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Block Preconditioning for Implicit Ocean Models

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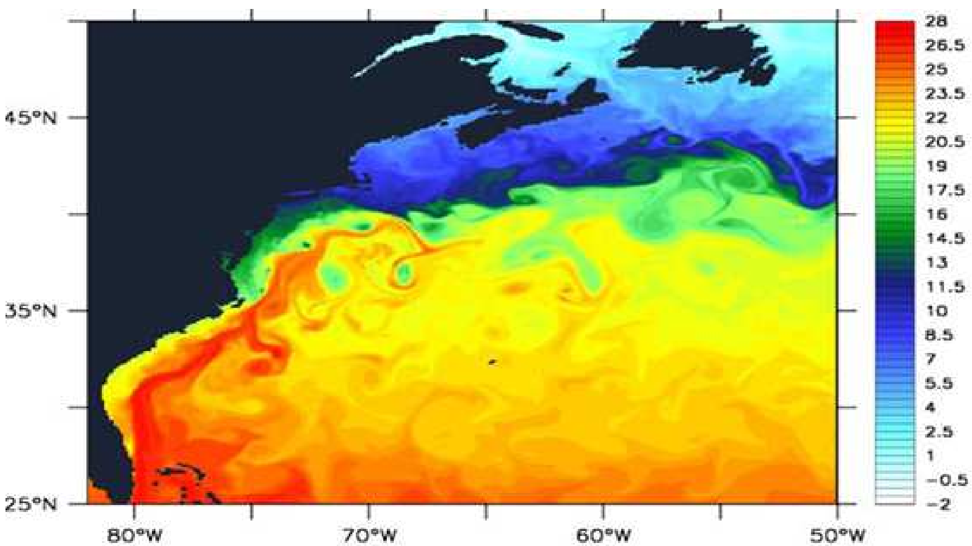
- LANL - Wilbert Weijer.
- U. Groningen - Fred Wubs, Jonas Theis.
- U. Utrecht - Henk Dijkstra, Erik Bernsen.

Outline

- Introduction to Ocean Models & POP.
- The Coriolis Term.
- Pressure Coupling in POP.
- Conclusions & Future Work.

Ocean Modeling in POP

- Parallel Ocean Program(POP) is one of the models in the Community Climate System Model (CCSM).
- Physics of POP
  - Thin stratified fluid equations w/ hydrostatic and Boussinesq approximations.
  - Coupled temperature & salinity advection-diffusion.



Left: THCM, J. Thies; Right: POP website

Why Implicit?

- Ocean codes have historically been explicit or semi-implicit.
- Motivation # 1: Spin Up
  - To spin up the ocean requires time integration lasting for centuries.
  - This is, in effect, setup for another run, so the dynamics don't matter.
- Motivation # 2: Bifurcation analysis of steady states.
- Motivation # 3: Transient analysis above the CFL (see Dana Knoll; Wed 4:30pm).

POP Equations

$$\frac{\partial u}{\partial t} + \mathcal{C}_1(u) - \alpha_1 uv - \Omega v + \alpha_2 \frac{\partial \eta}{\partial \lambda} + \alpha_3 \frac{\partial p_{bc}(S,T)}{\partial \lambda} - D_1(u,v) = 0$$

$$\frac{\partial v}{\partial t} + \mathcal{C}_1(v) + \alpha_1 u^2 + \Omega u + \alpha_4 \frac{\partial \eta}{\partial \phi} + \alpha_5 \frac{\partial p_{bc}(S,T)}{\partial \phi} - D_2(u,v) = 0$$

$$\frac{\partial S}{\partial t} + \mathcal{C}_2(S) + \mathcal{C}_3(u,v,S) - D_3(S,T) = 0$$

$$\frac{\partial T}{\partial t} + \mathcal{C}_2(T) + \mathcal{C}_3(u,v,T) - D_3(S,T) = 0$$

$$\frac{\partial \eta}{\partial t} + \int_{-H}^{0} \left(\alpha_6 \frac{\partial u}{\partial \lambda} + \alpha_7 \frac{\partial v}{\partial \phi} \right) dz = 0$$

POP Equations

$$\frac{\partial u}{\partial t} + \mathcal{C}_1(u) - \alpha_1 uv - \Omega v + \alpha_2 \frac{\partial \eta}{\partial \lambda} + \alpha_3 \frac{\partial p_{bc}(S,T)}{\partial \lambda} - D_1(u,v) = 0$$

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Diffusion
Convection
Coriolis
Coupling #1
Coupling #2

POP Test Problem

- Sector
  - Rectangular (8 z nodes; number of x & y nodes vary).
  - Horizontally homogeneous thermal stratification
  - No forcing
  - Little fluid flow.

Outline

- Introduction to Ocean Models & POP.
- The Coriolis Term.
  - Analysis of 2D Coriolis-Diffusion.
  - Algebraic Multigrid (AMG).
  - Convection-Coriolis-Diffusion Problems in POP.
- Pressure Coupling in POP.
- Conclusions & Future Work.

The Coriolis Term (1)

- Consider the Coriolis-Diffusion equation:

$$\begin{bmatrix} -\Delta & \Omega \\ -\Omega & -\Delta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}.$$

- (Periodic) Fourier analysis gives spectral radius:

$$\rho_J = \left(\frac{16\cos^2(2\pi h) + \Omega^2 h^4}{16} \right)^{1/2}$$

which means it converges if:

$$\Omega < \frac{4}{h^2} |\sin(2\pi h)|.$$

The Coriolis Term (2)

$$\begin{bmatrix} -\Delta & \Omega \\ -\Omega & -\Delta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}.$$

- Problem: Ocean models have kilometer scale h.
- Solution: Ω is diagonal, so it can be block inverted.
- (Periodic) Fourier analysis shows Block(2) Jacobi is stable for any Ω and spectral radius,

$$\rho_B = \left(\frac{16 \cos^2(2\pi h)}{16 + \Omega^2 h^4} \right)^{1/2}$$

- In fact, larger $\Omega \Rightarrow$ *faster* convergence.

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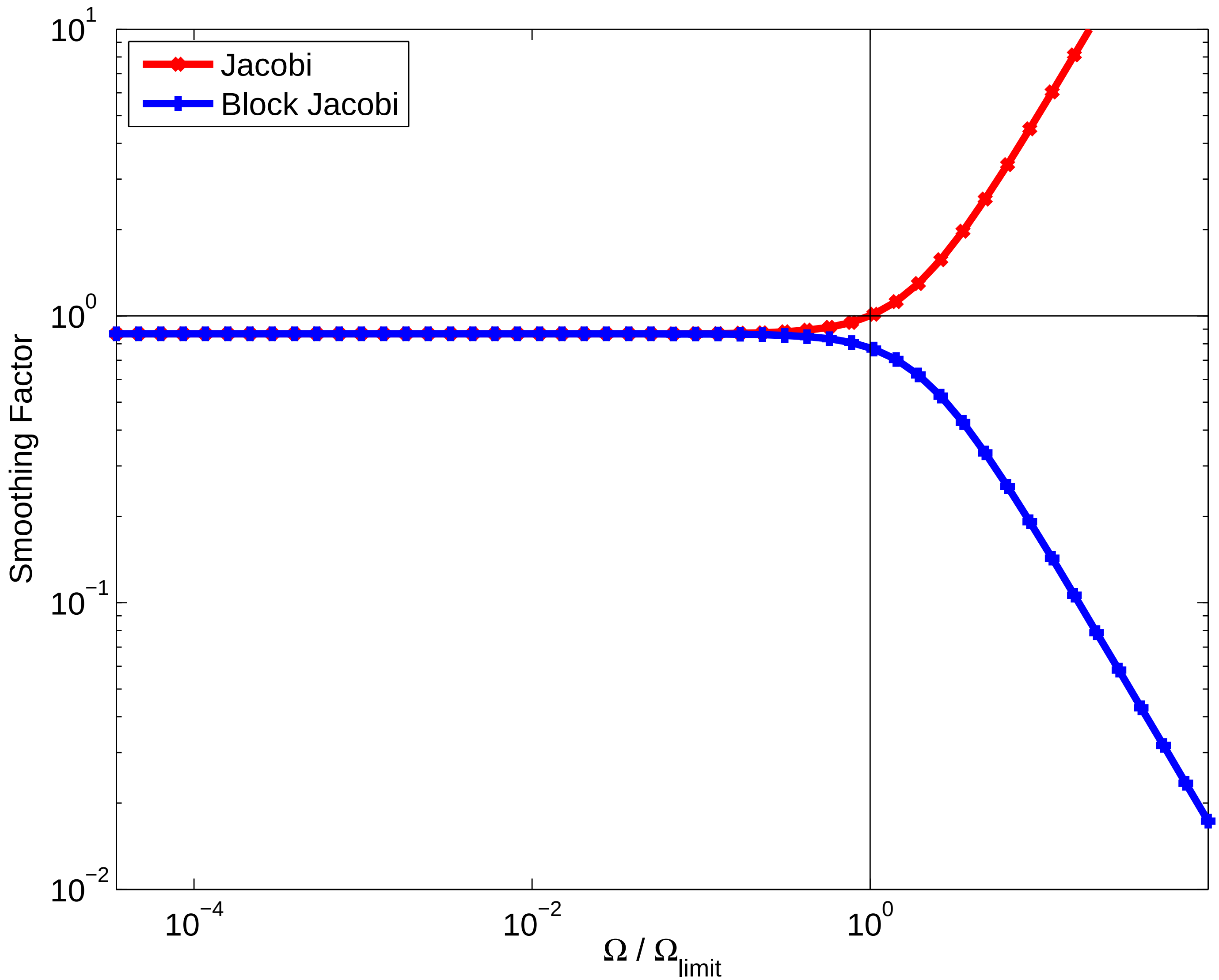
Smoothing Factor by Ω



Coriolis Term & Time

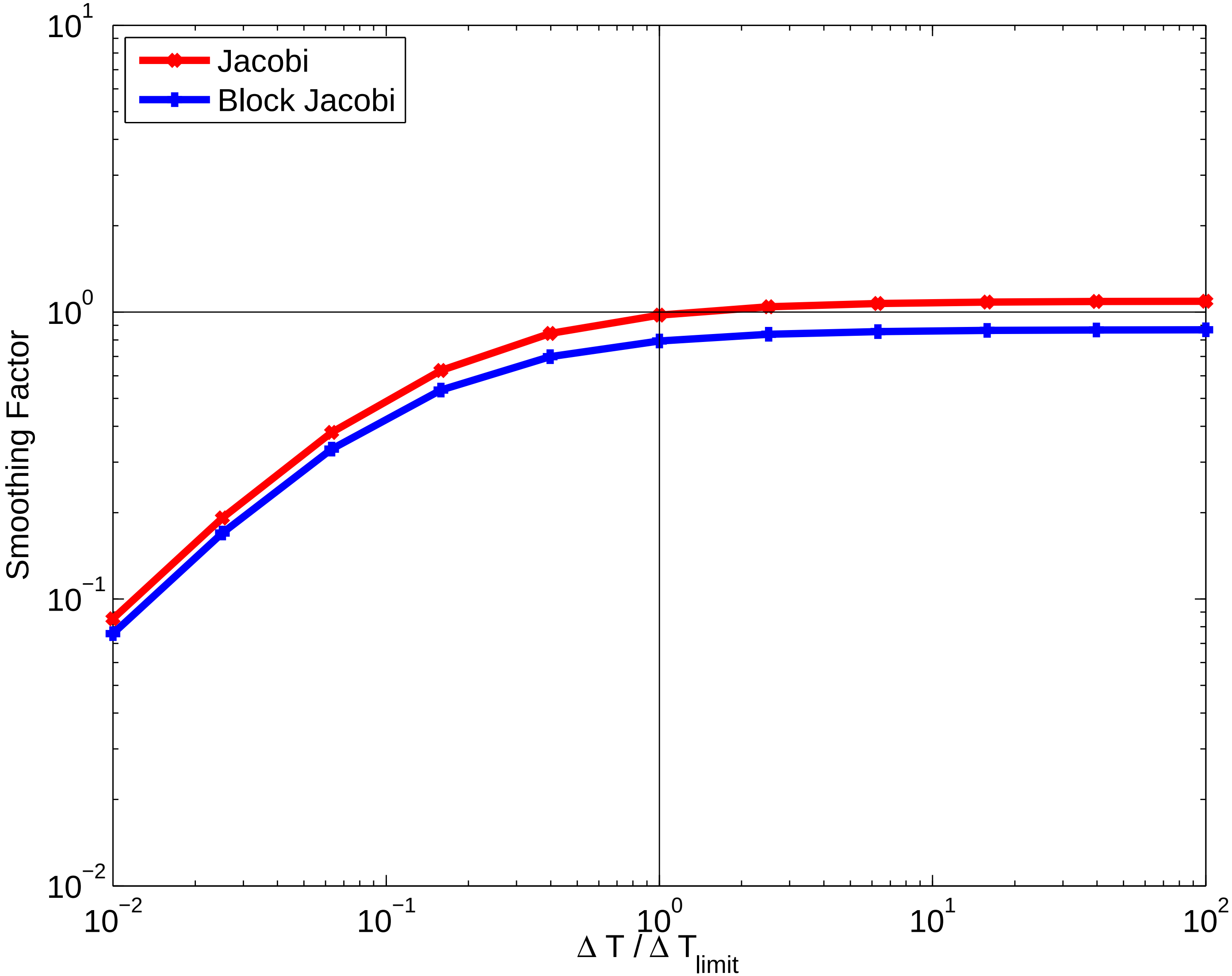
- Now consider the time-dependent version:

$$\begin{bmatrix} \frac{\partial}{\partial t} - \Delta & \Omega \\ -\Omega & \frac{\partial}{\partial t} - \Delta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}.$$

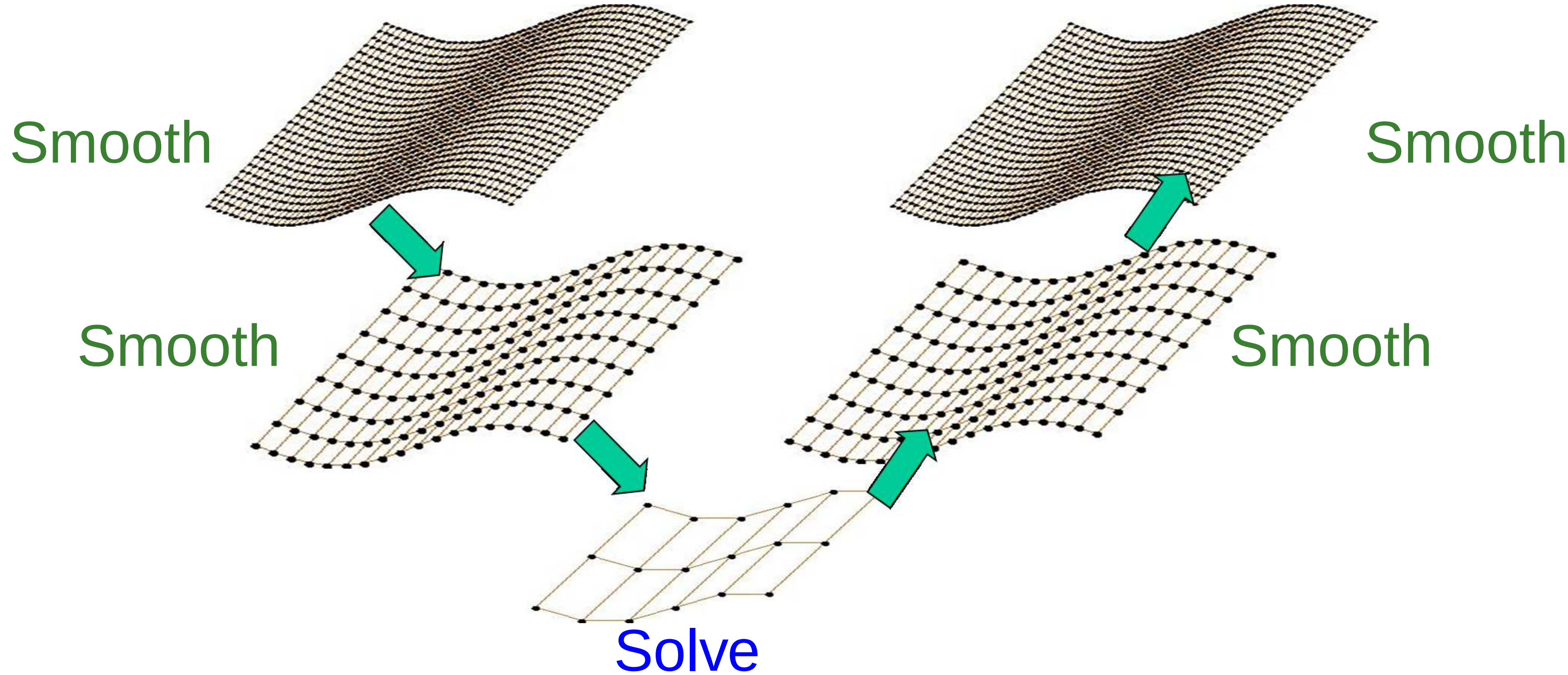
- This adds a little more stability…

$$\rho_J = \left(\frac{16\cos^2(2\pi h) + \Omega^2 h^4}{\left(4 + \frac{h^2}{\Delta t}\right)^2} \right)^{1/2}$$

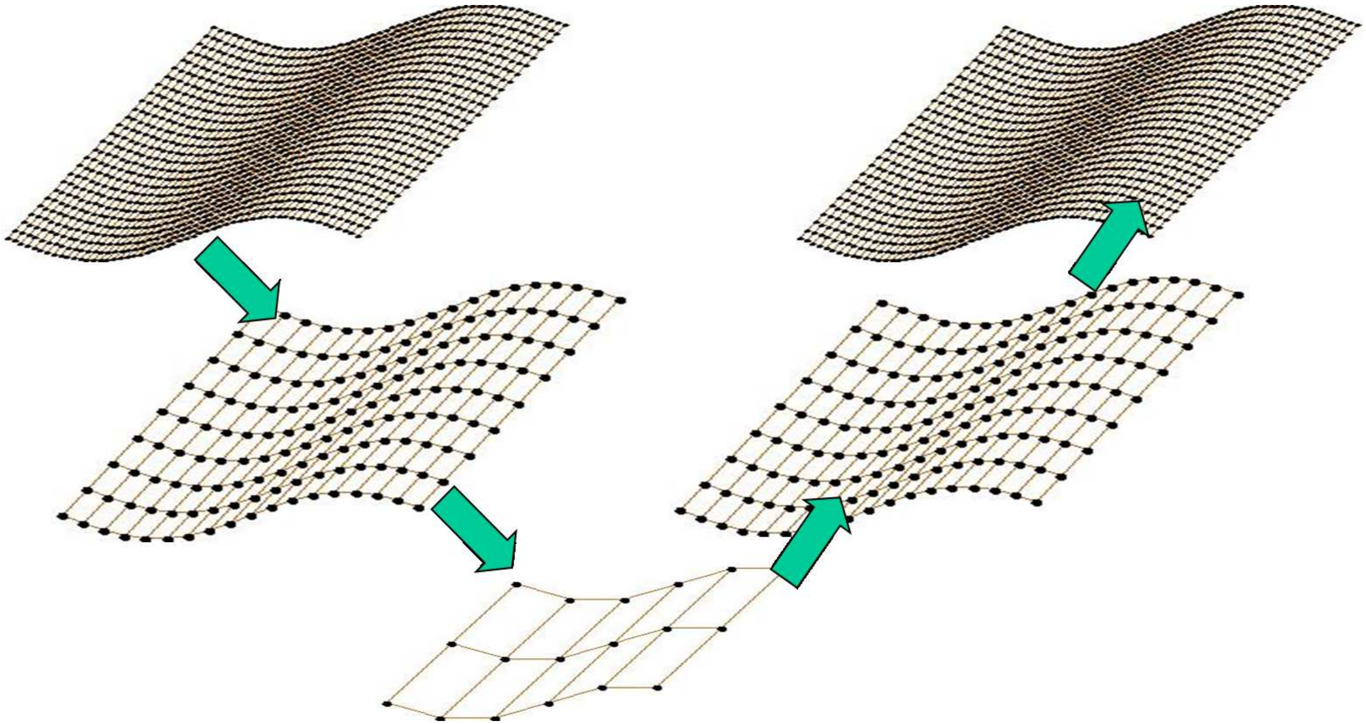
Smoothing Factor by Δt



AMG for Coriolis-Diffusion

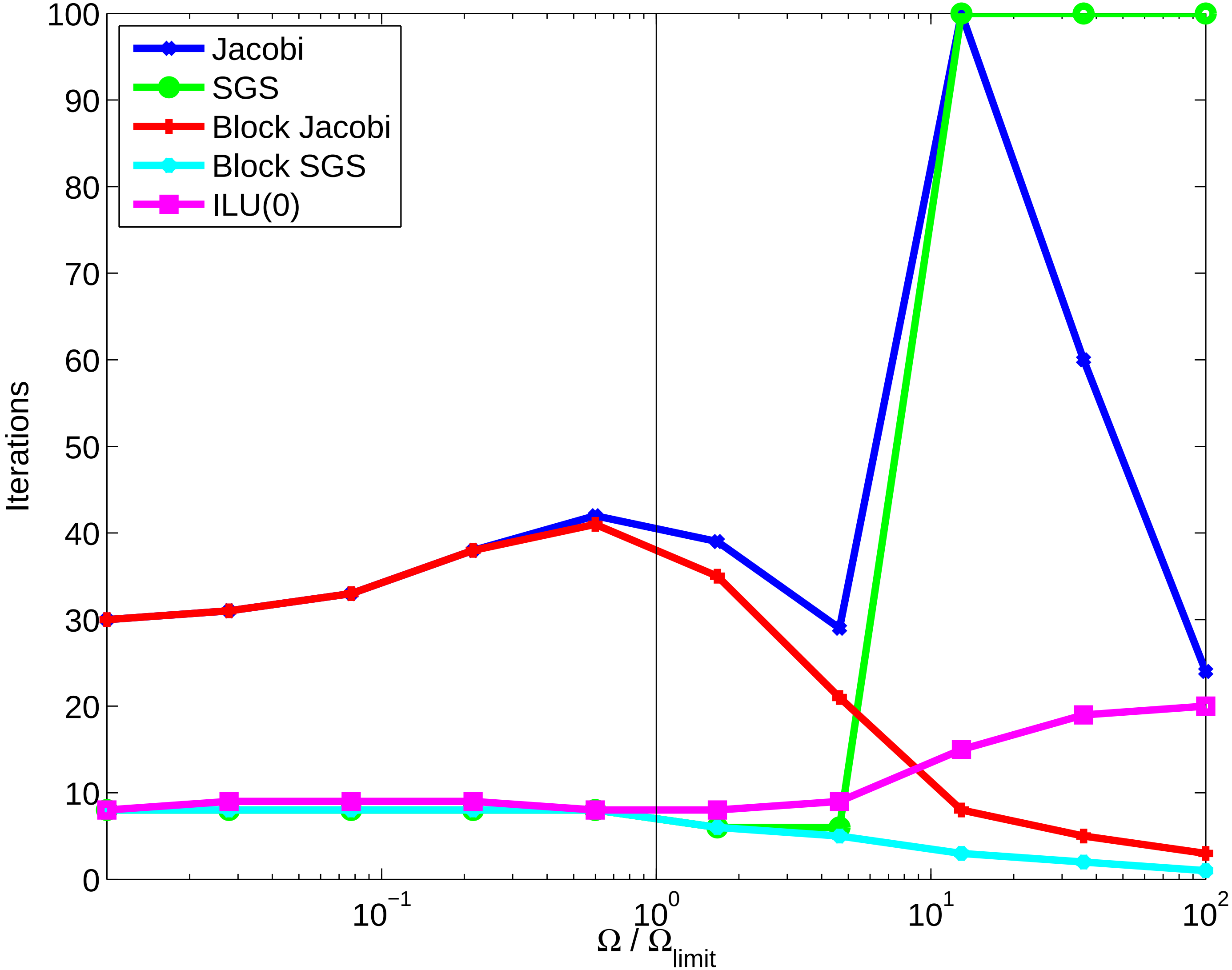


AMG for Coriolis-Diffusion



- Consider (Block) Jacobi, (Block) SGS and ILU(0) as smoothers to a 2-grid method.
- Coarse grid: Un-smoothed aggregation w/ 2 nullspace vectors.

2-Level AMG Convergence



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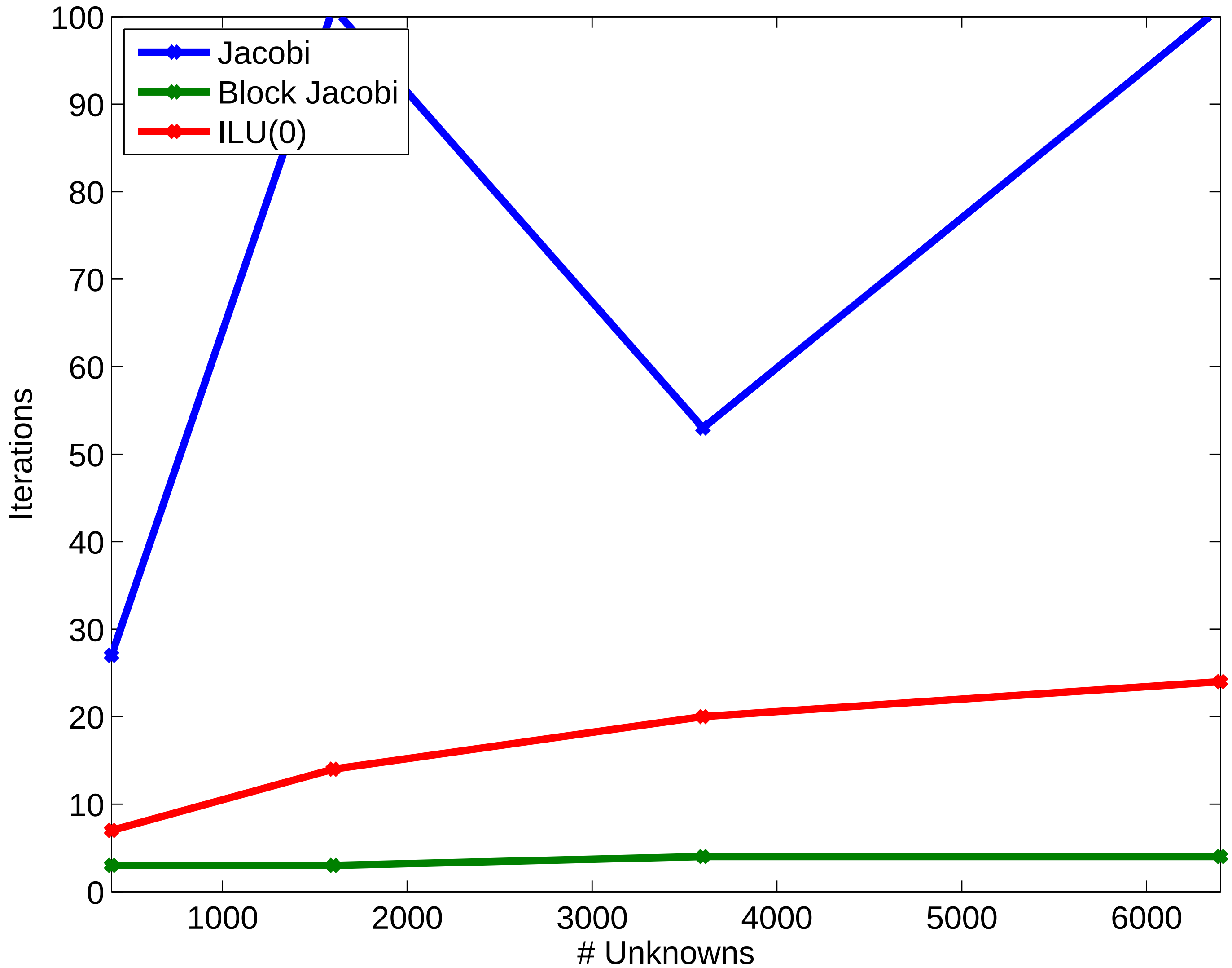
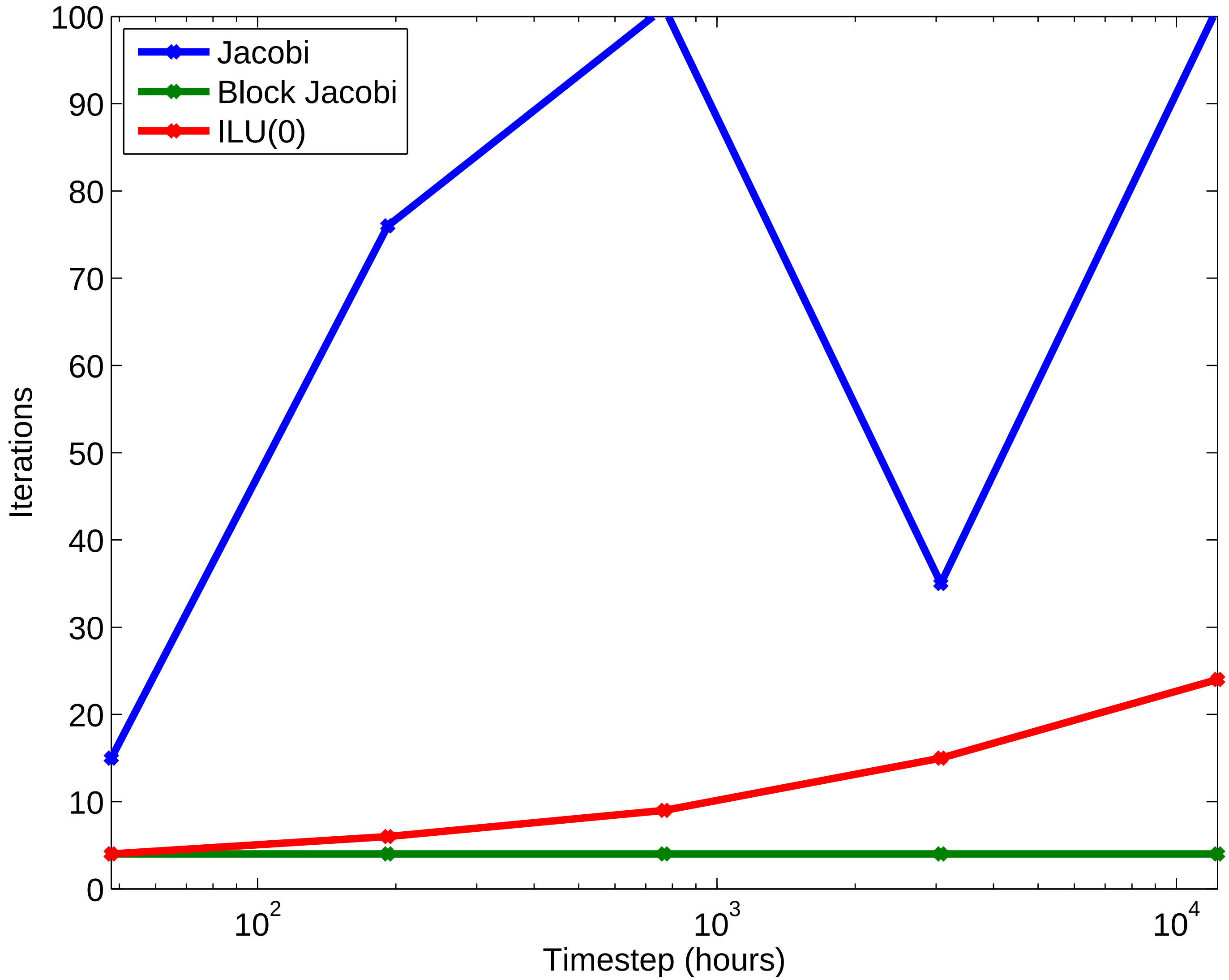
Sector: Δx Refinement



Sector: Large Δt



Outline

- Introduction to Ocean Models & POP.
- The Coriolis Term.
- Pressure Coupling in POP.
  - Schur Complement Preconditioners.
  - Why SIMPLE Won't Suffice.
  - A Quick Look at Probing.
- Conclusions & Future Work.

Schur Complements

- Factor a block 2×2 matrix:

$$\begin{bmatrix} A & B^T \\ C & D \end{bmatrix} = \begin{bmatrix} I & \\ CA^{-1} & I \end{bmatrix} \begin{bmatrix} A & \\ & S \end{bmatrix} \begin{bmatrix} I & A^{-1}B^T \\ & I \end{bmatrix}$$

  where $S = D - CA^{-1}B^T$.

- Approximate this factorization in order to precondition.
- Now we're left with two questions:
  - How to approximate A^{-1}? (To isolate effect of S^{-1}, we will use LU).
  - How to approximate S^{-1}?

SIMPLE & Block SIMPLE

- SIMPLE-like methods approximate $S = D - CA^{-1}B^T$, with

$$S = D - CF^{-1}B^T,$$

  where $F = \text{diag}(A)$.
- Wait a second!
  - Isn't SIMPLE's F a lot like a step of point Jacobi on the convection-diffusion-Coriolis block?
  - Didn't we just show that this is unstable for large Ω?
- We'd best look at a Block SIMPLE as well.

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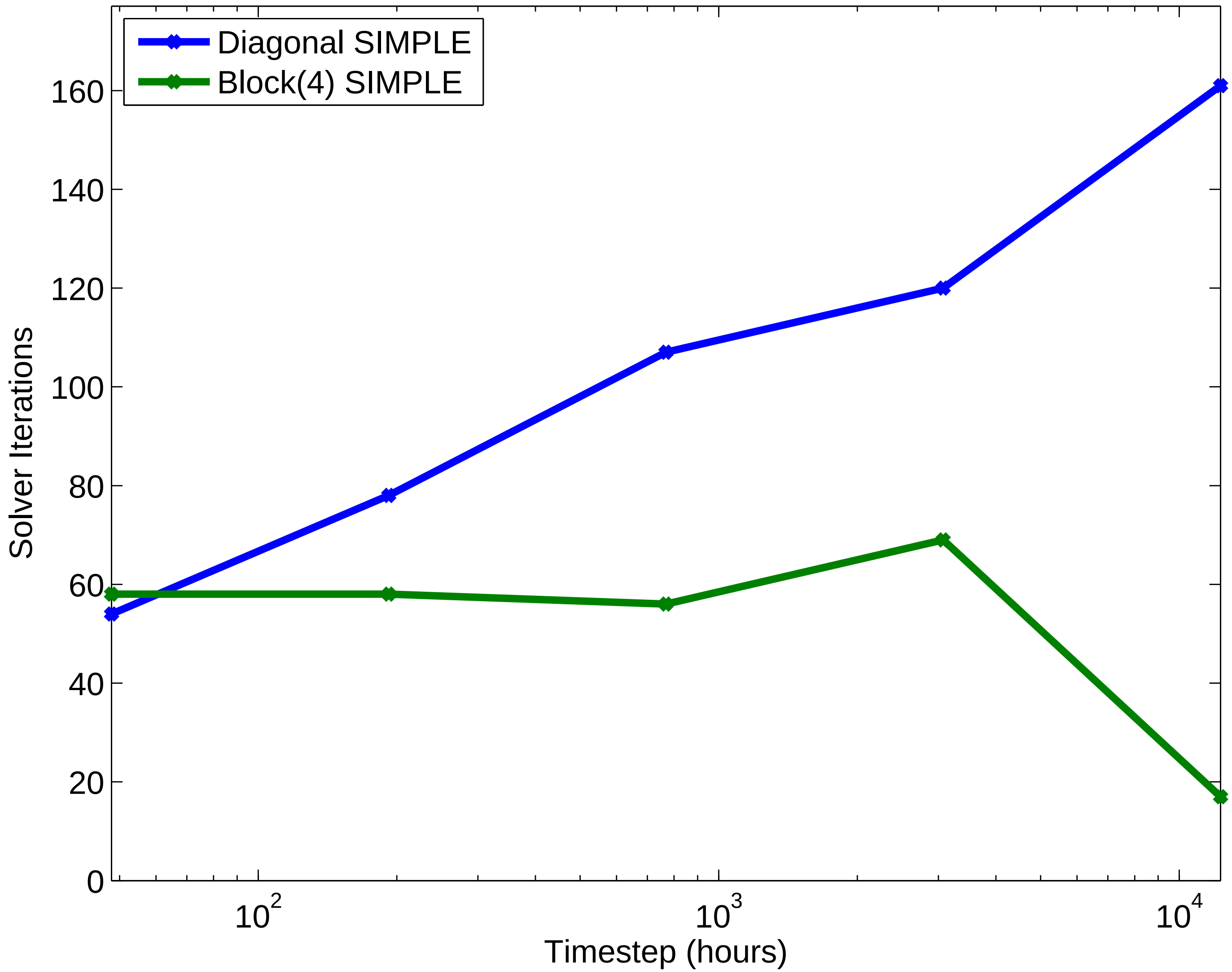
Sector: Large Δt



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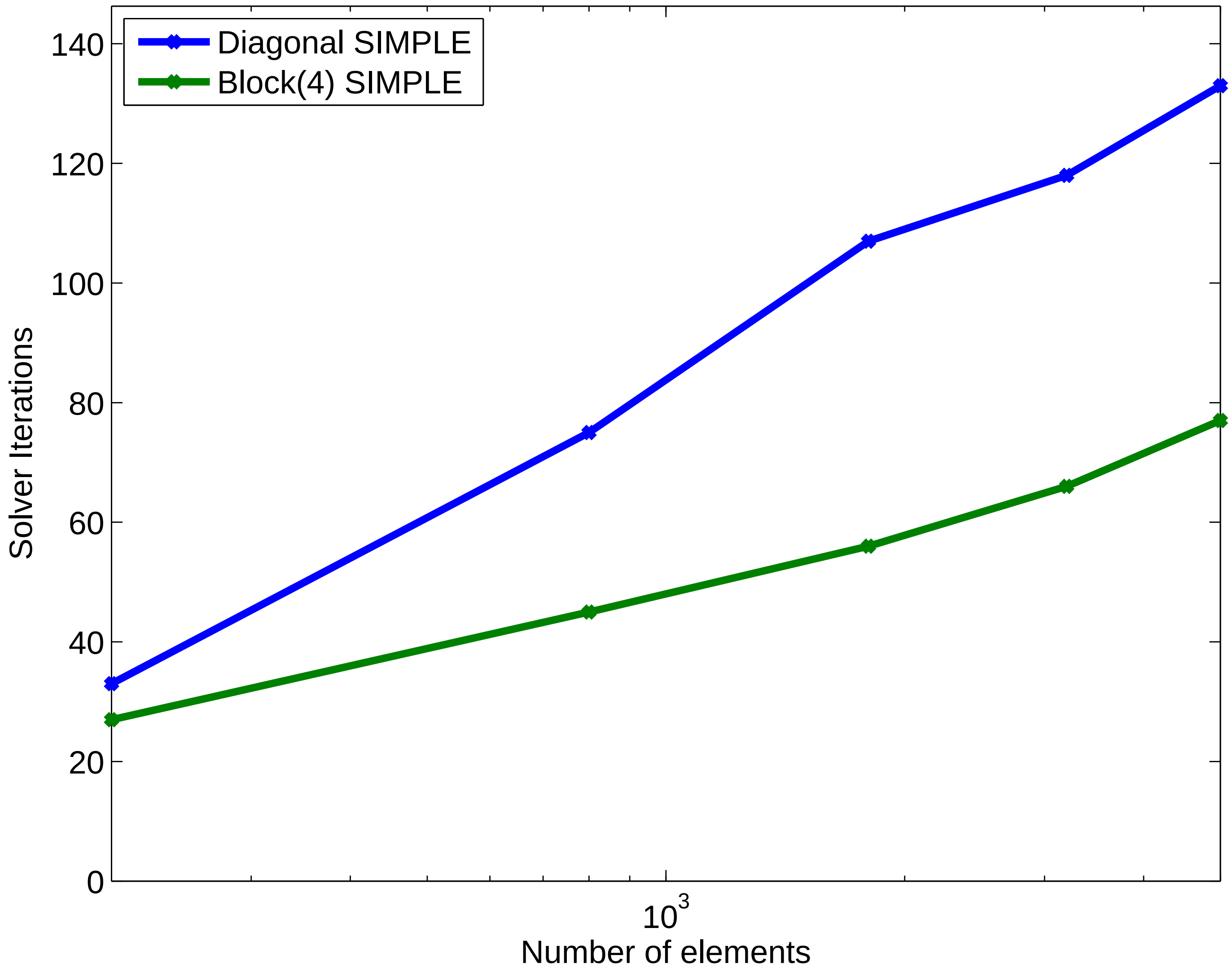

A Quick Look at Probing

- Even Block SIMPLE has problems as Δx gets small.
- Graph coloring-based approach: Probing.
- Idea # 1: Two columns with disjoint sparsity can be exactly probed by a single matvec.

Exact

$$\begin{bmatrix} X & X & & & & \\ X & X & X & & & \\ & X & X & X & & \\ & & X & X & X & \\ & & & X & X & X \\ & & & & X & X \end{bmatrix}$$

A Quick Look at Probing

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Exact

$$\begin{bmatrix} X & X & & & & \\ X & X & X & & & \\ & X & X & X & & \\ & & X & X & X & \\ & & & X & X & X \\ & & & & X & X \end{bmatrix}$$

Approximate

$$\begin{bmatrix} X & X & o & o & & \\ X & X & X & & o & \\ o & X & X & X & & o \\ o & & X & X & X & o \\ & o & & X & X & X \\ & & o & o & X & X \end{bmatrix}$$

A Quick Look at Probing

- Even Block SIMPLE has problems as Δx gets small.
- Graph coloring-based approach: Probing.
- Idea # 1: Two columns with disjoint sparsity can be exactly probed by a single matvec.
- Idea # 2: If we know the “large” parts of the matrix *a priori*, we can approximate the matrix by probing only those.
- Try two sparsity patterns:
  - CB - 9 point stencil.
  - $(CB)^2$ - 27 point stencil.

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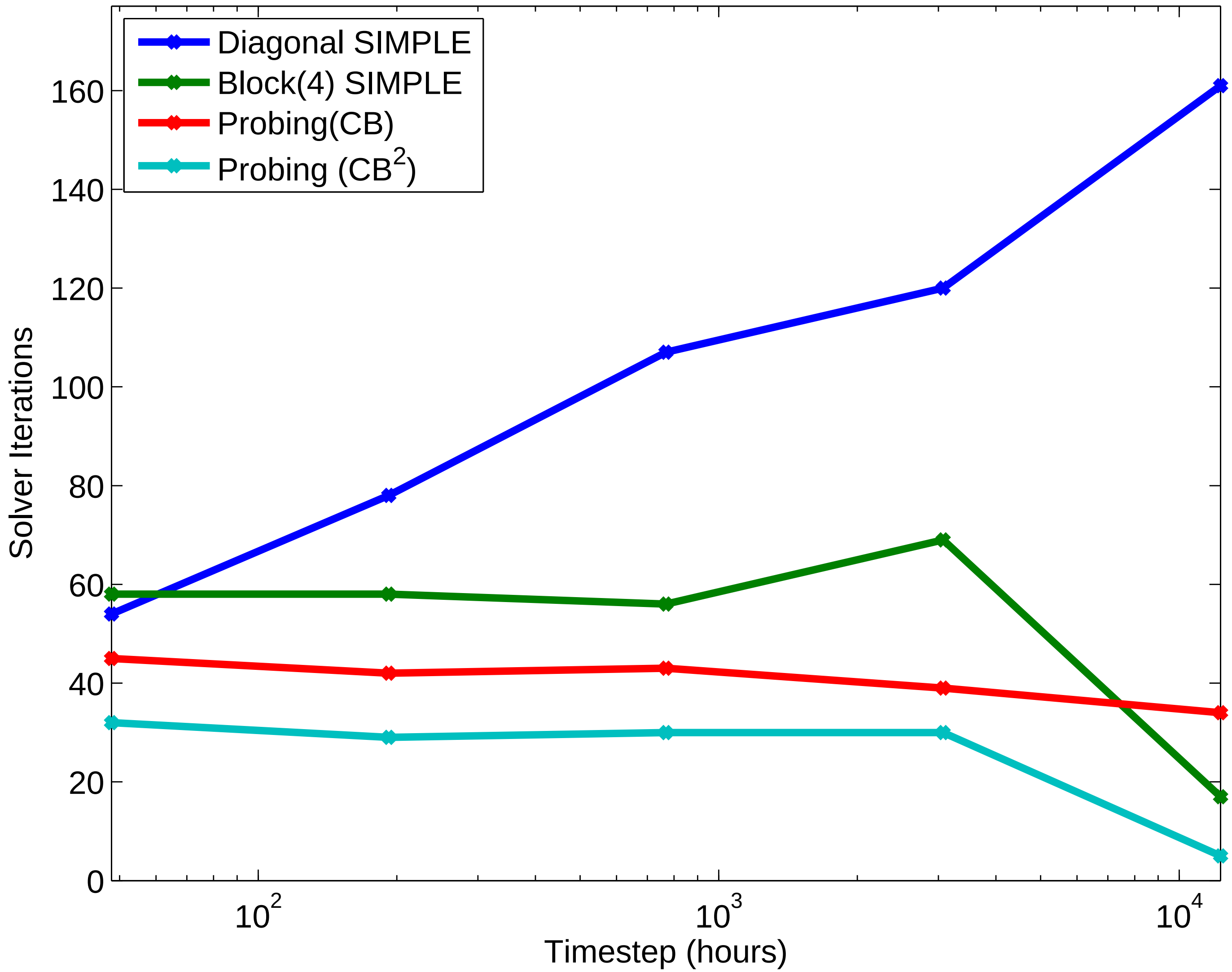
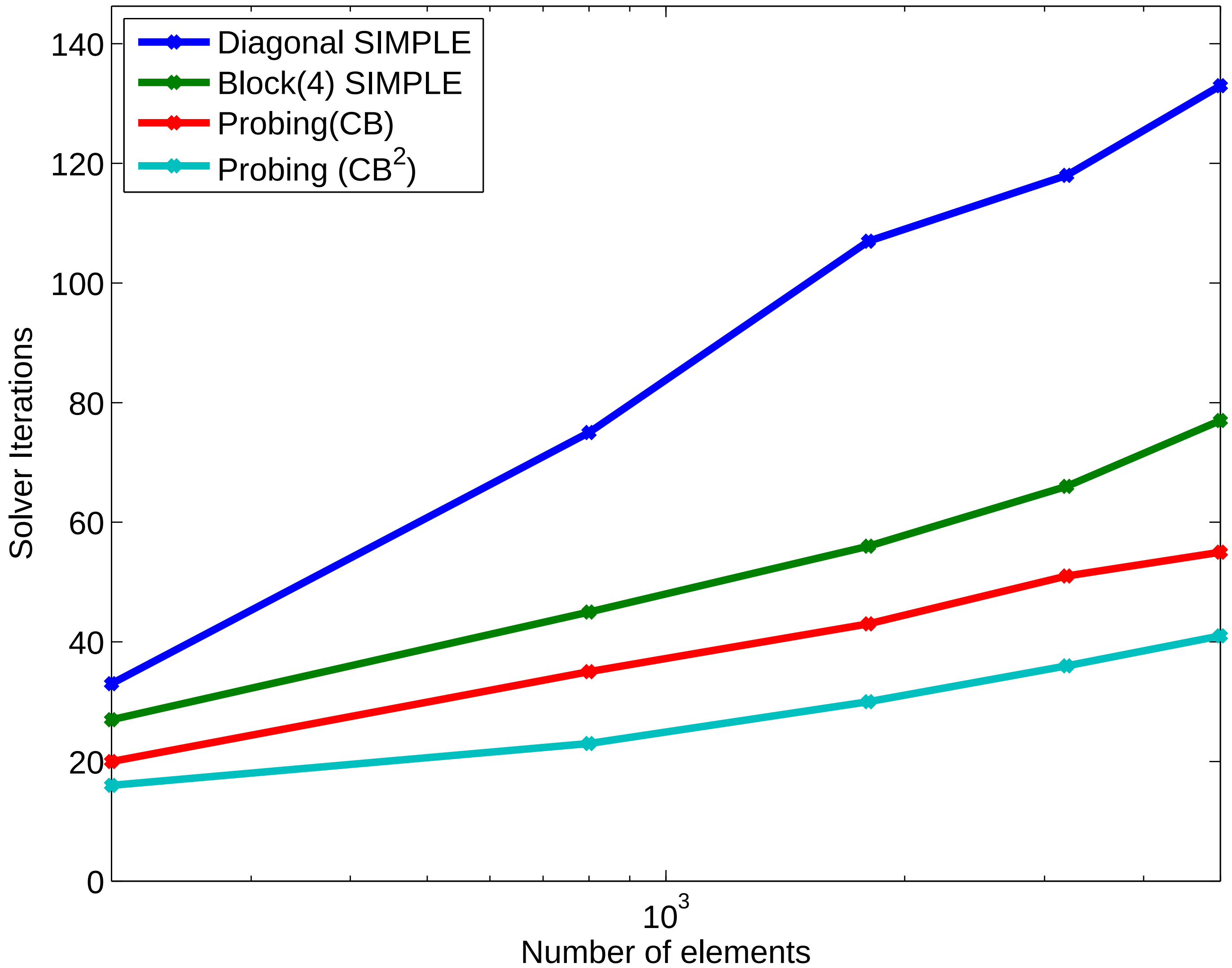
Sector: Large Δt



Sector: Δx Refinement



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Conclusions

- Block methods are important for Coriolis-Diffusion.
  - AMG + Block Jacobi / Block GS works well.
  - Convection may require more powerful block smoothers (Block ILU?).
- Schur complement must capture (1,1)'s block nature.
  - Block SIMPLE is OK even w/ large timesteps.
  - Probing can do somewhat better w/ fine meshes.
- Future work
  - Less mesh dependence.
  - Robust implementation in POP.
  - Harder & more realistic problems.

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