

# Modeling of the dynamic inelasticity of poly- and single crystal tantalum under ramp wave loading\*

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**\* Work sponsored by Sandia National Labs**

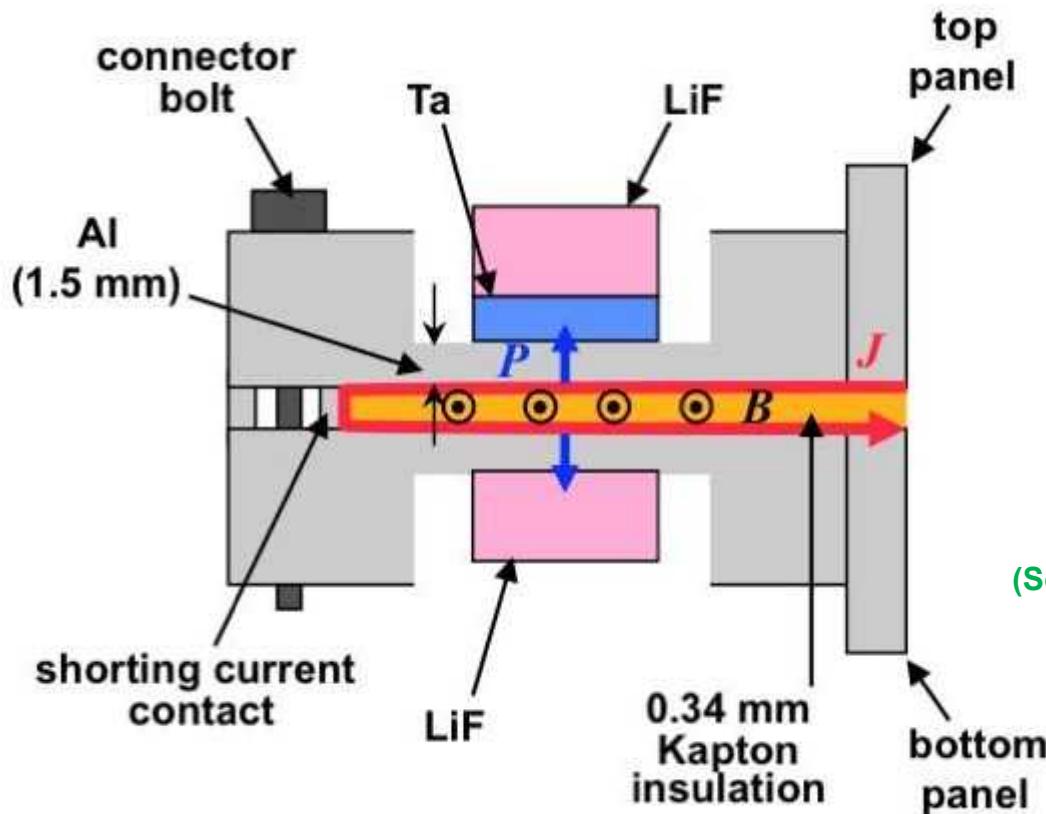
**Presented at the 4<sup>th</sup> Workshop on Ramp Compression**

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# Outline

- Experimental observations of poly- and single crystal tantalum behavior under ramp wave loading.
- Objectives and approach.
- Dislocation-mechanics based continuum material model for polycrystal.
- Simulation of the single crystal experiments and insights gained from the simulation.
- Extension of the polycrystal model to single crystals.
- Simulation of the single crystal experiments and insights gained from the simulation.
- Conclusion.

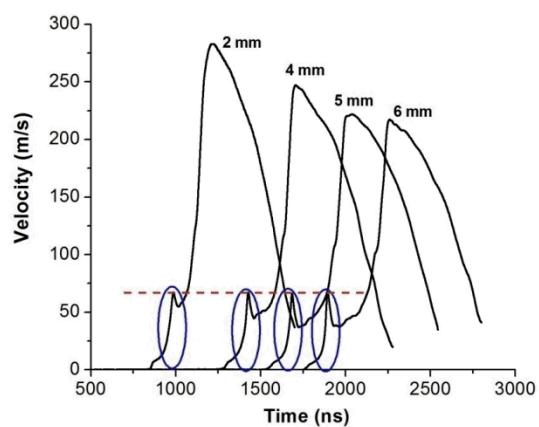
# Configuration Of The Ramp Wave Experiments On Veloce



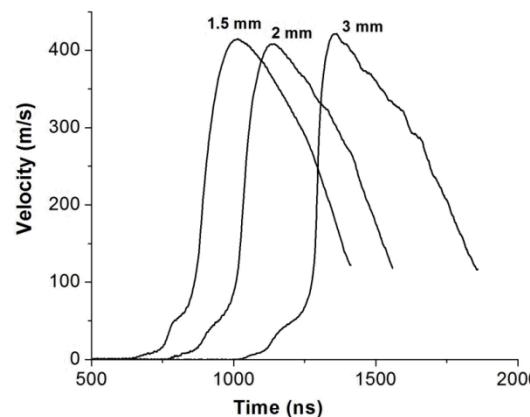
(Source: Ao et al. Rev. Sci. Instr. 2008)

generate a structured wave with finite risetime in the very high stress regime

# Polycrystal Tantalum Under Ramp Wave Loading



Annealed



Cold-Rolled (26%)

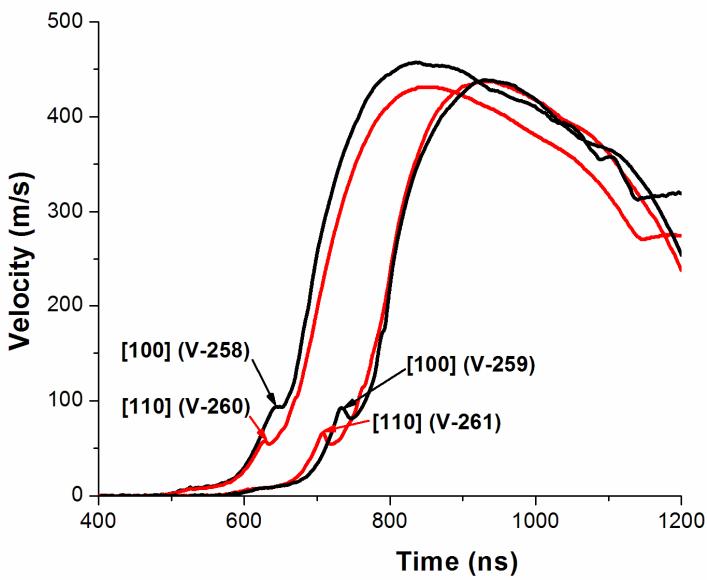
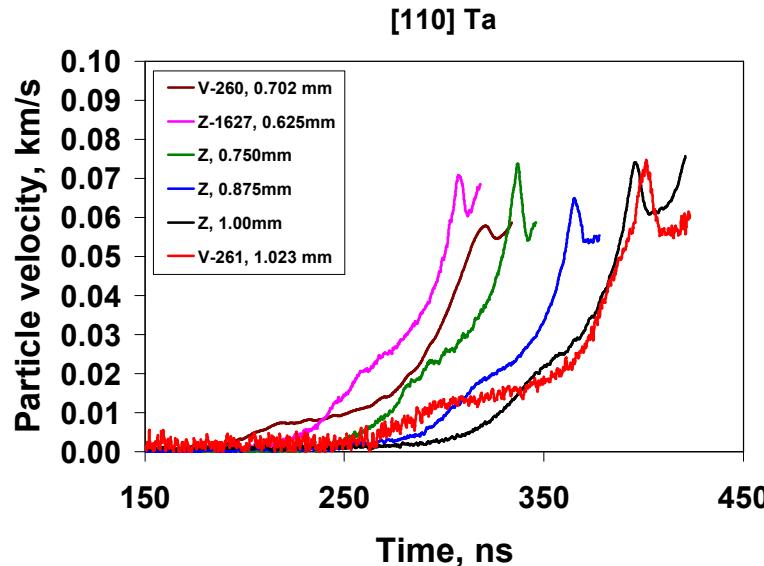
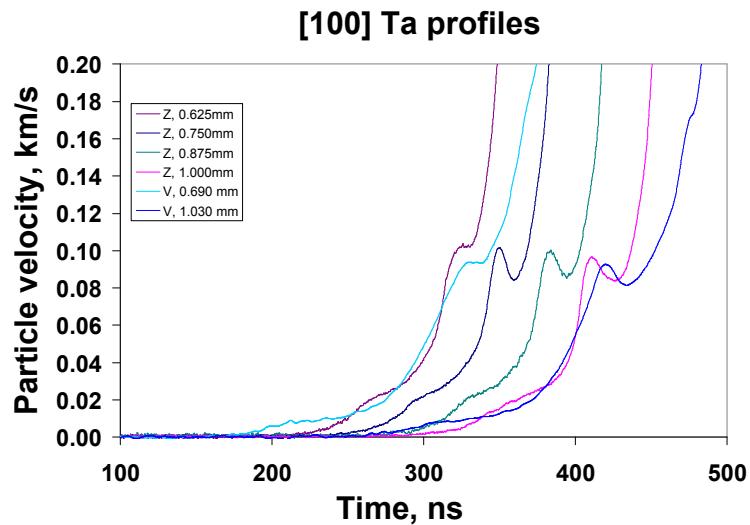
- Annealed samples:

- Elastic precursor showed a pronounced overshoot followed by a significant velocity drop or stress relaxation
- The extent of the velocity drop increased with the sample thickness.
- precursor amplitude showed very little attenuation, even with the significant stress relaxation behind the precursor

- Cold-Rolled:

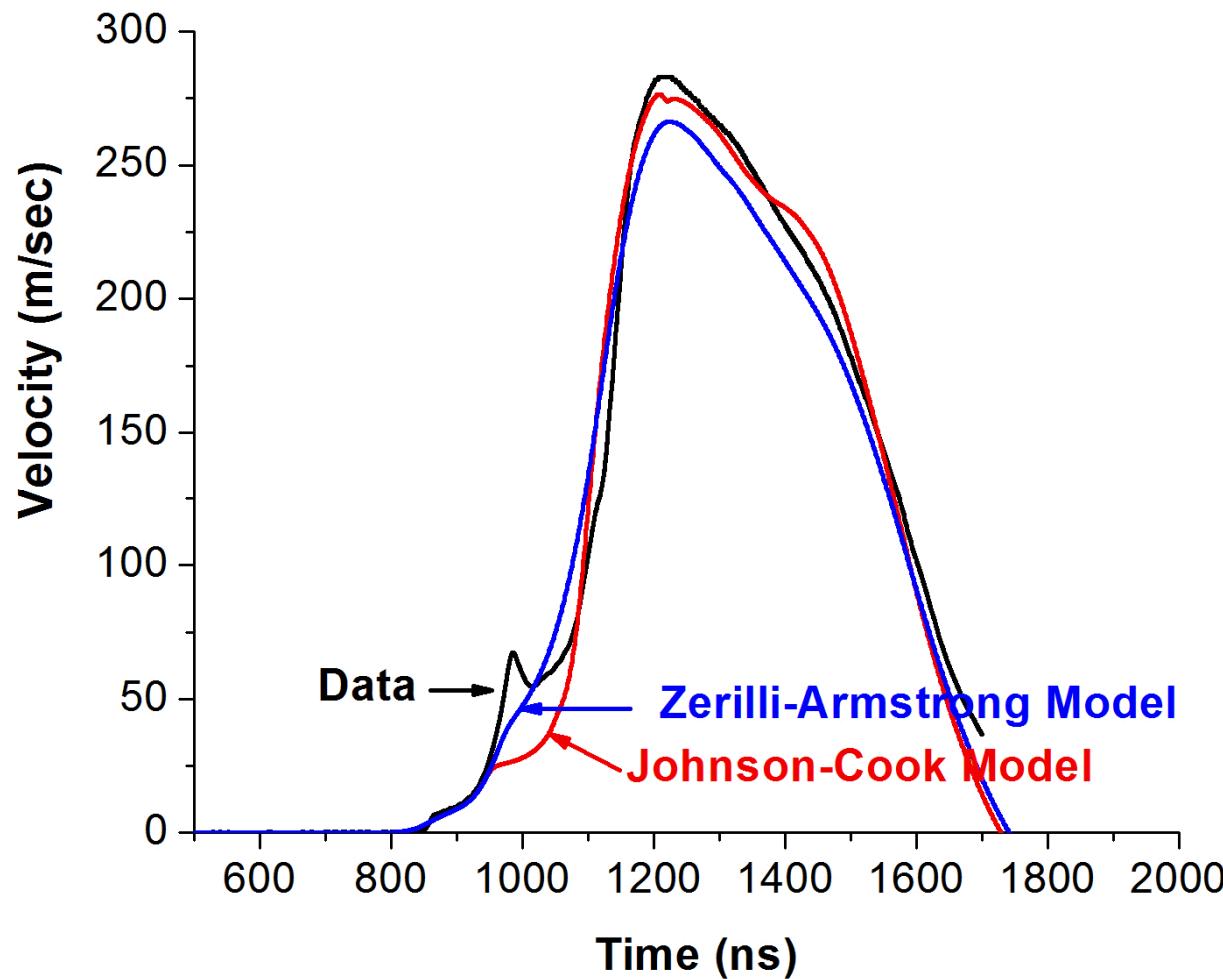
- More dispersed elastic precursor.
- no rapid velocity drop or stress relaxation was observed behind the precursor

# Single Crystal Tantalum Under Ramp Wave Loading



- Similar elastic precursor behavior as polycrystal.
- Strong orientation dependence
- [100] orientation showed slight decay of the precursor , but no apparent trend for [110] orientation.
- [100] orientation is more rate sensitive than [110].

# Comparison of Annealed Polycrystal With Some Existing Models



# Objectives and Approach

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## Objectives:

- To gain insights into the mechanical behavior of tantalum, particularly the elastic precursor behavior, and their implication on the deformation mechanisms for tantalum.
- To gain an understanding of the dynamic inelasticity of poly- and single crystal tantalum, including the material strength and its evolution.

## Approach:

- Develop a constitutive model that captures the material features observed experimentally.
- Use numerical simulation to gain additional insight into the inelastic behavior of tantalum.

# Thermomechanical Constitutive Relation

$$\dot{\sigma}_{ij}^e = B_{ijkl} \dot{\varepsilon}_{kl}^e - \rho \Gamma_{ij} \theta \dot{s} = B_{ijkl} (\dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^p) - \rho \Gamma_{ij} \theta \dot{s}$$

$$\left( B_{ijkl} = \frac{\partial \sigma_{ij}}{\partial \varepsilon_{kl}^e} \Big|_s, \quad \Gamma_{ij} = -\frac{1}{\rho \theta} \frac{\partial \sigma_{ij}}{\partial s} \Big|_{\varepsilon_{ij}^e} \right)$$

$$\dot{\theta} = -\theta \Gamma_{ij} \dot{\varepsilon}_{ij}^e + (\theta / C_v) \dot{s} = -\theta \Gamma_{ij} (\dot{\varepsilon}_{ij} - \dot{\varepsilon}_{ij}^p) + (\theta / C_v) \dot{s}$$

$$\dot{s} = \frac{1}{\rho \theta} (\sigma_{ij}^{e'} \dot{\varepsilon}_{ij}^p - q_{i,i})$$

$$q_i = -k \theta_{,i}$$

$$\dot{\gamma} = \dot{\gamma}_{mech} + \dot{\gamma}_{con}$$

$$\dot{\gamma}_{mech} = \frac{ds}{dt} - \left( -\frac{1}{\rho \theta} q_{i,i} \right) = \frac{\sigma_{ij}^{e'} \dot{\varepsilon}_{ij}^p}{\rho \theta} \geq 0$$

$$\dot{\gamma}_{con} = -\frac{1}{\rho \theta^2} q_i \theta_{,i} \geq 0$$

# Strength Model

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$$\dot{\sigma}'_{ij} = 2G\dot{\varepsilon}'_{ij} = 2G(\dot{\varepsilon}'_{ij} - \dot{\varepsilon}'^p_{ij})$$

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$$\dot{\varepsilon}'^p_{ij} = \bar{\dot{\varepsilon}}^p \left( \sigma'_{ij} / \left| \sigma'_{ij} \right| \right) \quad \text{and} \quad \bar{\dot{\varepsilon}}^p = A[\left| \sigma'_{ij} \right| - \sigma_{th}]^{1.5}$$

$$\dot{\varepsilon}^p = \left( \frac{2}{3} \dot{\varepsilon}'^p_{ij} \dot{\varepsilon}'^p_{ij} \right)^{1/2} : \text{effective plastic strain rate}$$

$$\left| \sigma'_{ij} \right| = \left( \frac{3}{2} \sigma'_{ij} \sigma'_{ij} \right)^{1/2} : \text{effective stress, a measure of material strength}$$

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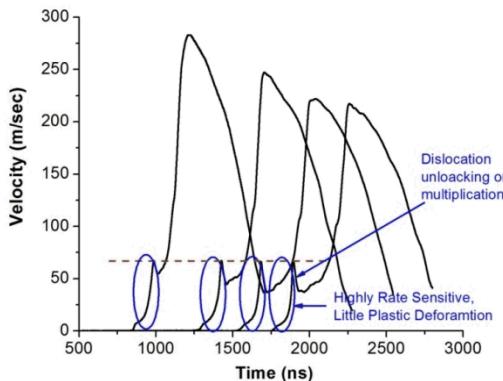
$$G = G_0 \left[ 1 + \left( \frac{G'_P}{G_0} \right) \frac{P}{\eta^{1/3}} + \left( \frac{G'_T}{G_0} \right) (T - T_0) \right]$$

$$\sigma_{th} = \sigma_{th_0} \left[ 1 + \beta (\bar{\dot{\varepsilon}}^p + \bar{\dot{\varepsilon}}^p_i) \right]^n \left[ 1 + \left( \frac{G'_P}{G_0} \right) \frac{P}{\eta^{1/3}} + \left( \frac{G'_T}{G_0} \right) (T - T_0) \right]$$

(Steinberg, Cochran, and Guinan, JAP, 1980)

# Association of The Rate Eqn. with Dislocation Motion

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- Little decay of precursor implies little plastic deformation during precursor loading
- Dislocation unlocking or multiplication at the precursor tip.

## Dislocation model (Orowan Eqn.):

$$\dot{\gamma}^p = b \rho_m v \quad \text{where} \quad \begin{aligned} \dot{\gamma}^p &: \text{plastic strain rate;} \\ b &: \text{Burgers vector;} \\ \rho_m &: \text{mobile dislocation density;} \\ v &: \text{stress dependent dislocation velocity} \end{aligned}$$

$$\dot{\varepsilon}^p = A[\sigma'_{ij} - \sigma_{th}]^{1.5} = b \rho_m \{c[\sigma'_{ij} - \sigma_{th}]^{1.5}\}$$

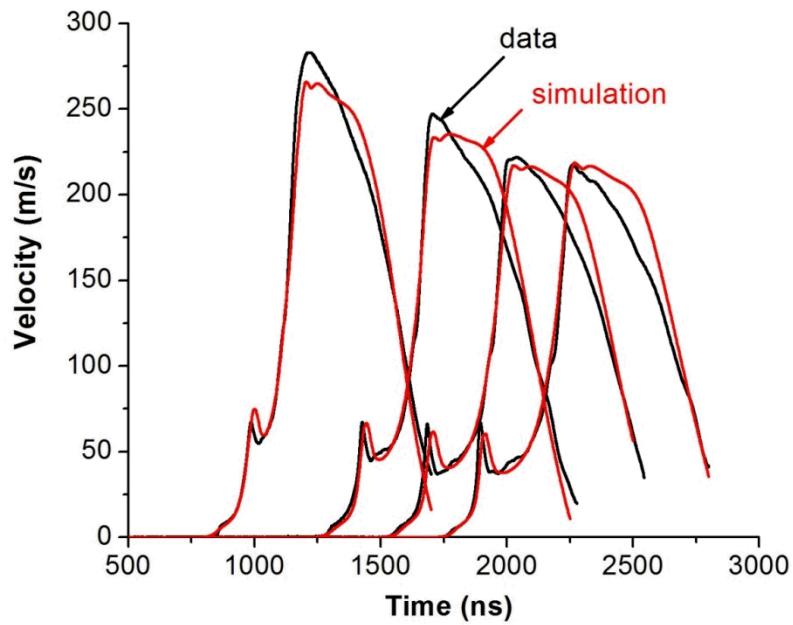
where

$$\rho_m = f \rho_t \quad \text{with } \rho_t \quad \text{being the total dislocation density}$$

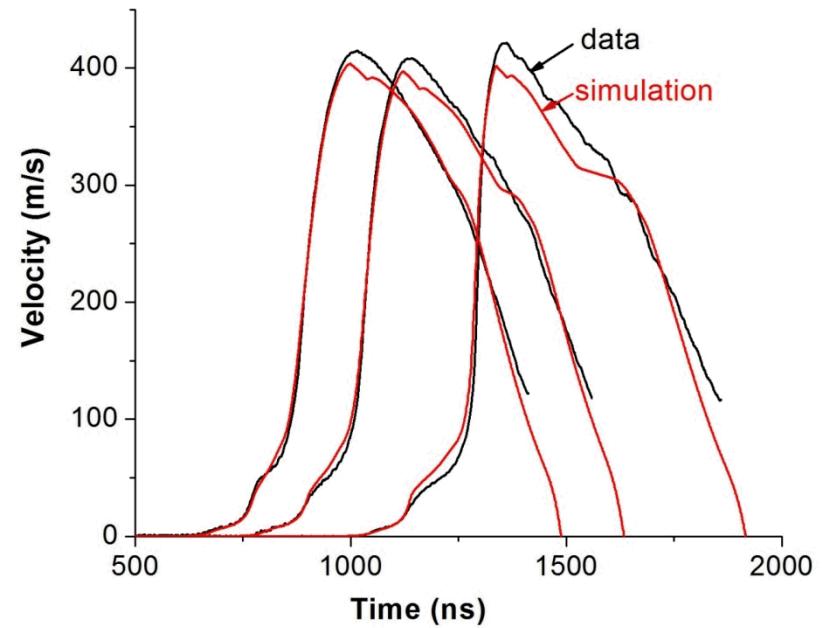
$$\rho_t = \rho_0 + C(\bar{\varepsilon}^p)^a \quad (\text{Hahn, Acta Met. 1962})$$

$$f = f_i + (f_f - f_i)(1 - e^{-\lambda \bar{\varepsilon}^p}) \quad (\text{Yoshida et. al., IJP, 2008})$$

# Simulated Polycrystal Velocity Histories



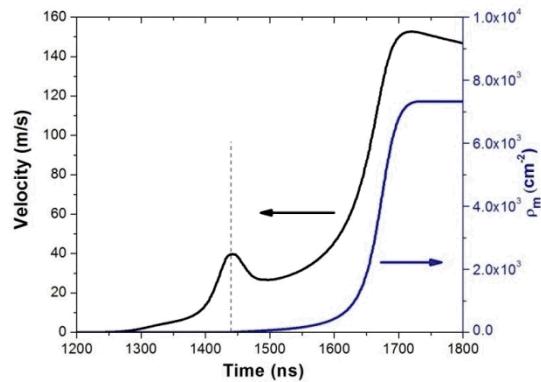
Annealed



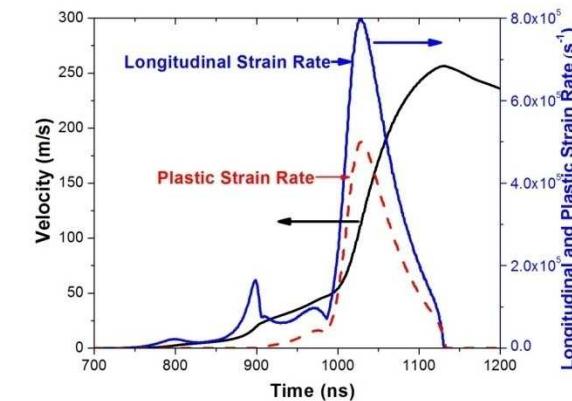
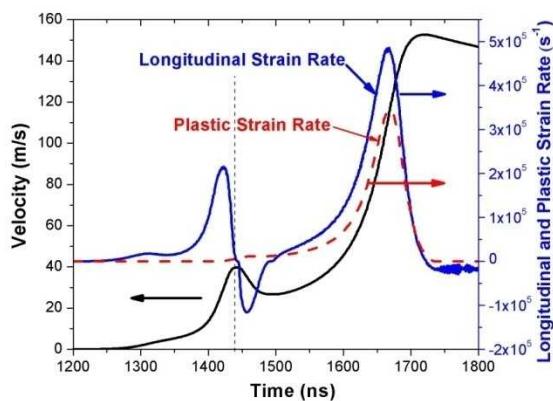
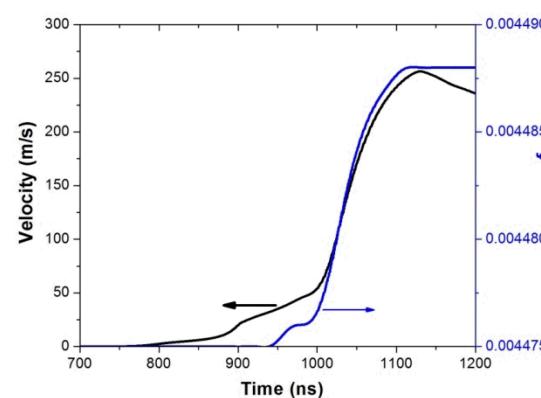
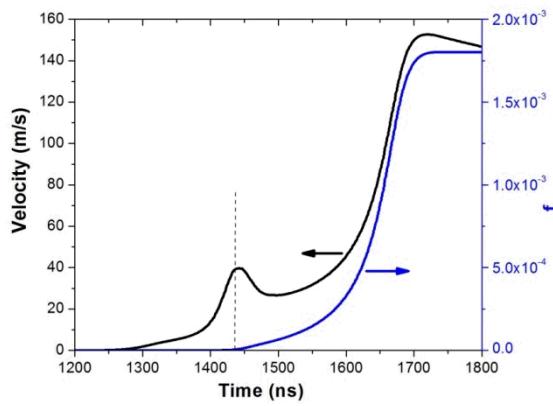
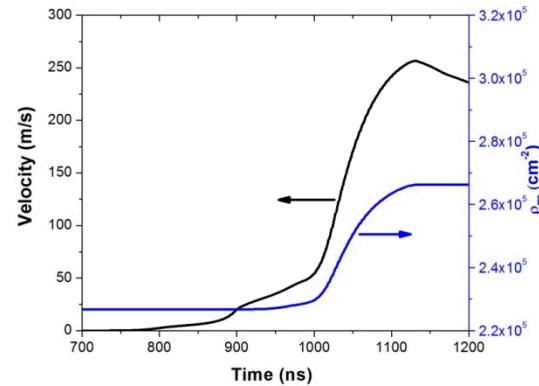
Cold-Rolled (26%)

# Annealed vs Cold Rolled

## Annealed

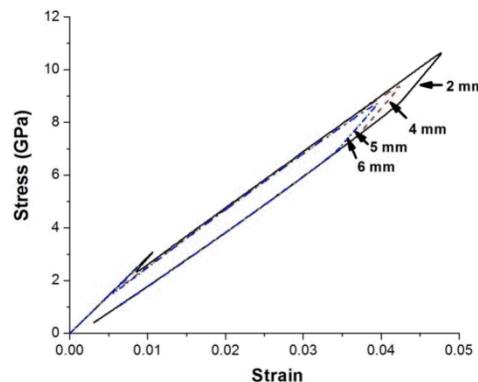
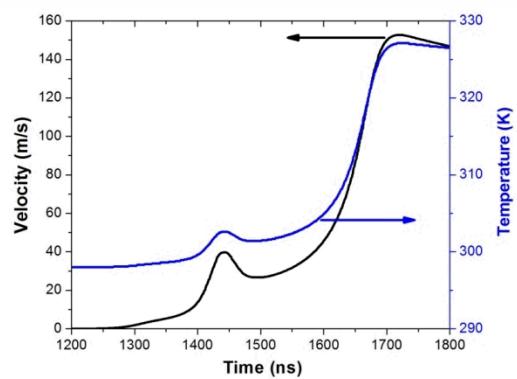
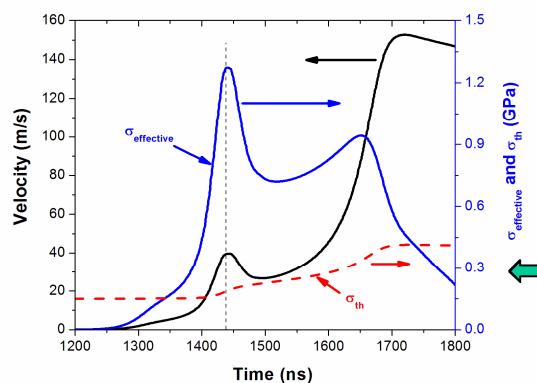


## Cold-Rolled

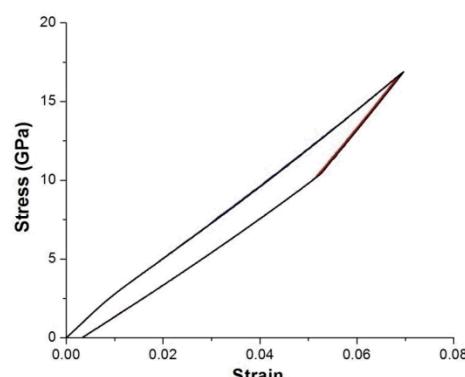
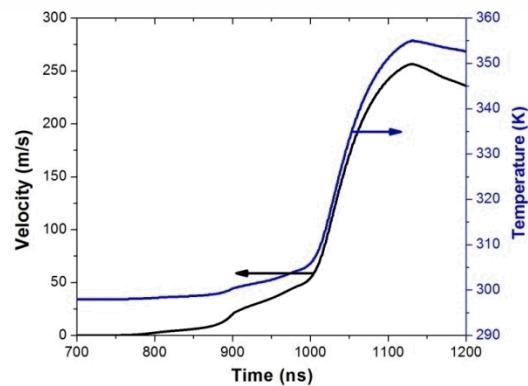
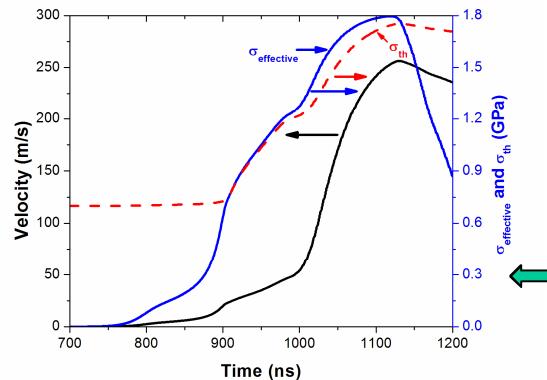


# Annealed vs Cold Rolled

## Annealed



## Cold-Rolled



highly rate sensitive response

less rate sensitive response

# Simulation of Single Crystal Tantalum With Polycrystal Model

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- Use continuum model with different material constants for different orientations to describe the single crystal behavior.
- The model is used as a data analysis tool to estimate the material properties and their evolution.

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$$\dot{\varepsilon}^p = A[\sigma'_{ij} - \sigma_{th}]^{1.5} = b\rho_m \{c[\sigma'_{ij} - \sigma_{th}]^2\} \quad (\dot{\gamma}^p = b\rho_m v)$$

where  $\rho_m = f\rho_t$  with  $\rho_t$  being the total dislocation density

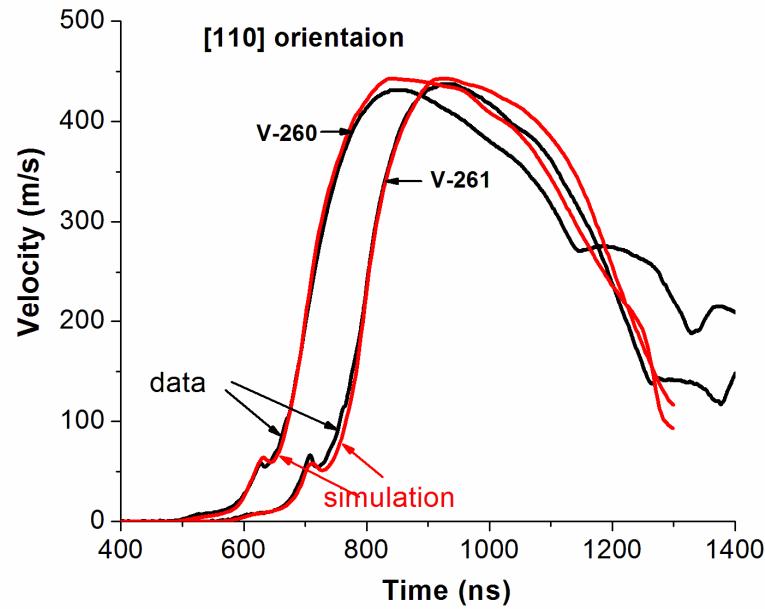
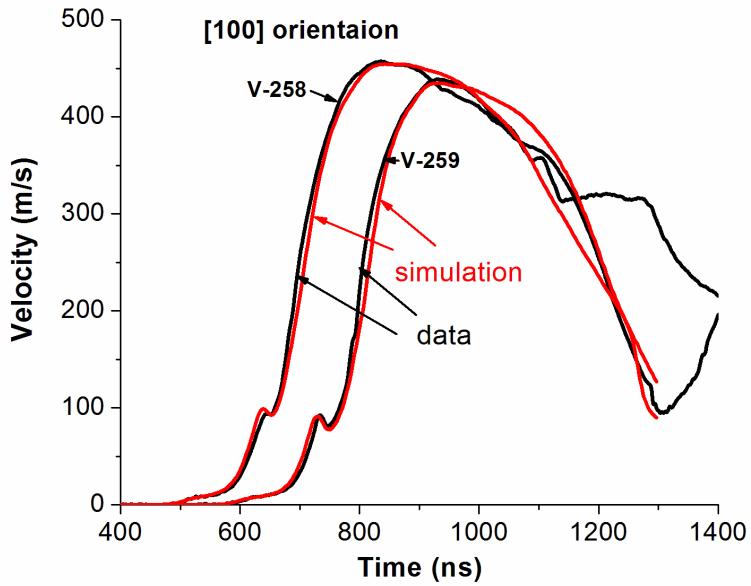
$$\rho_t = \rho_0 + C(\bar{\varepsilon}^p)^a \quad (\text{Hahn, Acta Met. 1962})$$

$$f = f_i + (f_f - f_i)(1 - e^{-\lambda \bar{\varepsilon}^p}) \quad (\text{Yoshida et. al., IJP, 2008})$$

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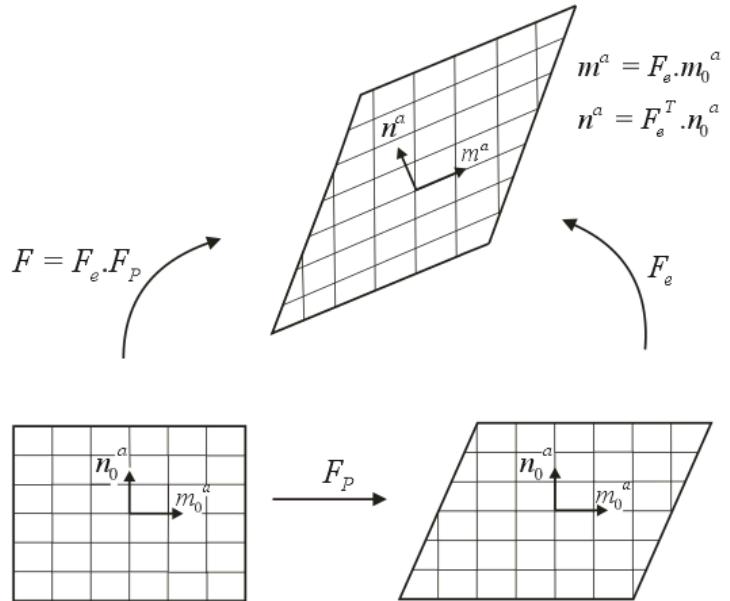
- All the material constants are kept the same as for polycrystal except for  $\lambda$
- $\lambda = 25$  for [100]; and  $\lambda = 60$  for [110] orientation
- Smaller  $\lambda$  results in a more rate sensitive behavior

# Simulation of Single Crystal Tantalum With Polycrystal Model



What is the physical justification for different rate sensitivity for different orientations?

# Single Crystal Model - Kinematics



$$\tau^\alpha = \underline{\mathbf{m}}_0^\alpha \cdot \mathcal{C}^e \cdot \mathcal{S} \cdot \underline{\mathbf{n}}_0^\alpha$$

$$\overline{\mathcal{L}}^p = \sum_{\alpha} \dot{\gamma}^\alpha \underline{\mathbf{m}}_0^\alpha \otimes \underline{\mathbf{n}}_0^\alpha$$

## Single Crystal Model – Constitutive Relation

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$$\dot{\gamma}^\alpha = b \rho_m^\alpha \bar{v}^\alpha = b \rho_m^\alpha \{ \beta [(\tau^\alpha - \tau_{th}^\alpha) / \tau_{th}^\alpha]^\mu \}$$

$$\rho_m^\alpha = f^\alpha \rho_t^\alpha$$

$$f^\alpha = f_0^\alpha + (f_s^\alpha - f_0^\alpha)(1 - e^{-\lambda^\alpha \gamma^\alpha})$$

$$\rho_t^\alpha = \rho_{t0}^\alpha + \kappa (\gamma^\alpha)^\eta$$

$$\tau_{th}^\alpha = \tau_0^\alpha (1 + \omega \gamma)^\zeta$$

# Single Crystal Model – Slip Systems

Slip System Considered:  $\{110\}<111>$  and  $\{112\}<111>$

$\{112\}<111>$  possesses twinning/anti-twinning asymmetry

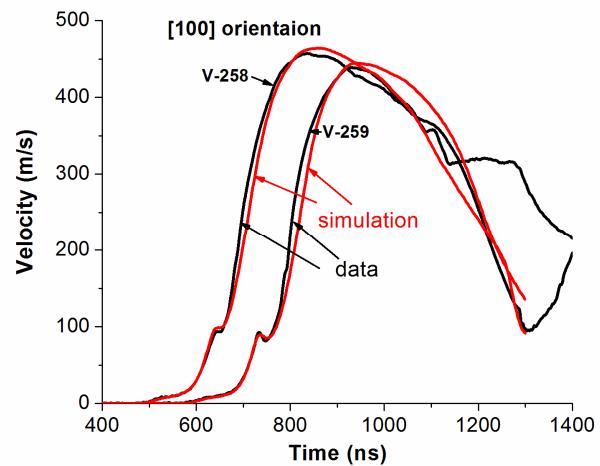
1	$(0\bar{1}1)[111]$	13	$(1\bar{2}1)[111]$
2	$(1\bar{1}0)[111]$	14	$(\bar{2}11)[111]$
3	$(10\bar{1})[111]$	15	$(11\bar{2})[111]$
4	$(01\bar{1})[\bar{1}11]$	16	$(\bar{1}1\bar{2})[\bar{1}11]$
5	$(\bar{1}0\bar{1})[\bar{1}11]$	17	$(211)[\bar{1}11]$
6	$(1\bar{1}0)[\bar{1}11]$	18	$(\bar{1}21)[\bar{1}11]$
7	$(011)[\bar{1}\bar{1}1]$	19	$(\bar{1}21)[\bar{1}\bar{1}1]$
8	$(\bar{1}10)[\bar{1}\bar{1}1]$	20	$(2\bar{1}1)[\bar{1}\bar{1}1]$
9	$(\bar{1}0\bar{1})[\bar{1}\bar{1}1]$	21	$(\bar{1}\bar{1}2)[\bar{1}\bar{1}1]$
10	$(0\bar{1}\bar{1})[1\bar{1}1]$	22	$(1\bar{1}2)[1\bar{1}1]$
11	$(10\bar{1})[1\bar{1}1]$	23	$(\bar{2}\bar{1}1)[1\bar{1}1]$
12	$(110)[1\bar{1}1]$	24	$(121)[1\bar{1}1]$

$\lambda = 15$  for the above slip systems

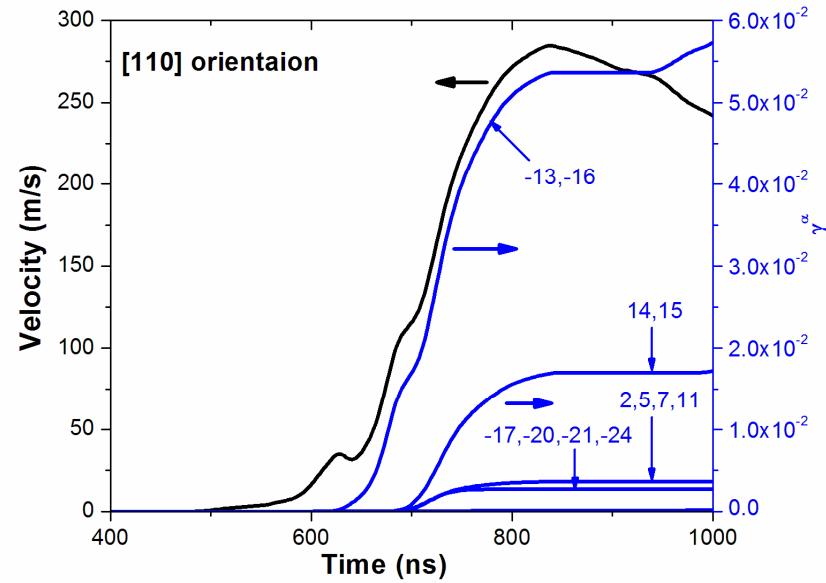
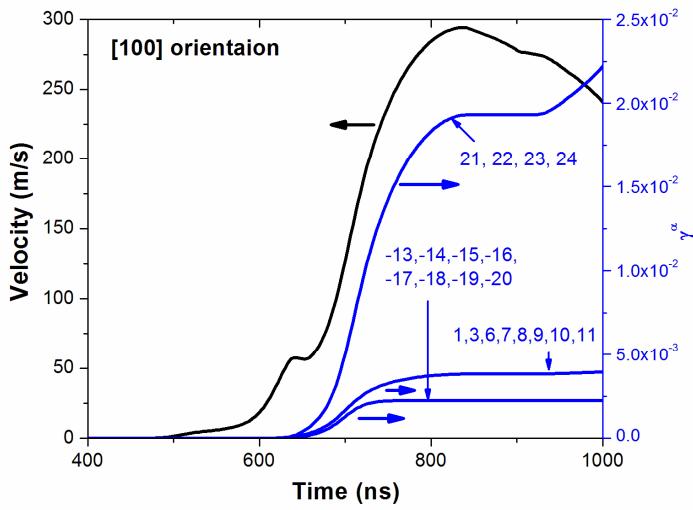
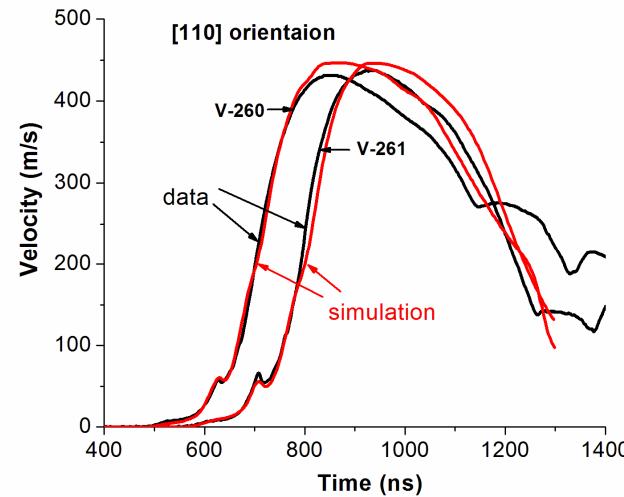
$\lambda = 50$  for the  $\{112\}<111>$  system along the twinning direction

# Simulation of Single Crystal Tantalum

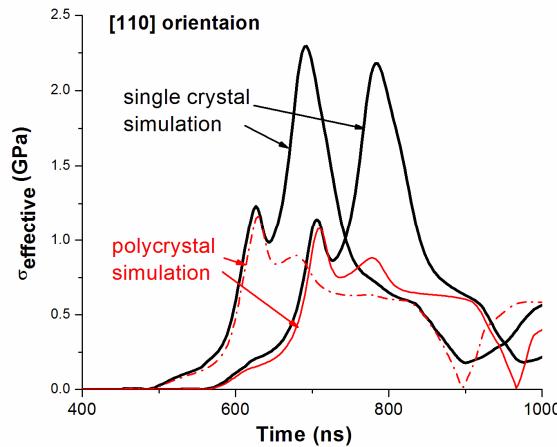
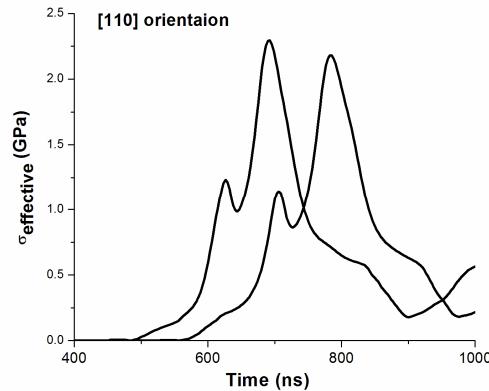
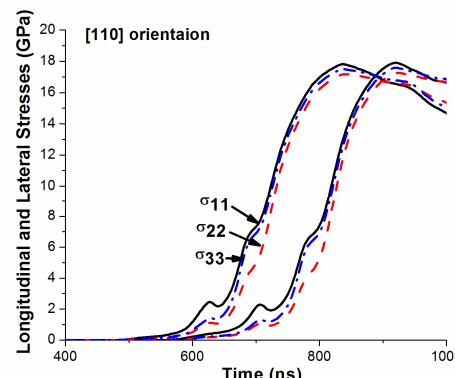
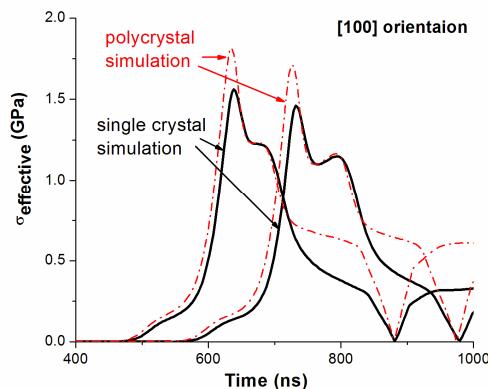
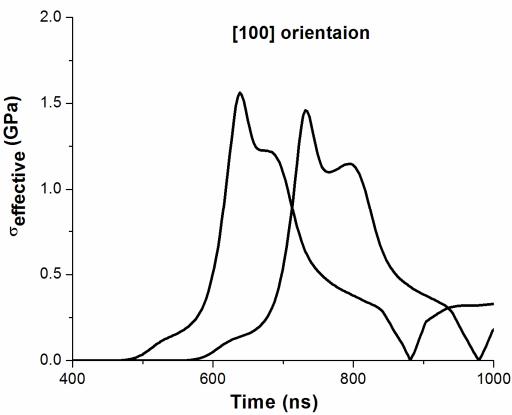
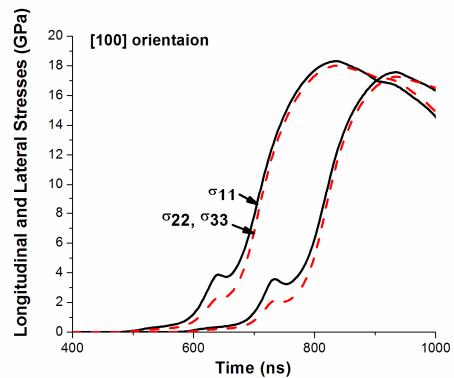
[100]



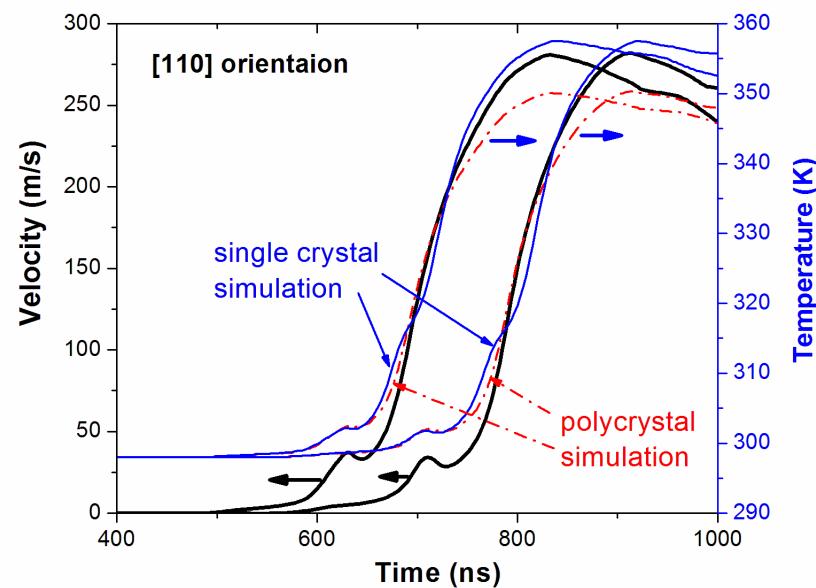
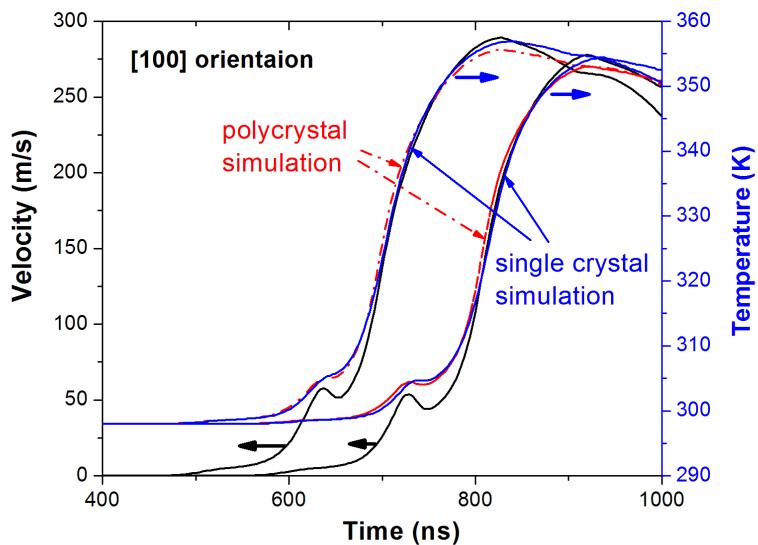
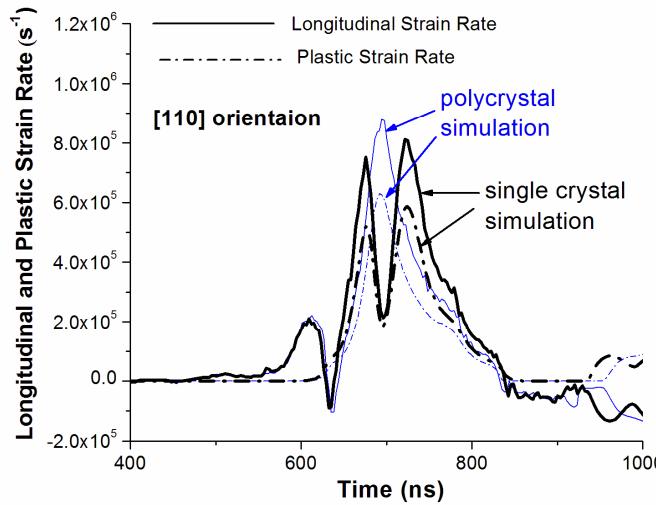
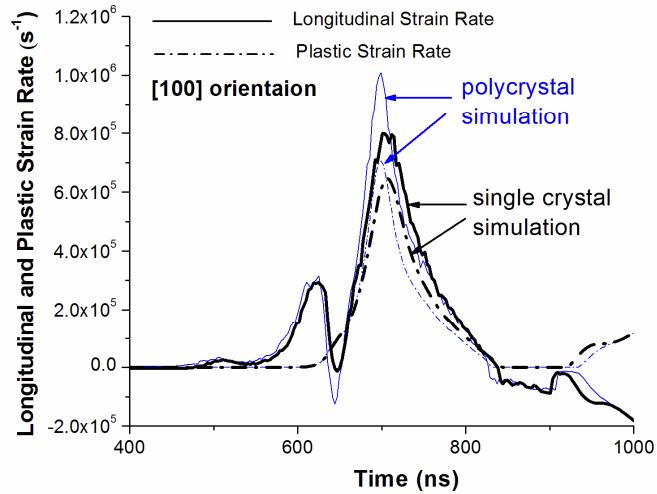
[110]



# Simulation of Single Crystal Tantalum



# Simulation of Single Crystal Tantalum



# Conclusions

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- A dislocation-mechanics based model yields consistent description of the behavior of both the poly- and single crystal tantalum under ramp wave loading.
- The various features of the observed tantalum behavior can be interpreted as a manifestation of the high rate sensitivity.
- Dislocation nucleation is used as a key mechanism for modeling the high rate sensitivity.
- On the microscopic level, the anisotropy of rate sensitivity is assumed to be attributed to the twinning/antitwinning asymmetry of the BCC crystals.