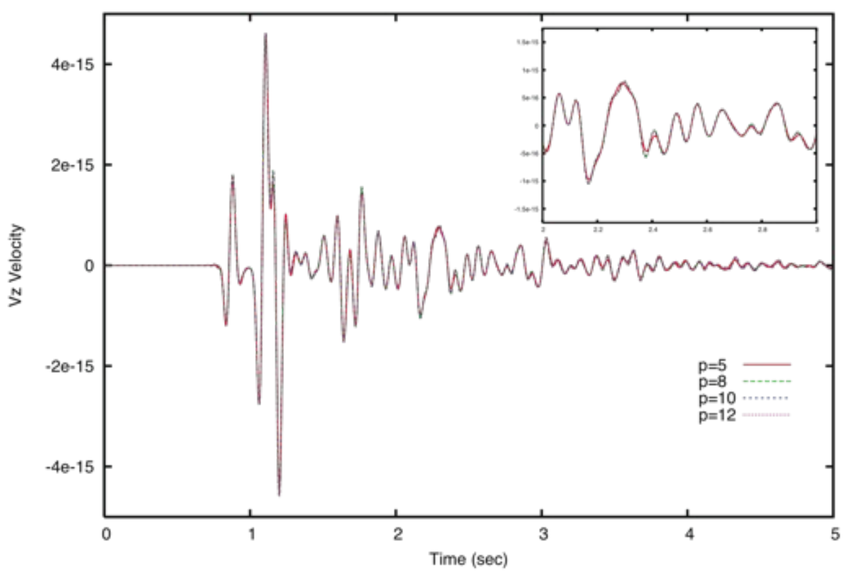
h = 20m



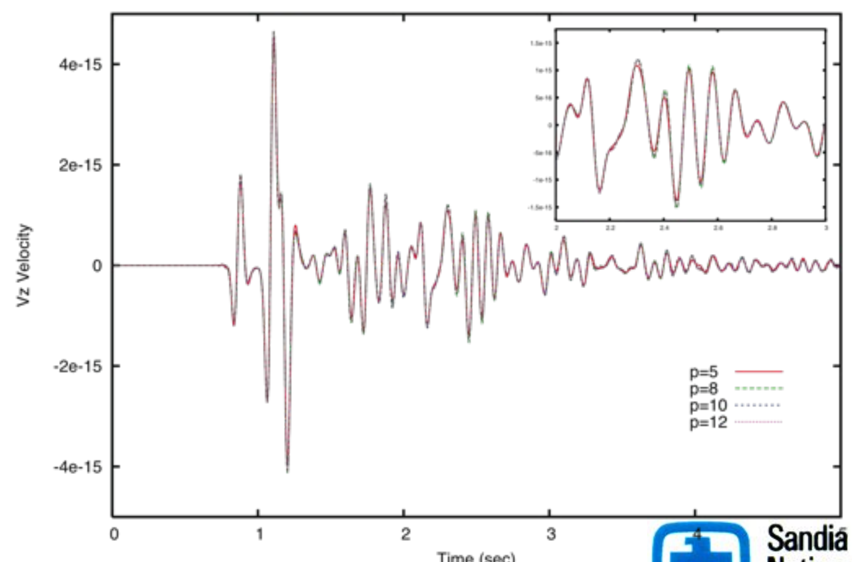
h = 100m, Mp = 0



h = 100m, Mp = 8



h = 100m, Mp = 0



h = 100m, Mp = 8

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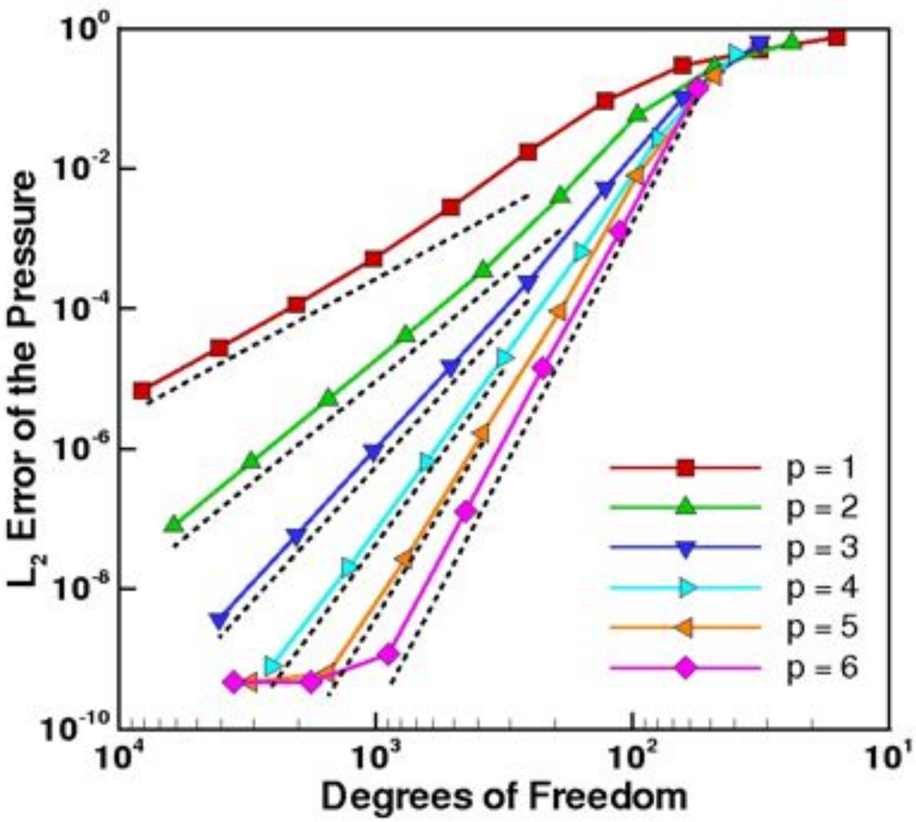
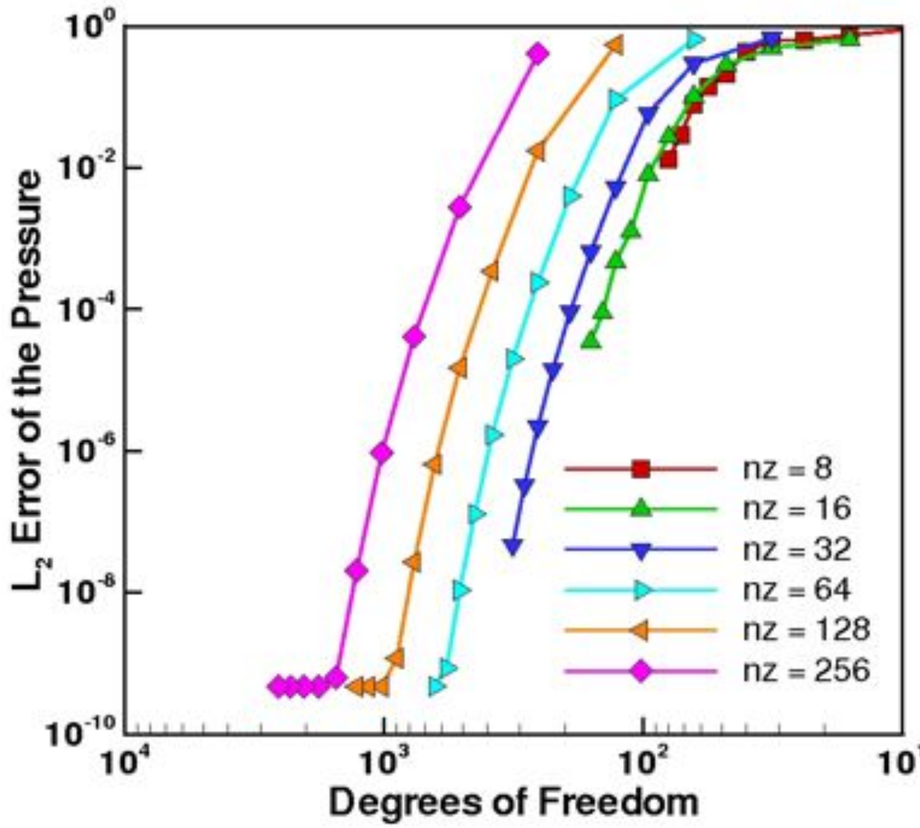
Code Verification

<u>Mesh Refinement</u>



<u>Polynomial Refinement</u>

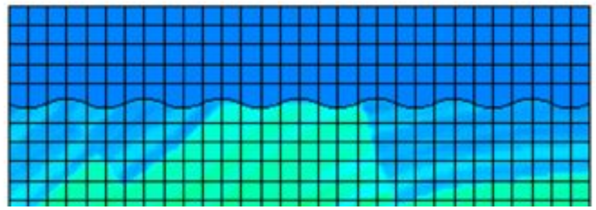


See: Ober, Collis, van Bloemen Waanders, Marcinkovich, SEG 2009.

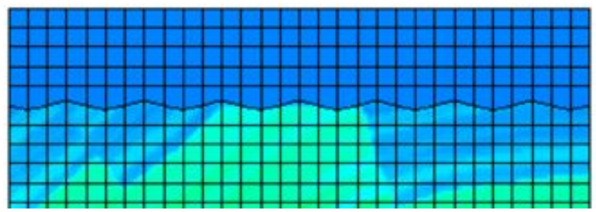
Topology Capturing

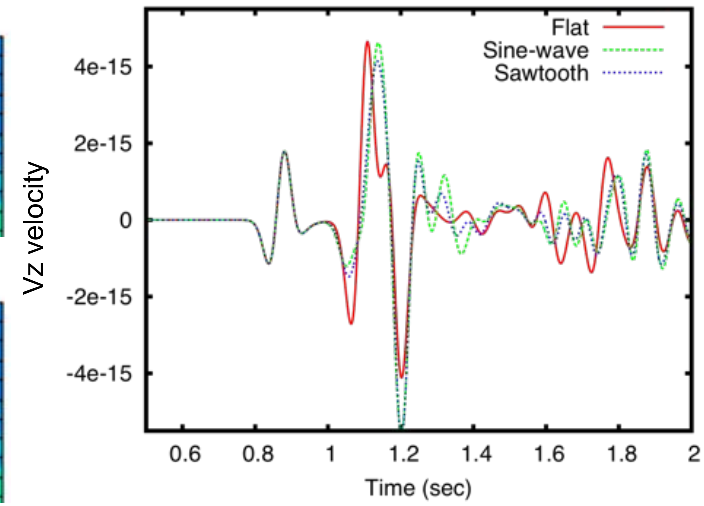
- **For example:**
  - Ocean floor
  - Faults
  - Salt structures
  - Even ocean waves…
- **Elastic and Acoustic**

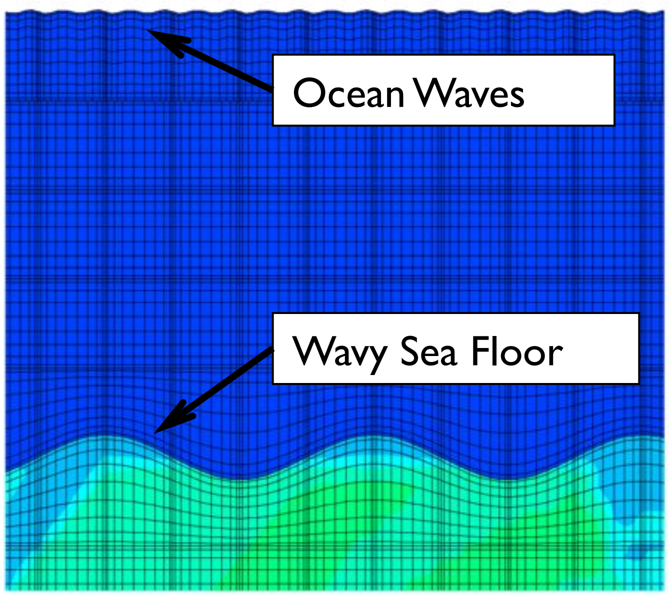
Sinusoidal Ocean Bottom



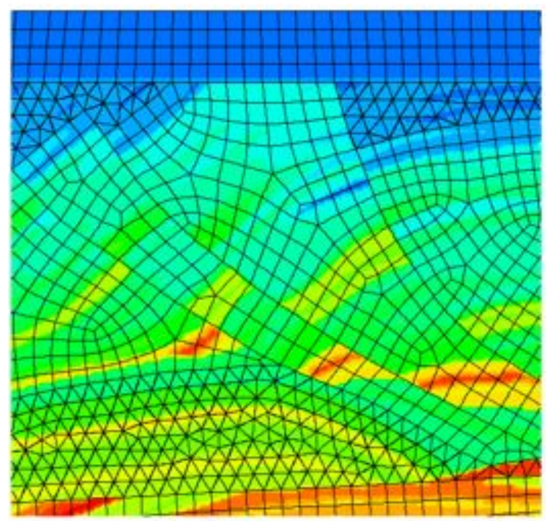
Sawtooth Ocean Bottom



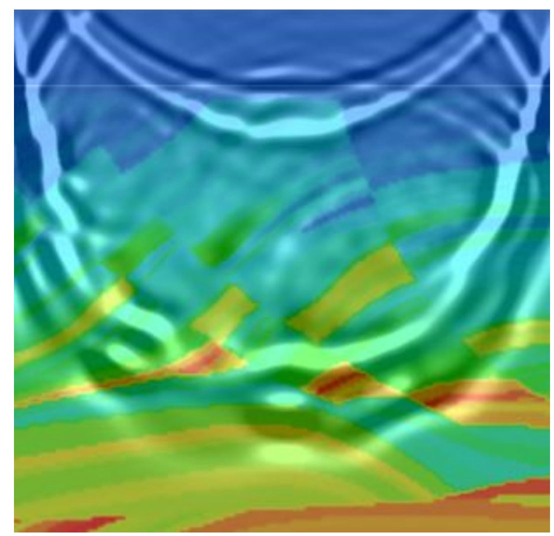




Hybrid Mesh, h = 100 m

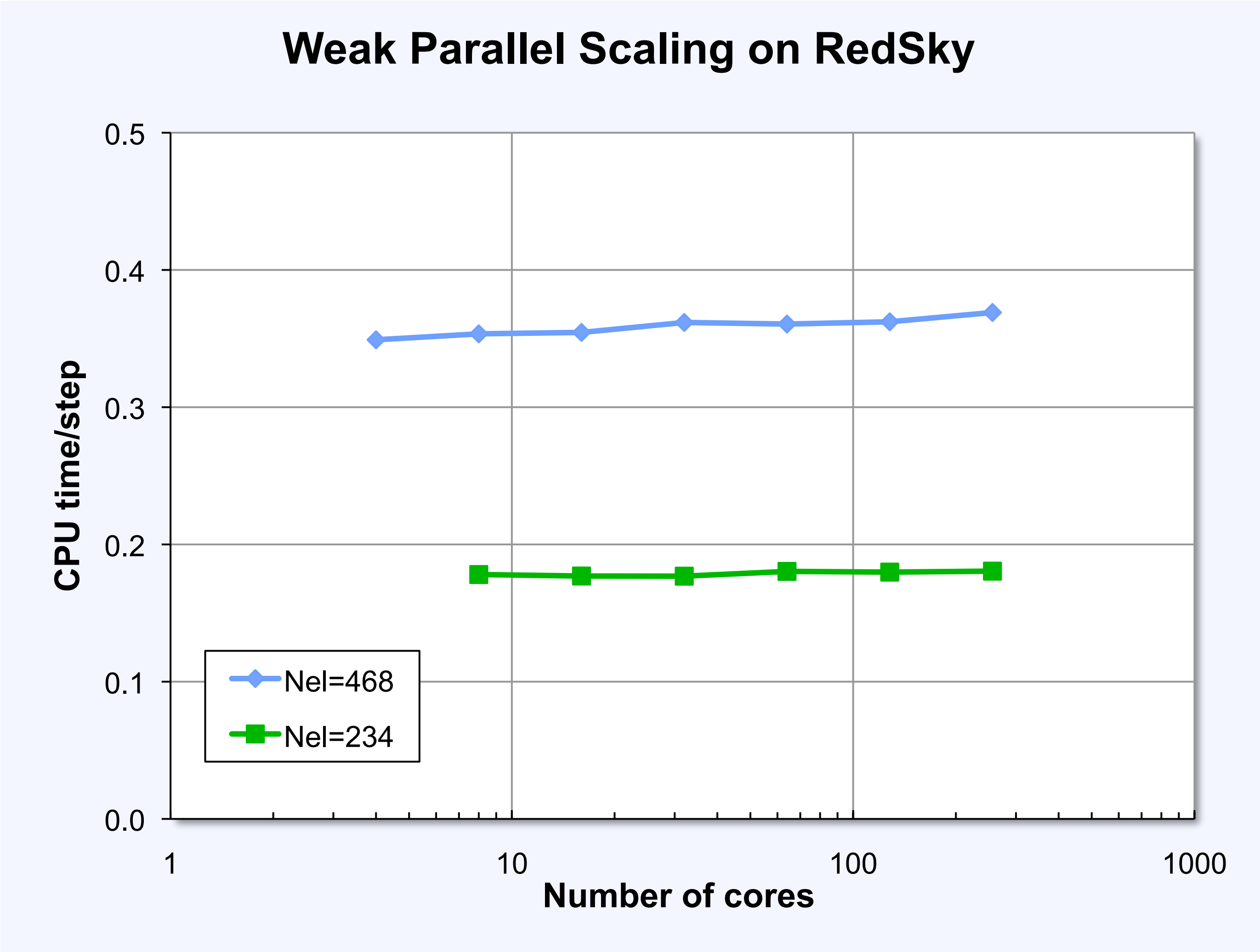


Pressure Wavefield, t=1.25 s



See: Smith, Collis, Ober, Overfelt, Schwaiger, SEG 2010.

Ichos Performance



Parallel efficiency = 84%

Seismic Inversion

Inversion Formulation

- PDE constrained optimization problem

$$\min_{\beta \in \mathcal{B}} \; \frac{1}{2} \sum_{r=1}^{N_r} \int_0^T \int_\Omega \xi_r(\mathbf{x}) \left(p(\mathbf{x},t) - \sum_{s=1}^{N_s} \omega_s \tilde{p}(\mathbf{x},t) \right)^2 \, d\mathbf{x} \, dt$$

- Subject to acoustic wave equations in conservative form

$$\begin{aligned} \beta \frac{\partial p}{\partial t} + \nabla \cdot \mathbf{v} &= \beta \phi \quad \text{in } \Omega \times (0,T] \\ \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla p &= \mathbf{0} \quad \text{in } \Omega \times (0,T] \\ p(\mathbf{x},0) &= 0 \quad \text{for } \mathbf{x} \in \Omega \\ \mathbf{v}(\mathbf{x},0) &= 0 \quad \text{for } \mathbf{x} \in \Omega \end{aligned}$$

- Approach
  - Simultaneous Source Inversion
    - » Krebs et al. (2009)
    - » Phase Encoding – Romero et al. (2000)
    - » Speedup of 50x in 2D; ~2000x in 3D
  - Gradient-based optimization…

$$\begin{aligned} \beta &= 1/(\rho c^2) \text{ – compressibility} \\ \mathcal{B} &= \text{space of admissible media} \\ \rho &= \text{mass density} \\ c &= \text{wave speed} \\ \Omega &= \text{computational domain} \\ T &= \text{time horizon} \\ N_r &= \text{number of receivers} \\ N_s &= \text{number of sources} \\ \omega_s &\in \{-1,1\} \text{ – random phase encoding} \\ \phi &= \sum_{s=1}^{N_s} \omega_s w(t) \xi_s(\mathbf{x}) \text{ – encoded sources} \\ \tilde{p} &= \text{measured pressure data} \\ \xi_r, \xi_s &= \text{spatial kernel for receiver, source} \end{aligned}$$

Model Problem – Marmousi2*

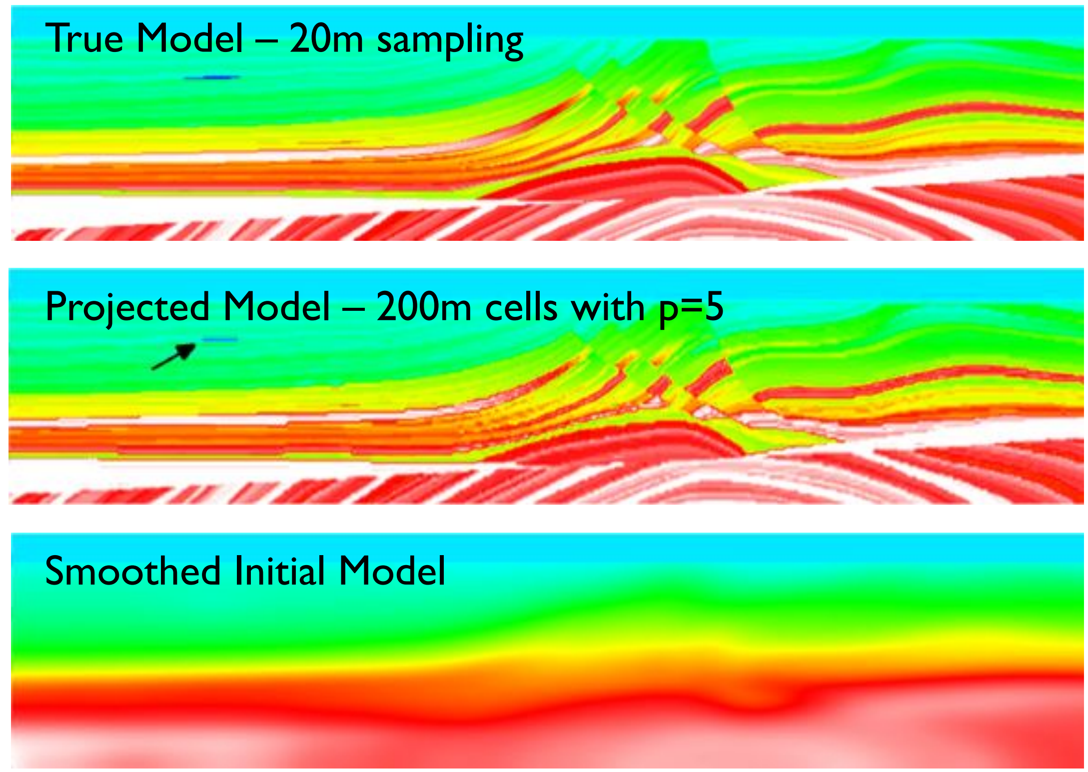
- **True Model**
  - 500m sponge on bottom
  - 500m sponge on ends
  - Fixed spread of receivers

    $x_r = r*200+500\ (0 \le r \le 75)$

    $y_r = 100\text{m}$
  - Uniformly spaced sources

    $x_s = s*1000\text{m}\ (1 \le s \le 15)$

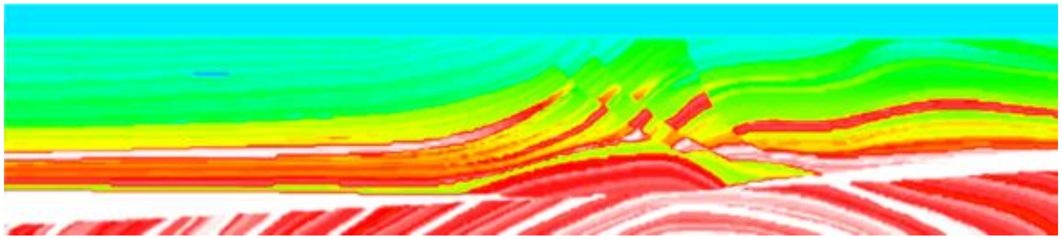
    $y_s = 300\text{m}$
- **Projected Model**
  - 80 x 20 =1600 elements = 57.6k dof
  - FD 20m cells = 160k dof
- **Smoothed Initial Model**
  - Damped least-squares method



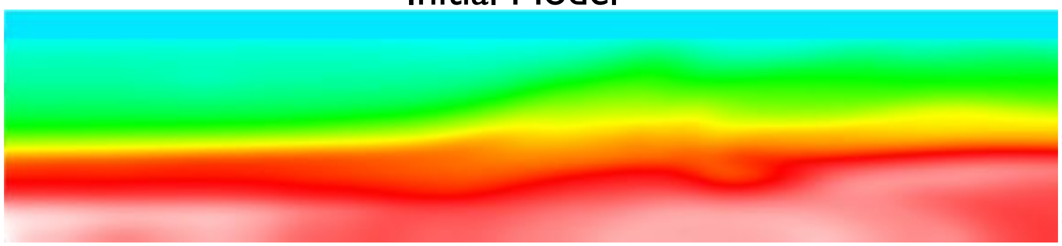
* Martin, G. S., R. Wiley, and K. J. Marfurt, 2006, Marmousi2: An Elastic Upgrade for Marmousi: The Leading Edge, **25, 156–166.**

Acoustic Inversion

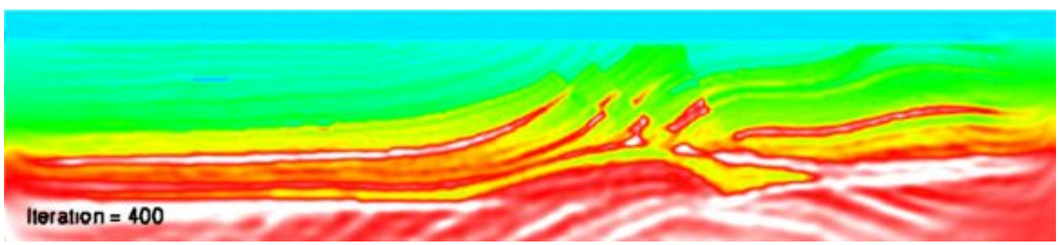
Marousi2: True Model (h=200m, p=5)



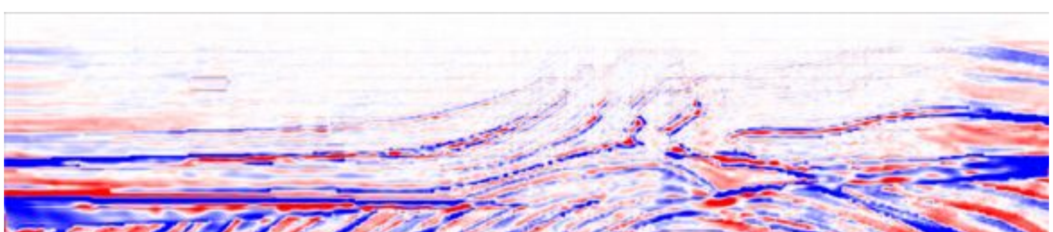
Initial Model



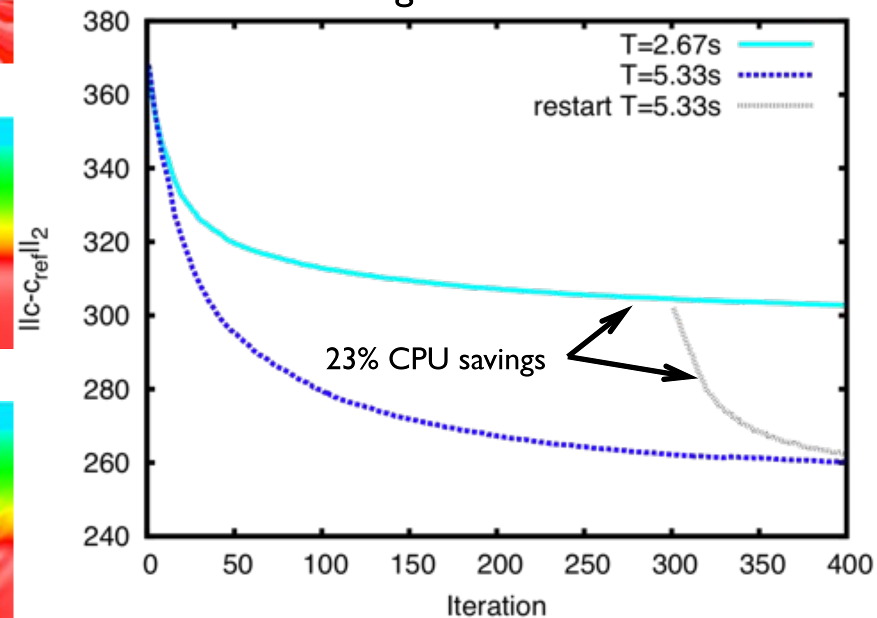
Inverted Model



Inverted - True



Convergence of Model Fit



See: Collis, Ober, van Bloemen Waanders, SEG 2010.

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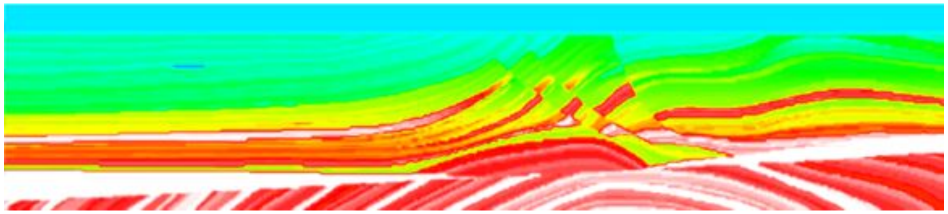
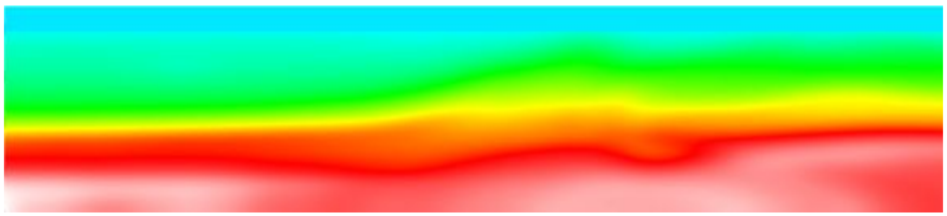
Acoustic Inversion

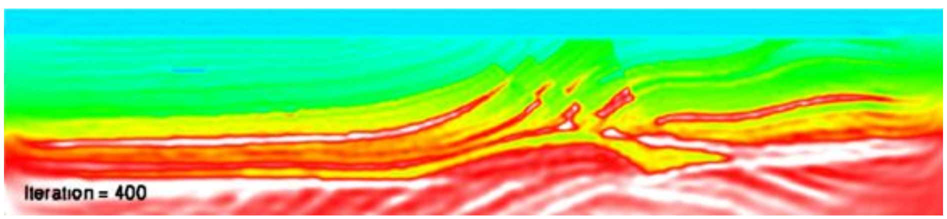
Marmousi2 True Model (h = 200m; p = 5)



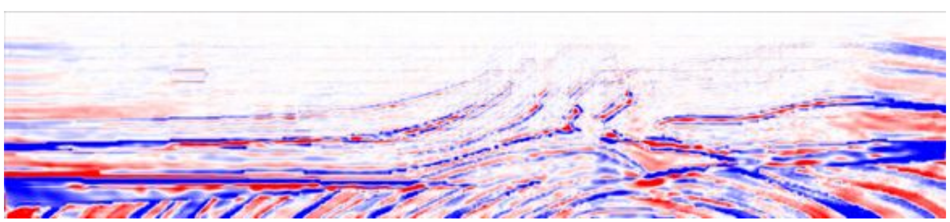
Initial Model



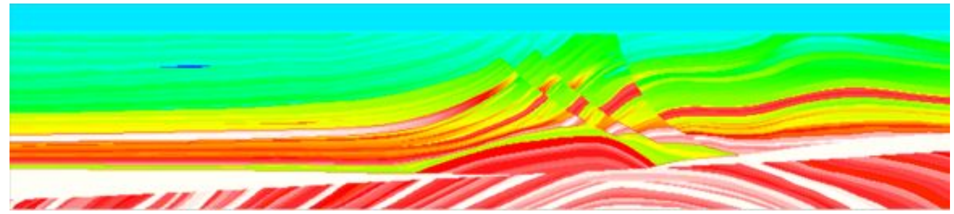
Inverted Model



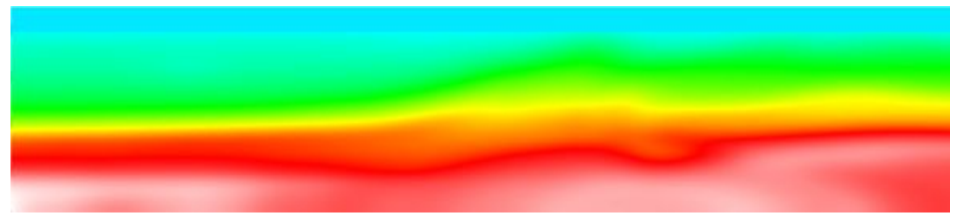
Inverted - True



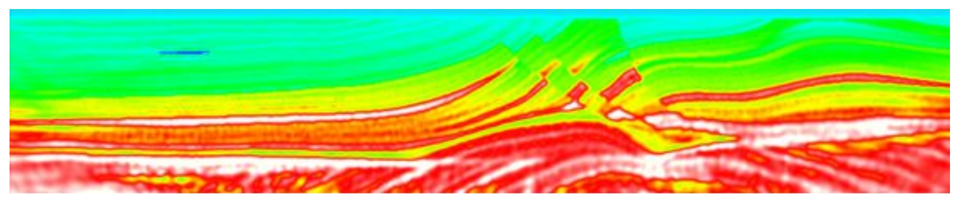
FD SSI – Krebs et al. (2009) (h = 20m; 8th order)



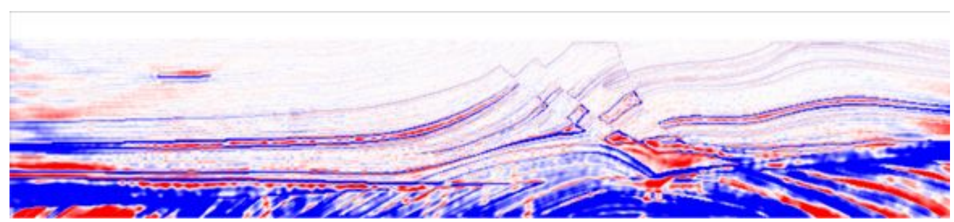
Initial Model



Inverted Model



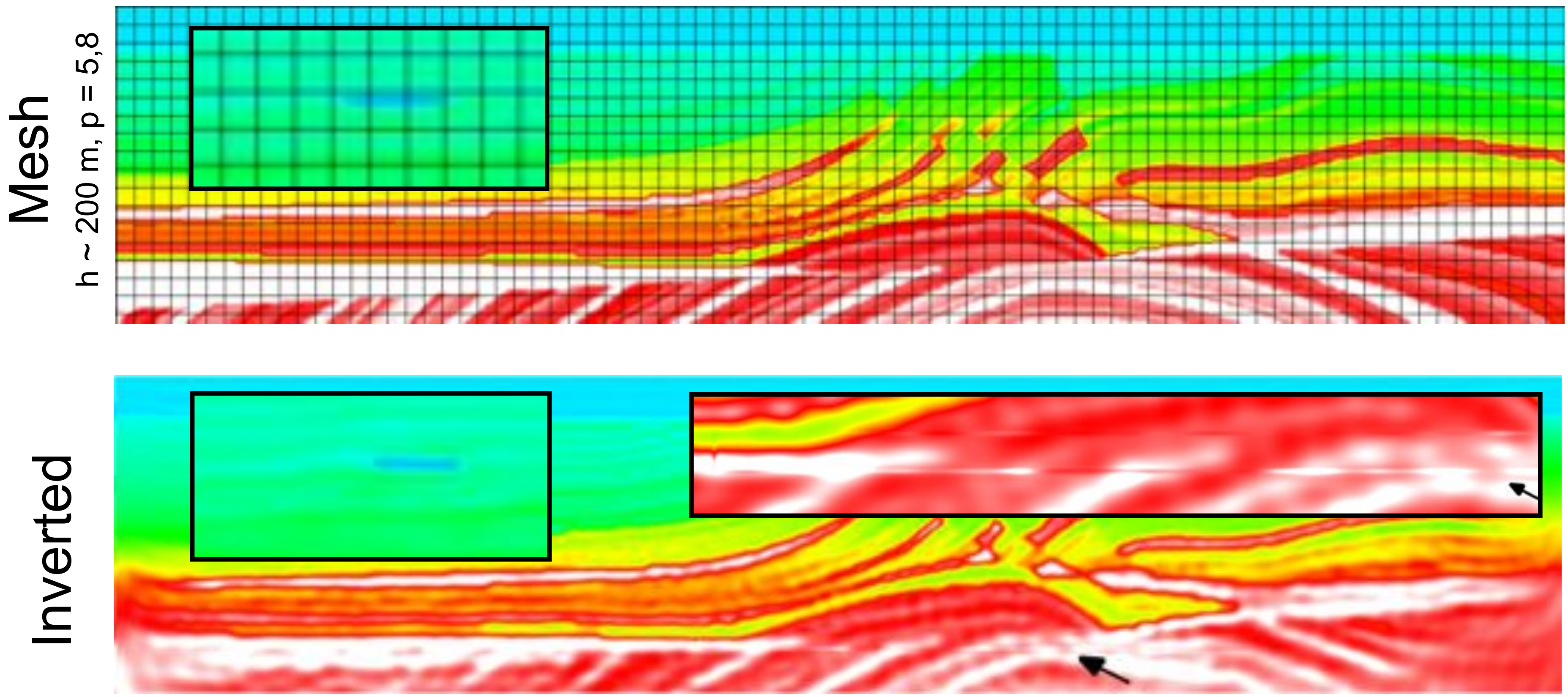
Inverted - True



Ichos Acoustic Inversion

15 sources at 1000m spacing, 76 receivers at 200m spacing

Uniform Mesh



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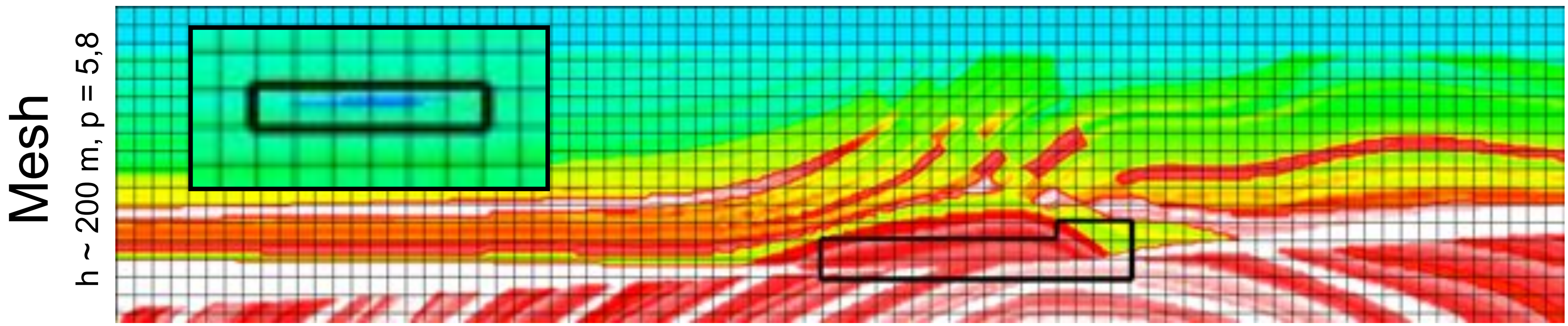
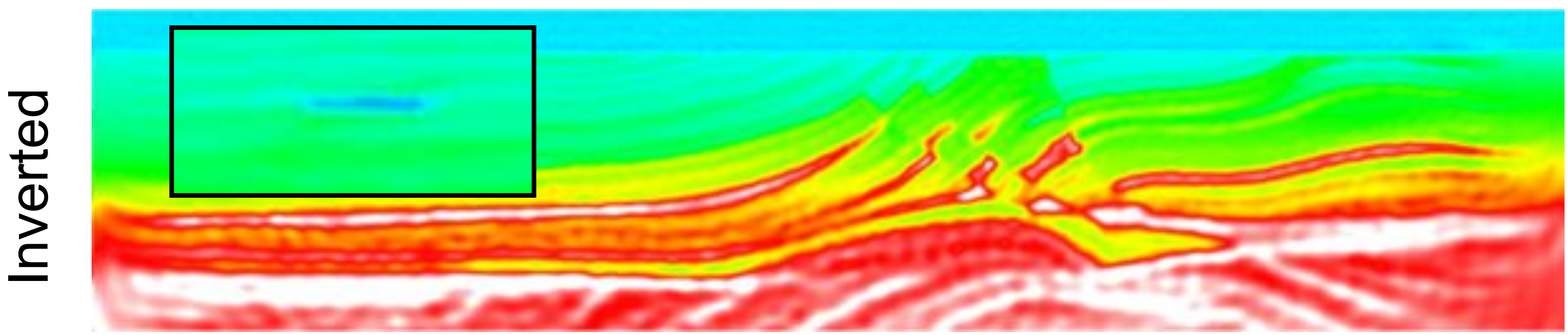
Ichos Acoustic Inversion

15 sources at 1000m spacing, 76 receivers at 200m spacing

Local Polynomial Refinement





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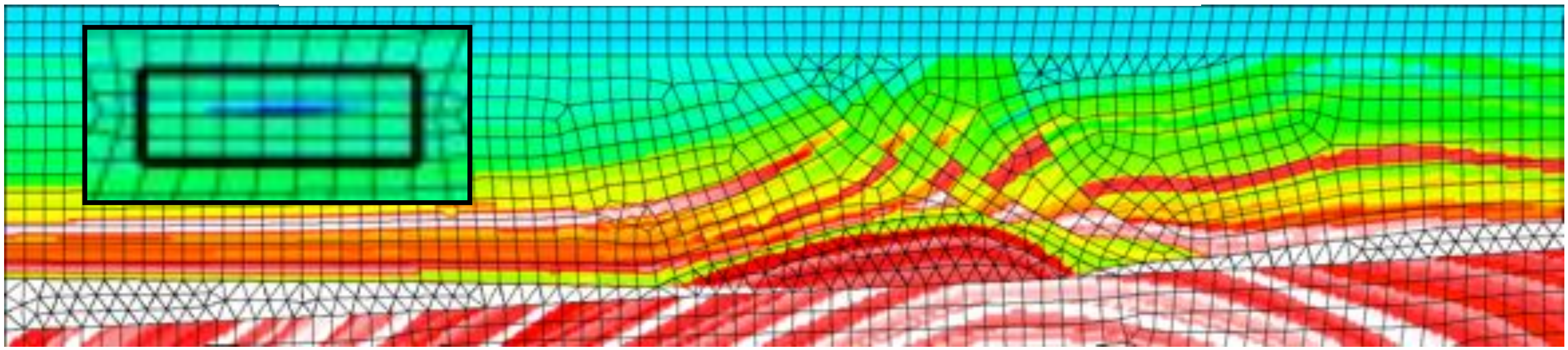
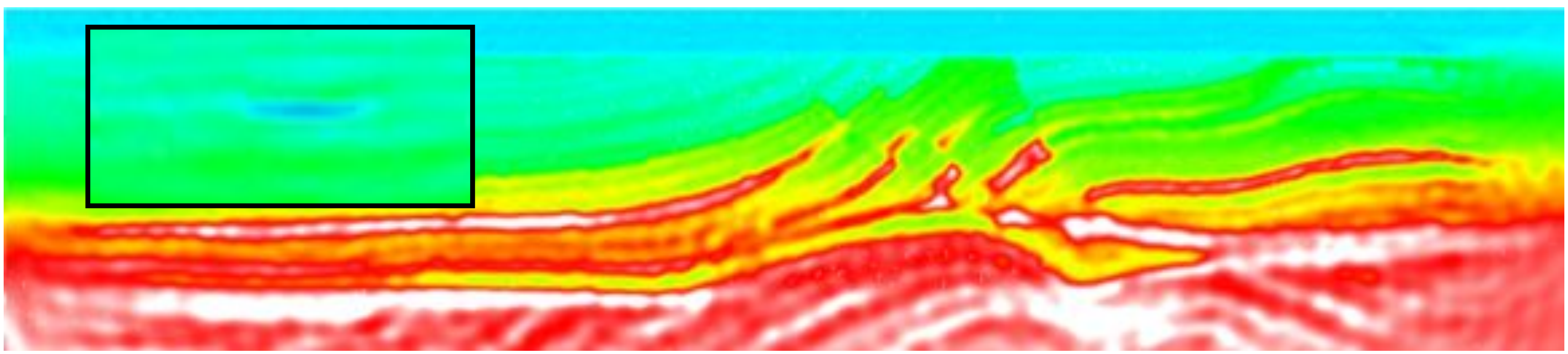
Unstructured Mesh

Mesh
h ~ 200 m, p = 5,8



Inverted



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Closing Comments

- Ichos demonstrates the potential of high-order DG methods
  - Based on flexible DGM toolkit + Trilinos + Dakota
  - Acoustic and Elastic wave propagation in 1d, 2d, and 3d
  - Infrastructure in place for gradient-based inversion in 1d, 2d and 3d
- Proof-of-principle inversion studies:
  - Uniform structured mesh: similar accuracy, fewer degrees of freedom
  - Local p-refinement improves representation of localized geological features
  - Unstructured meshes accurately capture discontinuous material interfaces (faults, ocean bottom, salt structures).
  - Simultaneous Source Inversion (SSI) offers significant algorithmic speedup
- Component-based approach: key to scalability, sustainability and agility…

Extras

Linear Elastodynamics

$$\rho(\mathbf{x})\frac{\partial v_i(\mathbf{x},t)}{\partial t} - \frac{\partial \sigma_{ij}(\mathbf{x},t)}{\partial x_j} = f_i(\mathbf{x},t) + \frac{\partial m^a_{ij}(\mathbf{x},t)}{\partial x_j},$$

$$\frac{\partial \sigma_{ij}(\mathbf{x},t)}{\partial t} - \lambda(\mathbf{x})\frac{\partial v_k(\mathbf{x},t)}{\partial x_k}\delta_{ij} - \mu(\mathbf{x})\left[\frac{\partial v_i(\mathbf{x},t)}{\partial x_j} + \frac{\partial v_j(\mathbf{x},t)}{\partial x_i}\right] = \frac{\partial m^s_{ij}(\mathbf{x},t)}{\partial t},$$

Stress tensor: $\sigma_{ij}(\mathbf{x},t)$
Particle velocity: $v_i(\mathbf{x},t)$

Force vector: $f_i(\mathbf{x},t)$
Moment tensor: $m_{ij}(\mathbf{x},t)$

Mass density: $\rho(\mathbf{x})$
Lame's first parameter: $\lambda(\mathbf{x}) = \rho(V_p^2 - 2V_s^2)$
Shear modulus: $\mu(\mathbf{x}) = \rho V_s^2$
Compressional wave speed: $V_p(\mathbf{x})$
Shear wave speed: $V_s(\mathbf{x})$

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Seismic Explosion

- Source composed with sum of shots

$$\phi(\mathbf{x},t) = \sum_{s=1}^{N_s} \omega_s w(t) \xi_s(\mathbf{x})$$

- Encoding $\omega_s \in \{-1, 1\}$
- Ricker wavelet

$$w(t) = (1 - 2\pi^2 f_p^2 (t - t_0)) \exp(-\pi^2 f_p^2 (t - t_0)^2)$$

- Gaussian spatial "ball" or Delta function

$$\xi_s(\mathbf{x}) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^N \exp\left(\frac{|\mathbf{x} - \mathbf{x}_s|^2}{2\sigma^2}\right)$$

$$\xi_s(\mathbf{x}) = \delta(\mathbf{x} - \mathbf{x}_s)$$