SAND2013-9837P

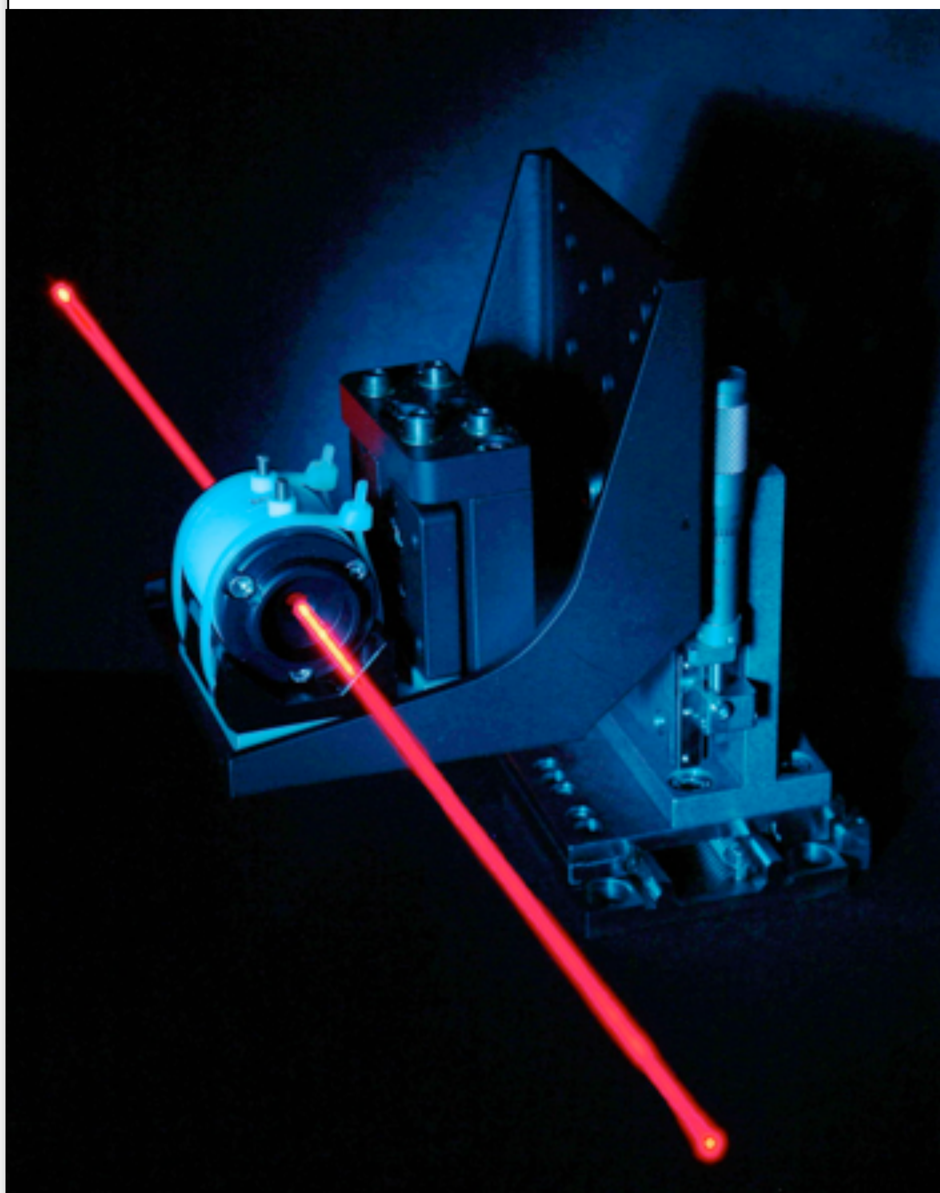
Robust, self-consistent, closed-form quantum tomography

John King Gamble
Sandia National Laboratories

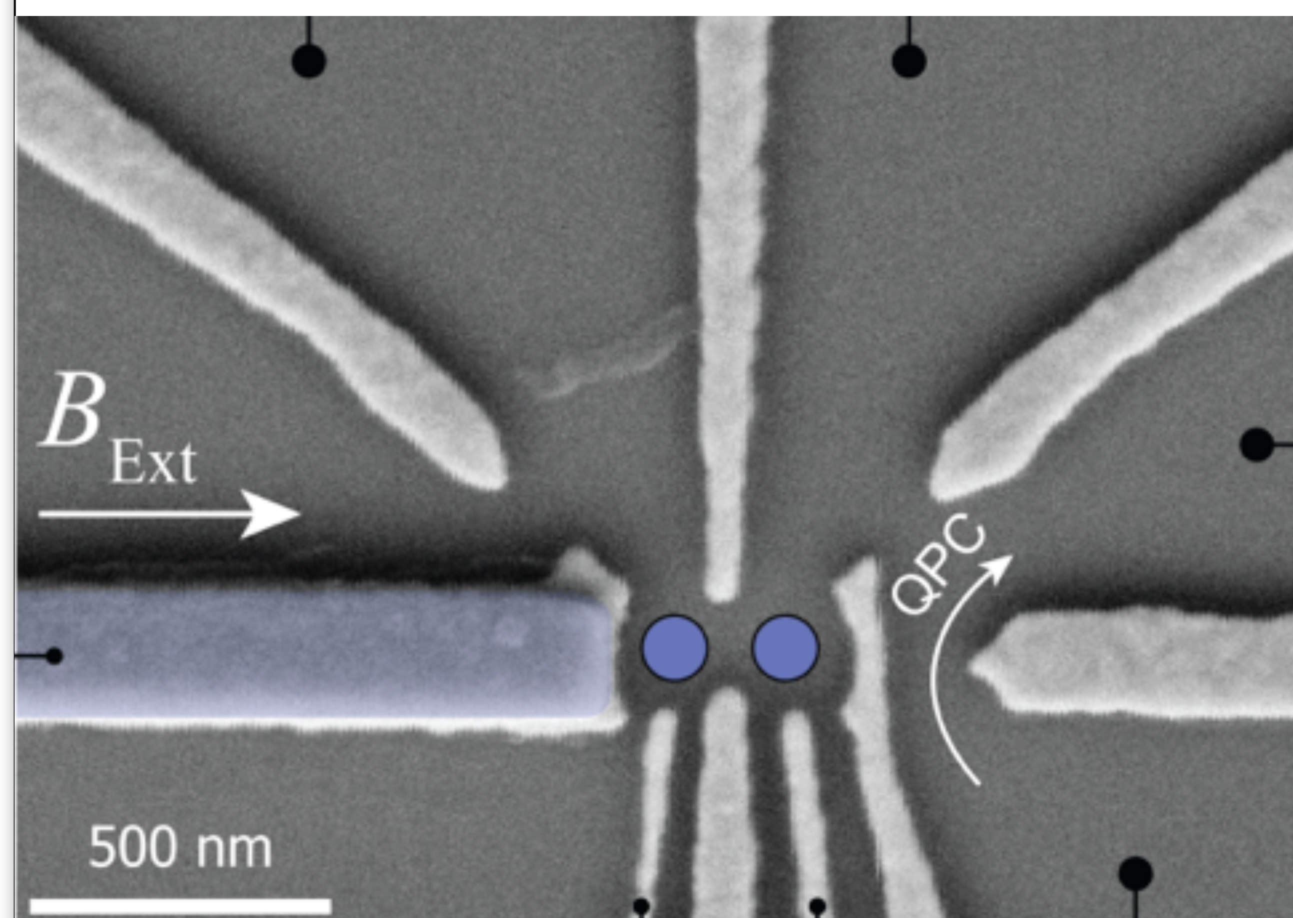
arXiv:1310.4492

Quantum tomography

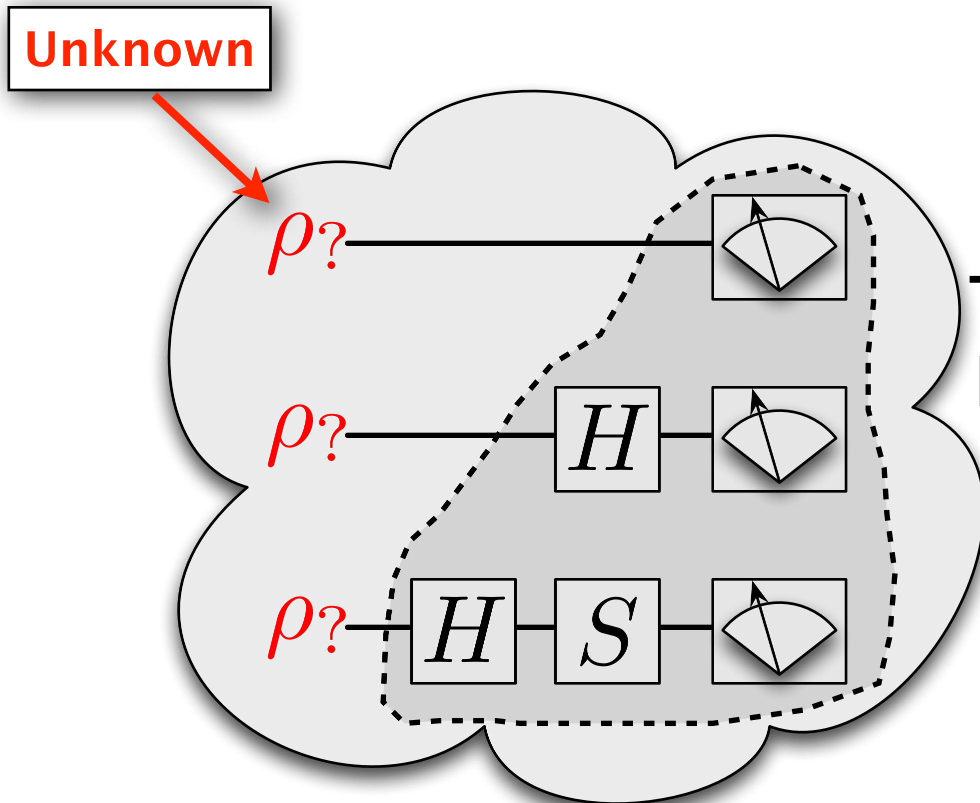
Invented here



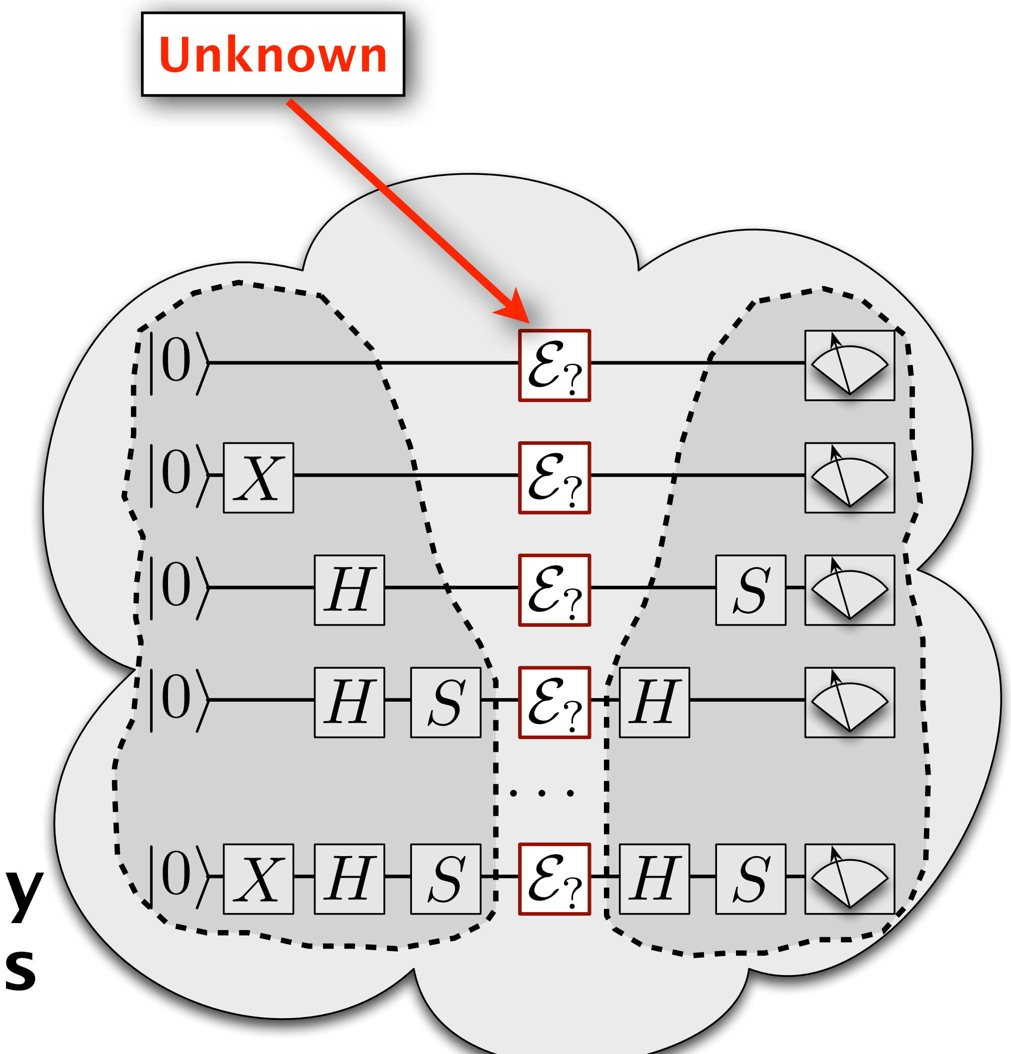
We need it here



The problem with tomography

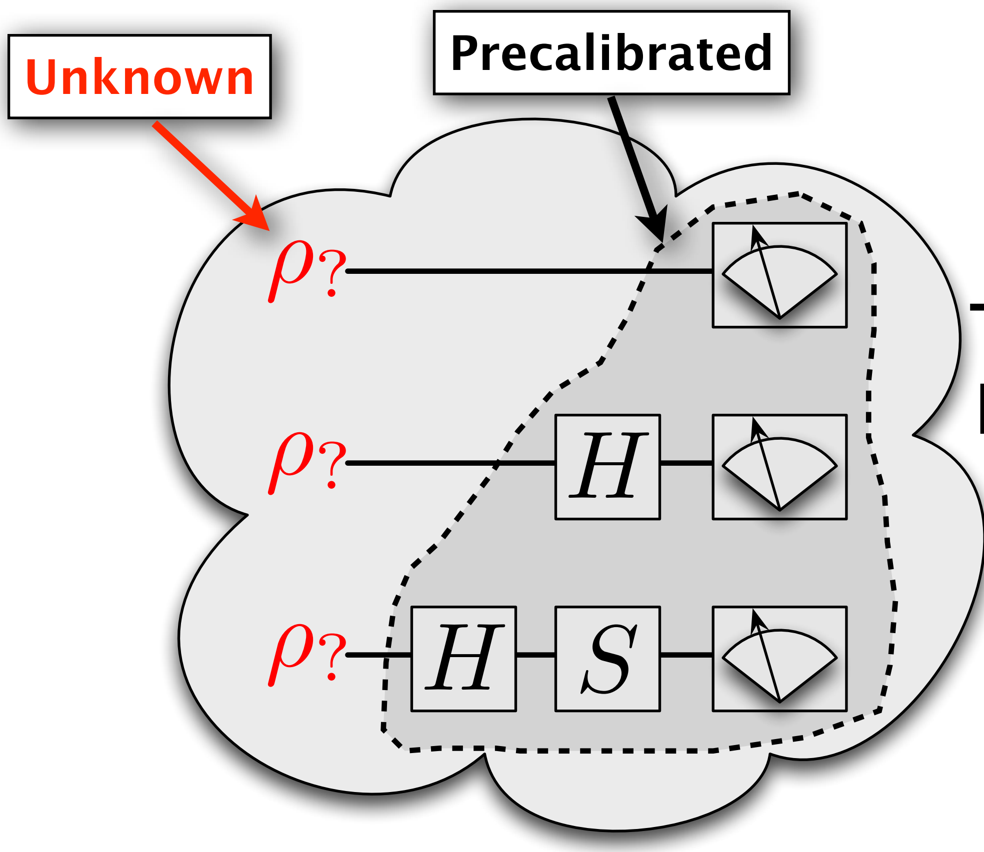


State Tomography Experiments



Process Tomography Experiments

The problem with tomography



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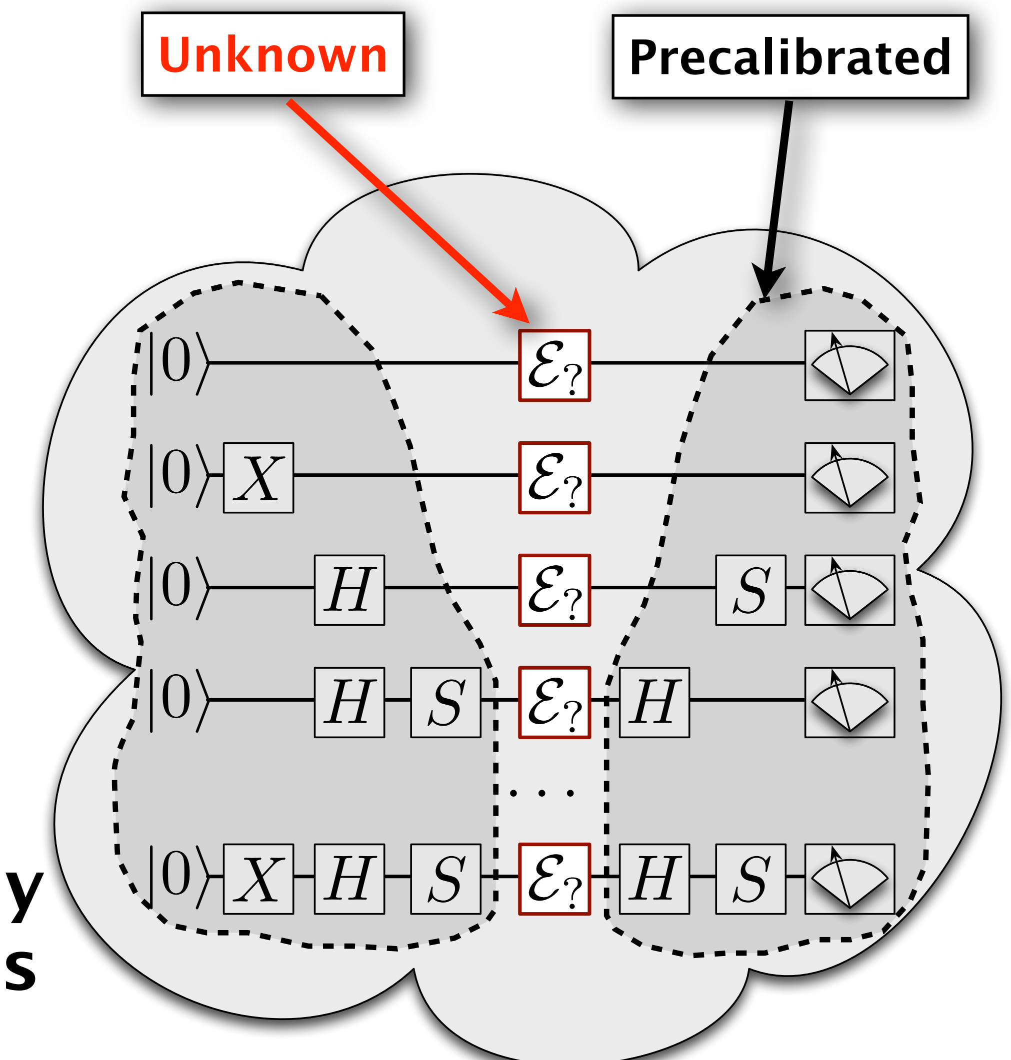

Process Tomography Experiments

Realistic model of a quantum experiment

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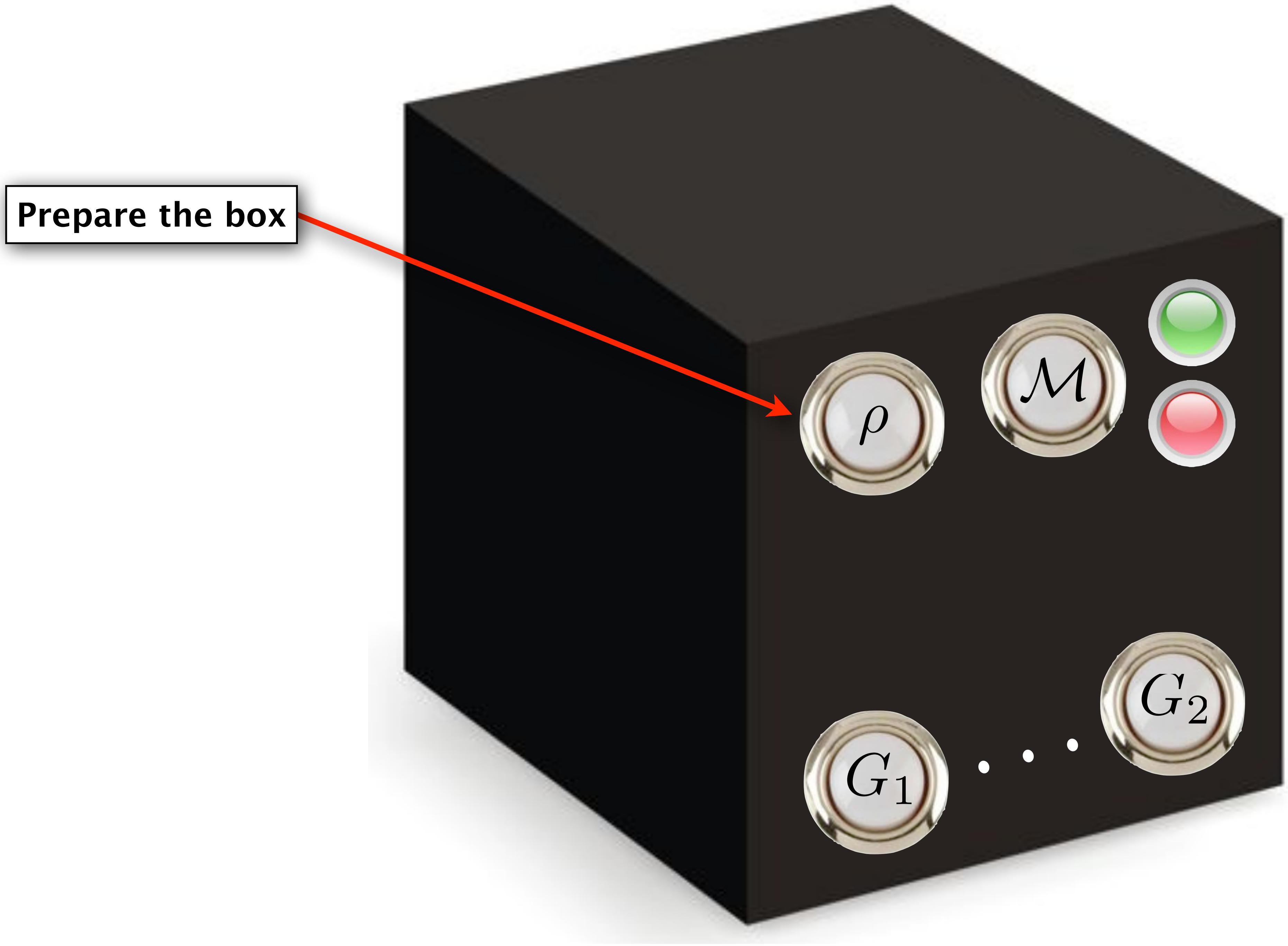
Realistic model of a quantum experiment



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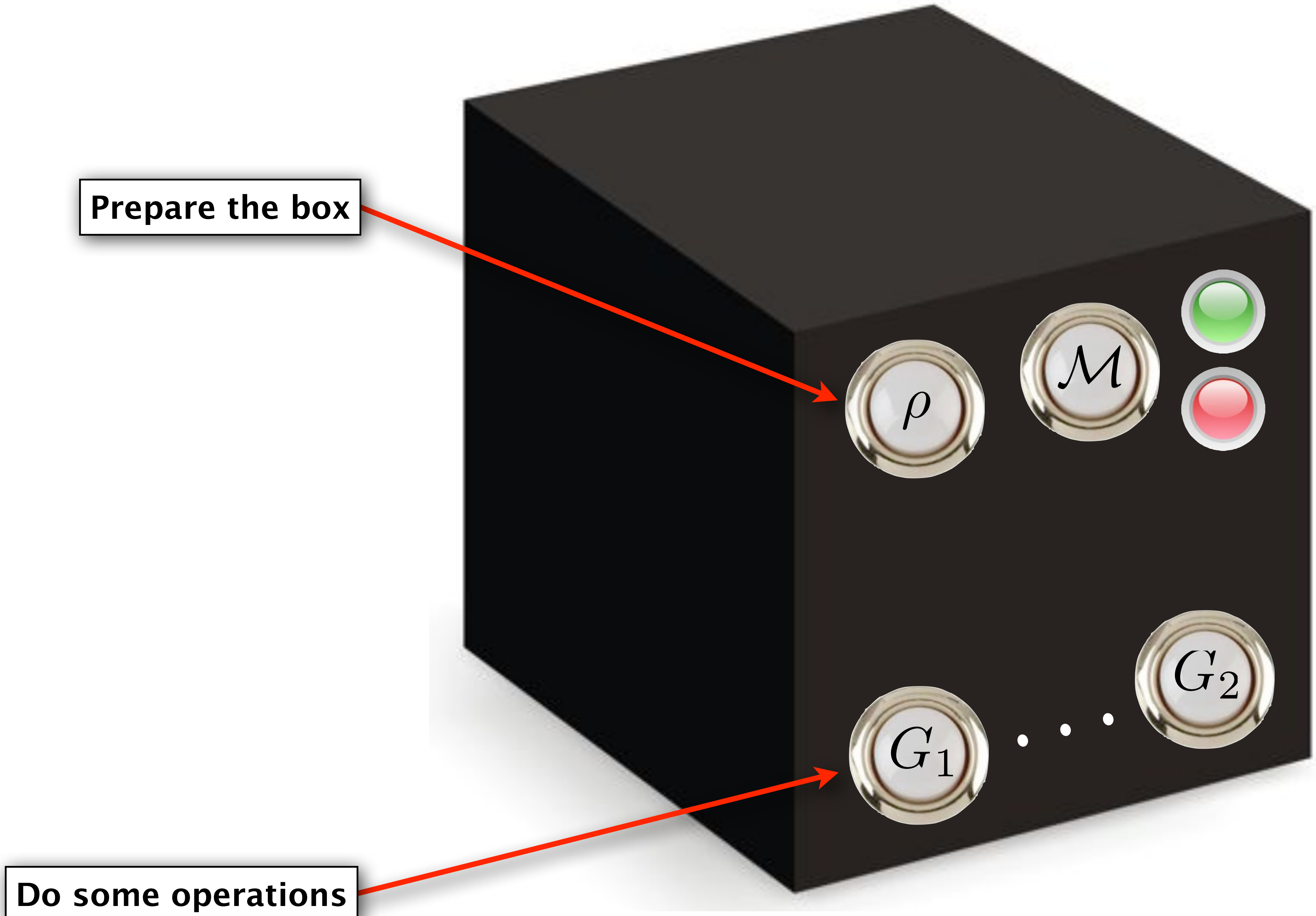

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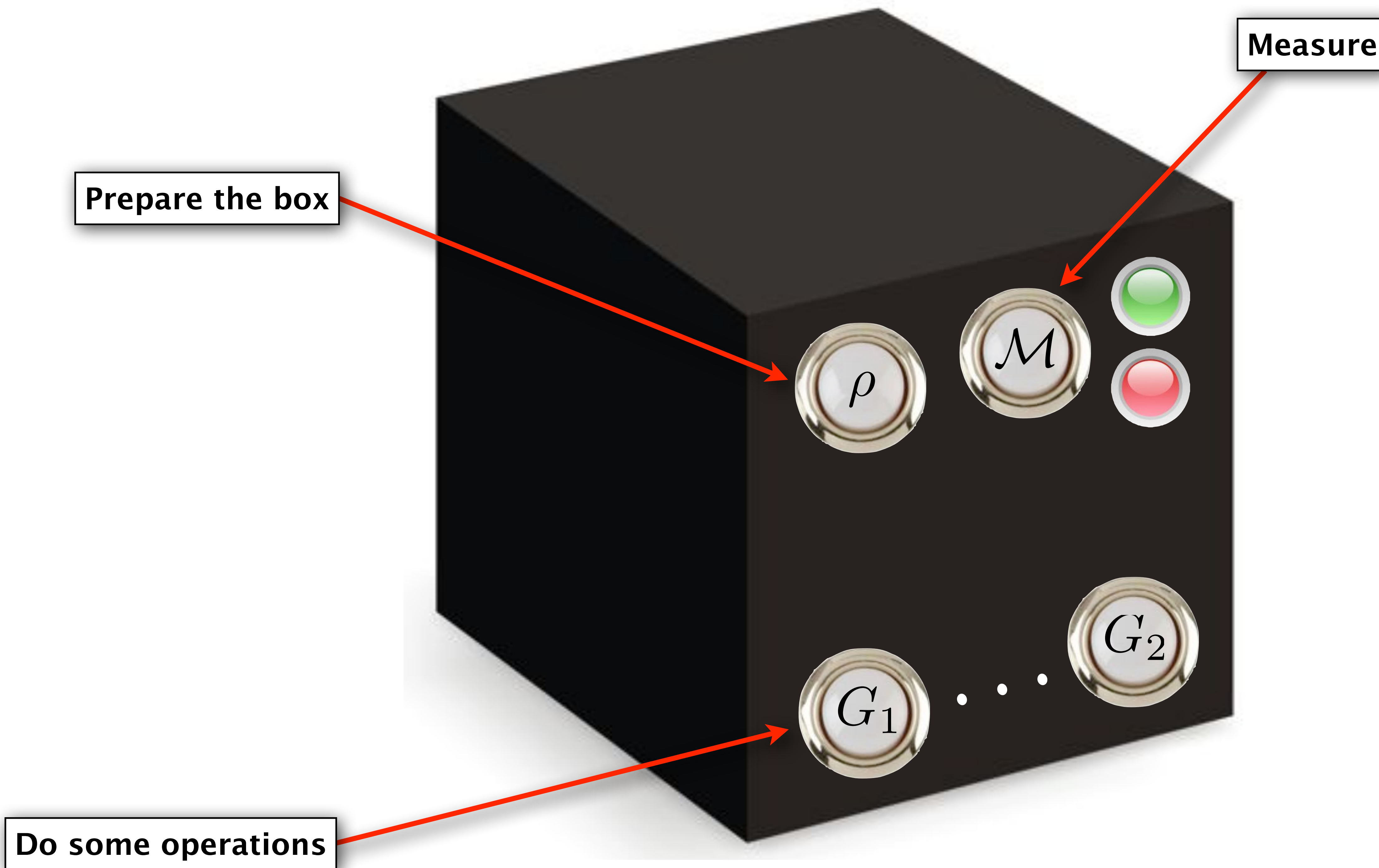

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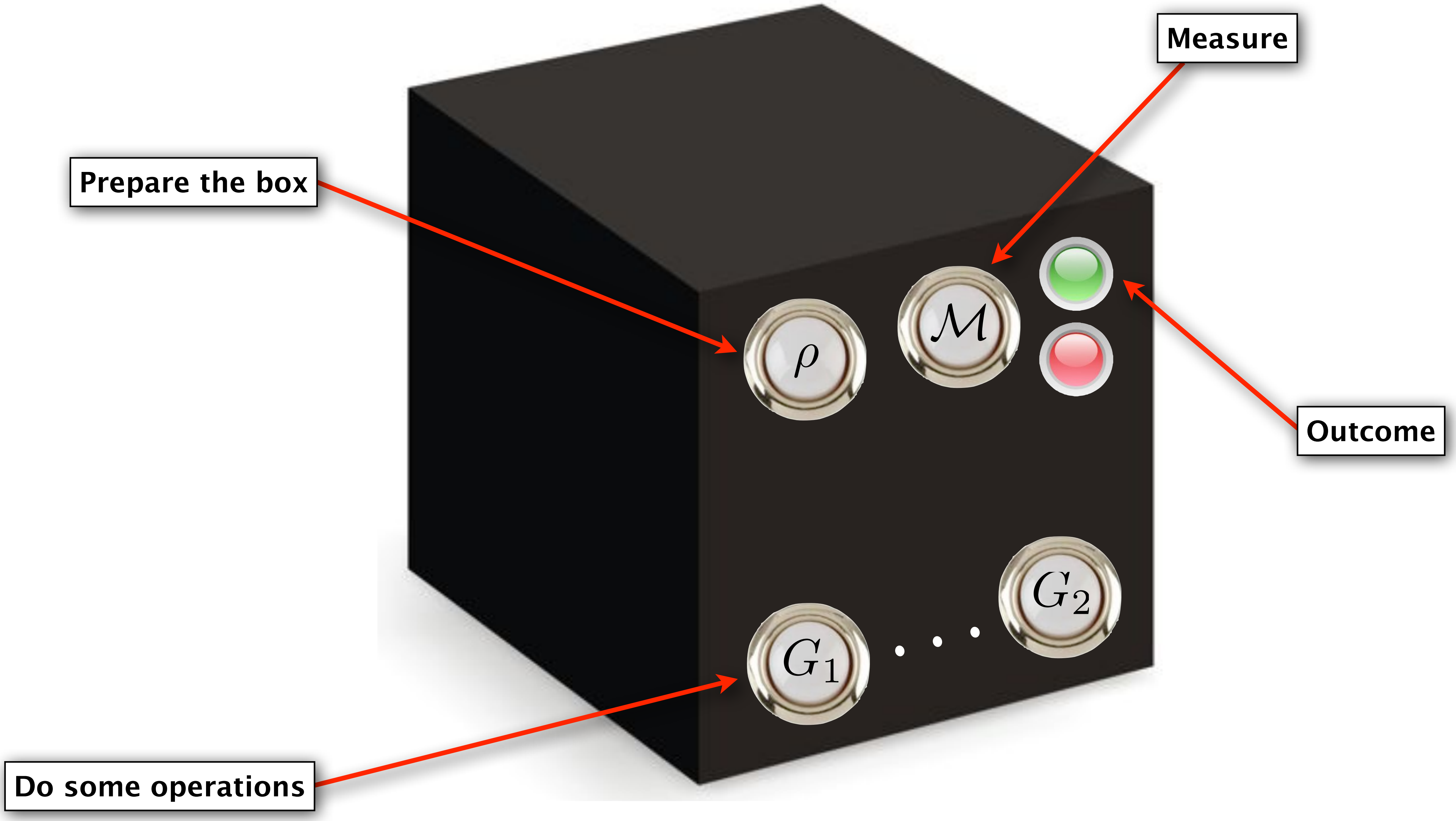
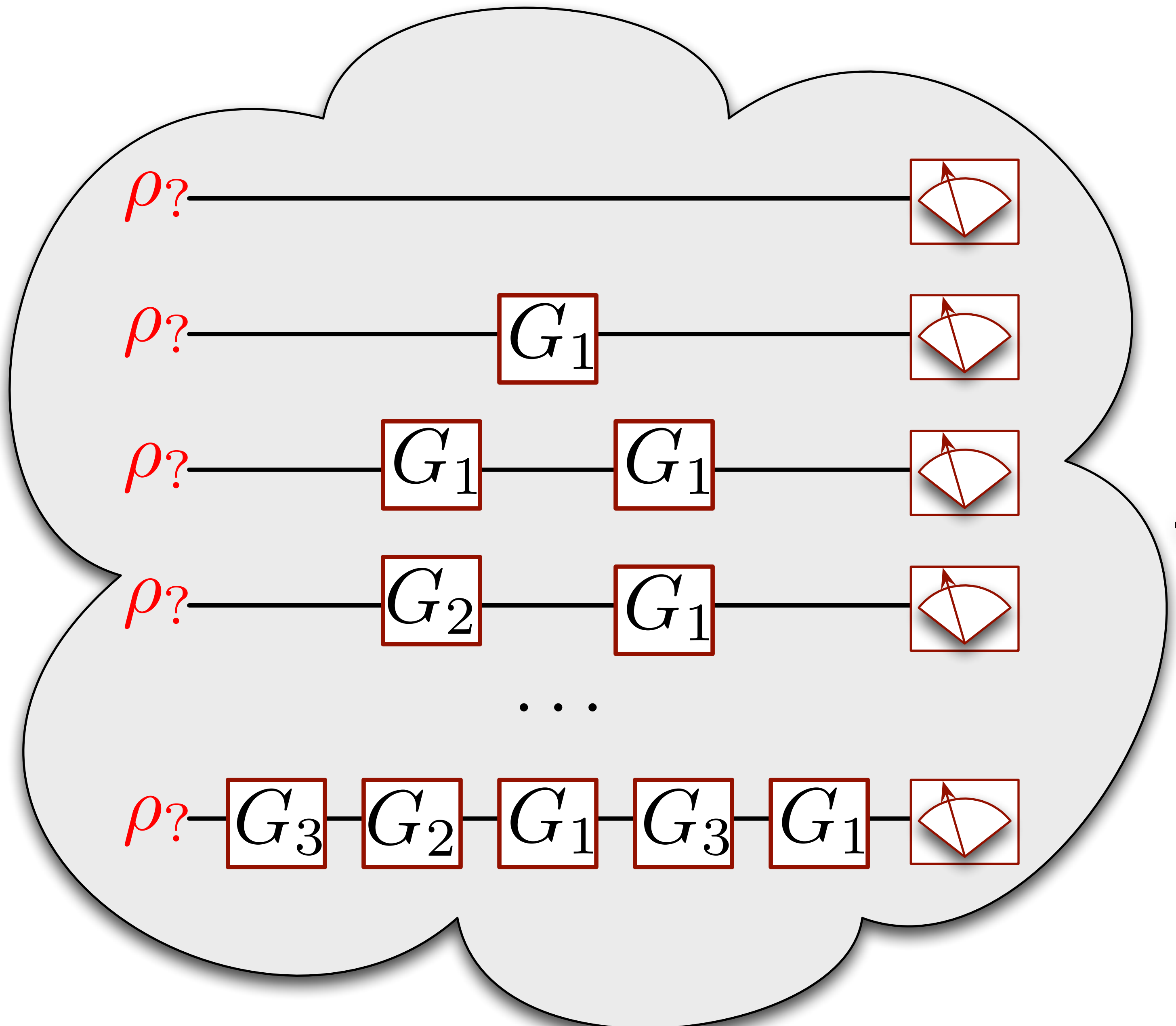


Gate set tomography



Estimate the entire gate set once.

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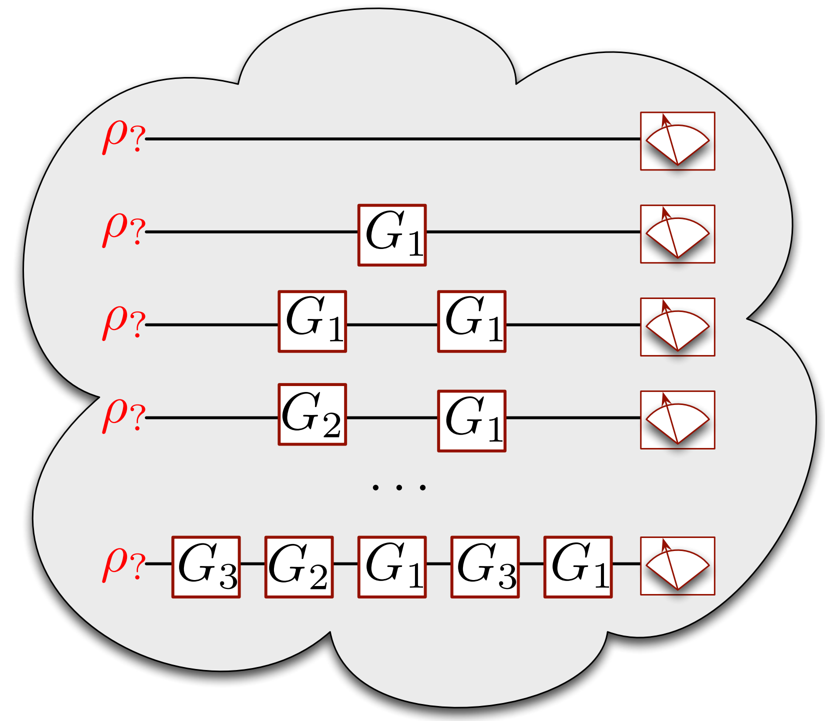
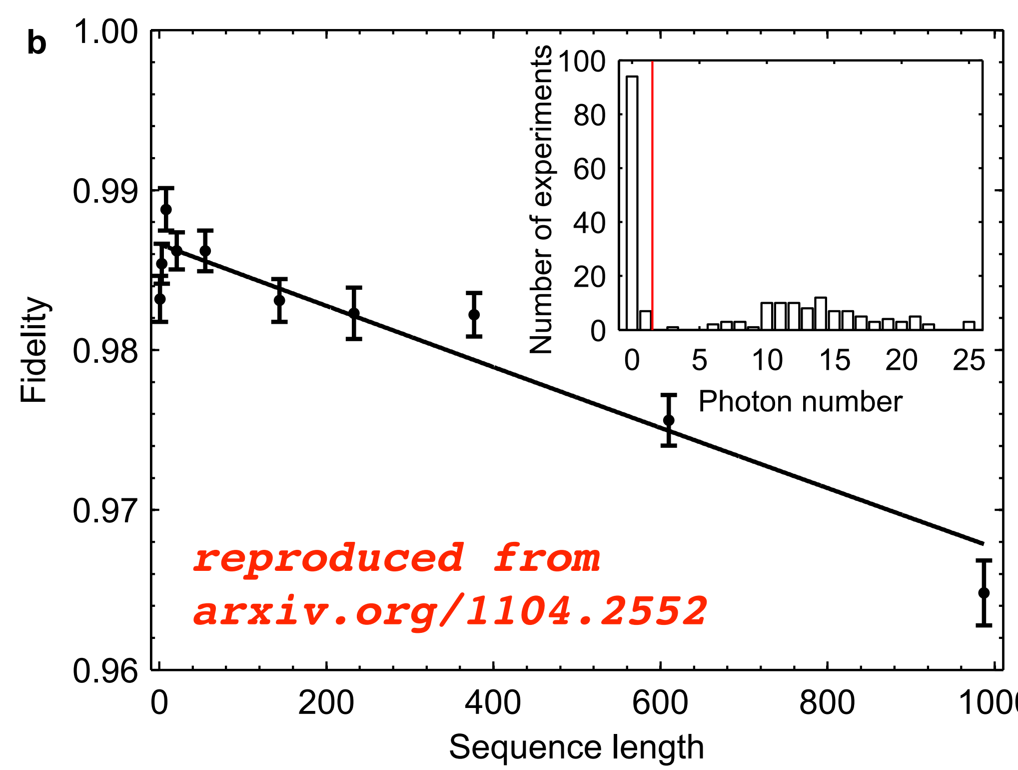
Alternate strategy: randomized benchmarking

- Assume nothing about prep/ measurements.
- Assume that dynamical gates are pretty good Clifford operations.
- Do many <u>random</u> gate sequences, all of which <u>would</u> perform the identity operation <u>**if**</u> the Clifford gates were perfect.
- Measure the rate of decay of success probability.
- Yields "per-gate error rate"... but provides absolutely <u>***no***</u> diagnostic/ debugging information.





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Randomized benchmarking vs. gate set tomography

- **RB and GST share a common foundation:**
  - robust to SPAM (preparation/measurement) error,
  - rely on [long] sequences of gates (unlike traditional tomography)
  - designed to test & verify real quantum hardware.

- RB is simpler than GST. Much easier to crunch the data.

- But **RB provides no diagnostic info** (just a per-gate error rate). **GST tells us <u>exactly</u> what operations are being performed**.

- RB requires that sequences of gates be chosen uniformly at random: "error rate" is an average over all strings.

- GST only requires a fixed # of sequences (~20 for 1 qubit) to identify the gates exactly -- and they can be chosen to optimize performance.

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Gate set tomography: necessities

1. Preparation in a consistent (unknown) state ρ.
2. At least 2 different (unknown) noncommuting operations $\{G_k\}$.
3. Some measurement $\{E_m\}$ -- perhaps just 2 outcomes $\{E, \mathbb{1}-E\}$.
4. A quorum of different experiments, corresponding to distinct gate strings:

$$S_i = G_{i_1} \circ G_{i_2} \circ \ldots \circ G_{i_L}$$

5. Enough repetitions to estimate:

$$\begin{aligned} Pr(E_m|\rho, S_i) &= \mathrm{Tr}\left[E_m S_i[\rho]\right] \\ &= \langle\langle E_m | G_{k_1} \circ G_{k_2} \circ \ldots \circ G_{k_L} | \rho \rangle\rangle \end{aligned}$$

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Linear inversion gate set tomography

Want to get: $|\rho\rangle, \langle E|, G_1, G_2, G_3, G_4$

Physically observable : $\langle E| X |\rho\rangle$

$$\tilde{J}_i = \langle E| G_i |\rho\rangle$$

Linear inversion gate set tomography

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(1) Compute the following observable quantities:

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(2) Solve for the quantities of interest:

$$\tilde{I}^{-1}\tilde{J} = B^{-1}|\rho\rangle \simeq |\rho\rangle \quad \tilde{J}^T = \langle E| B \simeq \langle E|$$

$$\tilde{I}^{-1}\tilde{G}_k = B^{-1} G_k B \simeq G_k$$

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$$B = \sum_k G_k |\rho\rangle\langle k| \qquad \tilde{I} = AB$$

$$A = \sum_j |j\rangle\langle E| G_j \qquad \tilde{G}_k = A G_k B$$

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'Gauge' freedom:

$$\langle E| B \left(B^{-1} X B\right) B^{-1} |\rho\rangle = \langle E| X |\rho\rangle$$

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A word on the gauge: gates are relational

- Inability to estimate gauge variables is ~~a real problem~~.
  utterly irrelevant!

- Parameters with no observable consequences are "ghosts".
  Any model (description of gates) that describes a sufficiently rich set of experiments will predict future experiments (e.g. quantum circuits) equally well.

- Absence of a reference frame is its own solution -- any set of gates that acts like {H, T, CNOT} in all circumstances is {H, T, CNOT}!

- Gate-set tomography is akin to randomized benchmarking -- to benchmark circuit elements, use them -- but:
  (1) more powerful because we keep track of which string was applied,
  (2) more robust -- no need for precalibrated Clifford gates.

- But if we can't write down unique gates, how do we ever compare to anything?

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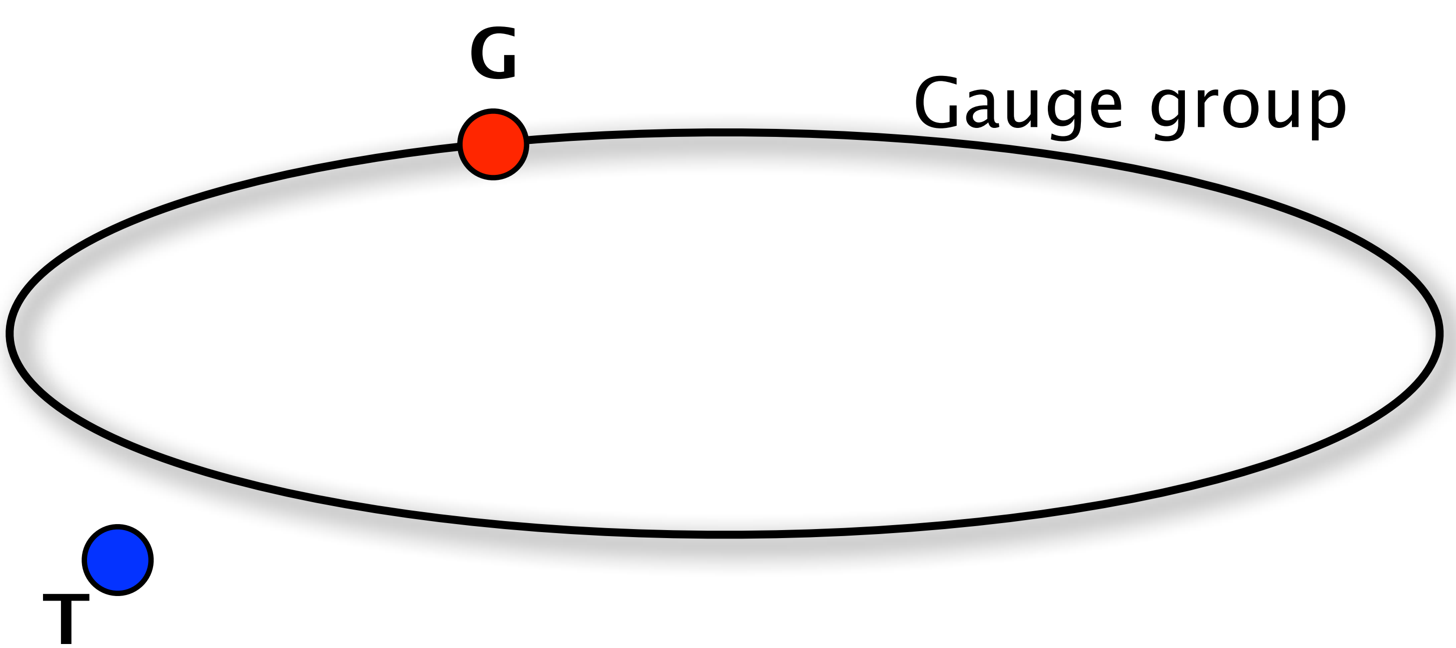
Gauge optimization

- Given a target gate set, compute the gauge that gets us closest:

$$\text{Distance} = \sum_i \left|\left|B^{-1}G_iB - T_i\right|\right|_2$$

- This problem scales well in the size of the gate set: we only need to find one matrix, regardless of the number of gates.
- We use the BFGS nonlinear optimization technique, starting from B=I, and get easy convergence.



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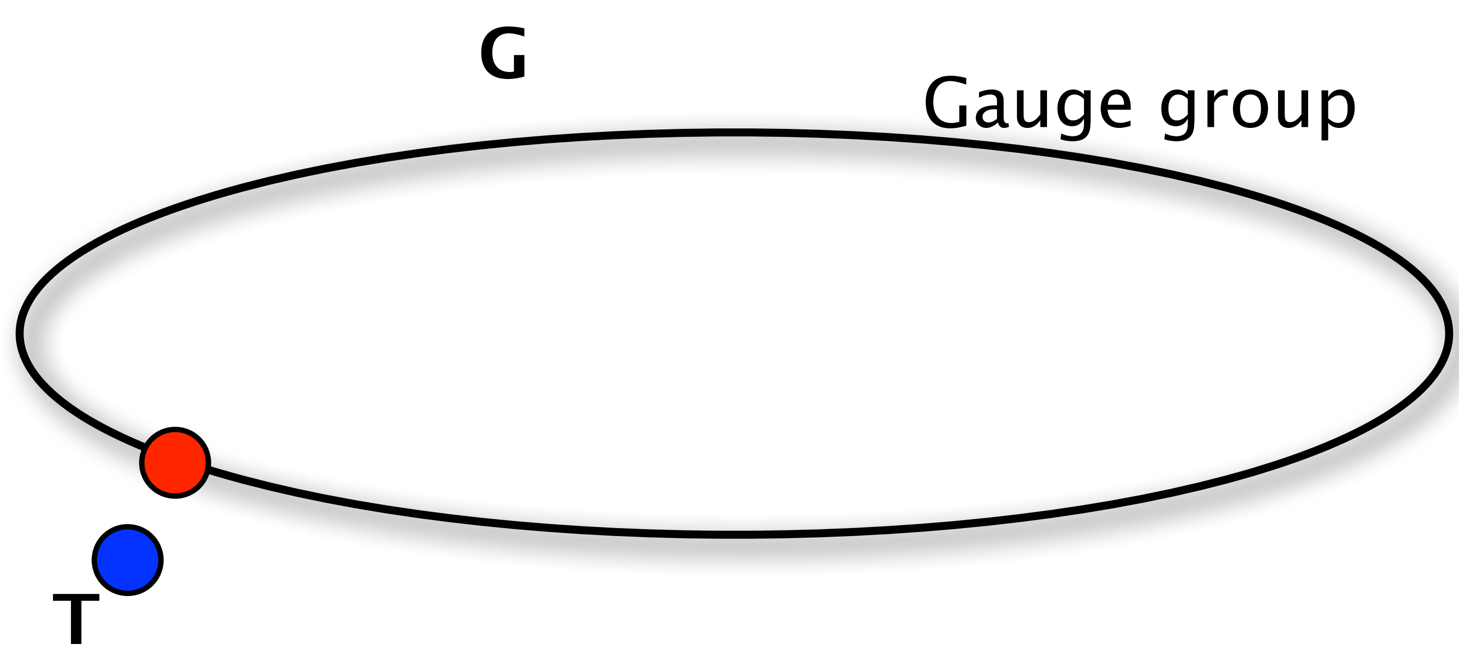
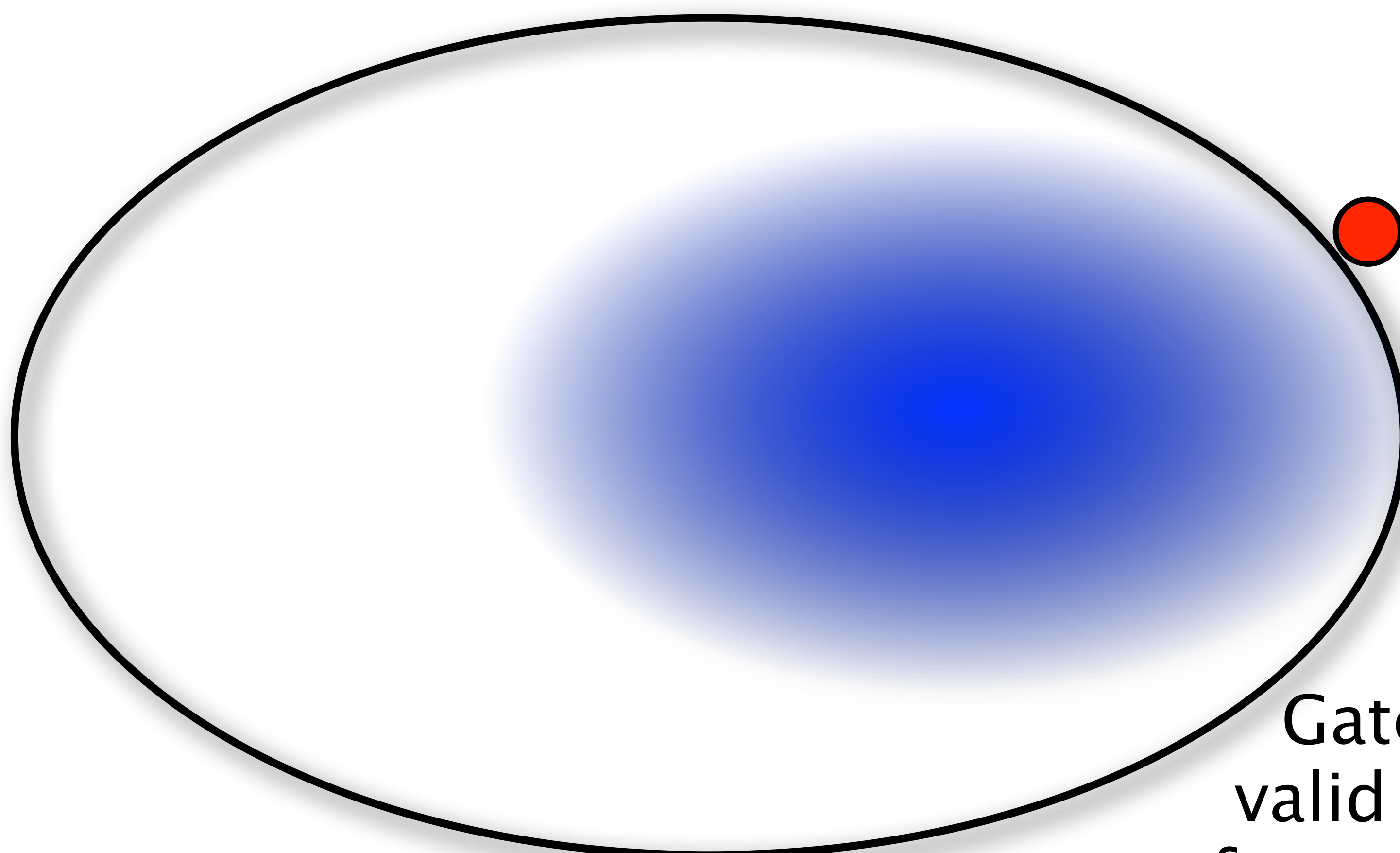


Refining using maximum-likelihood estimation

- Use linear GST as a seed to start maximum likelihood estimation.
- Problem: LGST gives gates that are not completely positive -> negative probabilities!
- Two-stage optimization:
  (1) Downhill simplex to find the closest feasible point.
  (2) BFGS to maximize the likelihood.



Gates that give valid probabilities for a given data set

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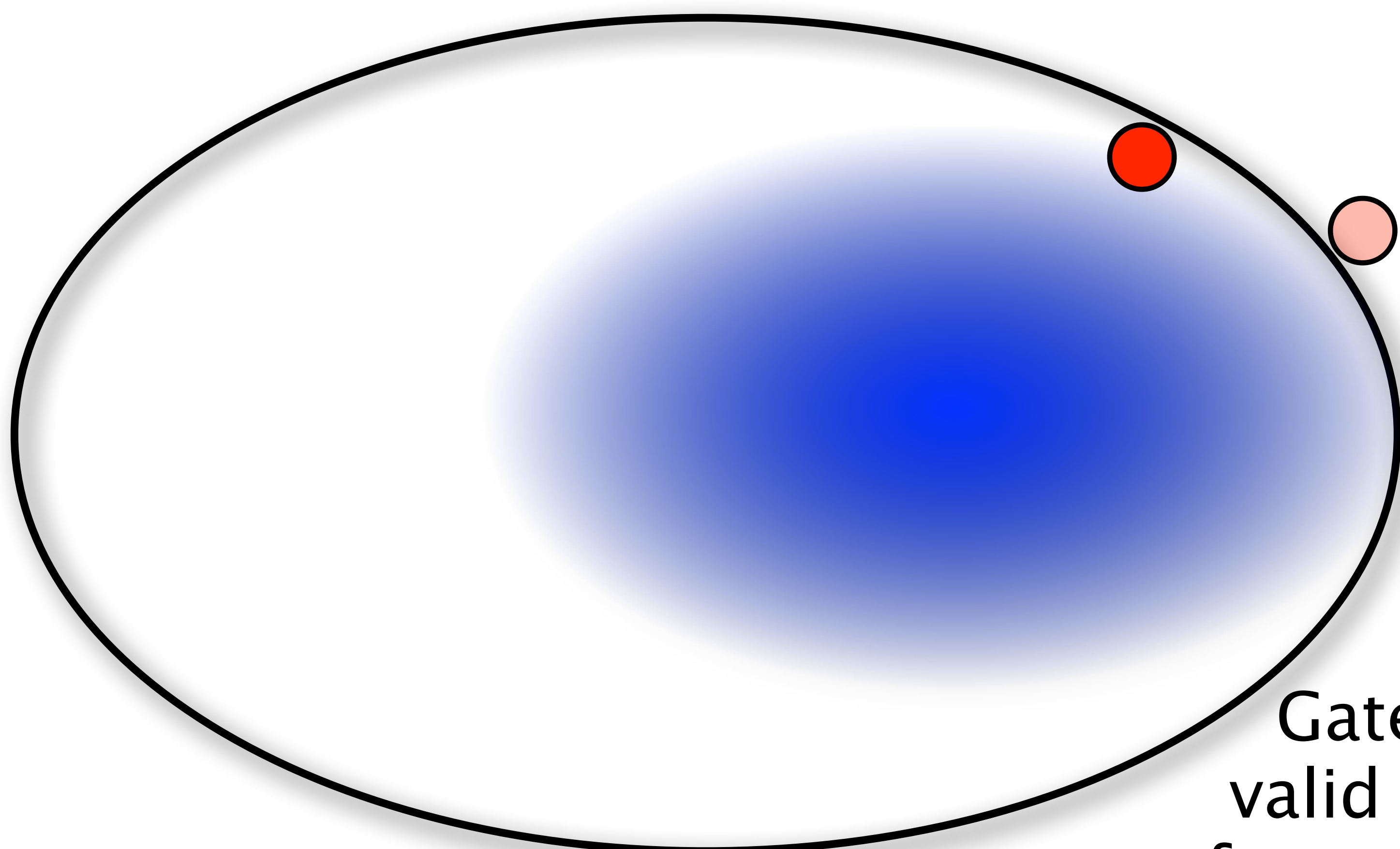

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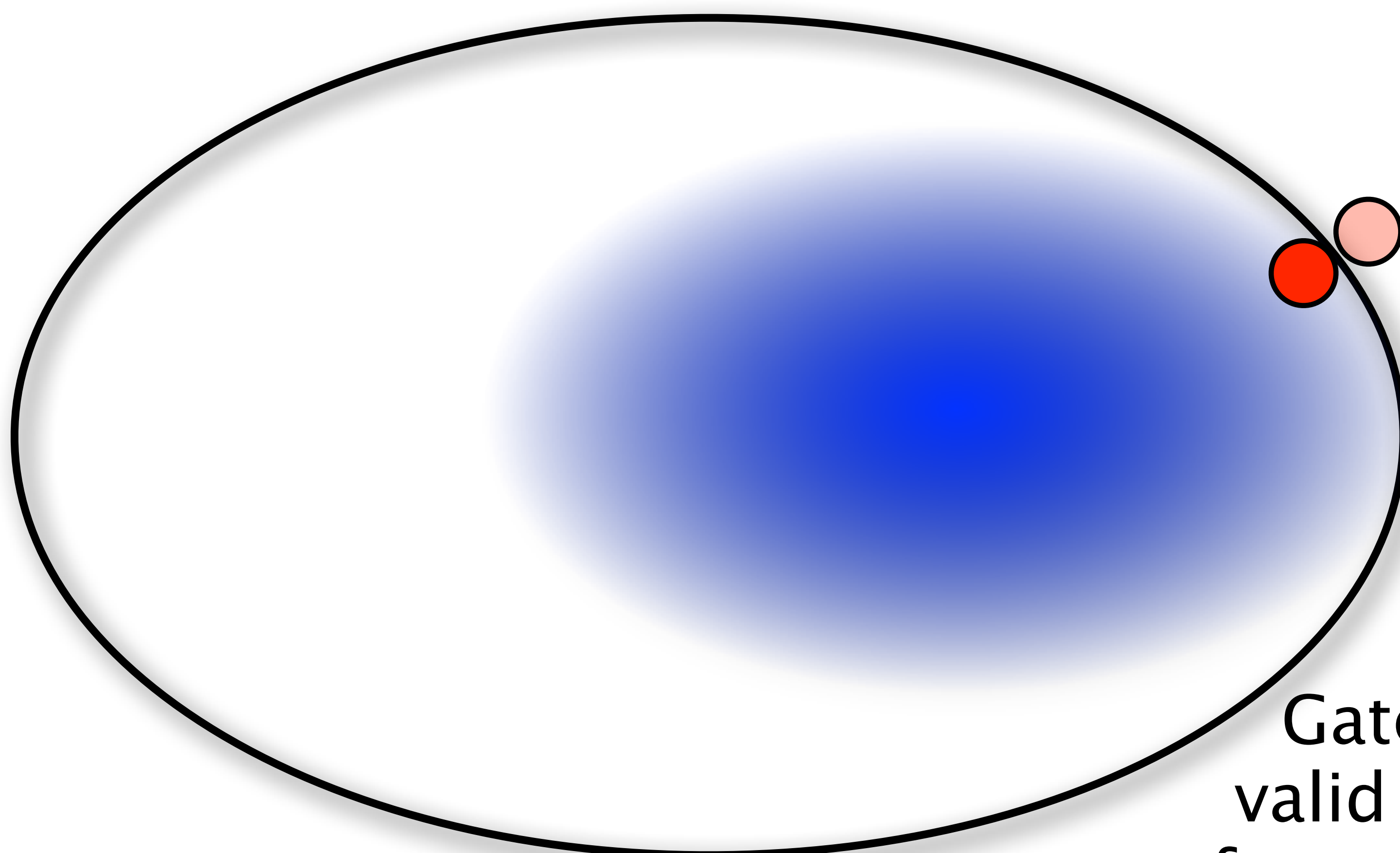

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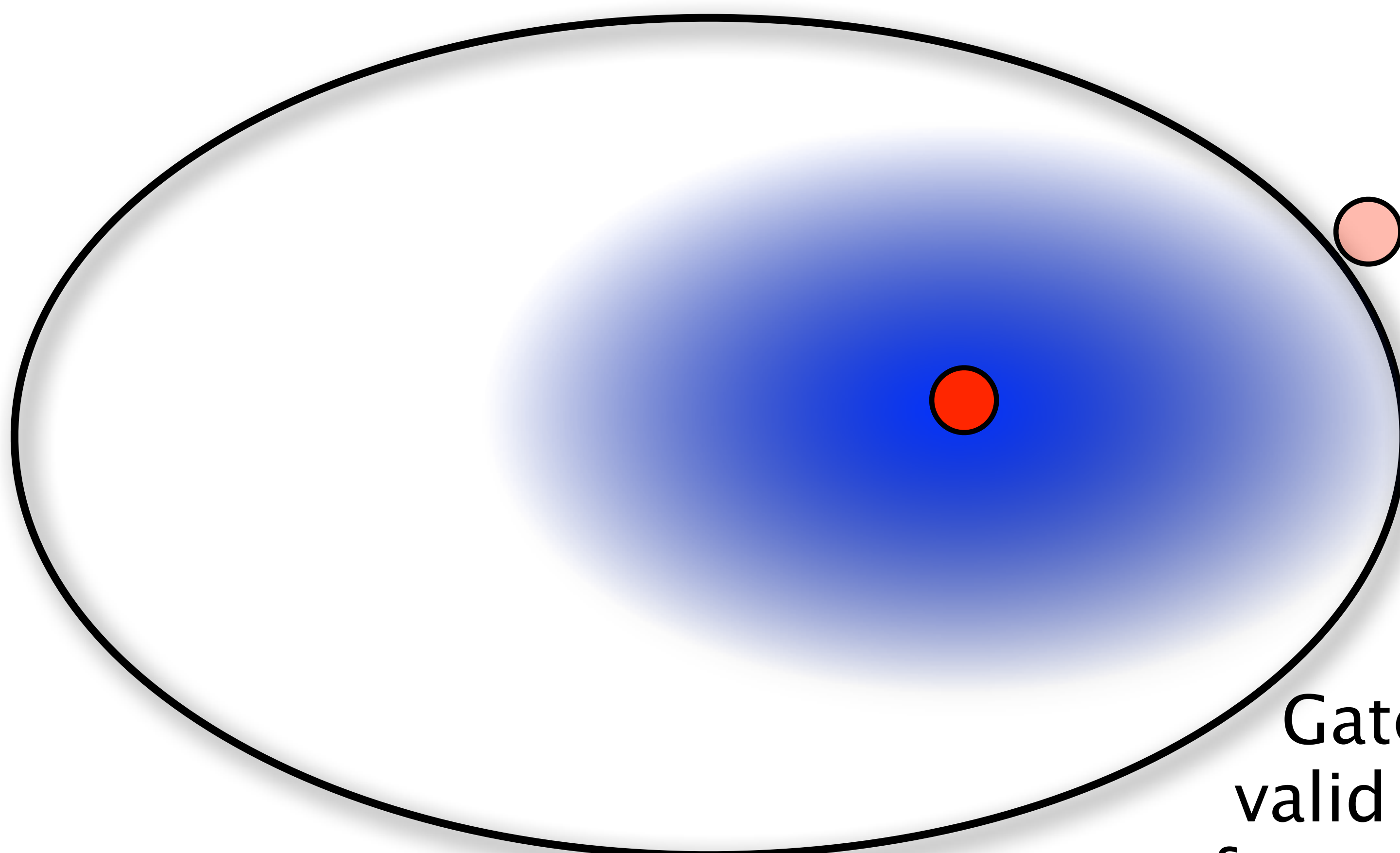
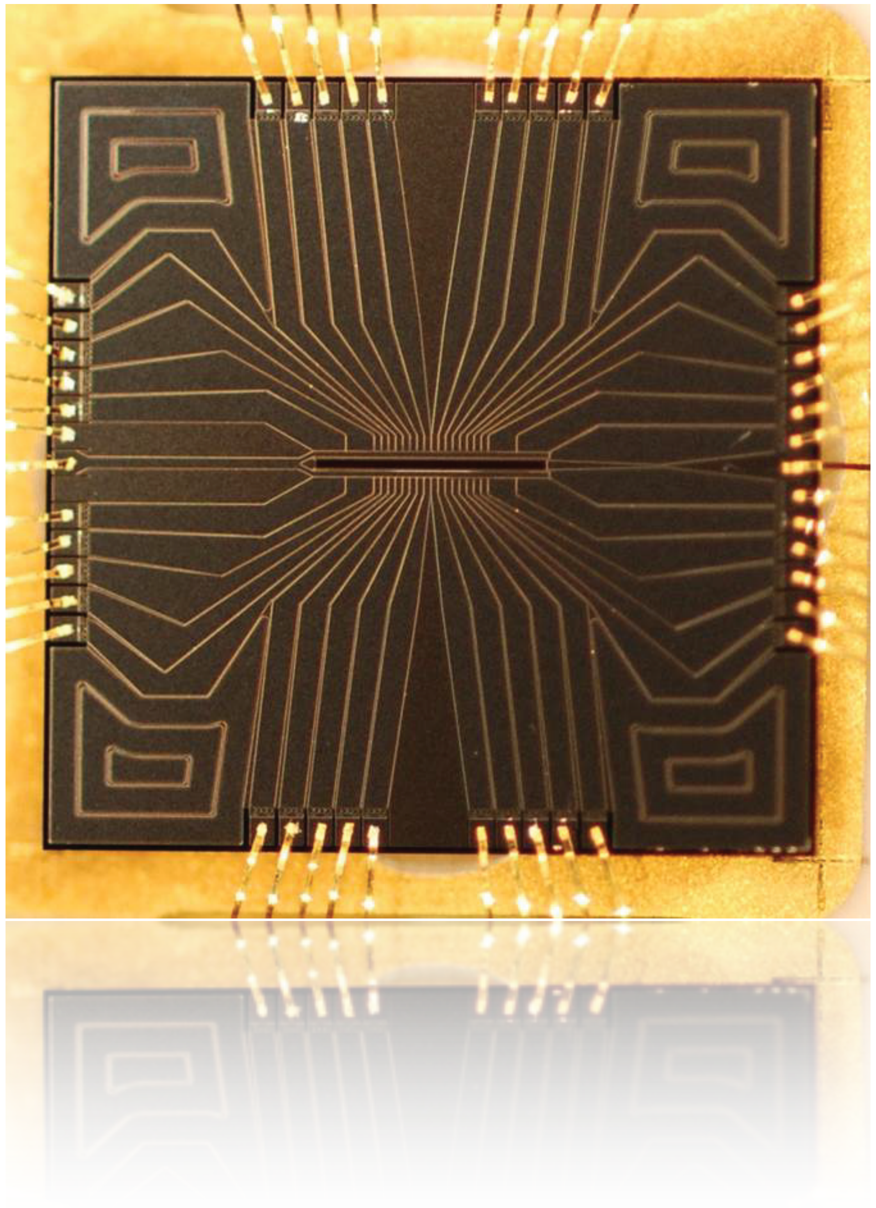


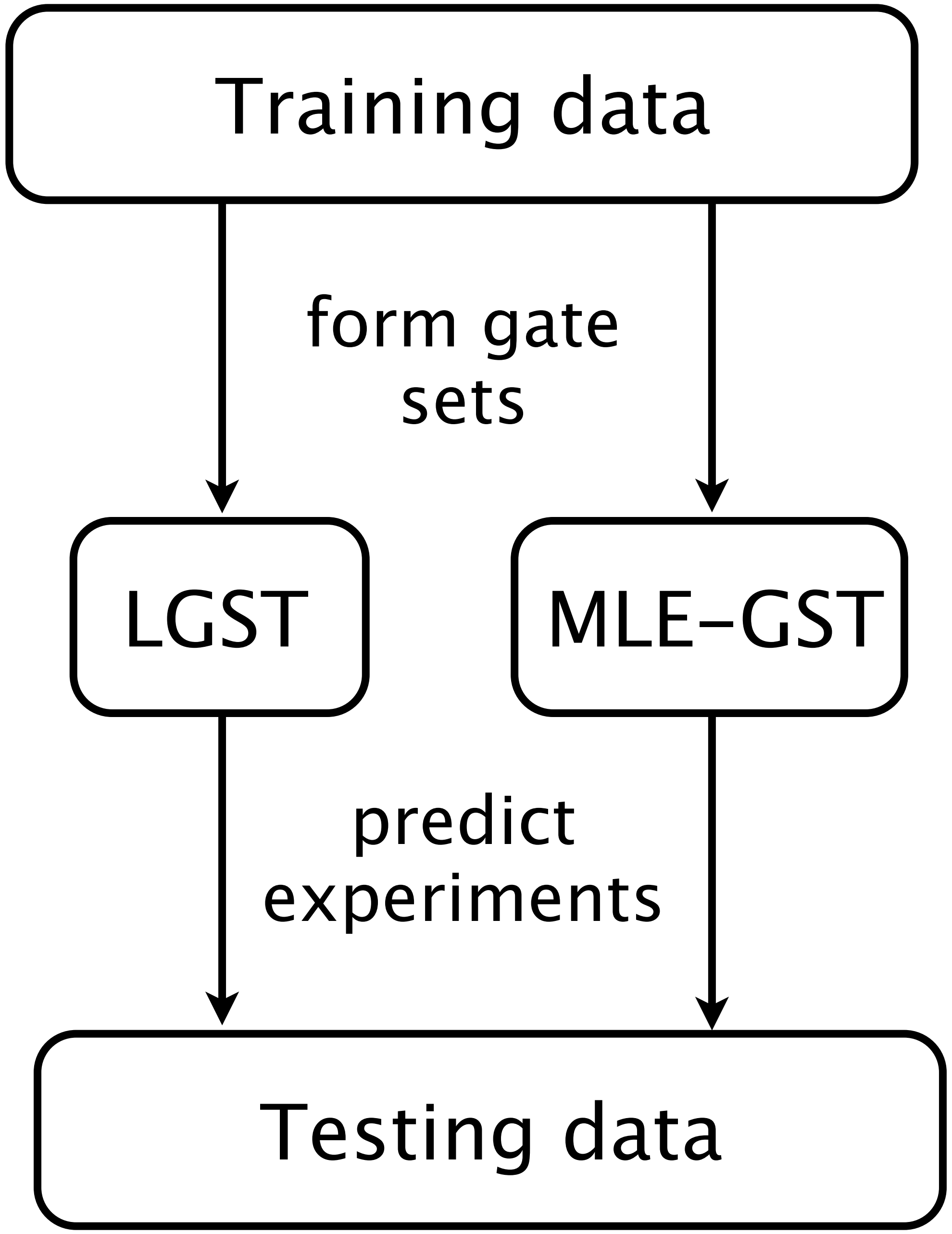
Gates that give valid probabilities for a given data set

Experimental setup

- Used a linear surface electrode ion trap, made and measured at Sandia.
- These experiments feature a single, $^{171}Yb^{+}$ atom in the center of the trap.
- Qubit encoded in the hyperfine clock states of the $^{2}S_{1/2}$ ground state.
- Microwave radiation resonant with the 12.6428 MHz qubit level spacing is used for control.
- π−pulses require ~ 50 μs.



Assessing goodness of fit



- We want to assess how well our tomography predicts future events.
- Divide experiments into 'training' and 'testing' – examine how well we do at predicting the testing data.
- Define a 'score' that is related to the likelihood of the the model predicting the testing data.
- We used a training data set of 1066 gate sequences, with each measurement repeated 1900 times.
- Our testing set consisted of ten distinct pulse sequences of length 100 and all their partial sequences, with each measurement repeated 950 times.

Tomographic gate estimates

	LGST estimate ($\hat{G}_k$)	Target (T_k)
ρ	$\begin{pmatrix} 0.0099 & 0.0104+0.0007i \\ h.c. & 0.9901 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
E	$\begin{pmatrix} 0.9879 & 0.0182-0.0023i \\ h.c. & 0.0121 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
G_1	$\begin{pmatrix} 0.9977 & -0.0219 & -0.0204 & 0.0024 \\ -0.0152 & 0.9657 & 0.017 & 0.0291 \\ 0.0031 & 0.0627 & 1.0172 & 0.0335 \\ 0.001 & 0.0065 & 0.0335 & 0.9915 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
G_2	$\begin{pmatrix} 0.9974 & -0.048 & -0.0304 & 0.0161 \\ -0.0077 & 0.9538 & -0.0033 & -0.0045 \\ -0.0113 & 0.0332 & 0.0066 & -1.0044 \\ -0.0029 & 0.0042 & 1.0099 & 0.0284 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
G_3	$\begin{pmatrix} 0.9923 & -0.0163 & -0.0066 & 0.001 \\ -0.0049 & -0.0087 & -0.0087 & 0.9839 \\ 0.0124 & -0.0082 & 1.0136 & -0.0017 \\ -0.0074 & -0.9797 & 0.0043 & 0.0025 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$
G_4	$\begin{pmatrix} 0.9991 & -0.0291 & 0.0028 & 0.0194 \\ 0.0096 & 0.9796 & -0.0049 & 0.0013 \\ 0.0083 & -0.0211 & -1.0494 & -0.0632 \\ -0.0091 & -0.0123 & -0.0427 & -1.0012 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

Tomographic gate estimates

	ML estimate (short dataset)	ML estimate (long dataset)	Target gates
ρ	$\begin{pmatrix} 0.0099 & 0.0077-0.0046i \\ h.c. & 0.9901 \end{pmatrix}$	$\begin{pmatrix} 0.0092 & -0.0017+0.0088i \\ h.c. & 0.9908 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
E	$\begin{pmatrix} 0.9911 & 0.0166-0.0006i \\ h.c. & 0.0089 \end{pmatrix}$	$\begin{pmatrix} 0.988 & 0.0019+0.0089i \\ h.c. & 0.012 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
G_1	$\begin{pmatrix} 1.0019 & -0.0128 & -0.0198 & -0.0002 \\ -0.0066 & 0.9775 & -0.0118 & 0.0122 \\ 0.0041 & 0.0842 & 1.0138 & 0.0073 \\ -0.0035 & -0.013 & 0.0075 & 0.9969 \end{pmatrix}$	$\begin{pmatrix} 1.0001 & -0 & 0.0003 & 0.0001 \\ 0.0001 & 0.9994 & -0.0003 & -0 \\ -0.0001 & 0.0006 & 0.999 & -0.0003 \\ -0 & -0.0001 & 0.0002 & 0.9998 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
G_2	$\begin{pmatrix} 1.0017 & -0.0276 & -0.0276 & -0.0048 \\ -0.0193 & 0.9582 & -0.0076 & -0.0127 \\ -0.0134 & 0.043 & 0.0082 & -0.9987 \\ -0.0072 & 0.002 & 1.0069 & 0.0192 \end{pmatrix}$	$\begin{pmatrix} 1 & -0.0001 & -0.0045 & -0.0005 \\ 0 & 0.9994 & -0.006 & -0.0018 \\ -0.005 & -0.0112 & -0.0064 & -0.9991 \\ 0.0006 & 0.0063 & 0.9993 & 0.0143 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
G_3	$\begin{pmatrix} 0.99 & -0.0114 & 0.0083 & 0.0044 \\ -0.0082 & -0.0141 & -0.0045 & 0.9892 \\ 0.0121 & -0.0044 & 1.0056 & -0.0059 \\ -0.0001 & -0.9848 & 0.0017 & -0.0016 \end{pmatrix}$	$\begin{pmatrix} 1.0001 & 0.0033 & 0.0001 & 0.0049 \\ 0.0033 & -0.0001 & -0.0005 & 0.9992 \\ -0.0002 & -0.0024 & 0.9995 & -0.0161 \\ -0.0019 & -0.9989 & 0.0179 & 0.0085 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$
G_4	$\begin{pmatrix} 0.9983 & -0.0217 & 0.0127 & 0.0142 \\ -0.0039 & 0.9745 & 0.0034 & 0.0077 \\ -0.0004 & -0.0145 & -1.0473 & -0.0323 \\ -0.014 & -0.0167 & -0.0072 & -1.0024 \end{pmatrix}$	$\begin{pmatrix} 1.0001 & -0 & 0.0062 & 0.0028 \\ -0 & 0.9997 & 0.0127 & 0.0022 \\ 0.0066 & 0.0164 & -0.9976 & 0.0065 \\ -0.004 & -0.0004 & -0.0066 & -0.9981 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

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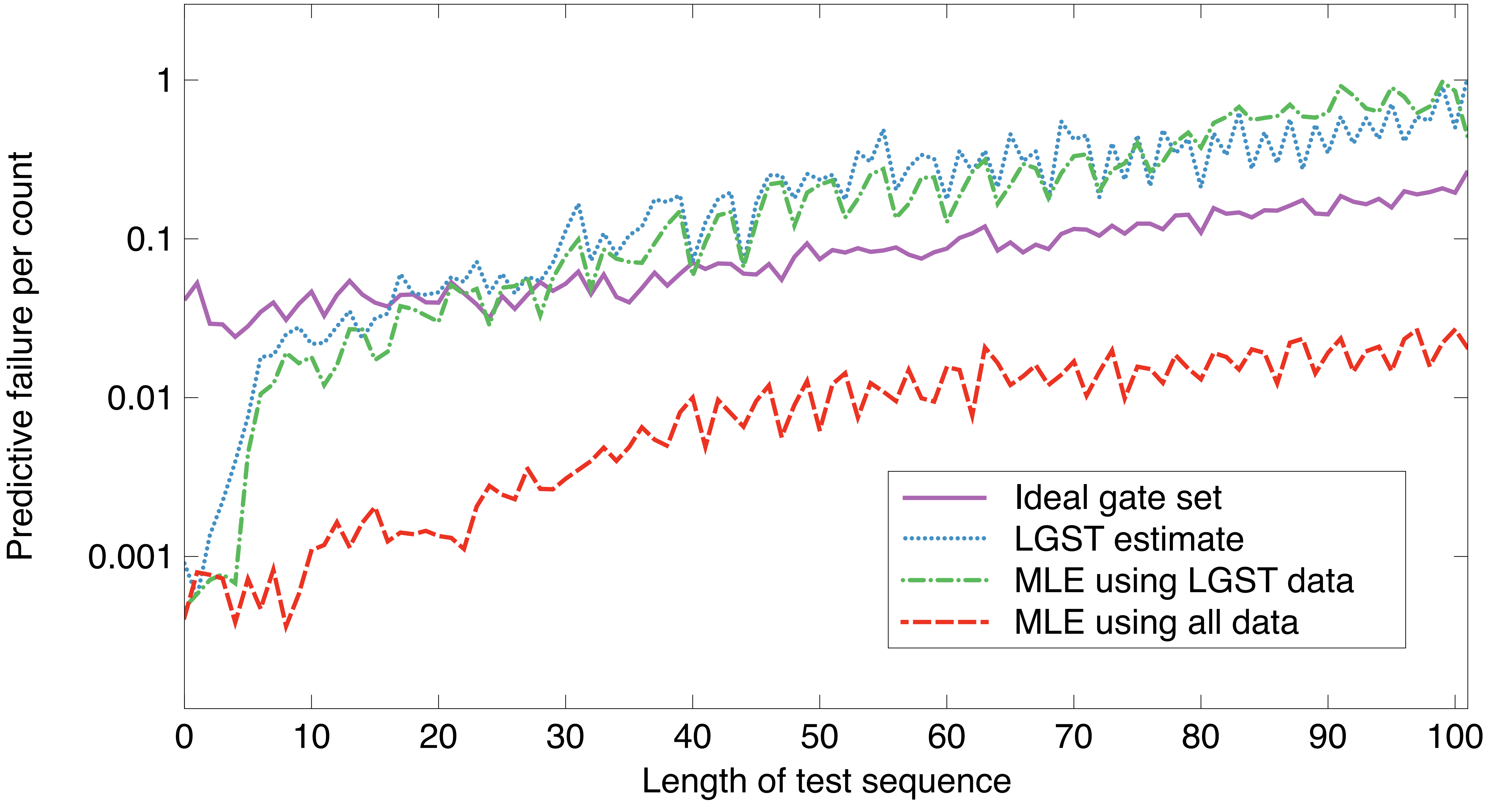
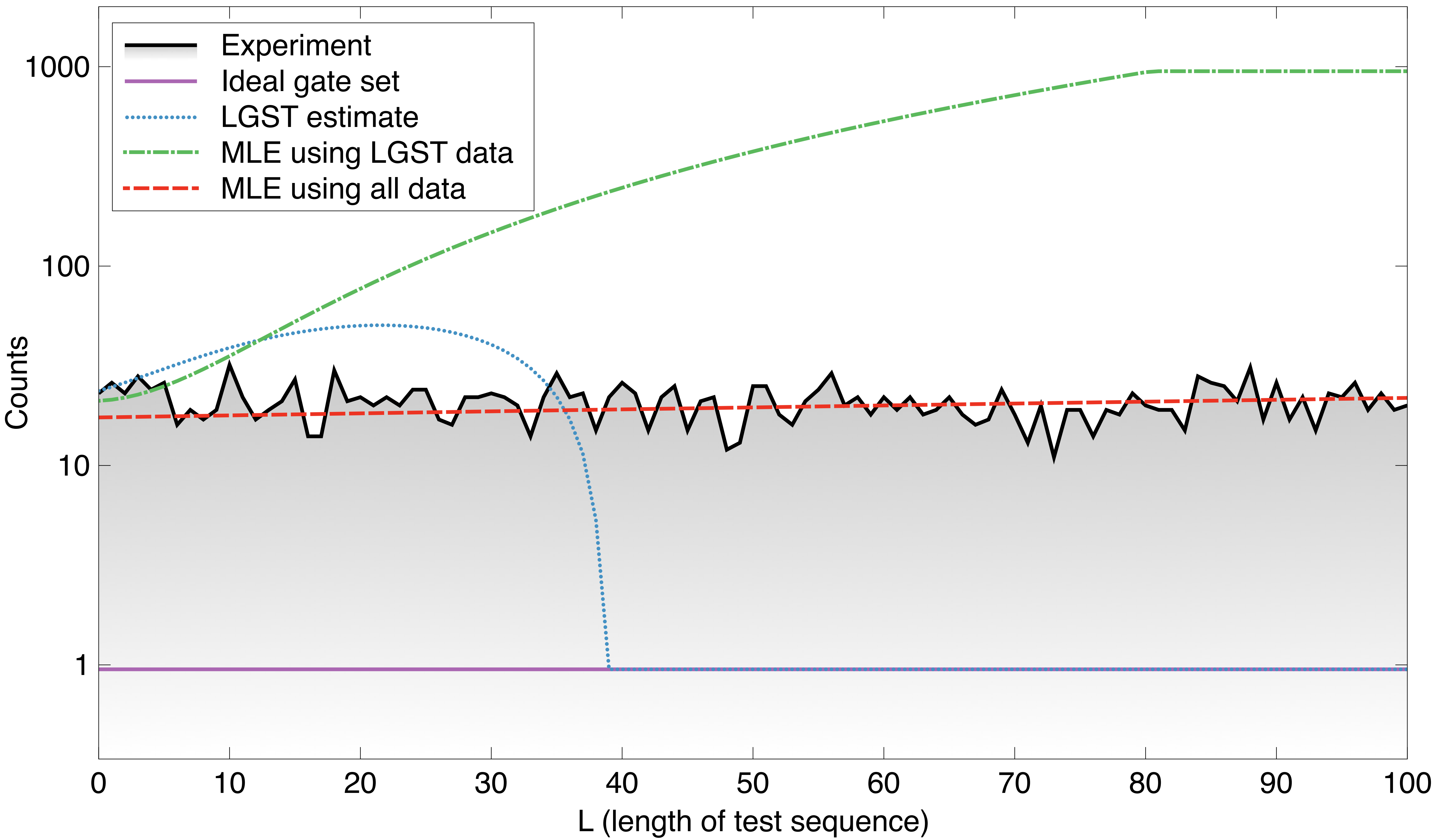
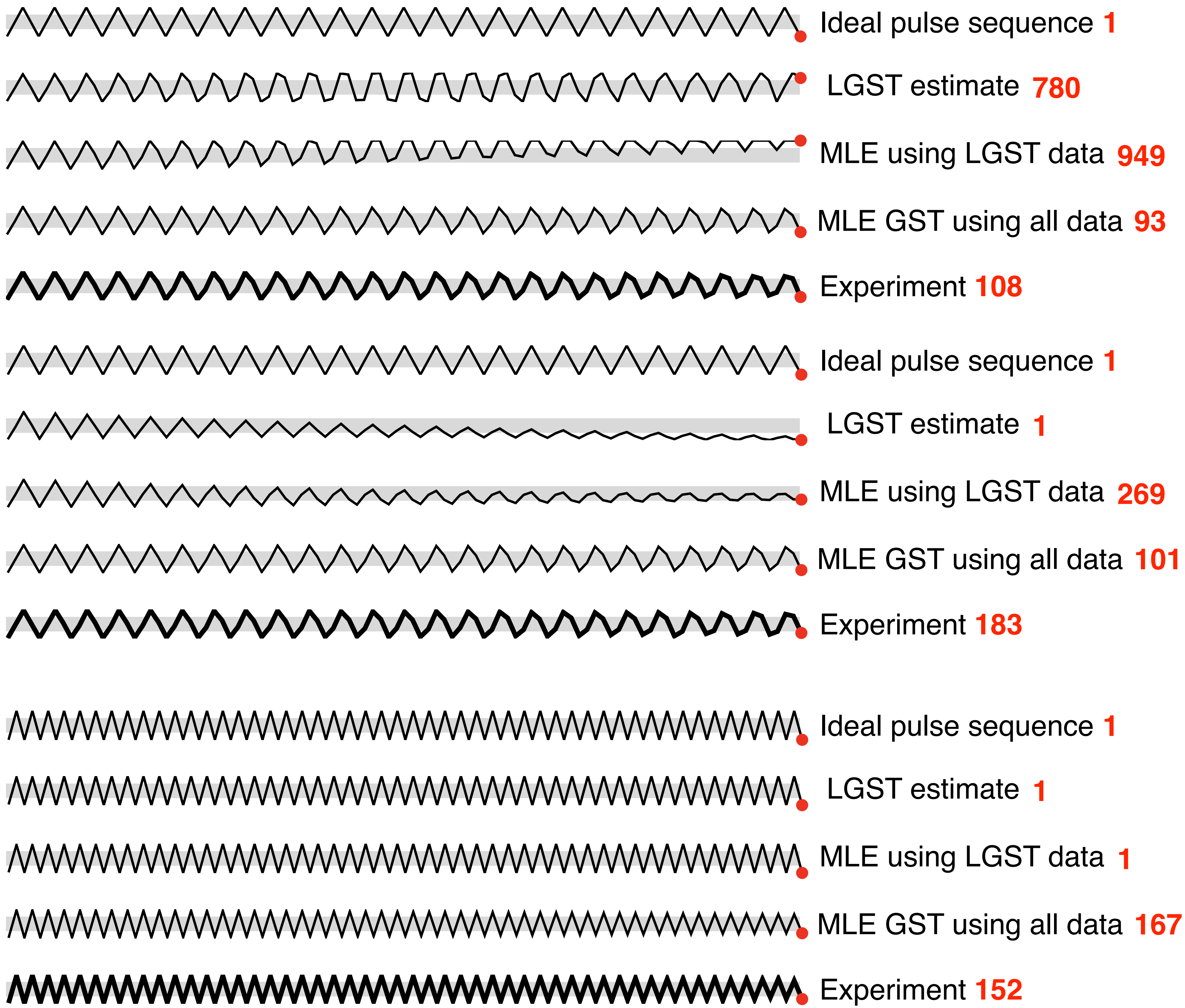
Scoring the gate estimates



Repeated identity applications



Rabi oscillation experiments



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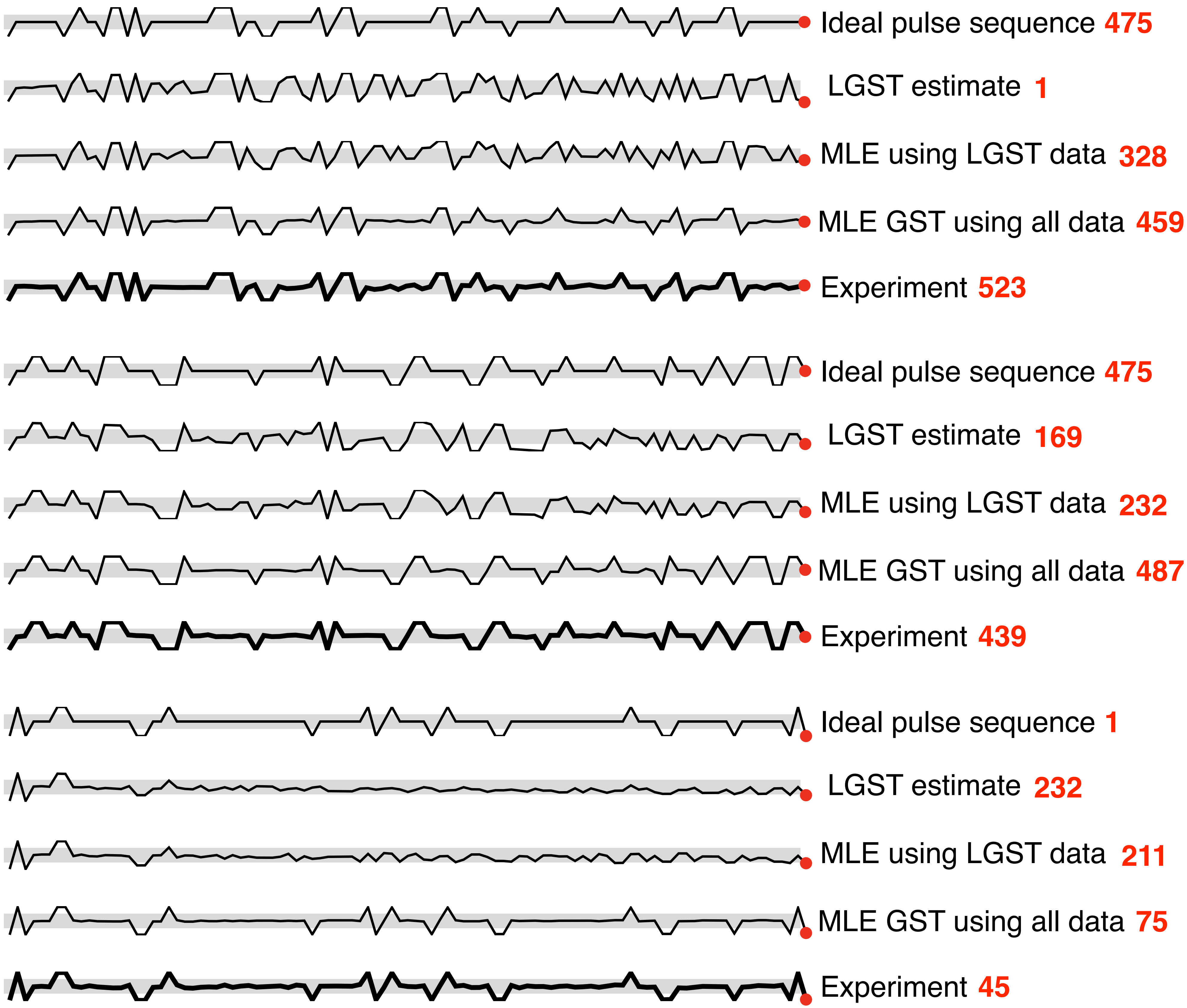
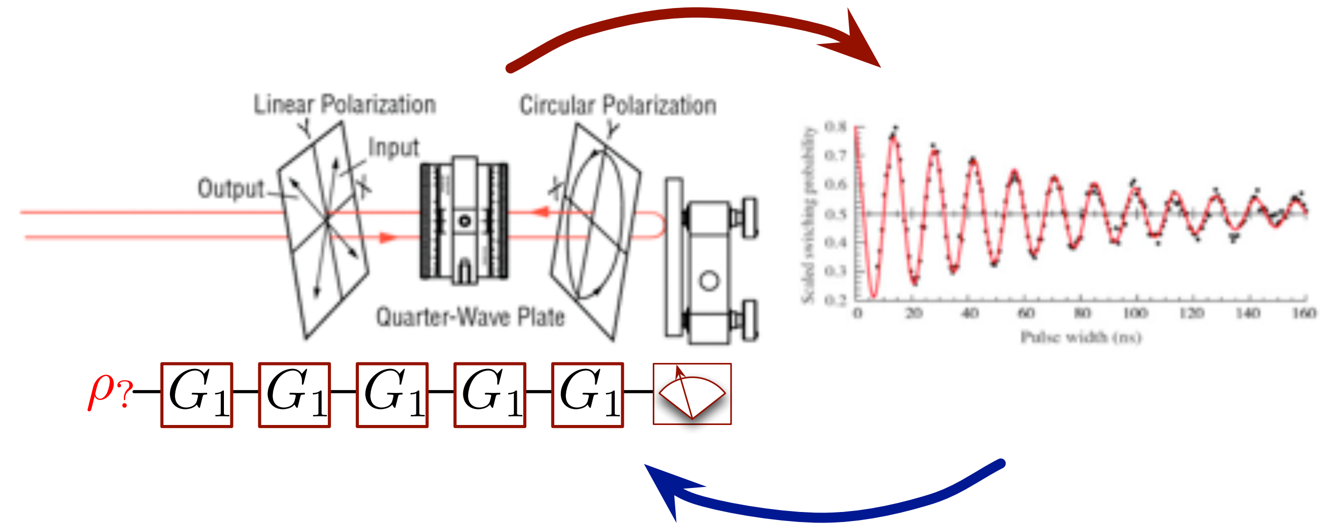
Irregular pulse sequence experiments



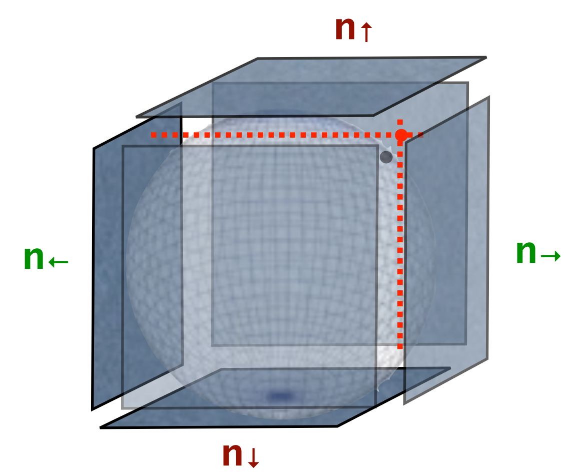
Why gate set tomography must work

- Traditional tomography:
  (1) Calibrate essential gates using “other” methods that incorporate nonlinearity.

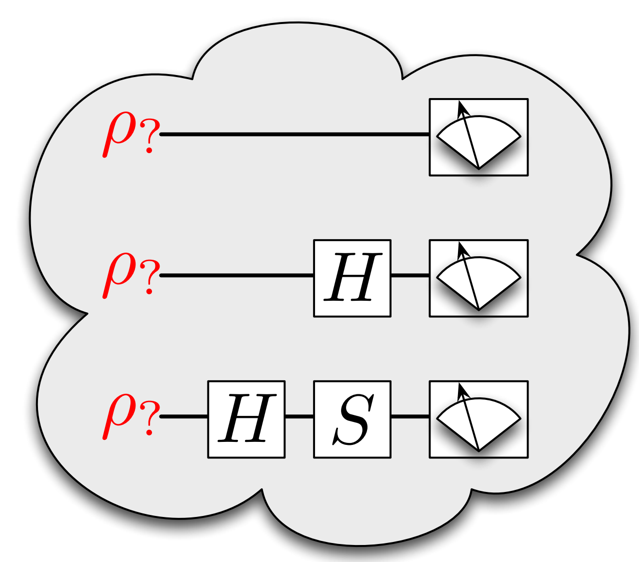
  (2) Use those calibrated resources to do tomography (linear).



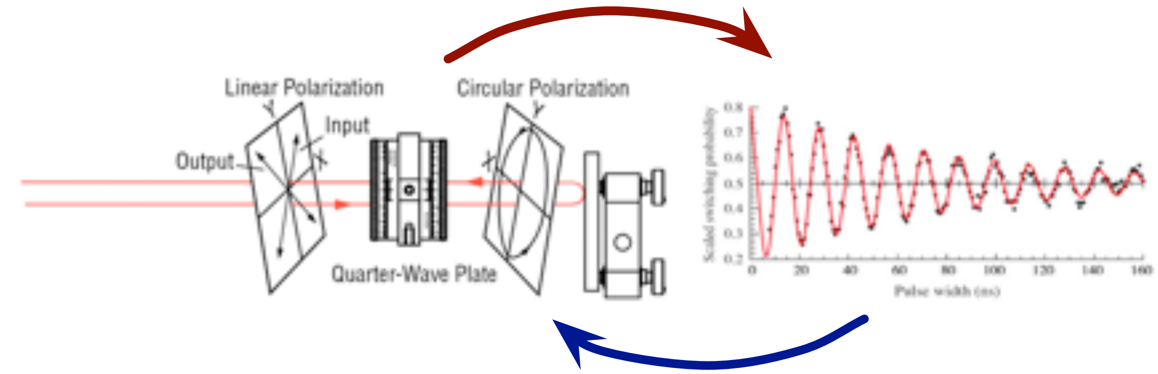
- So all the information:
  (1) must be present in the entire experimental chain;
  (2) can be extracted using (even suboptimal) statistical analysis.



- Gate-set tomography is just the obvious step of integrating these parts to get (potentially huge) improvements in efficiency and precision.



Adaptive gate set tomography

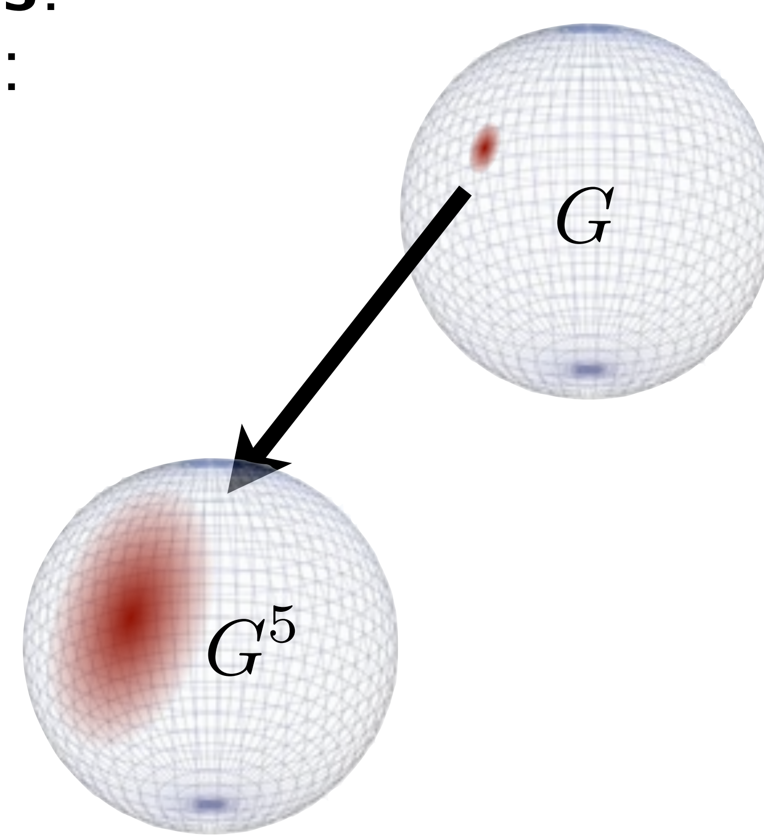


Three reasons for adaptivity:

1. Gate calibration traditionally involves feedback (adjust, measure, rinse, lather, repeat...)

2. **Same reason as for state tomography**: error metrics are fidelity-like, and **zero probabilities are easier to estimate.**
=> prescription: do experiments that yield Pr ≈ {0,1}
=> improvement: quadratic, N samples => $N^{1/2}$ samples.

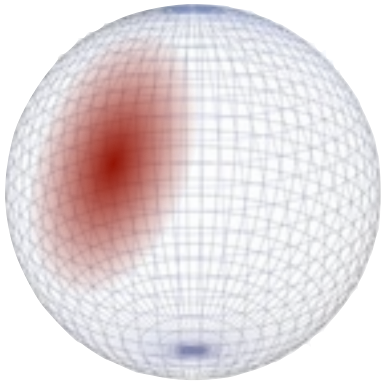
3. **Studying G^n for large n "blows up" small regions**:
=> improvement: **exponential**, N expts. => logN:
=> requires adaptivity to choose the experiments (sequences) that will efficiently resolve the current uncertainty region at each stage.



Speculative scheme for choosing gate strings

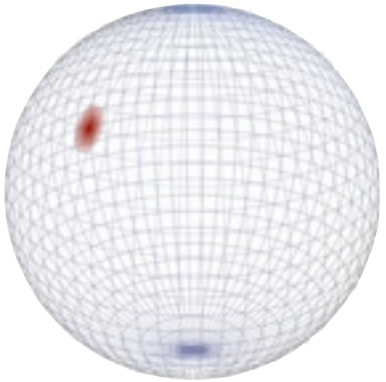
1. Perform a quorum of short-sequence experiments (e.g. all sequences of up to L=3..6 gates).

2. Get preliminary confidence region from likelihood function.



3. Generate candidate sequences of ~L^2 gates (how? something clever -- genetic algorithm?).

4. Select sequences that are expected to yield data that sharpen the likelihood function for all plausible states (in CR).



5. Iterate.

We are looking for beta testers

- We want to extend and develop the GST framework to all quantum information hardware, not just ion traps.
- If you do experimental quantum information processing and want to leverage GST, we would love to hear from you.
- Contact me (jkgambl@sandia.gov) for more information.

Thanks!

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Experimental work:
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