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Proposal to the American Institute of Mathematics for workshop on *Coherent quantum feedback control*

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Accurate control of physical systems is an essential element of any technology, and as technological building blocks shrink to the nanoscale and smaller, the control of quantum-mechanical degrees of freedom is becoming an important capability. Fueled by the prospects of building quantum technologies such as quantum computers and quantum communication devices, the theory for controlling quantum systems – quantum control – has progressed immensely over the past decade [1, 2]. Amidst this progress, one of the most exciting results that has emerged is the identification of an entirely new regime for controlling physical systems: coherent quantum feedback control (CQFC) [3–7].

Feedback control is a critical enabling element of today's technologies; most modern complex devices have some form of feedback within their operation. For example, the stability of airplanes, the precision of modern fabrication processes, and the operation of optical drives are all a result of feedback control. Feedback control in the quantum domain is also powerful [8–10] and has been demonstrated on several physical platforms in the laboratory [11–15]. However, in most cases until now, the controller has been treated a classical system.[1] That is, the system being controlled – be it quantum (e.g. an atom) or classical (e.g. an airplane) – is measured and these measurement signals are processed by a conventional computer to decide upon what feedback to apply to the system. The distinguishing feature of the CQFC model of control is that the controller and the system being controlled are both quantum mechanical; see the accompanying figure for a schematic comparison of the two forms of quantum feedback.

Coherent quantum feedback control is a new framework that holds tremendous promise as a model for controlling physical systems. Since both the system and controller are quantum mechanical, there is no intervening measurement to communicate between the two and one can in principle avoid undesirable disturbances introduced by so-called measurement backaction. This could be a tremendous advantage for designing stabilization and noise suppressing devices in the quantum domain [6, 7]. Another advantage of CQFC is its practicality; due to the difference in timescales between fast quantum phenomena and relatively slow classical electronic processing, classical controllers often cannot effectively control quantum systems in real-time as the temporal latency in the control loop is too great. However, in CQFC the controller and system are both quantum mechanical and therefore can evolve at commensurate timescales, eliminating this loop latency problem. For these reasons it is believed that CQFC is a more powerful model than conventional (quantum or classical) feedback, but this is as yet unproven.

I. THE STATUS OF COHERENT QUANTUM FEEDBACK CONTROL

Despite the potential of CQFC as a powerful model of control, little is known about the dynamics of such control systems. In fact, it is unclear how to answer even basic control theoretic questions in this domain, such as: is the CQFC system controllable, or stable?

Prior work in the CQFC field has been disparate and has been performed by researchers in physics, mathematical physics, and control theory. Consequently, there exists no unifying description of the power and limitations of CQFC. A natural mathematical description for quantum control systems connected through Markov fields has emerged – quantum stochastic differential equations (QSDEs) – and progress has been made in solving linear versions of such QSDEs [5, 16, 17]. But a path forward for solving, or even approximating, the dynamics of non-linear QSDEs is currently lacking. Furthermore, it is pivotal to establish a solvable mathematical framework for describing general coherent quantum feedback control systems that contain a combination of Markov field connections and direct Hamiltonian couplings between subsystems.

On the applied front, there have been designs for coherent feedback controllers from the control theory and physics communities for particular objectives (e.g. quantum error correction [18, 19]) but it is unknown how to generalize these results for synthesizing controllers for other tasks. In fact, the absence of a constructive formalism for designing controllers to achieve a prescribed control goal is a critical issue in CQFC. While classical control theory possesses many mathematical techniques for attacking this controller synthesis problem, what are the analogous techniques in

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[1] By classical system, we mean a system for which it is unnecessary to invoke quantum mechanics to describe accurately.

the CQFC domain? The mathematical control theory community has made some progress on this question within the sub-domain of linear coherent feedback systems – e.g. Refs. [5, 17] – but these results are not extendable to general systems. Formulating techniques for controller synthesis will be critical to designing and implementing coherent feedback controllers for achieving prescribed goals.

II. THE AIM WORKSHOP

The proposed AIM workshop will serve to assemble CQFC researchers from various fields (applied mathematics, control theory, physics, and engineering) in a focused, interactive environment, with the intent of addressing some of the pressing open questions in this nascent interdisciplinary research domain. We believe that such a workshop will be tremendously beneficial to the development of the field, and see AIM as the unique venue that can support such a small, interactive, interdisciplinary workshop.

The principal goal of the workshop will be to initiate a program for developing the mathematical and theoretical framework that underlies CQFC systems relevant for real experiments. This framework will seek to unify the existing diverse models of CQFC independently developed in engineering, physics, and mathematics. The workshop will be structured around four driving questions/themes:

1. What are the control tasks for which coherent feedback control is verifiably superior to other forms of control (including measurement-based feedback control, learning control, and open-loop control)?

2. What tools from mathematics and non-linear systems control theory can be used to solve the controller synthesis problem in CQFC? What new tools are necessary?

3. Identify the most promising research directions in the field of CQFC, integrating the perspectives of researchers from the variety of disciplines involved.

4. What are the mathematical advances necessary to realistically model experimental applications of CQFC? What are the most promising experimental platforms for demonstrating, developing and testing coherent feedback control theory?

We will seek the attendance of many of the prominent researchers in the field of CQFC for the workshop, including Profs. Hideo Mabuchi (Stanford University), Matthew James (Australian National University), and Viacheslav Belavkin (Nottingham University). In addition, we will ensure that several junior researchers also have the opportunity to attend and participate in discussions. We expect an attendance of about 25-28 people.

We propose that the workshop talks and discussion be organized according to the following topics: (i) physical and mathematical foundations of coherent quantum feedback control, (ii) control theoretic aspects of the synthesis problem for CQFC, and development of tools for synthesis of controllers for non-linear systems, (iii) mathematical models relevant for applications and experimental implementations of CQFC, and (iv) alternative approaches to modeling CQFC systems. This themes outlined above will be explored within this organization, by way of targeted lectures and interactive discussions.

It should be noted that the proposed workshop has a very different focus and topic from the AIM workshop on quantum control held in June 2010 (*Control and optimization of open quantum systems for information processing*). This previous workshop was focused on applying and extending existing frameworks for quantum control to the task of quantum information processing. In contrast, the emphasis of the proposed workshop on CQFC is to develop mathematical techniques to describe an entirely new framework for controlling quantum systems. Applications of CQFC to quantum technologies will be discussed but the primary goal will be to formulate solvable mathematical methods for modeling CQFC systems and explore their limits. We emphasize that while the general field of quantum control is relatively mature, this newly discovered subfield of coherent quantum feedback is nascent, and the questions posed by CQFC are new enough that a small focused AIM workshop with interdisciplinary attendees could make significant progress towards solving real open problems.

We anticipate that the proposed workshop will lead to tangible outcomes that will guide research in the field of coherent quantum feedback control for several years. The collaborative development of a common mathematical framework for describing CQFC systems and for formulating broadly applicable synthesis tools for CQFC controllers will be the most important outcome from the workshop. The identification and rigorous quantification of the advantages and disadvantages of CQFC are important problems that will be addressed and we expect that significant progress will be made towards solving them. Finally, we anticipate that interactions during AIM workshop will result in the publication of several significant papers, and that the workshop will serve as an ideal venue for the establishment of collaborative, inter-disciplinary research efforts that will push the field of CQFC forward.

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