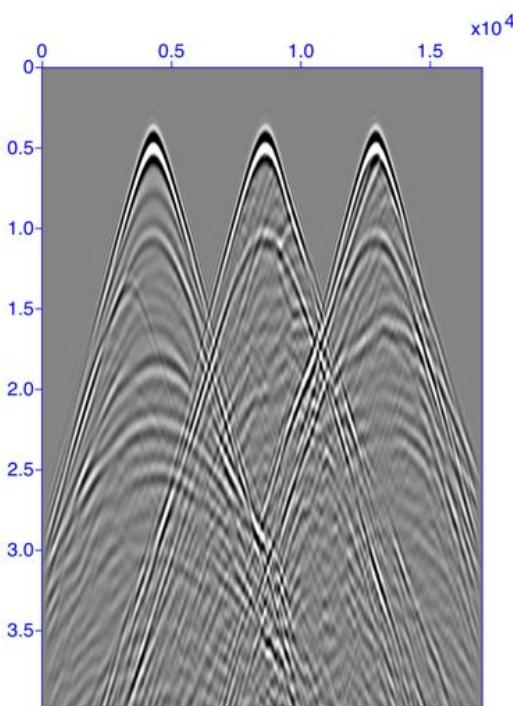




Discontinuous Galerkin for Wave Propagation and Inversion



S. Scott Collis

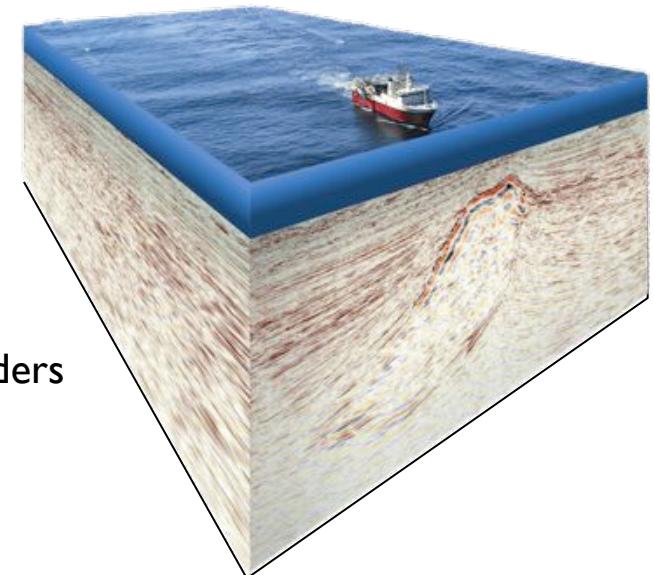
sscoll@sandia.gov, 505-284-1123

Sandia National Laboratories
Albuquerque, NM

Joint work with:

David Aldridge, Curt Ober,
James Overfelt, Thomas Smith,
Hans Schwaiger, Bart van Bloemen Waanders
and Joe Young

Presented at:
Oak Ridge National Laboratory
October 11, 2010

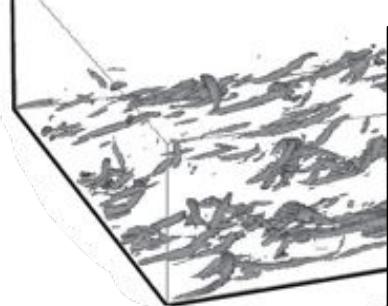


Approved by sponsor for release

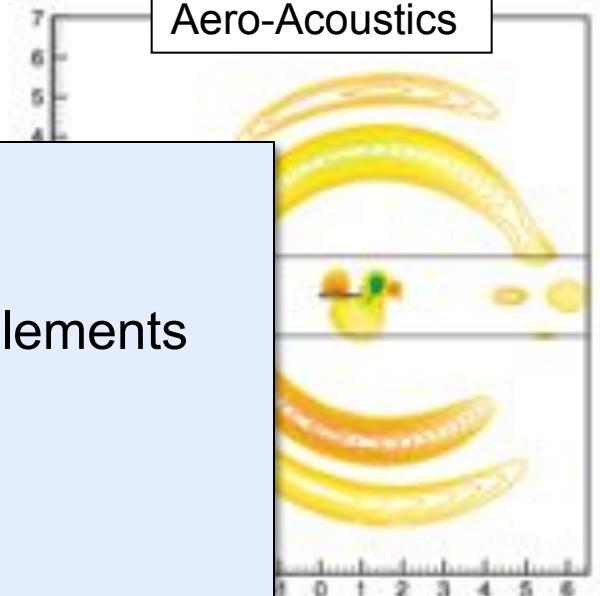
Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company,
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under contract DE-AC04-94AL85000.

Other things I've done...

Turbulence Control



Aero-Acoustics



What's in common:

- High-order numerical methods
 - Discontinuous Galerkin finite elements
 - Spectral methods
 - Finite difference
- Adjoint-based methods
 - Adjoint sensitivities
 - Optimal control
 - Inversion
- Multiscale variational methods
 - Large-eddy simulation
 - Discontinuity capturing

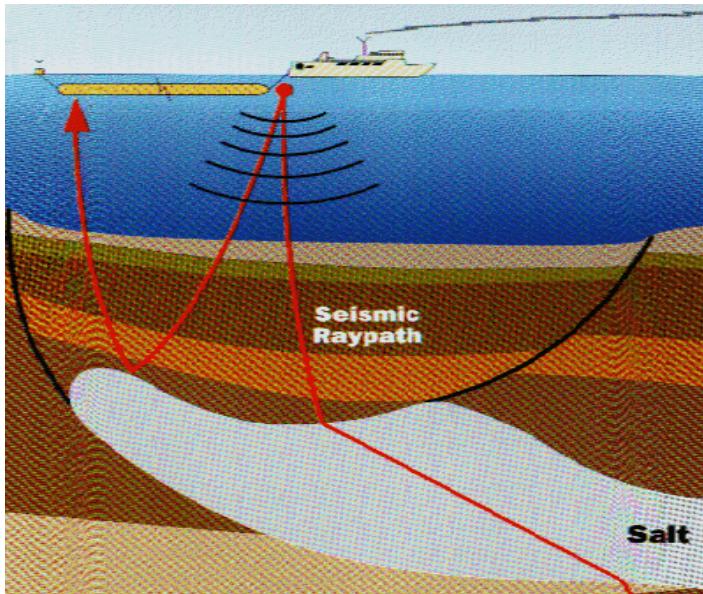
Receptivity & Transition

Multiscale LES



Full Waveform Inversion (FWI)

Seismic Experiment (“shot”)



- ★ 100,000's shots per survey
- ★ 1000's receivers per shot
- ★ 12 seconds at 1msec sampling

= 4.8 TB of data per survey!

- **Seismic Imaging (traditional)**

- Acoustic wave propagation
- “Manually” inverting for wave speed
- Primarily utilizes travel time
- Methods neglect waveform/amplitude
- ~20 exaFLOP (FD elastic)

- **Seismic Inversion (FWI)**

- Matching full waveform of the wavelets
- Acoustic, Elastic, Attenuation, Anisotropic
- Inverting for wave speeds and density $\mathcal{O}(10^8)$ parameters
- ~20,000 exaFLOP (FD elastic)

- **Interest to Sandia and DOE**

- Nuclear non-proliferation monitoring
- Underground structure identification
- Site characterization
(CO₂ sequestration and waste repositories)



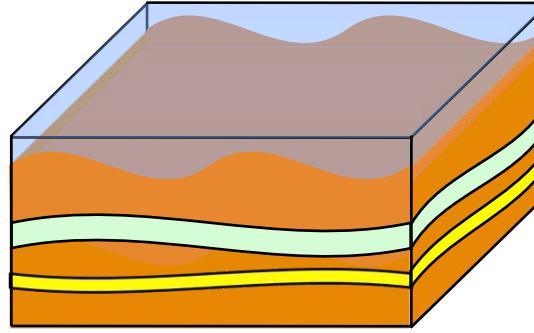
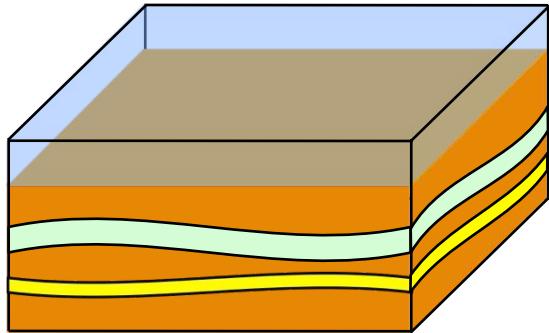
Project Goals

- Investigate use of unstructured Discontinuous Galerkin (DG) for seismic modeling & inversion
- Leverage DG capabilities for seismic modeling
 - Unstructured meshes can accurately capture discontinuous material interfaces (faults, ocean bottom, salt structures).
 - Local polynomial refinement enables improved resolution for localized geological features.
- Demonstrate DG for seismic inversion
 - Utilizing above advantages
 - Use Simultaneous Source Inversion (SSI), see Krebs et al. 2009
- Utilize algorithms from Trilinos and Dakota toolkits





Why discontinuous Galerkin?



- **Finite Difference**

- Uniform structured mesh
 - Hard to coarsen with depth
 - Hard to refine near targets
 - “Stair-step” interfaces
- Difficulty aligning mesh with
 - Surface topologies
 - Material interfaces
- Can high order be maintained?

- **Discontinuous Galerkin**

- Unstructured mesh
 - Can coarsen with depth
 - Can refine near targets
 - Exact interfaces
- Can match
 - Surface topologies
 - Material interfaces
- Provably high order

Discontinuous Galerkin

Strong form:

$$\mathbf{U}_{,t} + \mathbf{F}_{i,i} = \mathbf{S}, \quad \text{in } \Omega$$

$$\mathbf{U}(\mathbf{x}, 0) = \mathbf{U}_0(\mathbf{x}), \quad \text{at } t = 0$$

and appropriate boundary conditions on $\partial\Omega$.

Partition Ω into N subdomains Ω_e .

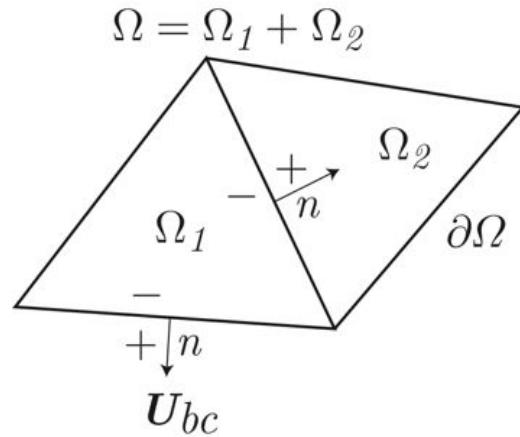
$$\int_{\Omega_e} \left(\mathbf{W}^T \mathbf{U}_{,t} - \mathbf{W}_{,i}^T \mathbf{F}_i \right) d\mathbf{x} + \int_{\partial\Omega_e} \mathbf{W}^T \mathbf{F}_n d\mathbf{s} = \int_{\Omega_e} \mathbf{W}^T \mathbf{S} d\mathbf{s}$$

Introduce numerical fluxes $\mathbf{F}_n(\mathbf{U}) \rightarrow \widehat{\mathbf{F}}_n(\mathbf{U}^-, \mathbf{U}^+)$ and sum over all elements

$$\sum_{e=1}^N \int_{\Omega_e} \left(\mathbf{W}^T \mathbf{U}_{,t} - \mathbf{W}_{,i}^T \mathbf{F}_i - \mathbf{W}^T \mathbf{S} \right) d\mathbf{x} + \int_{\partial\Omega_e} \mathbf{W}^T \widehat{\mathbf{F}}_n(\mathbf{U}^-, \mathbf{U}^+) d\mathbf{s} = 0$$

for all $\mathbf{W} \in \mathcal{V}$.

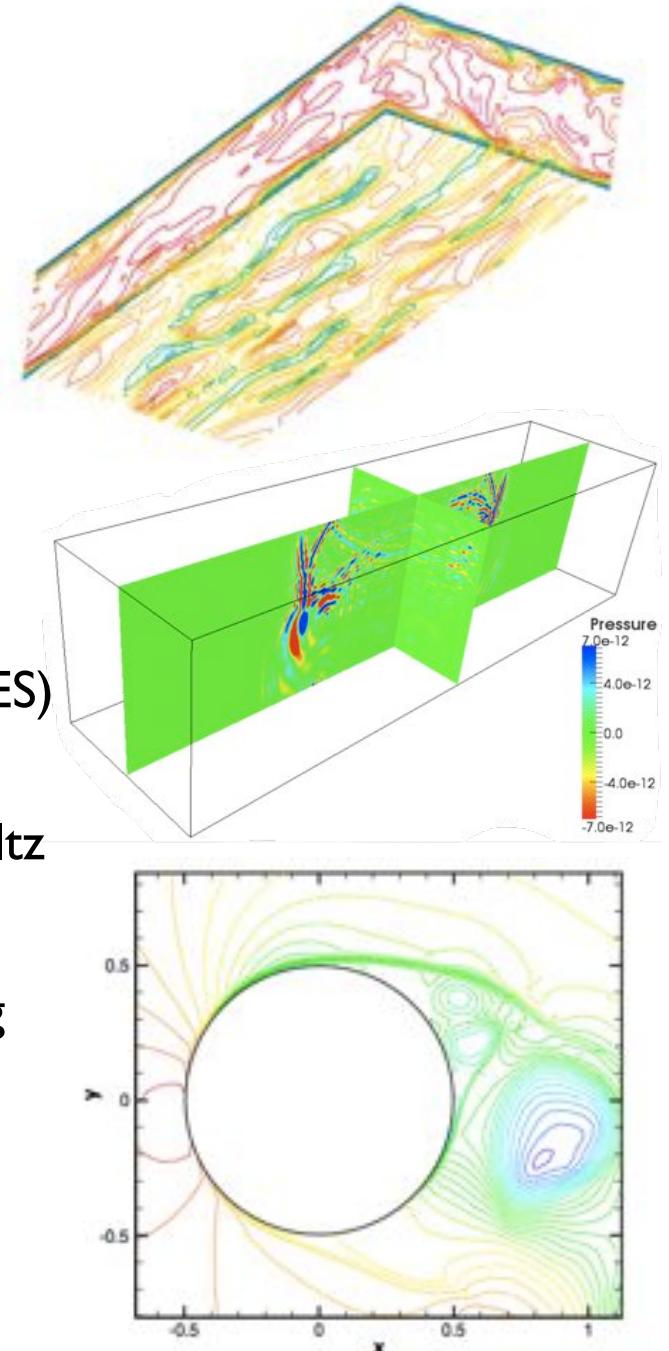
Benefits: High accuracy, unstructured, local hp -refinement, local conservation



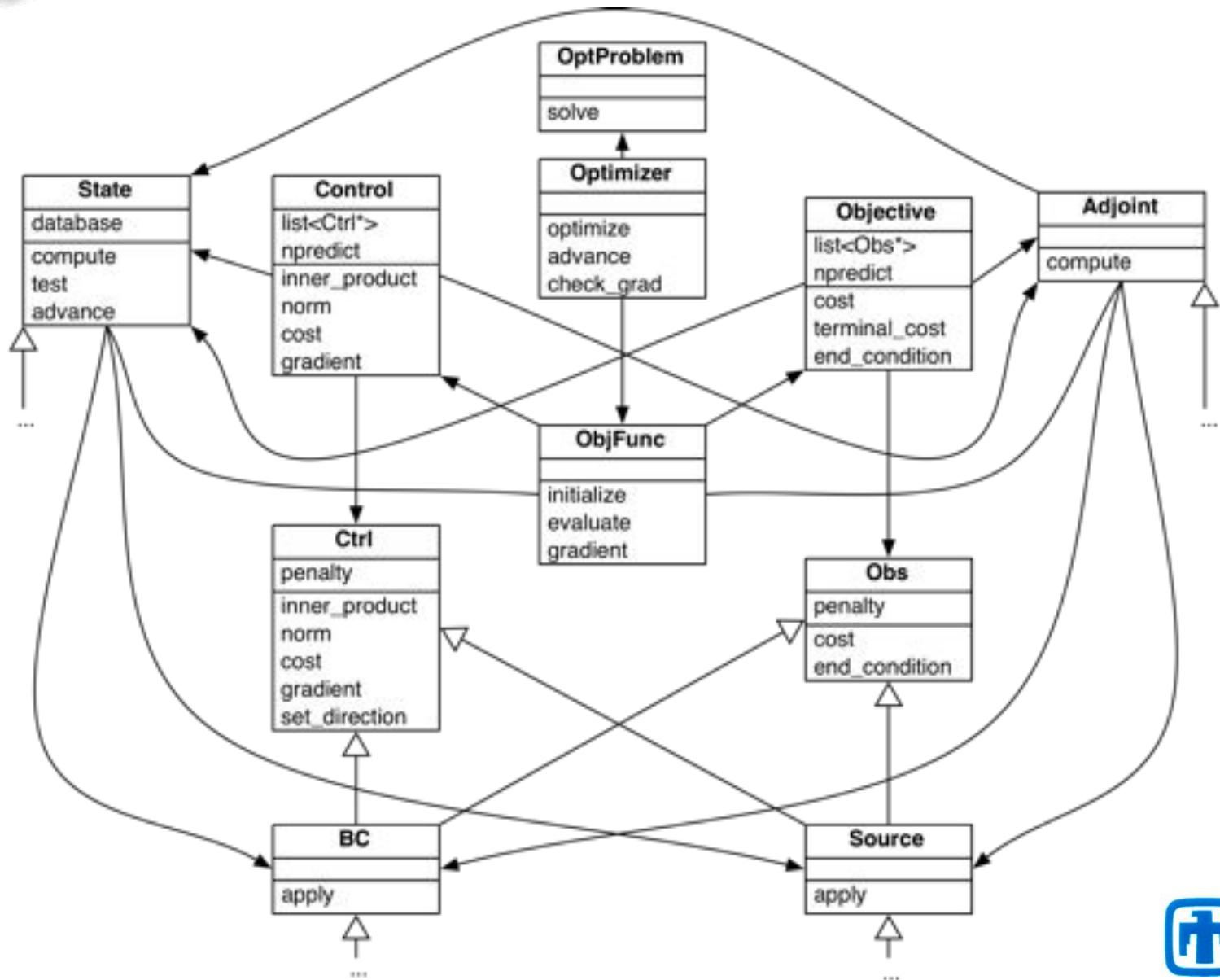


DGM Toolkit

- High-order on unstructured meshes
 - Line, Quad, Tri, Hex elements
- Supports local, p -refinement
- Object-oriented software design
- Physics independent: examples for
 - Compressible Euler & Navier-Stokes (with LES)
 - Incompressible Euler & Navier-Stokes
 - Advection-diffusion, Burgers, Darcy, Helmholtz
- Designed for adjoint-based optimization
 - Steady-state and transient with checkpointing
- MPI with MPI-IO
- Version 0.0 released open-source (Rice)
- Version 1.0 on the way...



DGM Inversion Infrastructure





Ichos

(Greek for sound or Tune)



• Discontinuous Galerkin

- Unstructured meshes
- Variable-order polynomial representation
 - » For both the solution and the media
- Local polynomial de-/refinement
- Curved and non-simplicial elements

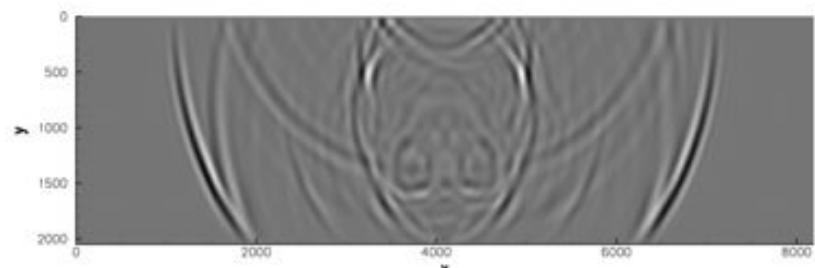
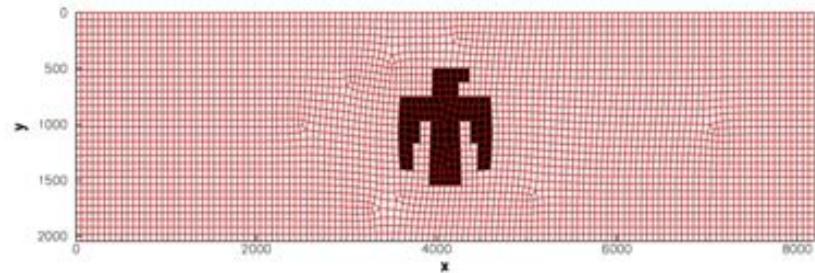
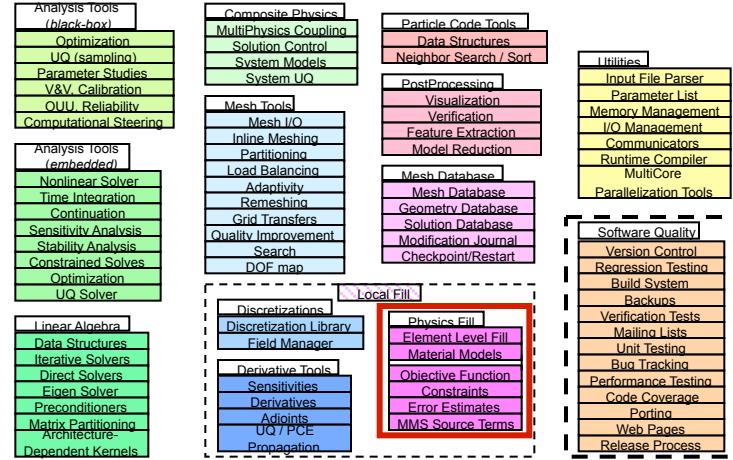
• Component Technology

- Built on DGM Toolkit
 - » Component-based software design for DG
- Agile Components (ModelEvaluator)
 - » Access to Trilinos (OptiPack) and Dakota
- Multiple physics (acoustic, elastic and attenuation)

• Optimization and Inversion

- Transient optimization
- Adjoint-based optimization/inversion
- Simultaneous Source Inversion

Agile Components





Project Management

- SVN/CVS repositories
- Mailman email lists
- Trac project web-site
- Hudson continuous testing
 - Linux
 - Mac
 - Clusters
- Full Doxygen documentation

Hudson

Hudson > ichos > ichos-build-lin

Project ichos-build-lin

Build the dgm and ichos executables for Linux

Build History (32)

Build	Timestamp	Result
#18	Oct 7, 2010 9:58:13 AM	OK
#17	Oct 7, 2010 8:37:44 AM	OK
#16	Oct 7, 2010 4:08:40 AM	OK
#15	Oct 6, 2010 4:09:37 AM	OK
#14	Oct 5, 2010 4:09:23 AM	OK
#13	Oct 4, 2010 4:09:17 AM	OK
#12	Oct 2, 2010 4:08:09 PM	OK
#11	Oct 1, 2010 4:08:36 AM	OK
#10	Sep 30, 2010 4:08:27 AM	OK
#9	Sep 29, 2010 4:08:13 AM	OK
#8	Sep 28, 2010 4:08:13 AM	OK
#7	Sep 22, 2010 4:08:11 AM	OK
#6	Sep 25, 2010 4:08:01 PM	OK
#5	Sep 24, 2010 4:07:26 PM	OK
#4	Sep 23, 2010 3:08:33 PM	OK
#3	Sep 24, 2010 3:04:04 PM	OK
#2	Sep 24, 2010 2:56:30 PM	OK
#1	Sep 24, 2010 2:49:32 PM	OK

Upstream Projects

- ichos-dgm

Downstream Projects

- ichos-build-linux-debug
- ichos-build-linux-parallel
- ichos-build-linux-x64

Permalinks

- Last Build (#18): 1 day, 5 hr ago
- Last stable build (#18): 1 day, 5 hr ago
- Last successful build (#18): 1 day, 5 hr ago

for all for Jenkins

Page generated: Oct 8, 2010 3:36:44 PM
Hudson ver. 3.372

Ichos Full-Waveform Inversion

logged in as scsll | Logout | Preferences | Help/Guide | About Trac | Search

Wiki | Timeline | Readme | Browse Source | View Tickets | New Ticket | Search | Admin | Start Page | Index | History | Last Change | Rename page

Project Statement:

Develop software and algorithms to perform large-scale full-wavefield inversion.

Areas of interest for research and development:

- Inversion methods and algorithms for large-scale seismic inversion
- Discontinuous-Galerkin methods
- Agile Components
- Inversion for anelastic

Some useful information

1. Basic Information
2. Teleconference

Main Page | Related Pages | Namespaces | Classes | Files | Q+ Search

Pidima Discontinuous Galerkin Application

Pidima - 1.0

Table of Contents

- Overview
- Optimization
- Directory Structure
- Executables: Drivers, Utilities and Scripts
- Users Guide
- Copyright
- Author
- Contributors

Overview

This is the Pidima application which is build on the DGM library which supports hybrid, one-, two- and three-dimensional discretizations for Adv-Diff, Burgers, Euler, LinEuler_quasi, Navier_Stokes, Euler3d, Navier_Stokes3d, Acoustic, ConAcoustic, AnAcoustic, Elastic, DarcyFlow, and PorousMediaequations. Note that not all PDEs can be solved at all spatial dimensions so see the particular module to learn more.

Pidima is designed using an object-oriented approach to PDE simulation and optimization/inversion. The class hierarchy begins at the highest level with the Problem class which defines a simple forward problem on a space-time Domain, Ω . One can also derive off of Problem to generate optimization, control, or error estimation Problems and examples of each are provided.

The next level in the hierarchy is the Domain class that holds a space-time domain that logically contains a prescribed set of physics. The physics must be an equivalent first-order PDE suitable for discontinuous Galerkin methods of the form

$$U_{,t} + F_{,x}(U) - F_{,x}^*(U) = S$$

where U is the solution vector, which in DGM notation is called a vField (short for vector Field). The convective flux in the x^1 direction is $F_{,x}(U)$ and the diffusive (or viscous) flux in the x^1 direction is $F_{,x}^*(U)$. Note that derivatives are denoted by subscripts following a comma with summation over repeated indices and $_{,t}$ denotes time.

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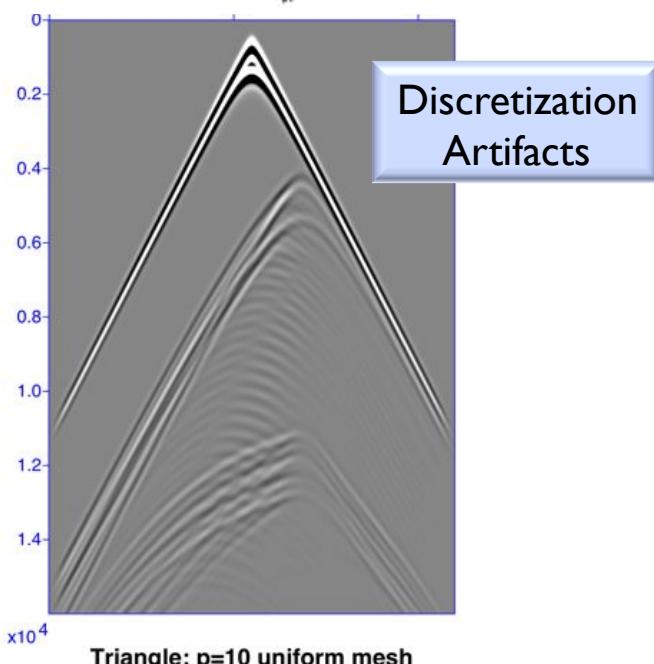
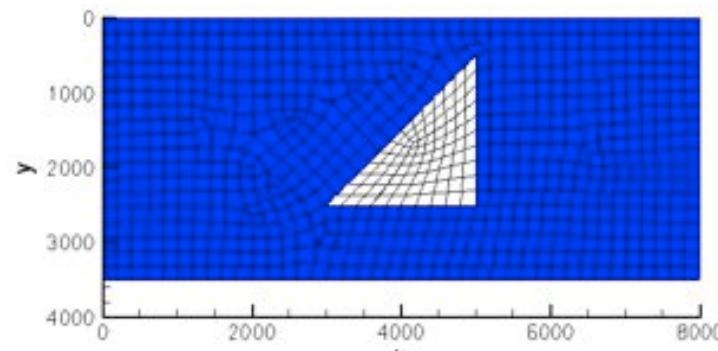
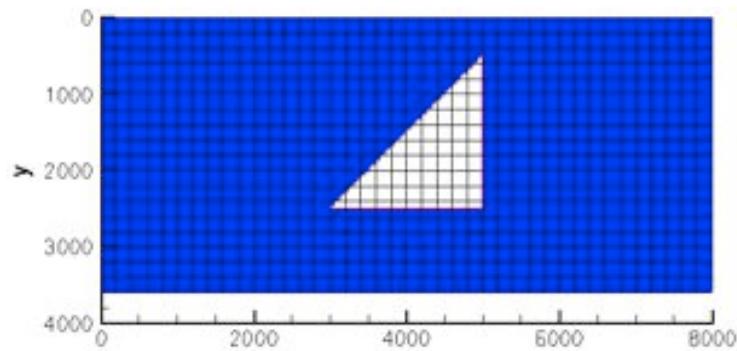


Forward Modeling

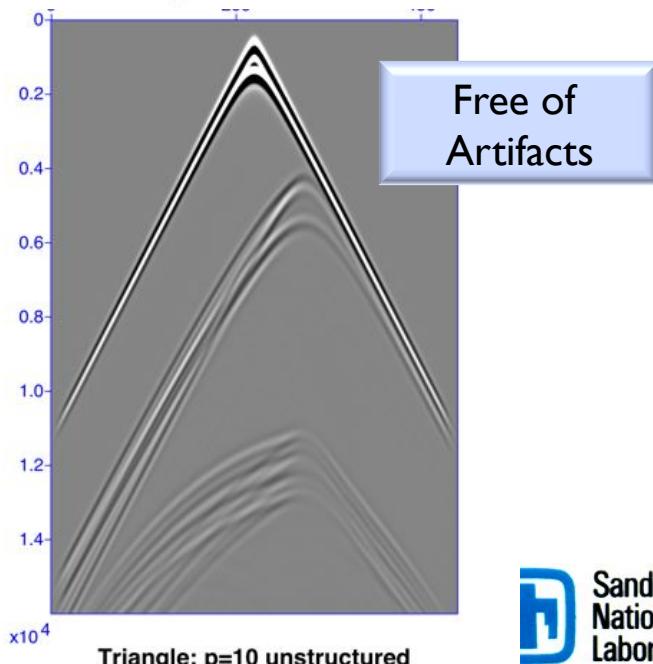


Discretization Challenges

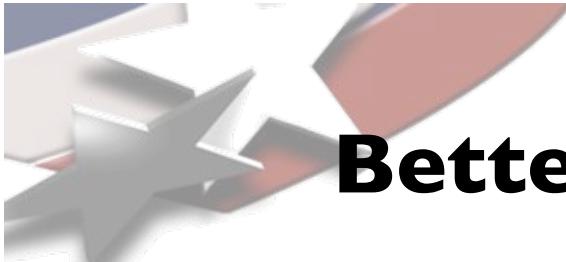
Traditional structured meshes have difficulty capturing geological features accurately



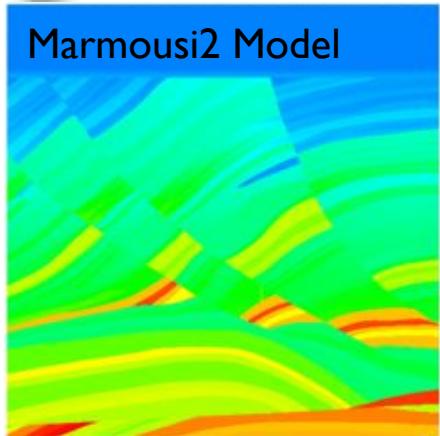
Discretization
Artifacts



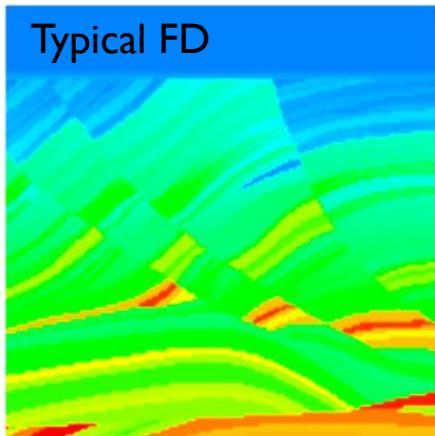
Free of
Artifacts



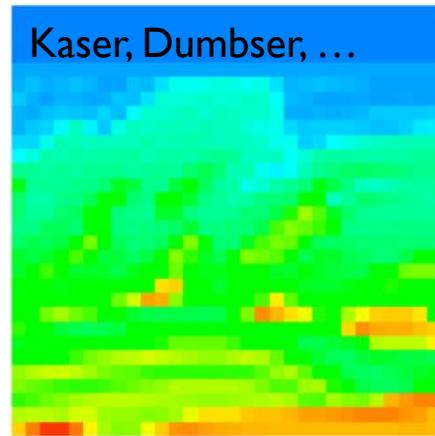
Better Media Representation



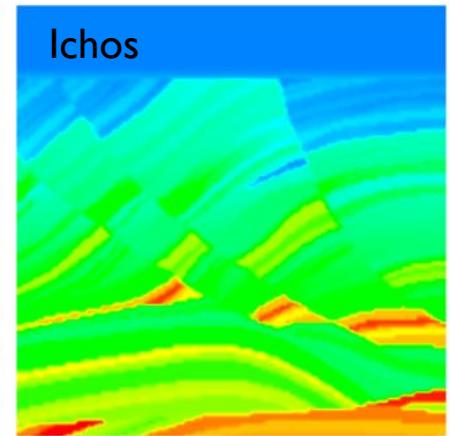
$h = 1.25m$



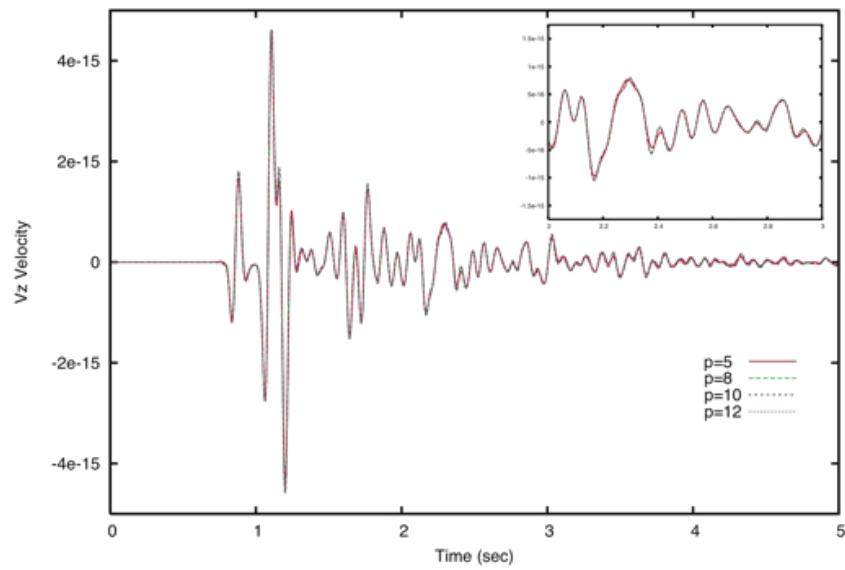
$h = 20m$



$h = 100m, Mp = 0$

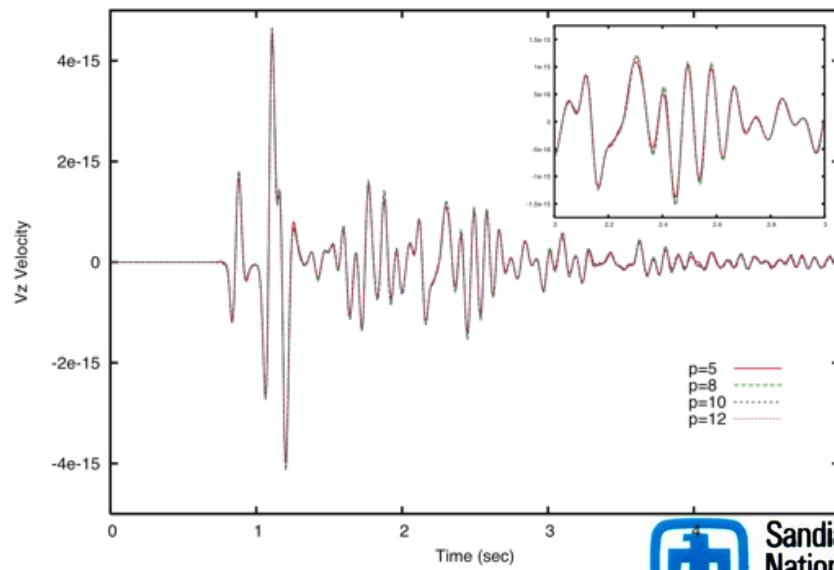


$h = 100m, Mp = 8$



13

$h = 100m, Mp = 0$



$h = 100m, Mp = 8$

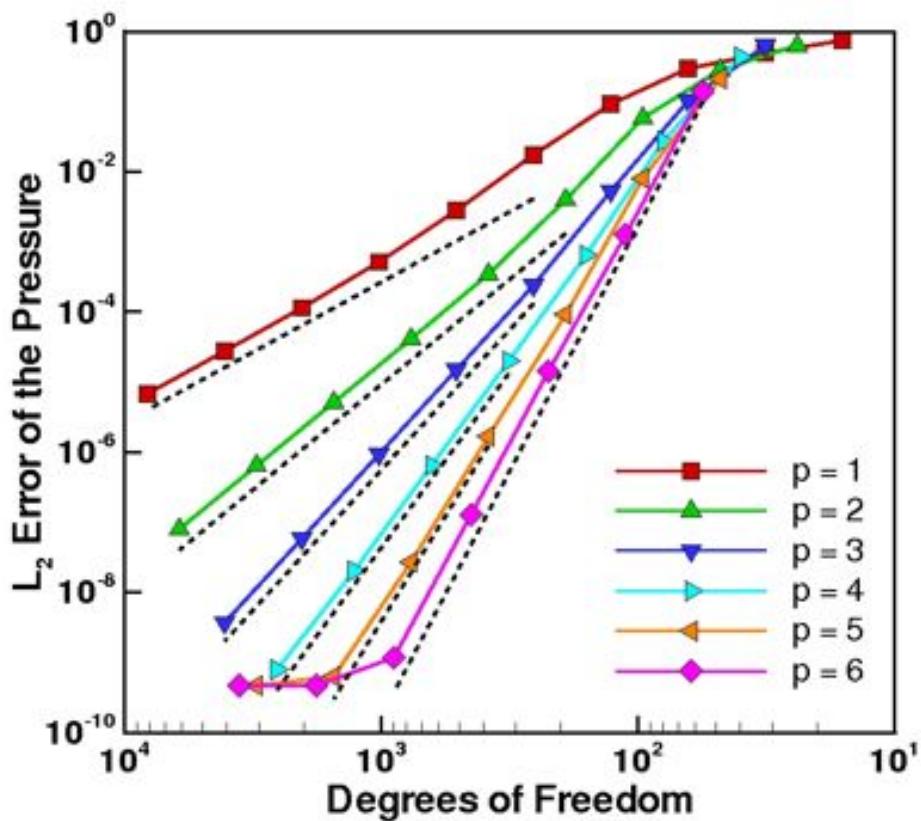


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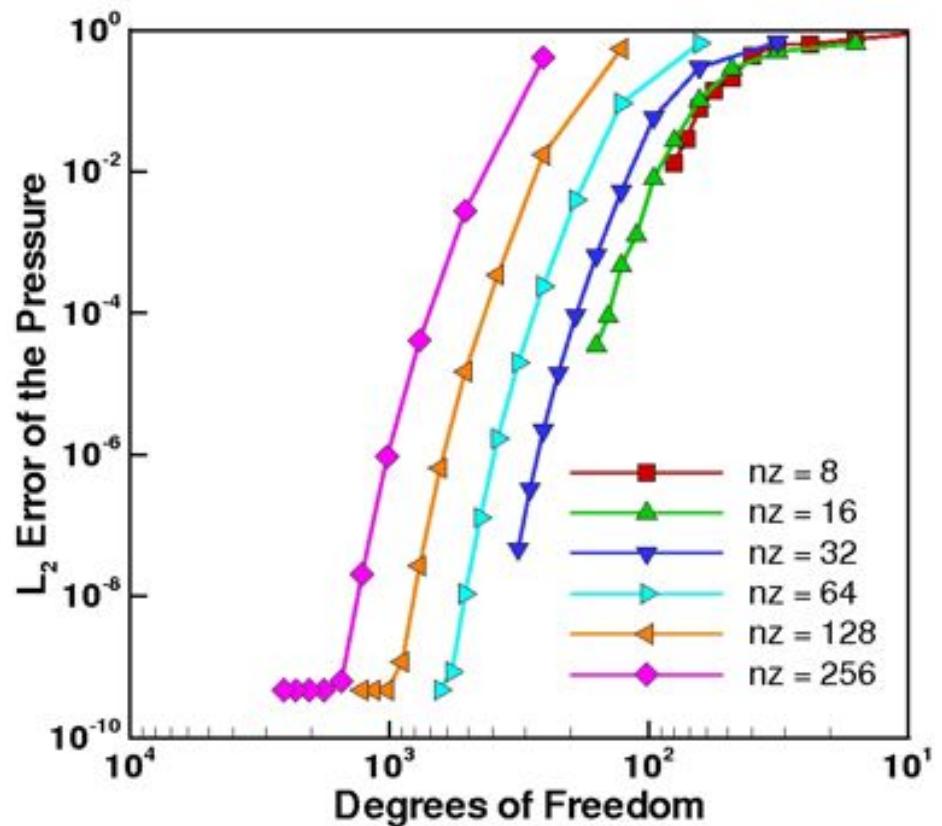


Code Verification

Mesh Refinement



Polynomial Refinement



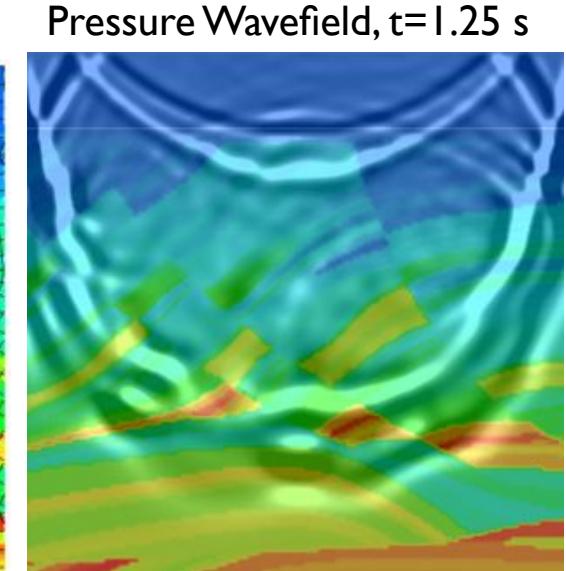
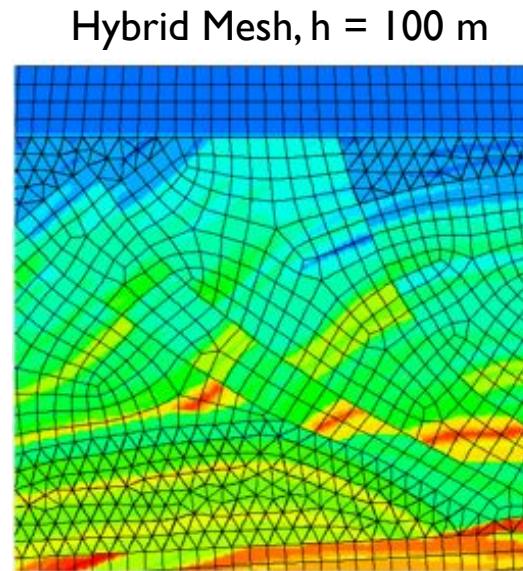
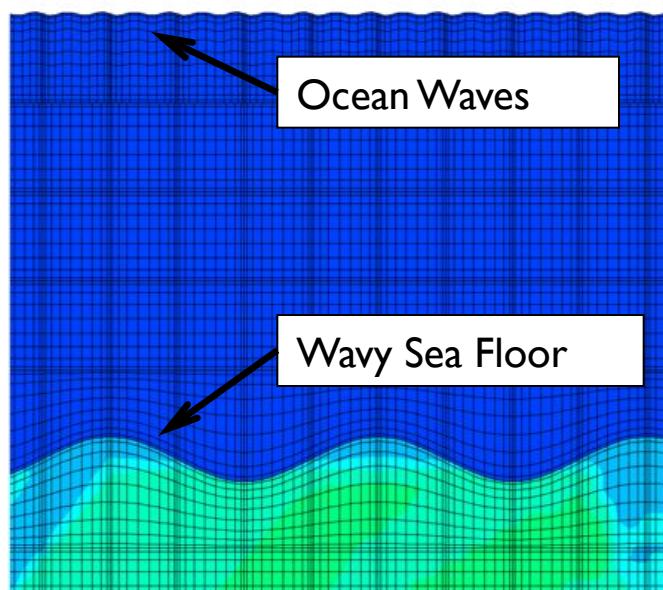
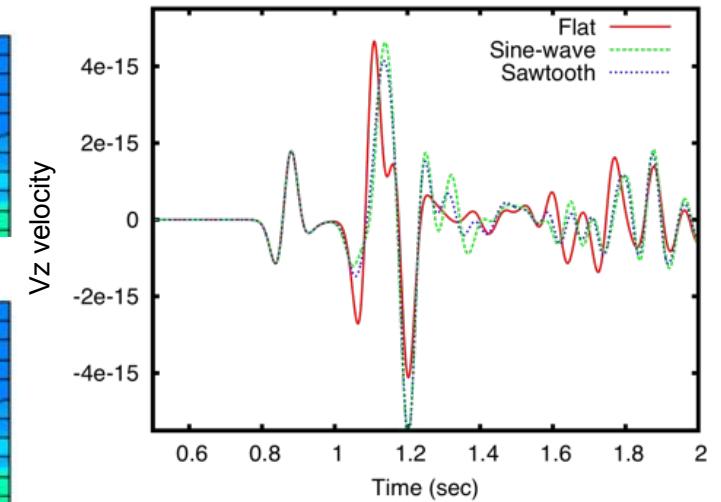
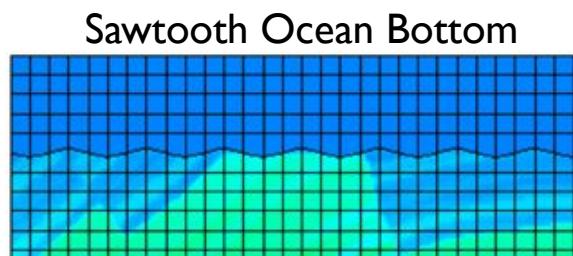
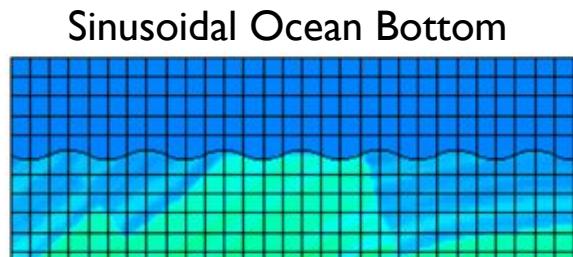
See: Ober, Collis, van Bloemen Waanders, Marcinkovich, SEG 2009.

Topology Capturing

- **For example:**

- Ocean floor
- Faults
- Salt structures
- Even ocean waves...

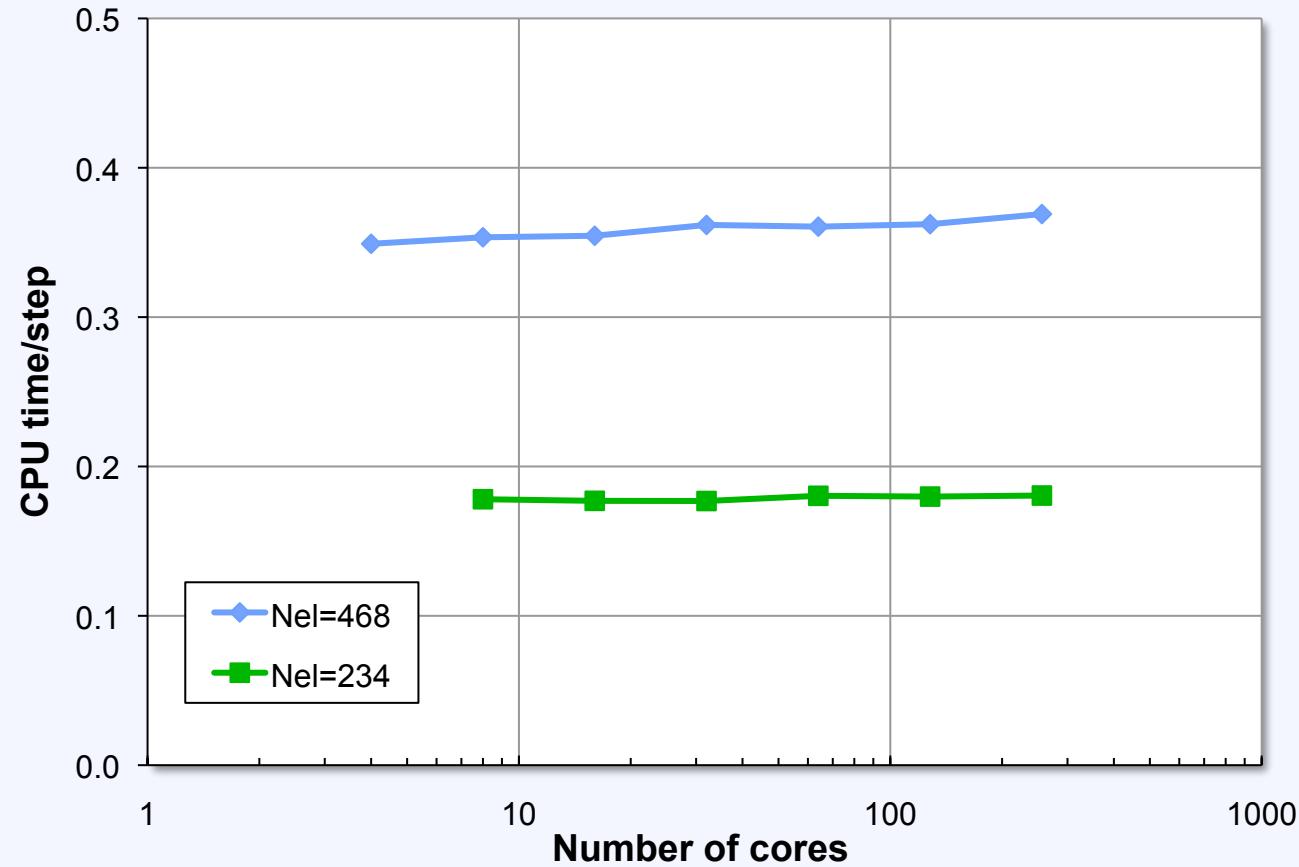
- **Elastic and Acoustic**



Ichos Performance



Weak Parallel Scaling on RedSky



Parallel efficiency = 84%



Seismic Inversion



Inversion Formulation

- PDE constrained optimization problem

$$\min_{\beta \in \mathcal{B}} \frac{1}{2} \sum_{r=1}^{N_r} \int_0^T \int_{\Omega} \xi_r(\mathbf{x}) \left(p(\mathbf{x}, t) - \sum_{s=1}^{N_s} \omega_s \tilde{p}(\mathbf{x}, t) \right)^2 d\mathbf{x} dt$$

- Subject to acoustic wave equations in conservative form

$$\beta \frac{\partial p}{\partial t} + \nabla \cdot \mathbf{v} = \beta \phi \quad \text{in } \Omega \times (0, T]$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \nabla p = \mathbf{0} \quad \text{in } \Omega \times (0, T]$$

$$p(\mathbf{x}, 0) = 0 \quad \text{for } \mathbf{x} \in \Omega$$

- Approach $\mathbf{v}(\mathbf{x}, 0) = 0$ for $\mathbf{x} \in \Omega$

- Simultaneous Source Inversion

- » Krebs et al. (2009)

- » Phase Encoding – Romero et al. (2000)

- » Speedup of 50x in 2D; ~2000x in 3D

- Gradient-based optimization...

$\beta = 1/(\rho c^2)$ – compressibility

\mathcal{B} = space of admissible media

ρ = mass density

c = wave speed

Ω = computational domain

T = time horizon

N_r = number of receivers

N_s = number of sources

$\omega_s \in \{-1, 1\}$ – random phase encoding

$\phi = \sum_{s=1}^{N_s} \omega_s w(t) \xi_s(\mathbf{x})$ – encoded sources

\tilde{p} = measured pressure data

ξ_r, ξ_s = spatial kernel for receiver, source



Model Problem – Marmousi2*

- **True Model**

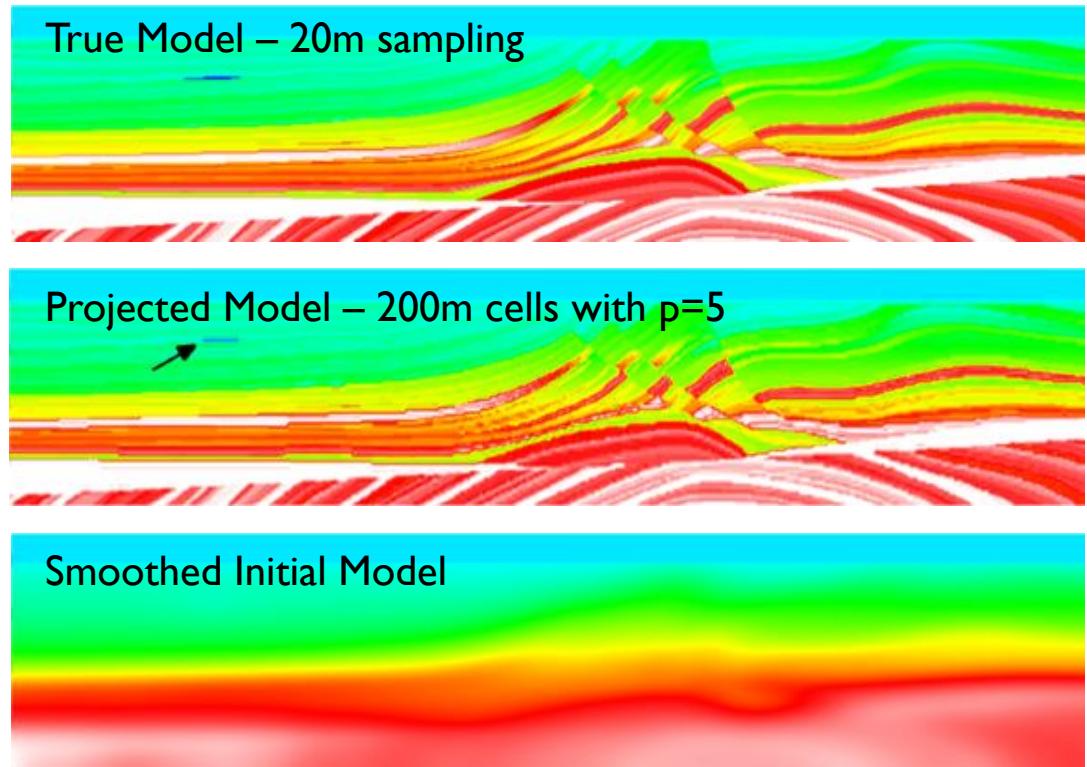
- 500m sponge on bottom
- 500m sponge on ends
- Fixed spread of receivers
 - $x_r = r*200+500 \ (0 \leq r \leq 75)$
 - $y_r = 100m$
- Uniformly spaced sources
 - $x_s = s*1000m \ (1 \leq s \leq 15)$
 - $y_s = 300m$

- **Projected Model**

- $80 \times 20 = 1600$ elements
- $= 57.6k$ dof
- FD 20m cells = 160k dof

- **Smoothed Initial Model**

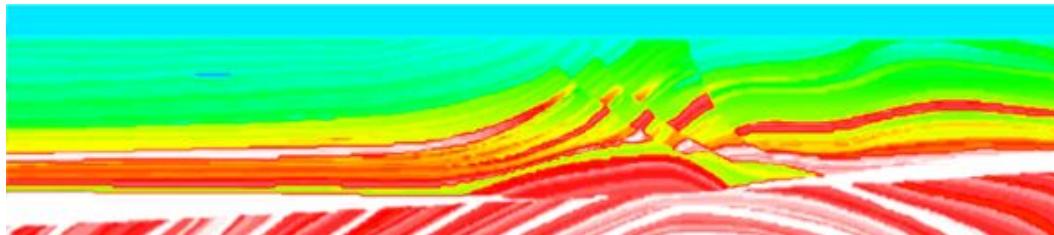
- Damped least-squares method



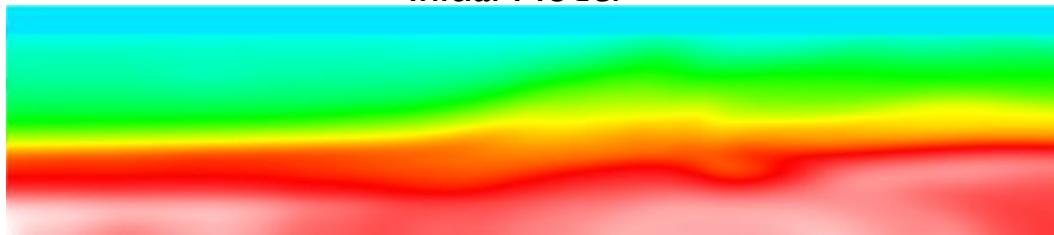
* Martin, G. S., R. Wiley, and K. J. Marfurt, 2006, Marmousi2: An Elastic Upgrade for Marmousi: The Leading Edge, **25**, 156–166.

Acoustic Inversion

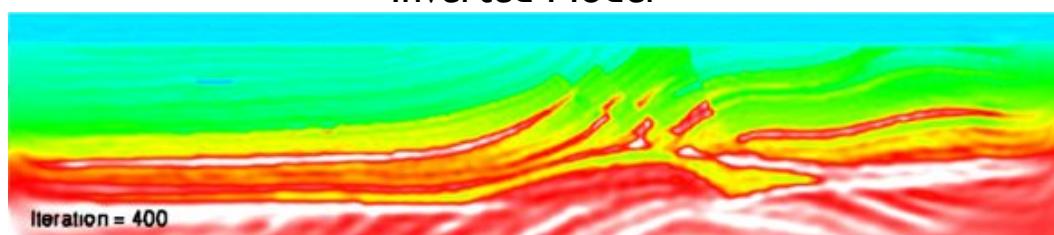
Marousi2: True Model ($h=200\text{m}$, $p=5$)



Initial Model

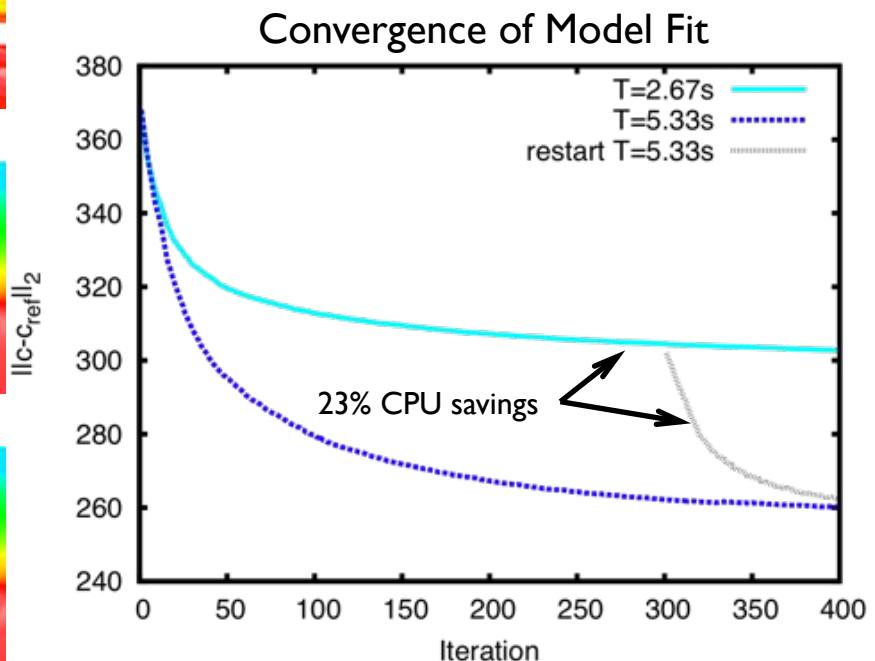
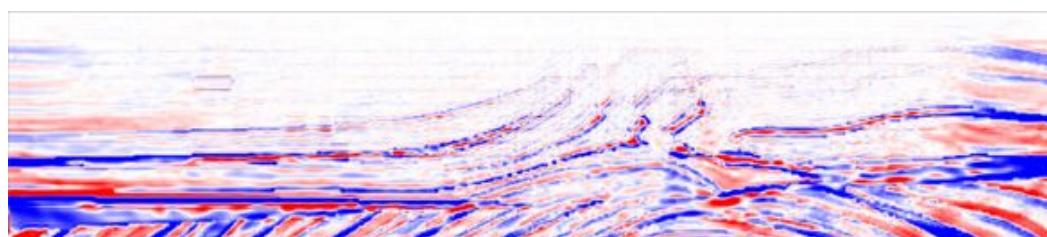


Inverted Model



Iteration = 400

Inverted - True

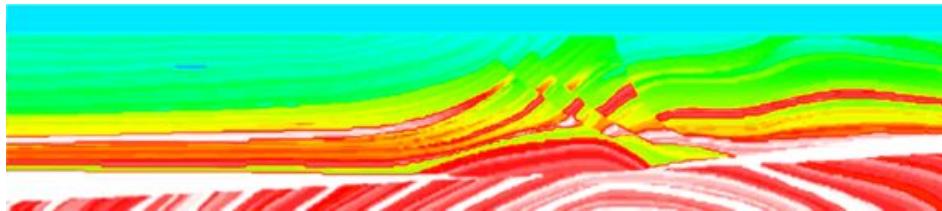


See: Collis, Ober, van Bloemen
Waanders, SEG 2010.

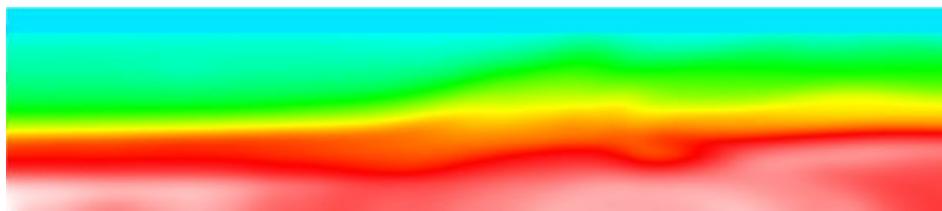


Acoustic Inversion

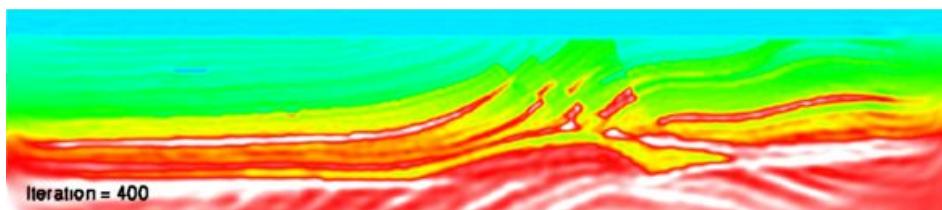
Marmousi2 True Model ($h = 200\text{m}$; $p = 5$)



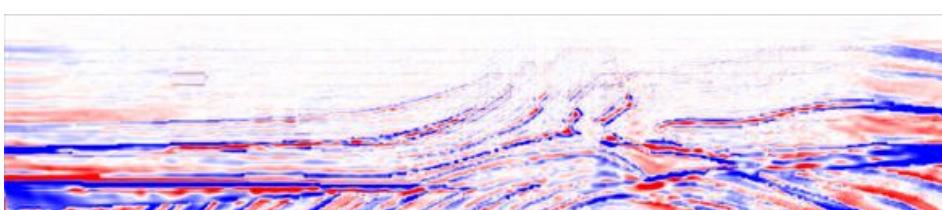
Initial Model



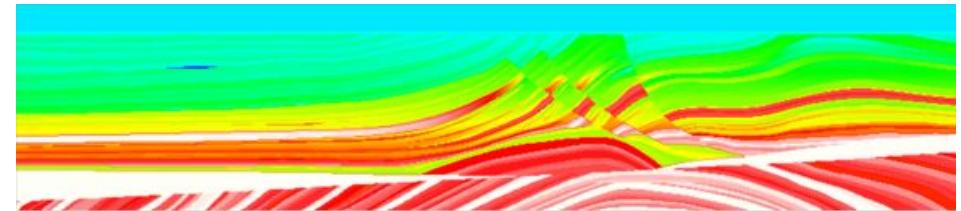
Inverted Model



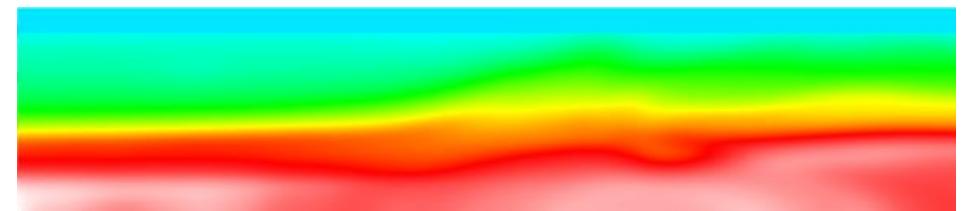
Inverted - True



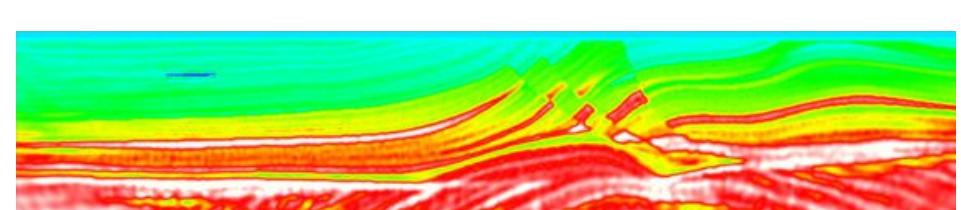
FD SSI – Krebs et al. (2009) ($h = 20\text{m}$; 8th order)



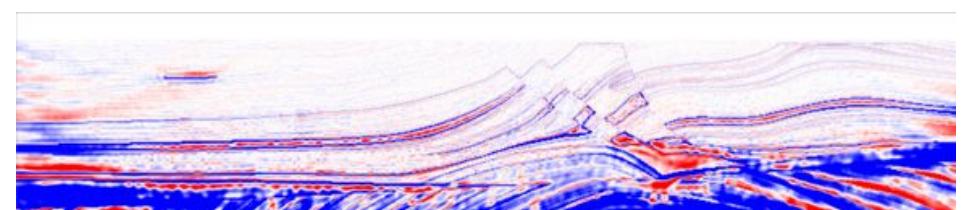
Initial Model

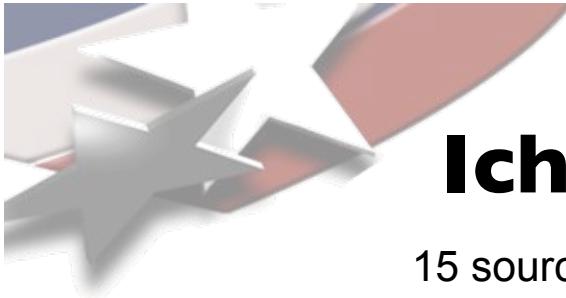


Inverted Model



Inverted - True

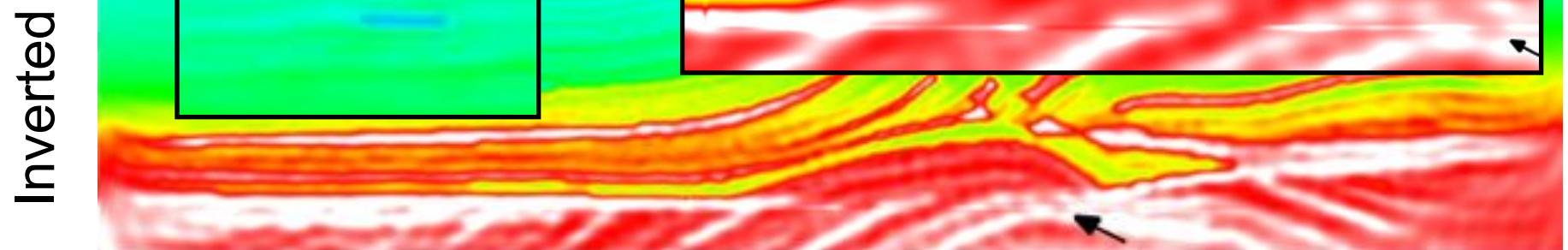
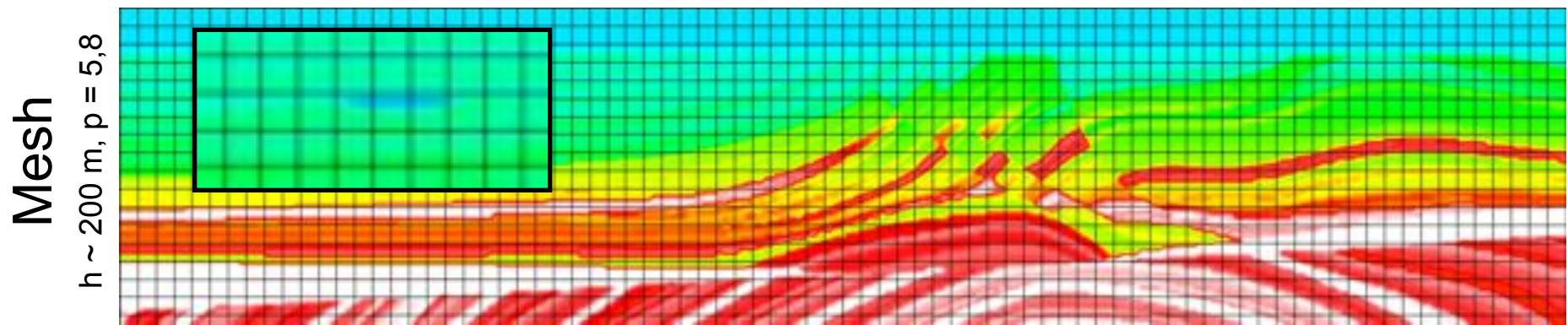




Ichos Acoustic Inversion

15 sources at 1000m spacing, 76 receivers at 200m spacing

Uniform Mesh

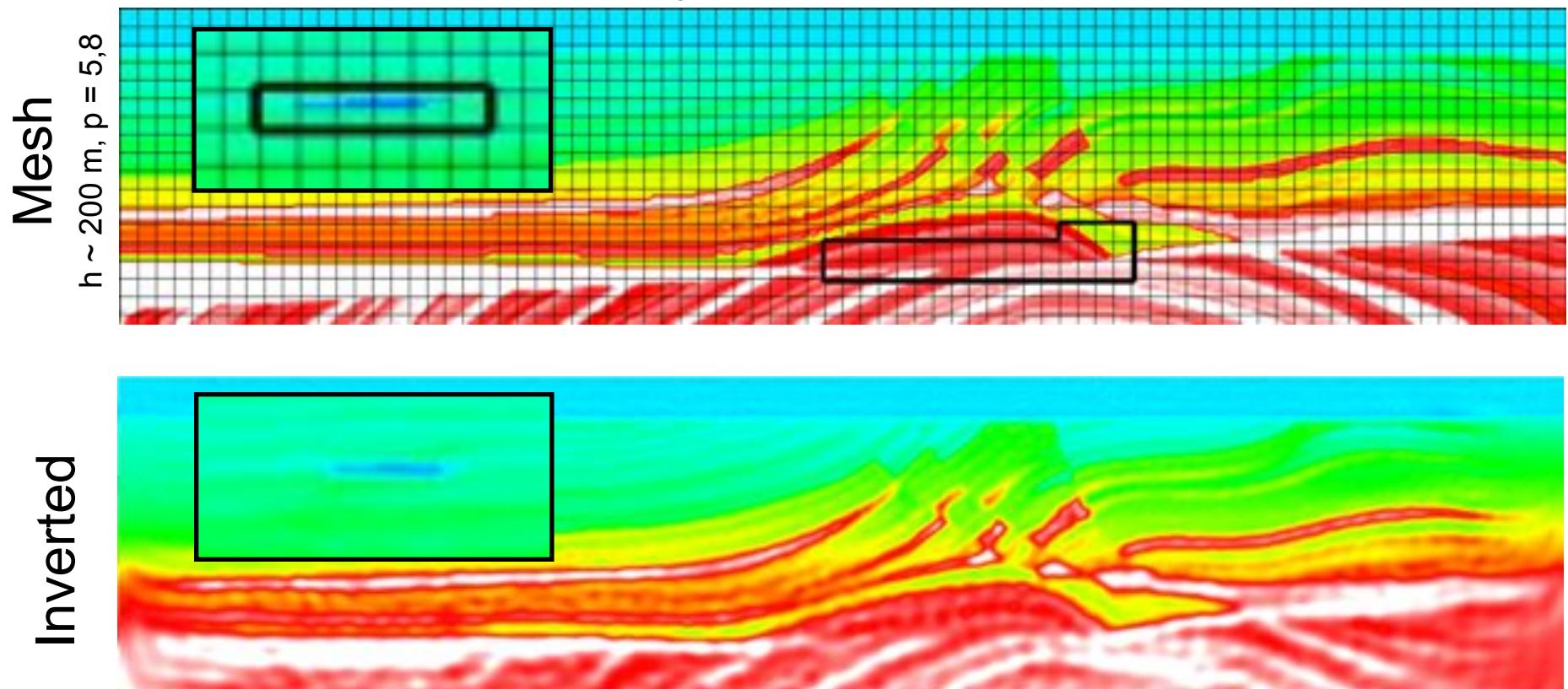


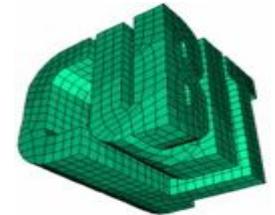


Ichos Acoustic Inversion

15 sources at 1000m spacing, 76 receivers at 200m spacing

Local Polynomial Refinement

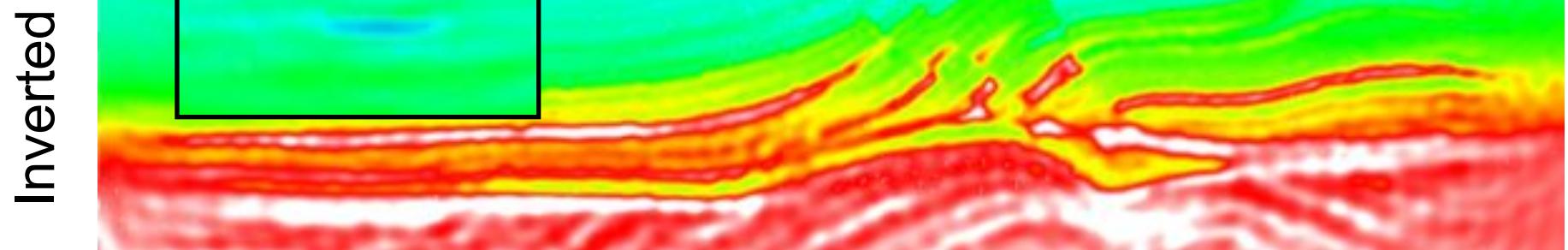
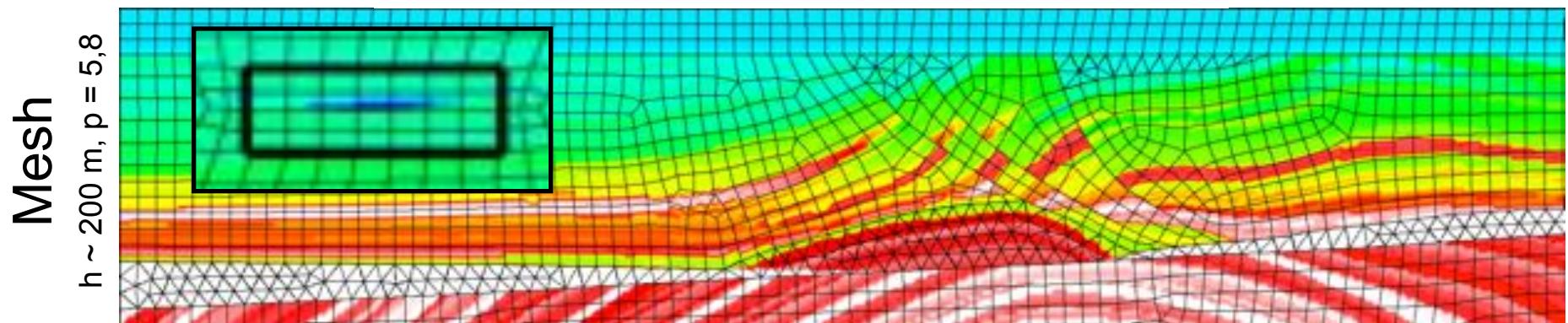




Ichos Acoustic Inversion

15 sources at 1000m spacing, 76 receivers at 200m spacing

Unstructured Mesh





Closing Comments

- Ichos demonstrates the potential of high-order DG methods
 - Based on flexible DGM toolkit + Trilinos + Dakota
 - Acoustic and Elastic wave propagation in 1d, 2d, and 3d
 - Infrastructure in place for gradient-based inversion in 1d, 2d and 3d
- Proof-of-principle inversion studies:
 - Uniform structured mesh: similar accuracy, fewer degrees of freedom
 - Local p-refinement improves representation of localized geological features
 - Unstructured meshes accurately capture discontinuous material interfaces (faults, ocean bottom, salt structures).
 - Simultaneous Source Inversion (SSI) offers significant algorithmic speedup
- Component-based approach: key to scalability, sustainability and agility...



Extras



Linear Elastodynamics

$$\rho(\mathbf{x}) \frac{\partial v_i(\mathbf{x}, t)}{\partial t} - \frac{\partial \sigma_{ij}(\mathbf{x}, t)}{\partial x_j} = f_i(\mathbf{x}, t) + \frac{\partial m_{ij}^a(\mathbf{x}, t)}{\partial x_j},$$

$$\frac{\partial \sigma_{ij}(\mathbf{x}, t)}{\partial t} - \lambda(\mathbf{x}) \frac{\partial v_k(\mathbf{x}, t)}{\partial x_k} \delta_{ij} - \mu(\mathbf{x}) \left[\frac{\partial v_i(\mathbf{x}, t)}{\partial x_j} + \frac{\partial v_j(\mathbf{x}, t)}{\partial x_i} \right] = \frac{\partial m_{ij}^s(\mathbf{x}, t)}{\partial t},$$

Stress tensor: $\sigma_{ij}(\mathbf{x}, t)$

Particle velocity: $v_i(\mathbf{x}, t)$

Force vector: $f_i(\mathbf{x}, t)$

Moment tensor: $m_{ij}(\mathbf{x}, t)$

Mass density: $\rho(\mathbf{x})$

Lame's first parameter: $\lambda(\mathbf{x}) = \rho(V_p^2 - 2V_s^2)$

Shear modulus: $\mu(\mathbf{x}) = \rho V_s^2$

Compressional wave speed: $V_p(\mathbf{x})$

Shear wave speed: $V_s(\mathbf{x})$



Seismic Explosion

- Source composed with sum of shots

$$\phi(\mathbf{x}, t) = \sum_{s=1}^{N_s} \omega_s w(t) \xi_s(\mathbf{x})$$

- Encoding $\omega_s \in \{-1, 1\}$

- Ricker wavelet

$$w(t) = (1 - 2\pi^2 f_p^2 (t - t_0)) \exp(-\pi^2 f_p^2 (t - t_0)^2)$$

- Gaussian spatial “ball” or Delta function

$$\xi_s(\mathbf{x}) = \left(\frac{1}{\sigma \sqrt{2\pi}} \right)^N \exp \left(\frac{|\mathbf{x} - \mathbf{x}_s|^2}{2\sigma^2} \right)$$

$$\xi_s(\mathbf{x}) = \delta(\mathbf{x} - \mathbf{x}_s)$$