Accelerated Simulation of Analog Systems Using Linear and Nonlinear Robust Compact Models

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Thanks to: Luca Daniel, Alex Megretski, Zohaib Mahmood, Yan Li, Tarek El-Moselhy -- MIT, Cambridge MA

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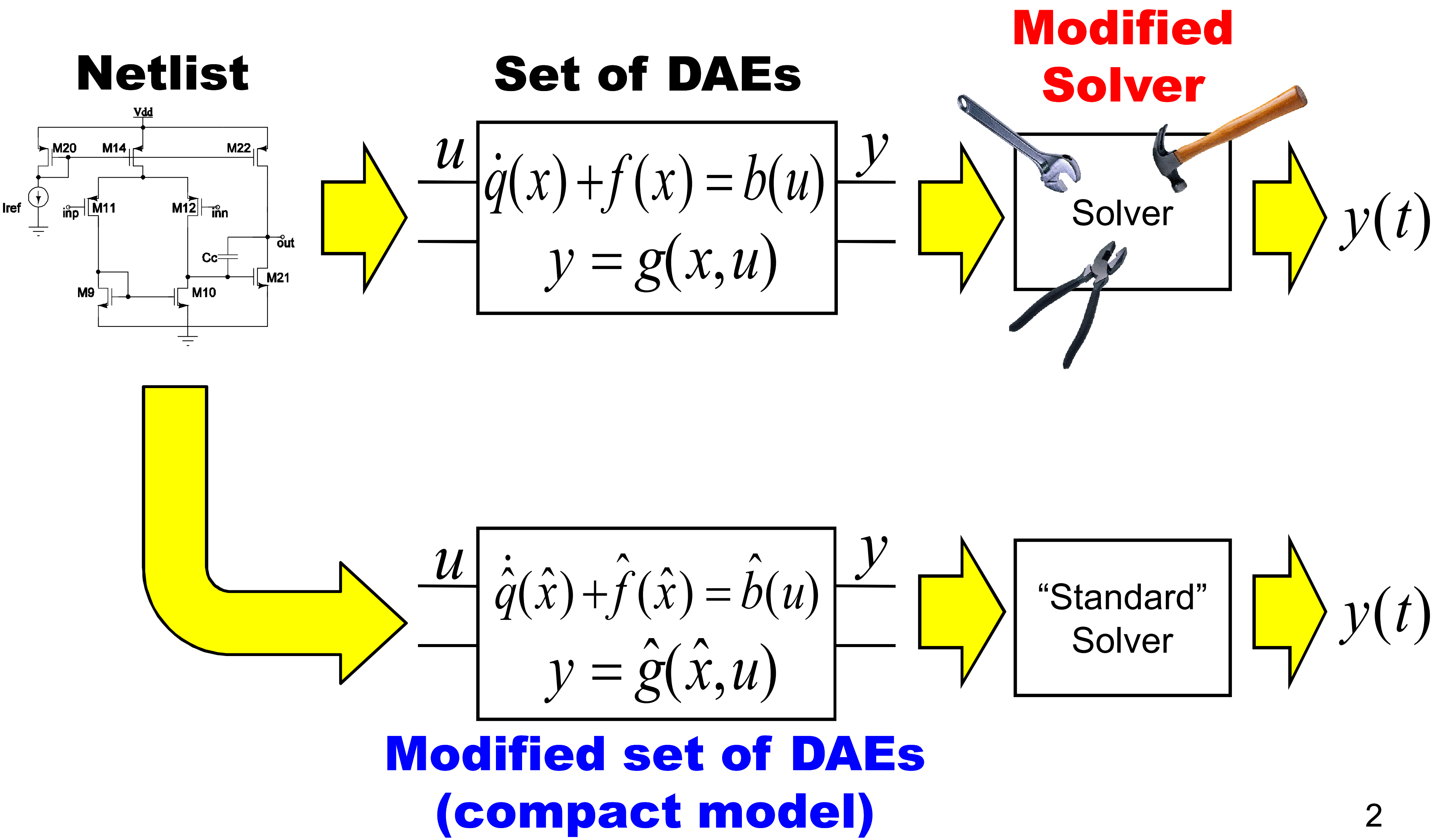
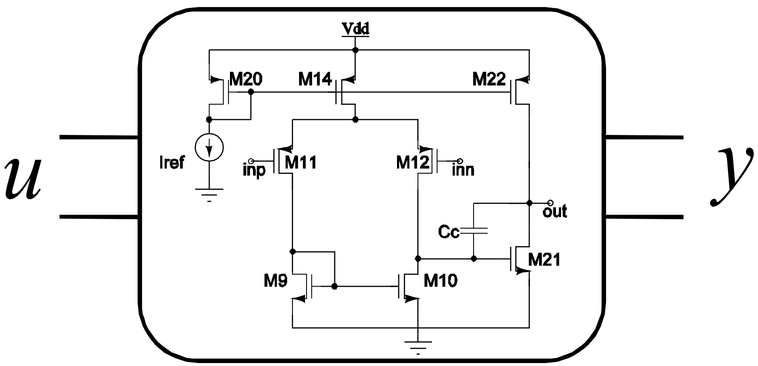
Accelerated Simulation of Analog Systems Using Linear and Nonlinear Robust Compact Models



Accelerated Simulation of Analog Systems Using Linear and Nonlinear Robust *Compact Models*



$$\dot{\hat{q}}(\hat{x}) + \hat{f}(\hat{x}) = \hat{b}(u)$$
$$y = \hat{g}(\hat{x}, u)$$

u y

Compact Model

- Port description of system
  - Device, block, circuit
- Fits into solver framework
  - System of ODEs or DAEs
- Reduced complexity
  - Faster function evaluations and system solves

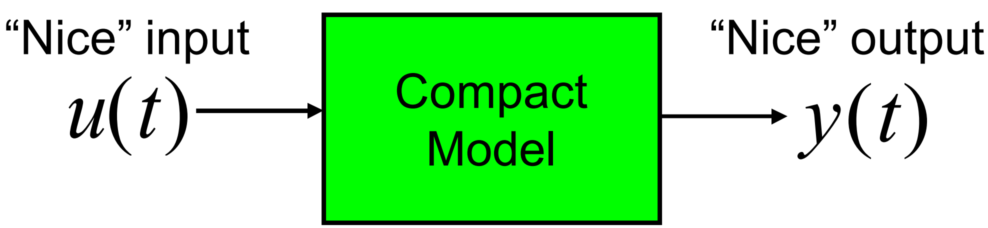
"Model Order Reduction"
"Reduced Order Model"

Accelerated Simulation of Analog Systems Using Linear and Nonlinear ***Robust*** Compact Models

Accurate

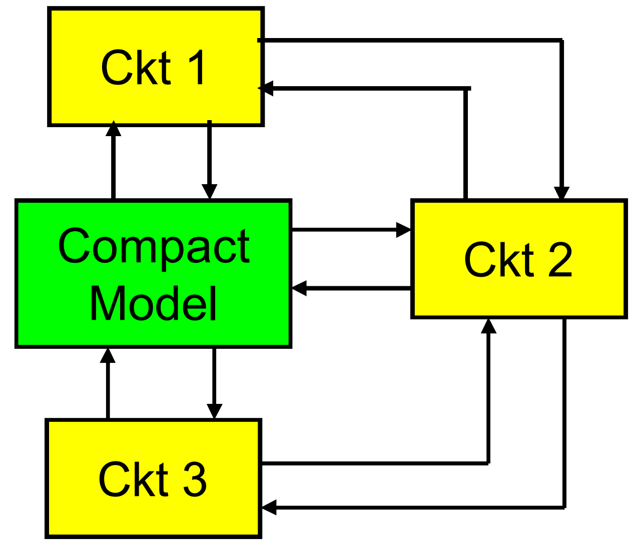
- Guarantee of accuracy between original model and compact model
- Wide range of inputs
  (amplitude, frequency, shape, …)

Stable

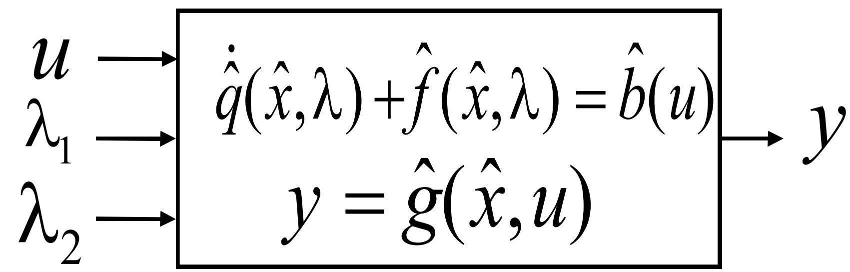


- Bounded input → bounded output
- Small input perturbation produces small output perturbation
- Difficult to enforce global stability via projection for nonlinear system

Passive

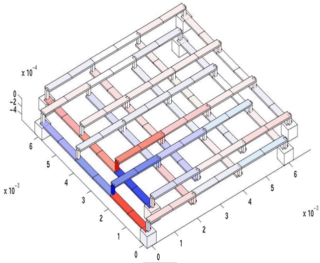


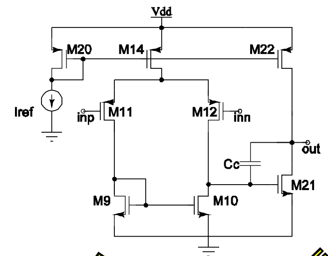
Parameterized



(e.g. design or manufacturing parameters)

Accelerated Simulation of *Analog Systems* Using Linear and Nonlinear Robust Compact Models





Input $u(t)$ → Transient Simulation → Output $y(t)$

RLC Model
R, L, C

Indefinite Linear System
$E\dot{x} = Ax + Bu$

Nonlinear System
$\dot{x} = f(x,u)$

Data Samples
$\{u(t), y(t)\}$

Need techniques not requiring special structure

Compact Model

Models useful for variety of applications

Transient

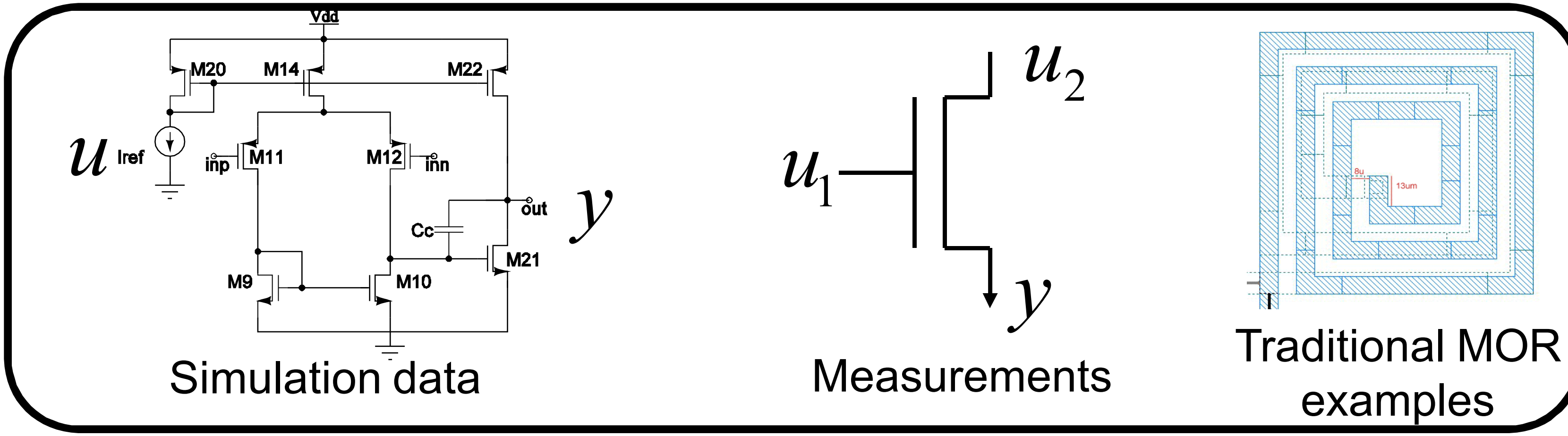
AC

Performance analysis

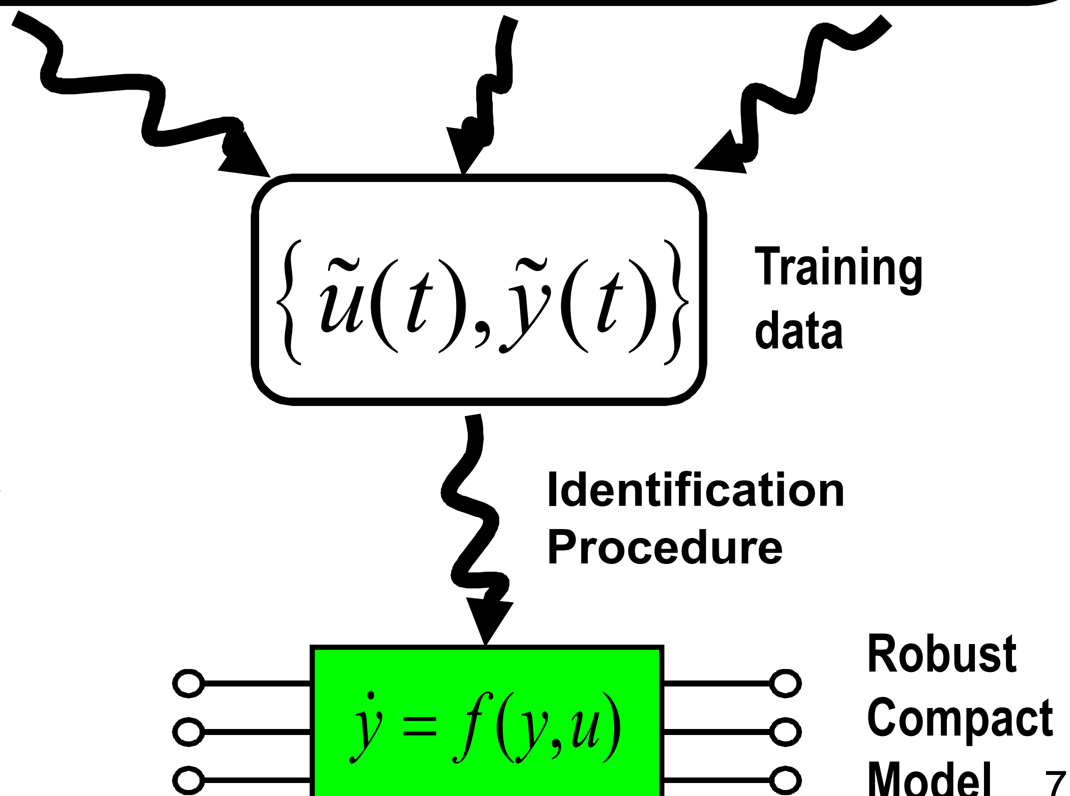
Outline

- Motivation
- System Identification (SYSID): Introduction
- SYSID: Theory
- SYSID: How to make it practical
- Examples

SYSID Intro (1): System Identification



- Solves MOR problems
  - Does *not* require system equations
  - Can enforce stability by construction



SYSID Intro (2): Model Terminology / Notation

		Continuous time	Discrete time
First order	**Explicit I/O system**	$\dot{y} = f(y,u)$	$y_t = f(y_{t-1}, u_{t-1})$
	Implicit I/O system	$F(\dot{y}, y, u) = 0$	$F(y_t, y_{t-1}, u_t) = 0$
Higher order	**Delays, derivatives**	$F(y, \dot{y}, \ddot{y}, \cdots, u, \dot{u}, \ddot{u}, \cdots) = 0$	$F(y_t, y_{t-1}, \cdots, y_{t-m}, u_t, u_{t-1}, \cdots, u_{t-k}) = 0$
	States	$F(\dot{x}, x, u) = 0$ $G(y, x) = 0$	$F(x_t, x_{t-1}, u_t) = 0$ $G(y_t, x_t) = 0$

f -- Explicit system

u -- Inputs

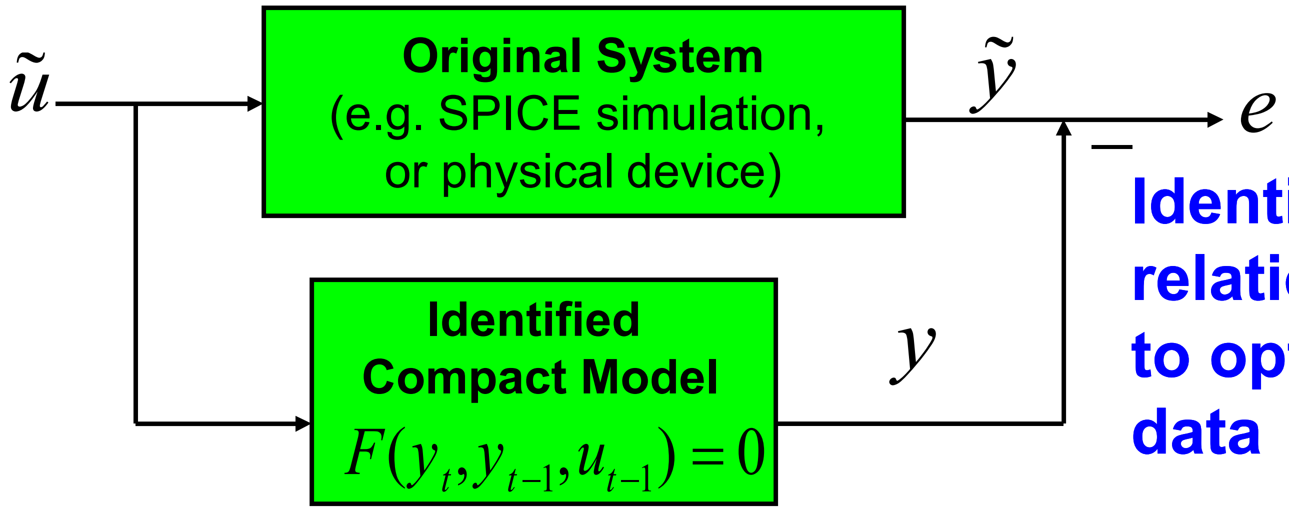
y -- Outputs

F -- Implicit system

(e.g. $F = \dot{y} - f(y,u) = 0$)

x -- States (optional)

SYSID Intro (3): Problem Statement



Identify dynamical relation $F(y_t, y_{t-1}, u_{t-1}) = 0$ **to optimally fit training data** $\{\tilde{u}_t, \tilde{y}_t\}$

System Identification…. Is it just Fitting?

$$\min_F \sum_t \left|\mathrm{E}(F, \tilde{y}, \tilde{u})\right|^2$$

$$\text{s.t. } \mathrm{Robustness}(F)$$

- What to minimize?
- Robustness constraint?
- How to describe F?

Optimization problem must be nice

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SYSID (1): Nonlinear Model Description

Implicit DT Models

$$F(y_t, y_{t-1}, u_t) = 0$$

Unlike projection, no equations to start from

Basis Functions

$$F(y_t, y_{t-1}, u_t) = \sum_j \alpha_j \phi_j(y_t, y_{t-1}, u_t)$$

Polynomial basis

$$\phi_j = y_t^{a_j} y_{t-1}^{b_j} u^{c_j}$$

Implicit model captures highly nonlinear behavior

For example,

$$y_t^2 - y_{t-1} + u_t^3 = 0 \Rightarrow y_t = \sqrt{y_{t-1} - u_t^3}$$

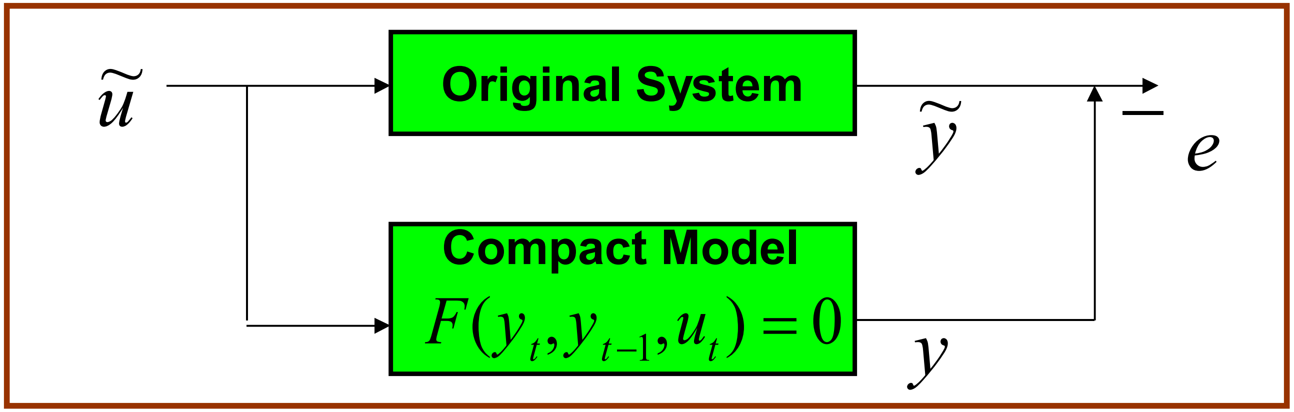
$$q(y_{t-1}) y_t - p(y_{t-1}, u_t) = 0 \Rightarrow y_t = \frac{p(y_{t-1}, u_t)}{q(y_{t-1})}$$

Rational Models

Later in the talk

Models with states, higher order systems, continuous time

SYSID (2): Explicit Error Minimization



Goal:

$$\text{minimize} \sum_t |e_t|^2$$

Assume explicit system

$$y_t = f(y_{t-1}, u_{t-1})$$

$$|e_t| = |\tilde{y}_t - y_t| = |\tilde{y}_t - f(y_{t-1}, u_t)|$$

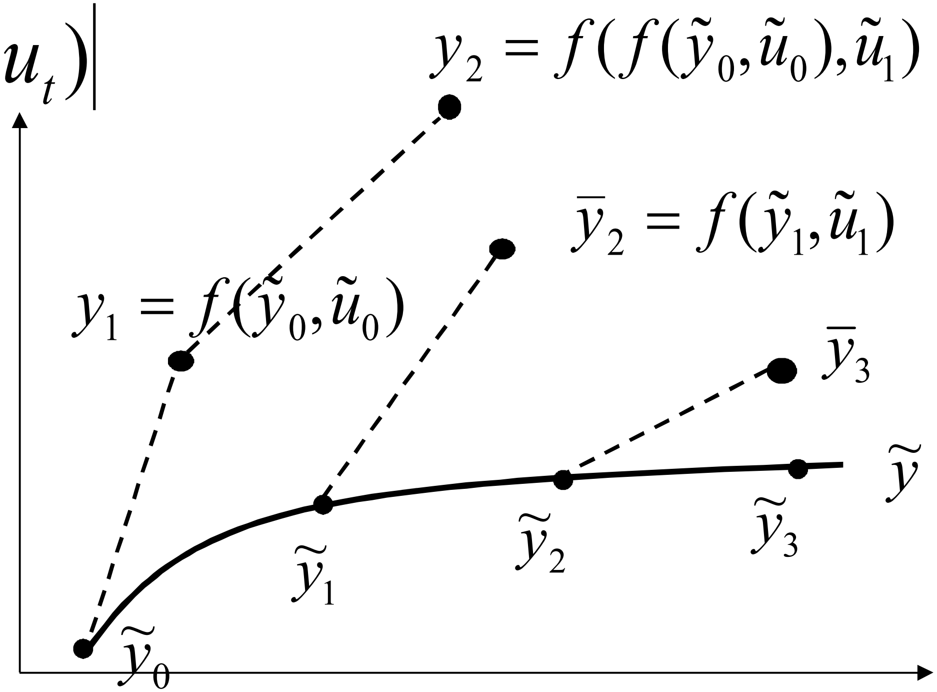
$$|e_1| = |\tilde{y}_1 - f(\tilde{y}_0, \tilde{u}_0)|$$

$$\cancel{|e_2| = |\tilde{y}_2 - f(f(\tilde{y}_0, \tilde{u}_0), \tilde{u}_1)|}$$

Nonlinear in unknown coefficients

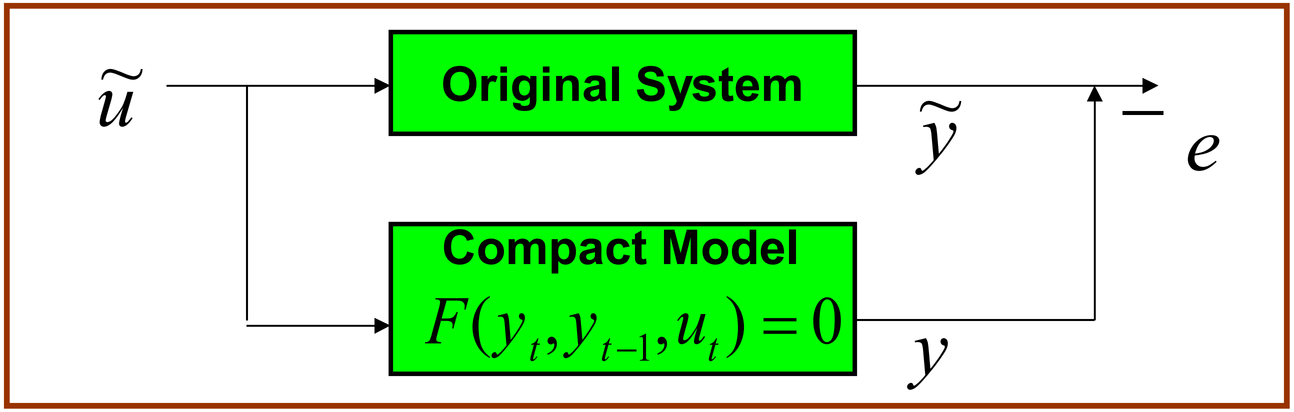
$$|e_2| = |\tilde{y}_2 - f(\tilde{y}_1, \tilde{u}_1)|$$

Can only minimize one-step errors



Must guarantee errors do not accumulate

SYSID (3): Implicit Error Minimization



Goal:

$$\text{minimize} \sum_t |e_t|^2$$

- For implicit system, minimize equation error (i.e. residual)

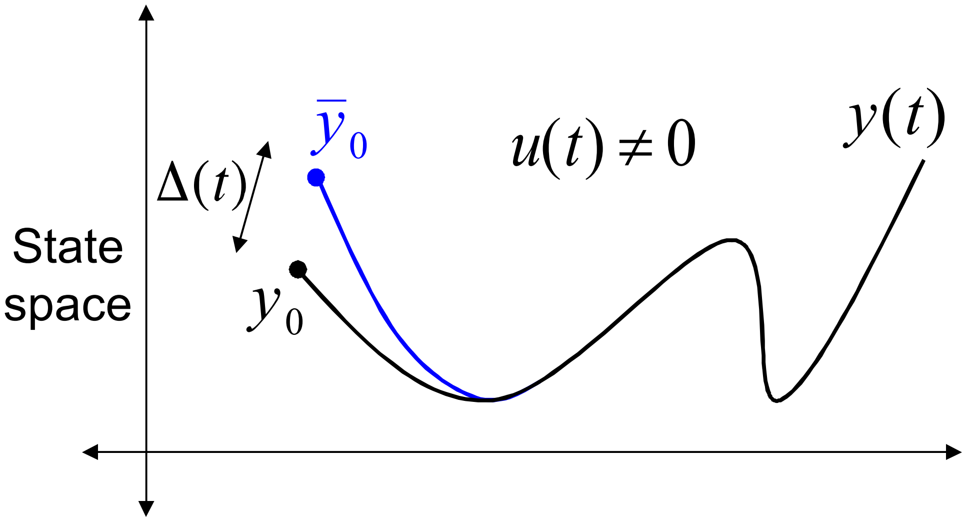
$$F(y_t, y_{t-1}, u_{t-1}) = 0 \quad \Rightarrow \quad \text{minimize} \sum_t \left|F(\tilde{y}_t, \tilde{y}_{t-1}, \tilde{u}_t)\right|^2$$

- Small equation error doesn't always imply small output error

$$\begin{aligned} Ax_1 &= b_1 \\ Ax_2 &= b_2 \end{aligned} \qquad \| b_1 - b_2 \| \text{ small} \;\not\Rightarrow\; \| x_1 - x_2 \| \text{ small}$$

Need relation between equation error and output error

SYSID (4): Incremental Stability



- Perturbations to solutions decay
- Errors do not accumulate

Incremental stability constraint

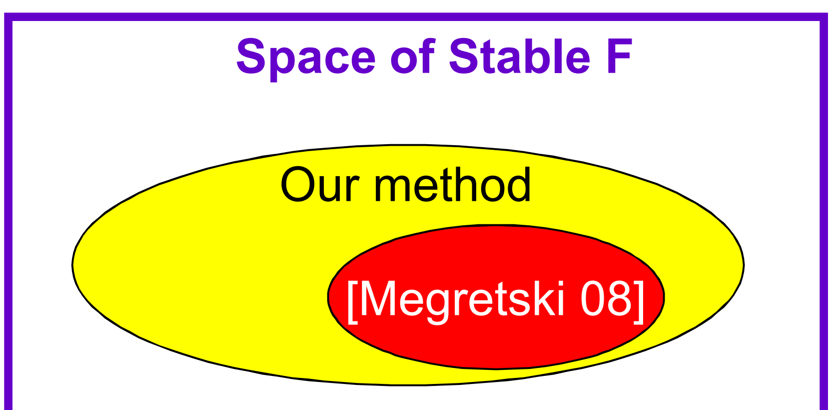
$$(y-\bar{y})^T\left(F(y,u)-F(\bar{y},u)\right) \geq h_+(y,\bar{y}) - h_-(y,\bar{y}) + |y-\bar{y}|^2$$

$h(y,\bar{y})$ -- Certificate of stability

One possible constraint – sufficient, but not necessary

- Equation error bounds output error

$$\sum_t \left|F(\tilde{y}_t,\tilde{y}_{t-1},\tilde{u})_t\right|^2 \geq \sum_t \left|y_t-\tilde{y}_t\right|^2$$



Modified incremental stability constraint
[Bond et al TCAD10]

$$-2\Delta_+^T \dot{F}(y,u)\Delta + h(y_+,\Delta_+) - h(y_-,\Delta_-) \leq -\left|\Delta_+\right|^2$$

- **Semidefinite constraint**
- **Tighter error bound**
- **Larger class of admissible stable models F**

SYSID (5): Identification Procedure

Optimization Problem

Robust equation error, upper bounds linearized output error

$$\min_{F,r,h} \sum_t r\left(\tilde{y}_t, \tilde{u}_t\right) \quad \text{subject to}$$

Robust dissipation constraint, implies local incremental stability

$$r_t + 2\Delta_0^T\left(F_t + \dot{F}_t\Delta\right) - |\Delta|^2 + h_- - h_+ \geq 0, \qquad \forall t, \Delta$$

Optional constraint, enforces global incremental stability

$$2\Delta_0^T \dot{F}(y,u)\Delta - |\Delta|^2 + h_- - h_+ \geq 0, \qquad \forall u, y, \Delta$$

Semidefinite constraints

Inputs

$\left\{\tilde{u}_t, \tilde{y}_t\right\}$ - Training data set

$\Phi = \left\{\phi_j(y,u)\right\}$ - Set of basis functions describing F,r,h

Outputs

- **Coefficients defining**

F - Stable reduced model

h - Storage function, certifies stability

r - Error bound

$$F(y,u) = \sum_j \alpha_j \phi_j(y,u)$$

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SYSID (1): Complexity

- **Most general model**

$$F(\lambda, x_t, \cdots, x_{t-m}, u_t, \cdots, u_{t-k}) = 0 \qquad G(x_t, y_t) = 0$$

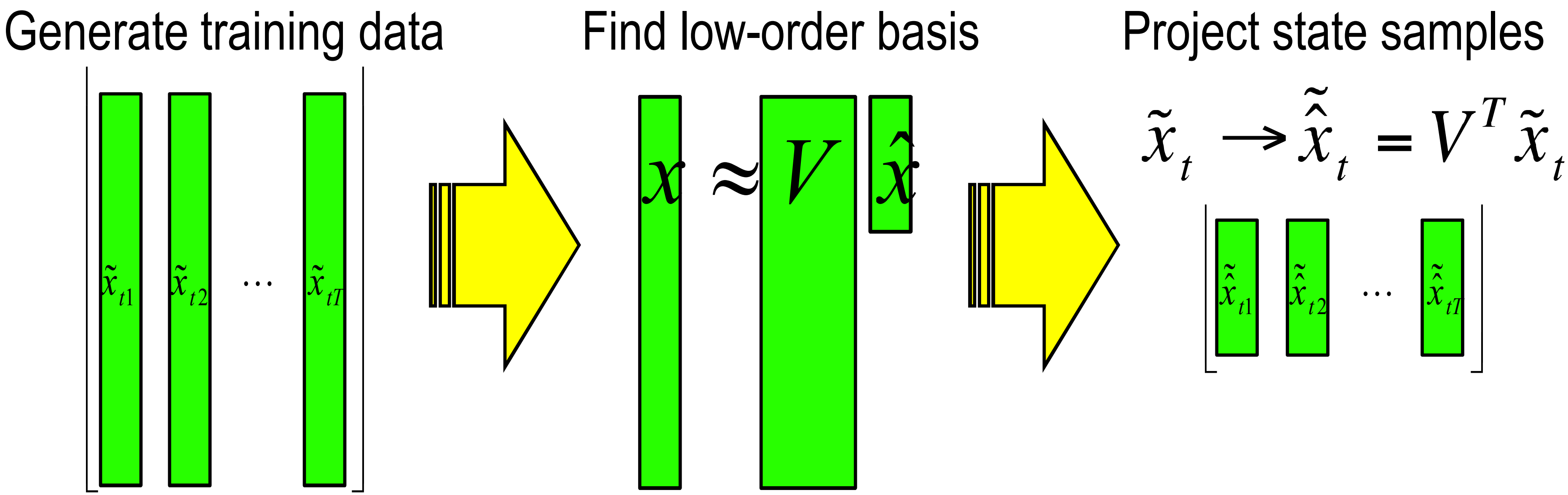
- **Complexity (# of coefficients) determined by**
  - Number of inputs, outputs, states, parameters
  - Number of delays (input and state memory)
  - Polynomial degrees

$$\phi = \phi\left(\lambda, x_t, \cdots, x_{t-m}, u_t, \cdots, u_{t-k}, y_t, \cdots, y_{t-l}\right) = \prod_{h,i,j,k,l,m,n,r,s} u_{t-i,n}^{p_j} x_{t-j,m}^{p_k} y_{t,s}^{p_h} \lambda_r^{p_l}$$

- **Need to keep total number of coefficients small**

SYSID (2): Projecting States

- Model with large number of states: $F(x_t, x_{t-1}, u_t) = 0$



- Fit to reduced set of states: $F(\hat{x}_t, \hat{x}_{t-1}, u_t) = 0$

- **Connections to traditional projection approaches**

$$U^T \tilde{F}(V\hat{x}, u) = 0 \qquad \min_F E(F(\hat{x}, u))$$

$$F(\hat{x}, u) = 0$$

- **Equations obtained by fitting to data rather than from residual orthogonality condition**

SYSID (3): Reduced Basis

- Model with large number of delays

$$F(\hat{x}_t, \cdots \hat{x}_{t-m}, u_t, \cdots, u_{t-k}) = 0$$

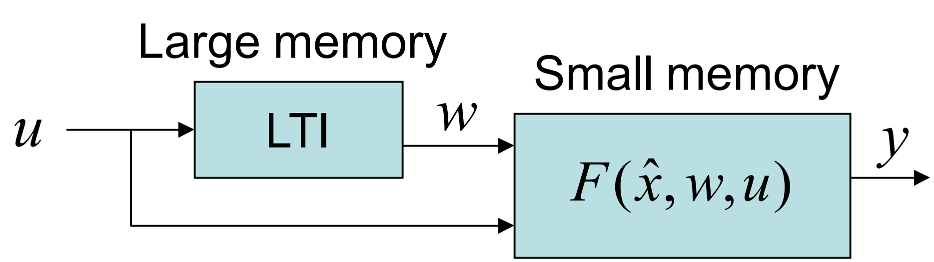
- Many basis terms for large polynomial degree

$$\left(\hat{x}_{t-1}, \cdots, \hat{x}_{t-m}, u_t, \cdots, u_{t-k}\right)^p$$

- Fit to reduced set of basis terms
  - Linear transformation of basis

$$u_t^2 + u_{t-1}^2 + u_{t-2}^2 + 2u_t u_{t-1} + 2u_t u_{t-2} + 2u_{t-1} u_{t-2} = \left(\overbrace{u_t + u_{t-1} + u_{t-2}}^{w_t}\right)^2$$

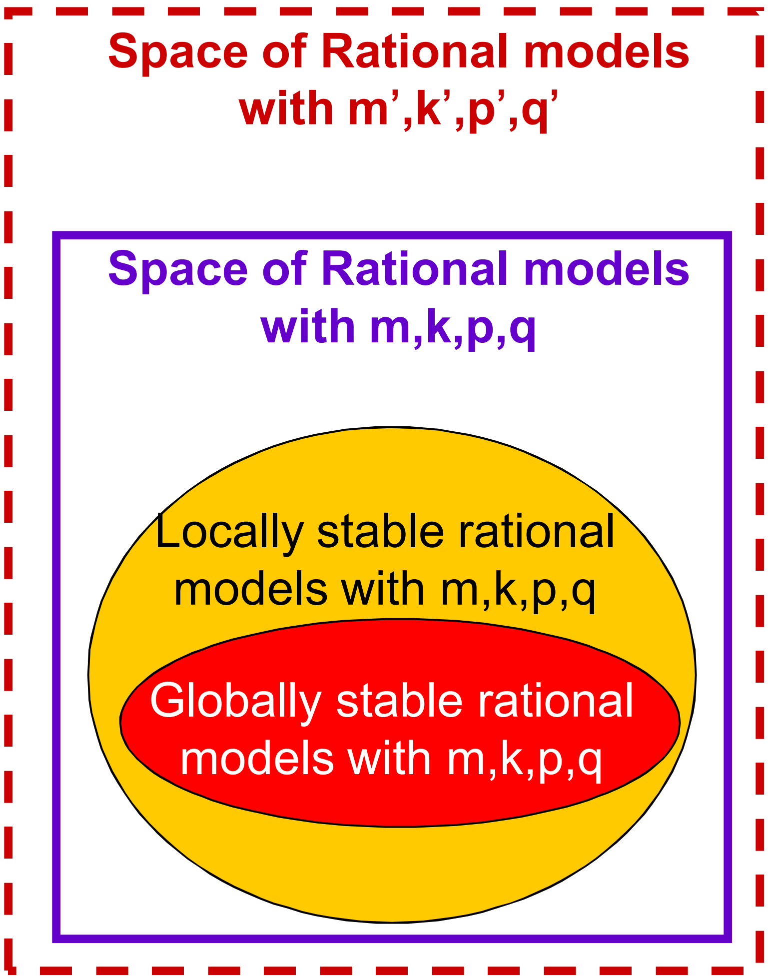
$$w_t = \sum_j \beta_j u_{t-j}$$

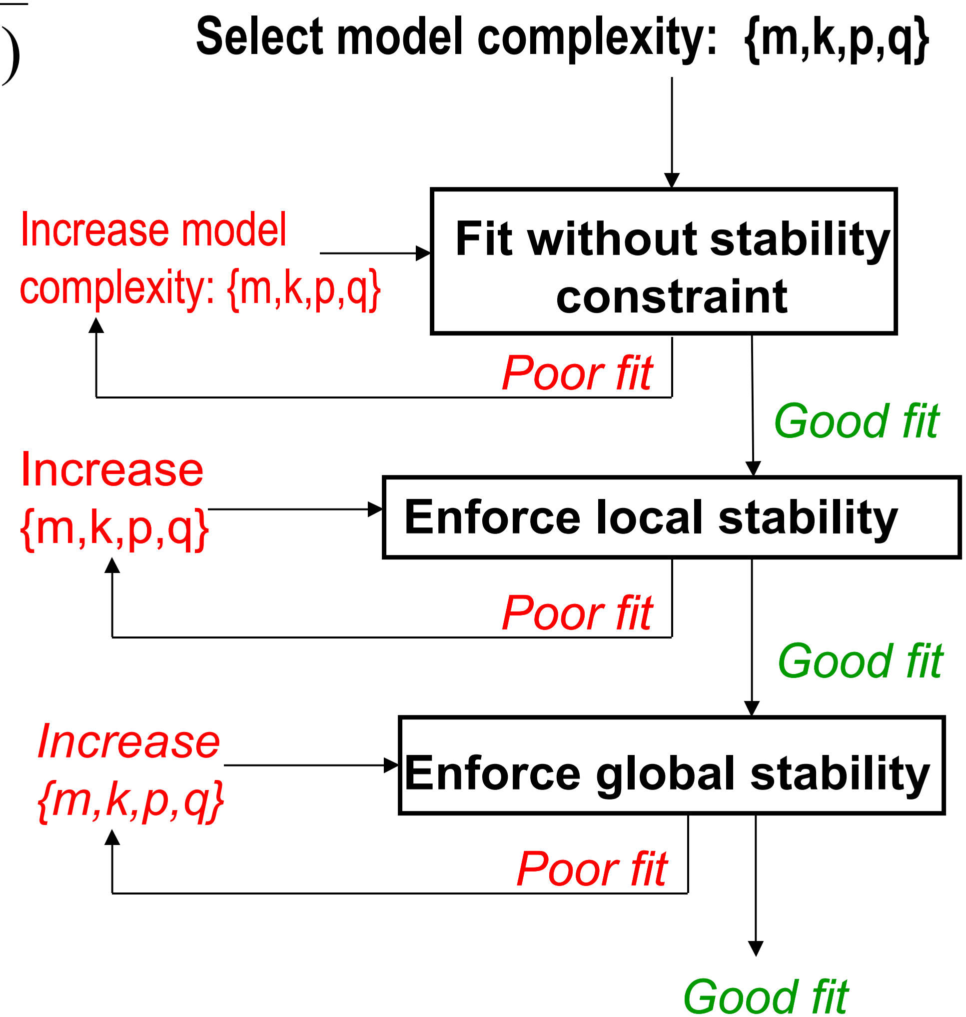


SYSID (4): Algorithm

Model complexity determined by m,k,p,q

$$y_t = \frac{p(y_{t-1},\cdots,y_{t-m},u_t,\cdots,u_{t-k})}{q(y_{t-1},\cdots,y_{t-m},u_t,\cdots,u_{t-k})}$$





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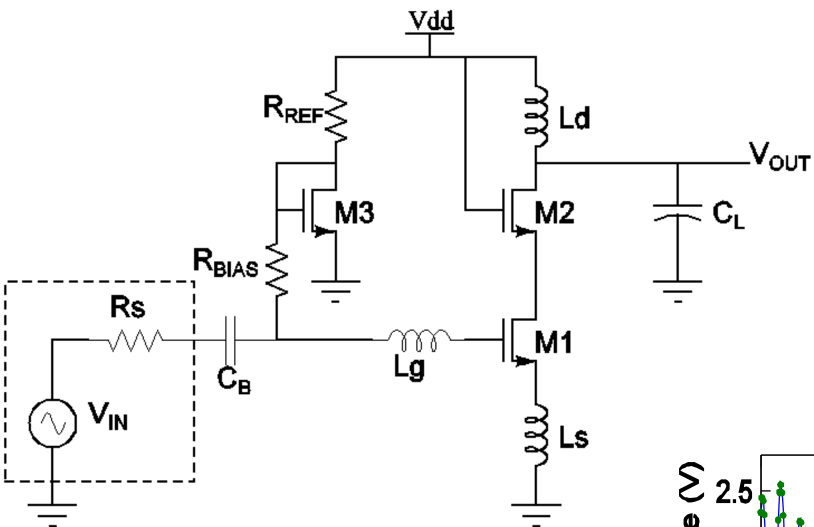
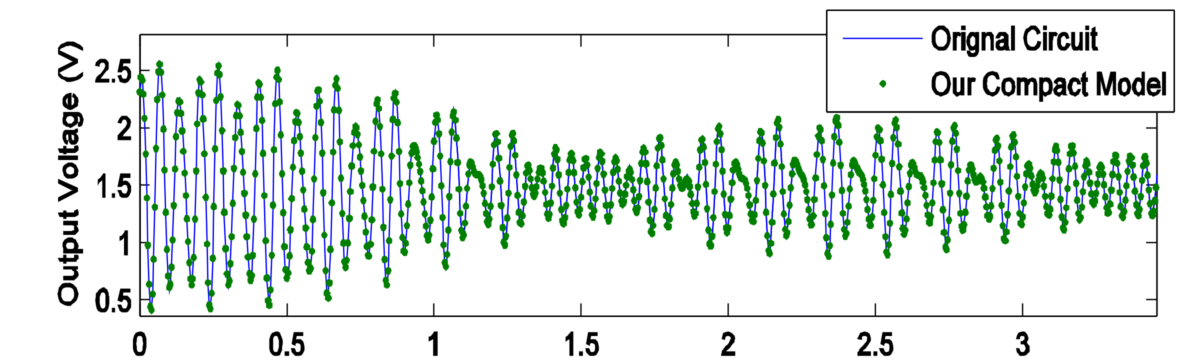
Example: Low Noise Amplifier (1)



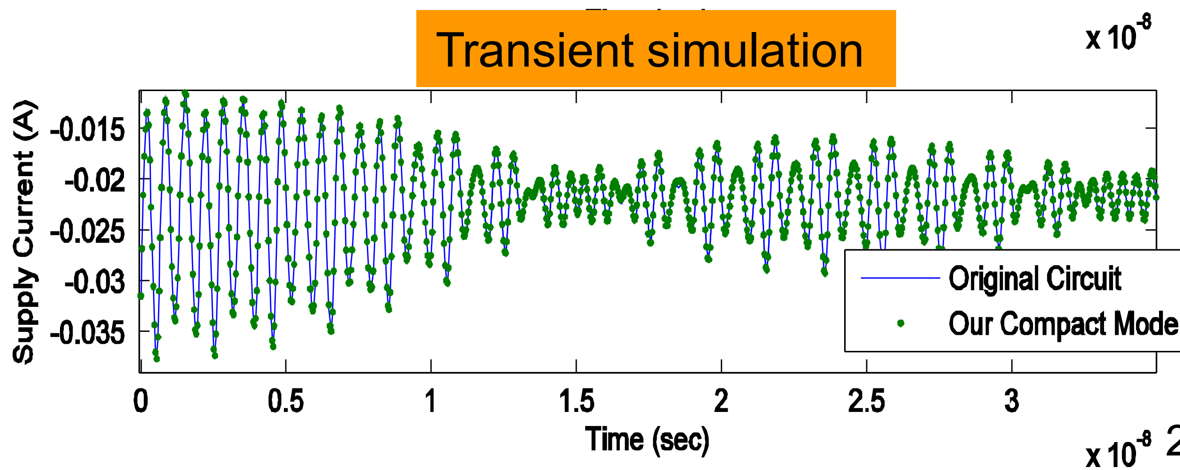
$$Q_2(y_{t-1}, u_t, u_{t-1}) y_t = p_3(y_{t-1}, u_t, u_{t-1})$$

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \qquad p = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

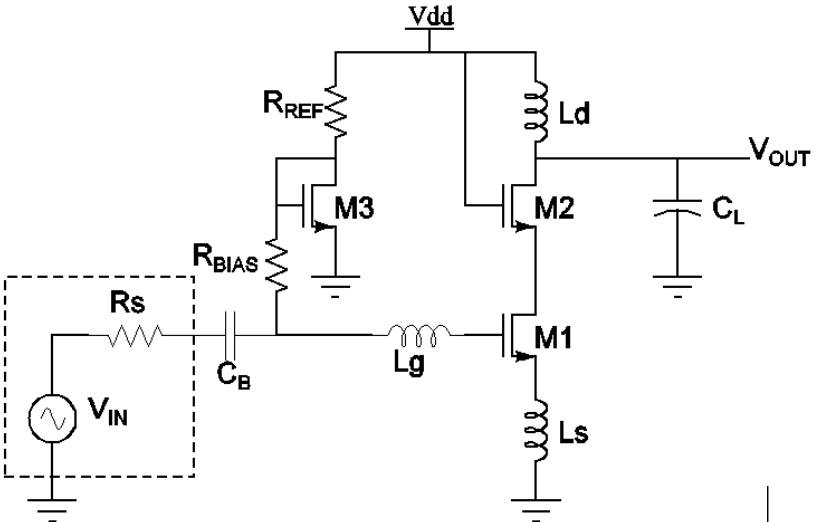
- Low noise amplifier
- Identified rational model with feedback
- Model has two outputs
- 102 coefficients describing model



Transient simulation



Example: Low Noise Amplifier (2)



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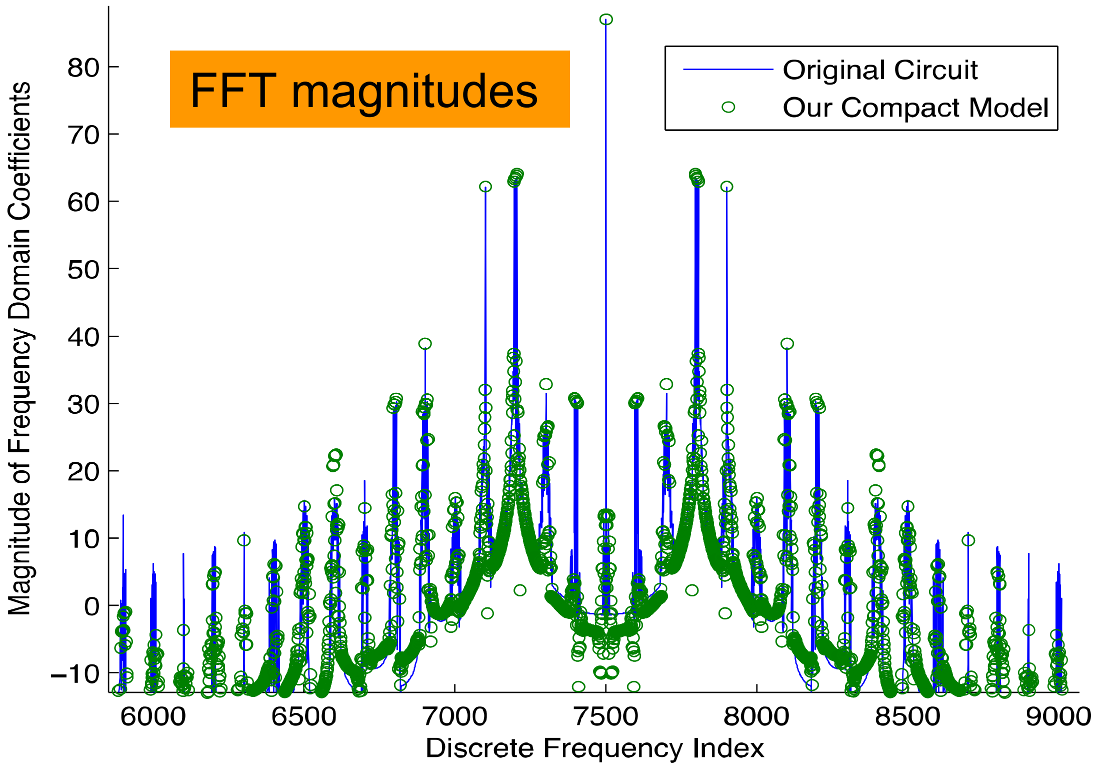
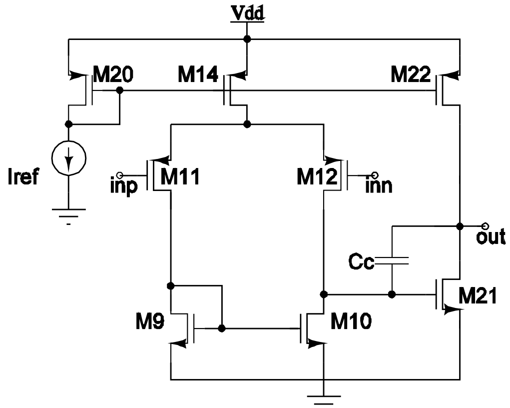


Example: Parameterized System – Opamp

Parameter λ



$$y_t = \frac{p_3(\lambda, y_{t-1}, u_t, u_{t-1})}{q_4(y_{t-1}, u_t, u_{t-1})}$$

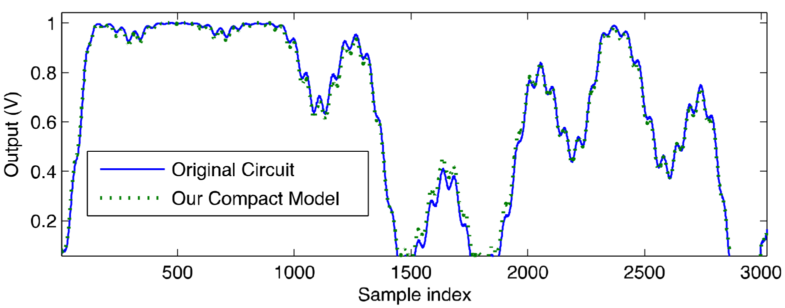
97 coefficients describing model

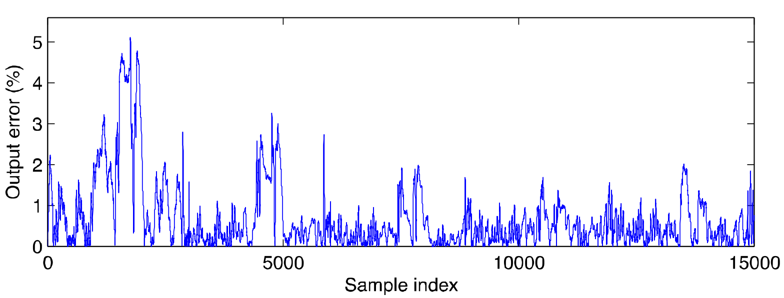
$$p = \alpha_0 + \alpha_1 u_t + \underbrace{\alpha_2}_{\alpha_{k,0} + \alpha_{k,1}\lambda + \alpha_{k,2}\lambda^2} y_{t-1} + \cdots + u_t^3$$

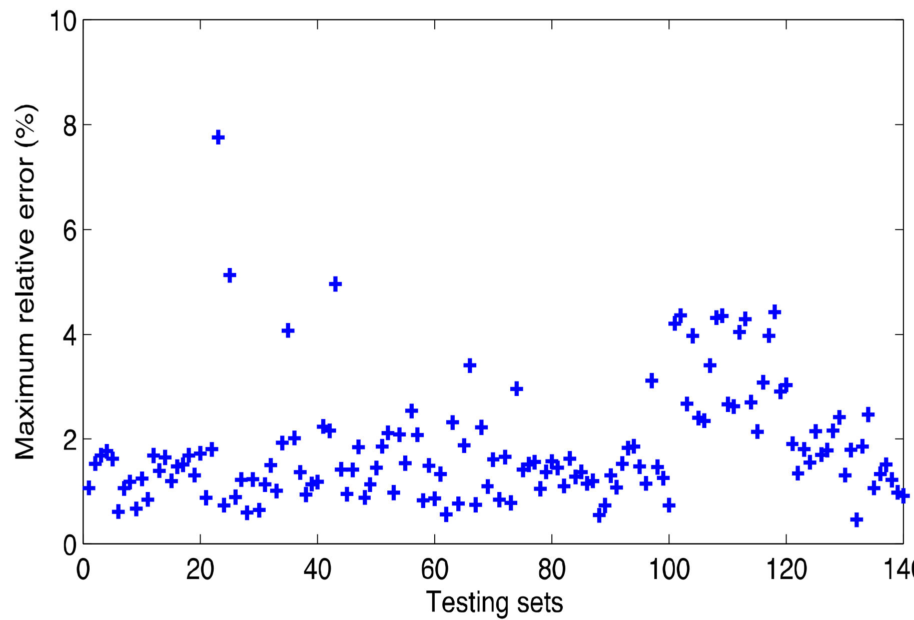
$$u(t) = \sum_{i=1}^{5} A_i \sin(2\pi f_i t + \phi_i)$$

Varied randomly

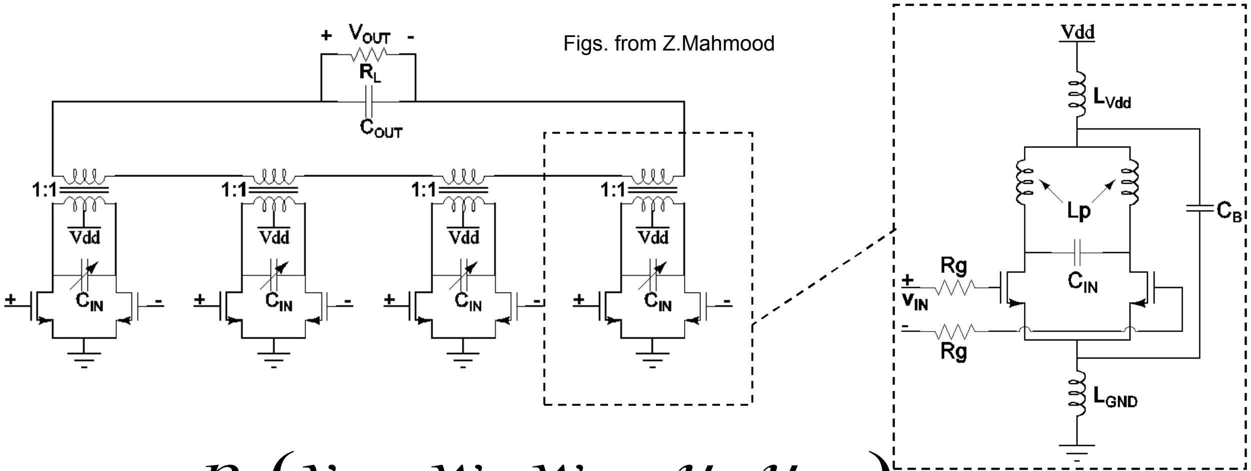
Tested on140 random pairs $\{\lambda_j, u_j(t)\}$





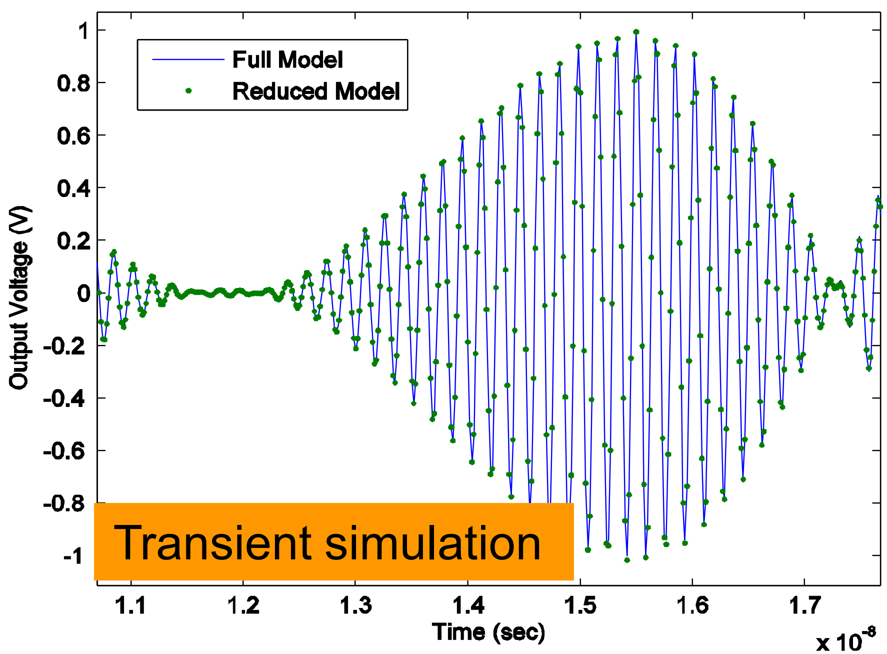


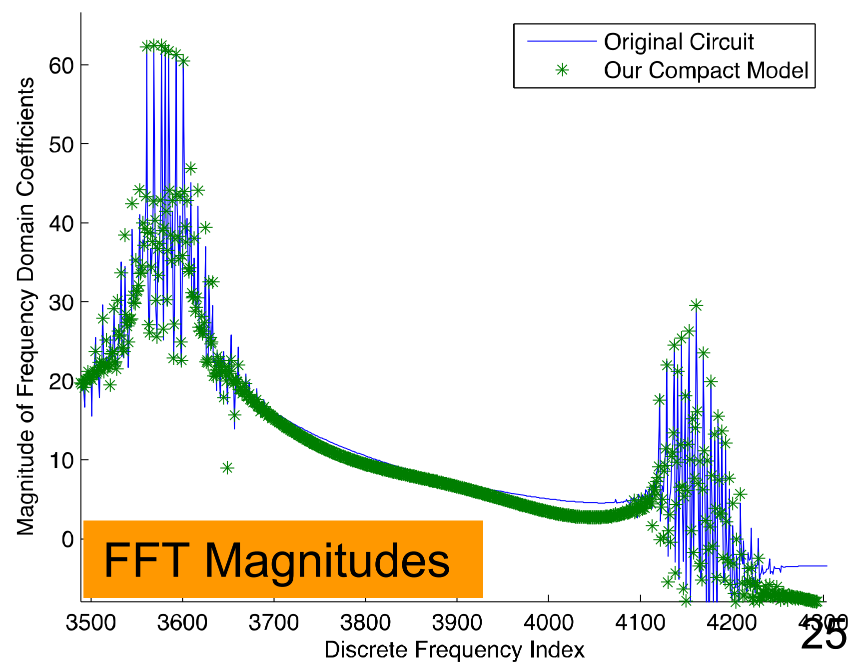
Example: Performance Analysis - Power Amplifier (1)



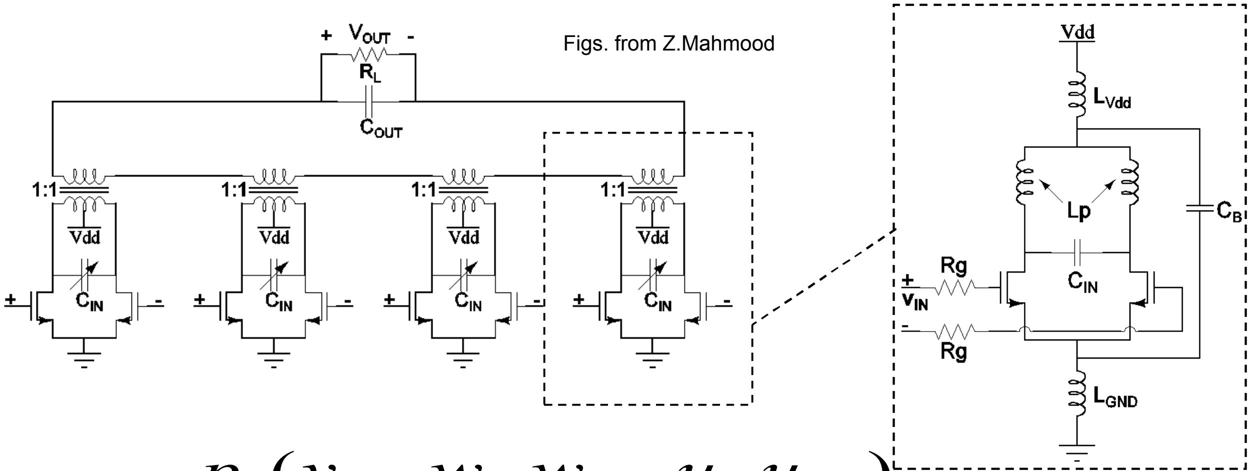
- Distributed power amplifier
- Identified rational model with feedback
- Fit also to LTI transformation of input
- 106 coefficients describing model

$$y_t = \frac{p_3(y_{t-1}, w_t, w_{t-1}, u_t, u_{t-1})}{q_4(y_{t-1}, w_t, w_{t-1}, u_t, u_{t-1})}$$





Example: Performance Analysis - Power Amplifier (2)



- Distributed power amplifier
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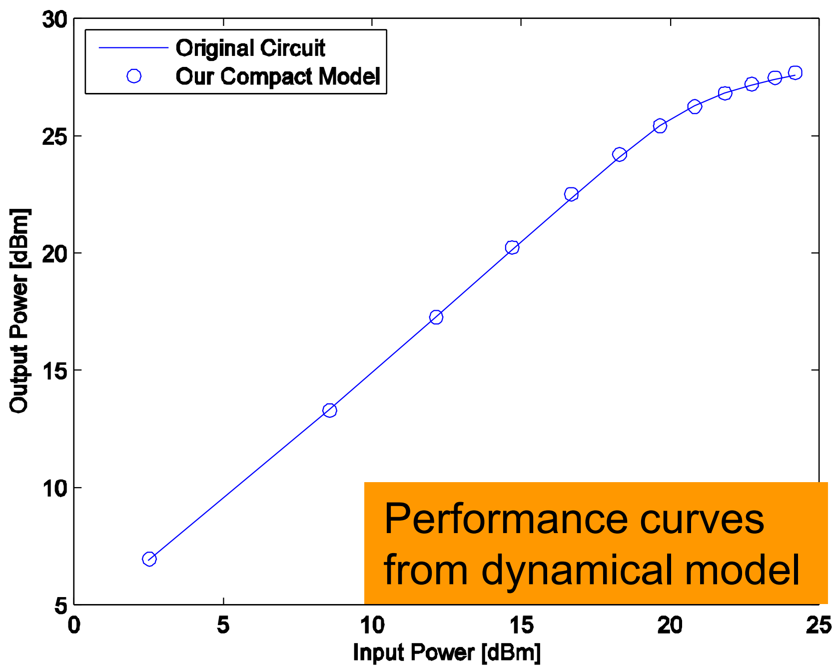
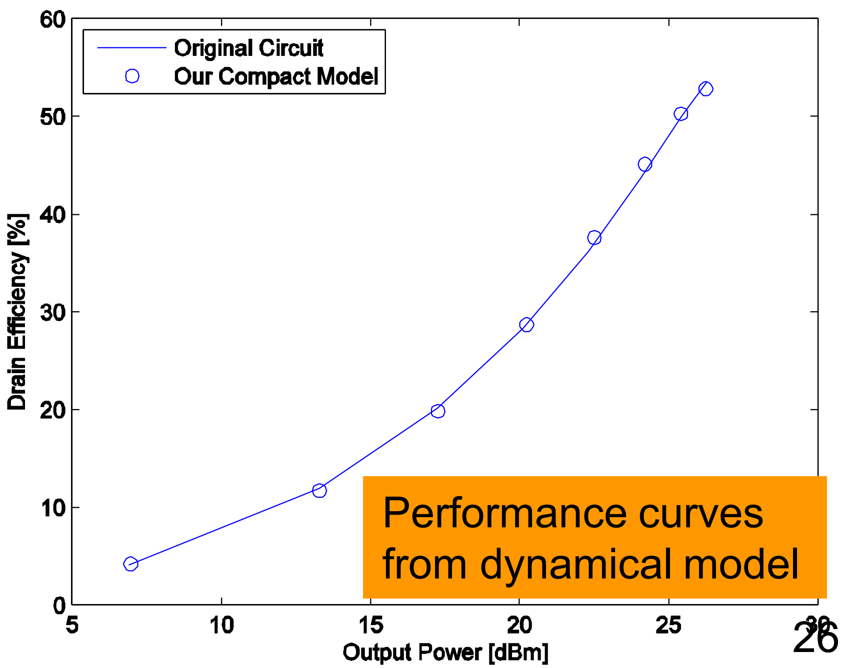
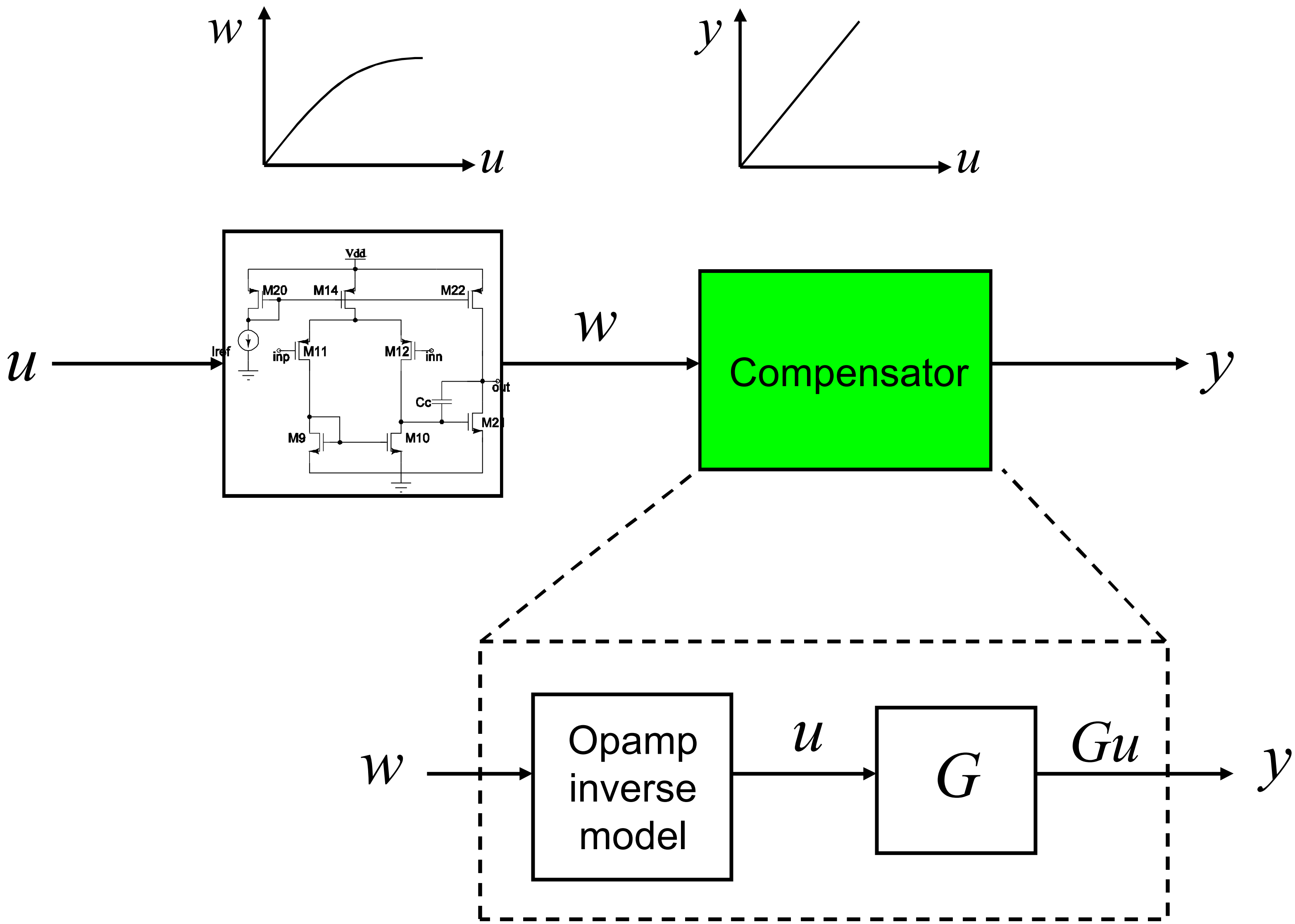


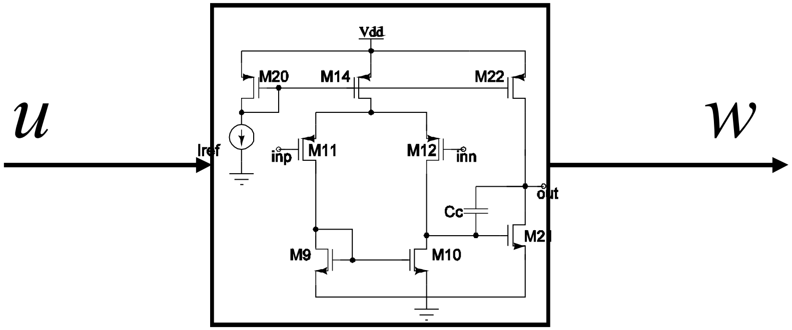


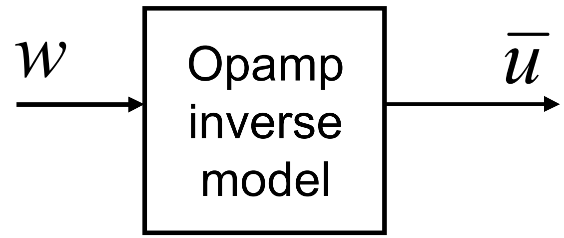
Example: Design - Opamp Compensator (1)

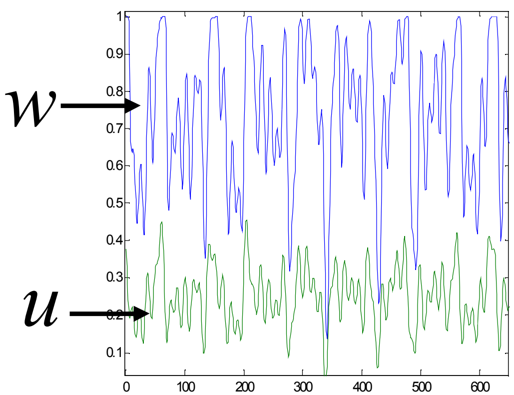


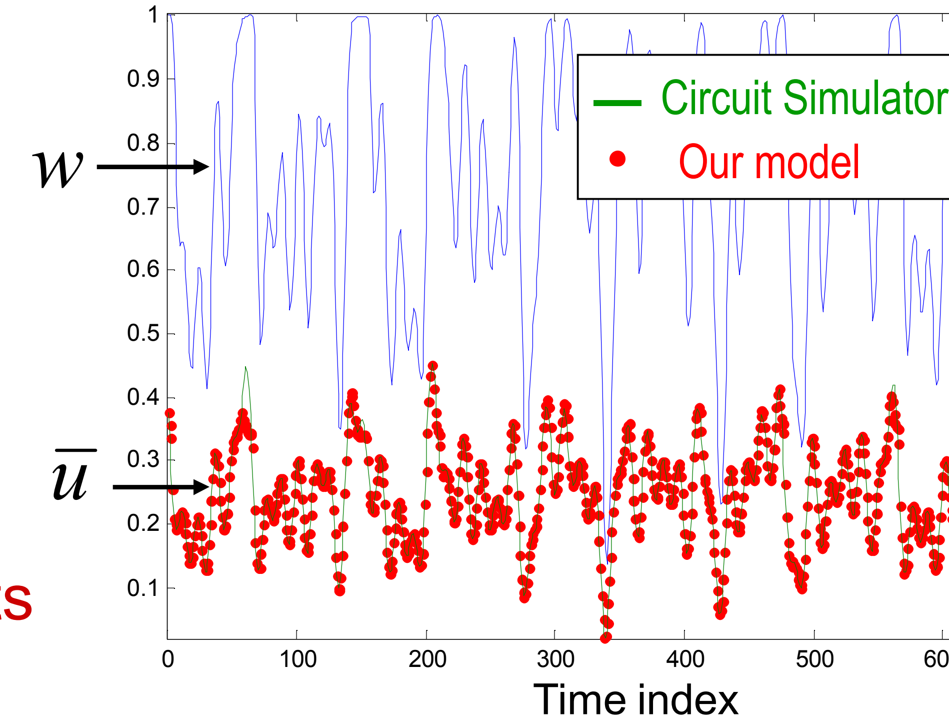
In collaboration with Yan Li, MIT

Example: Design - Opamp Compensator (2)





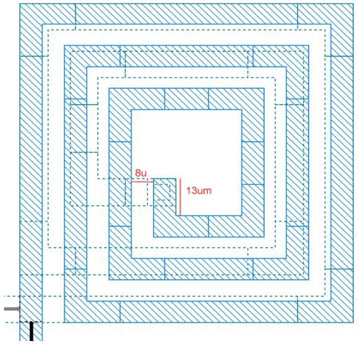




- Compare distortion in opamp and inverse outputs
- Compensator increases SDR by ~10dB

In collaboration with Yan Li, MIT

Example: Traditional LMOR Application - Inductor



$$E\dot{x} = Ax + Bu$$

Simulate original system

$$\{x, u\}$$

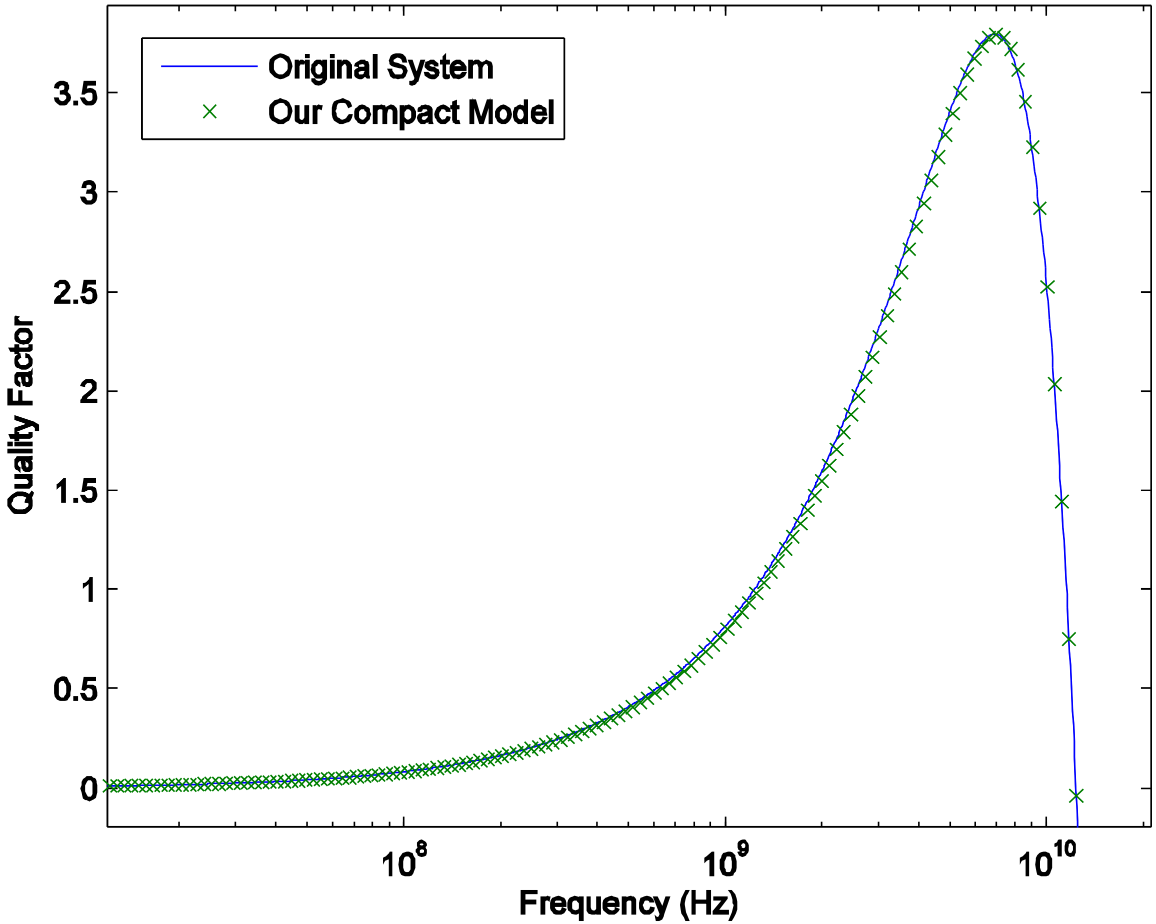
Project data

$$\{\hat{x}, u\}$$

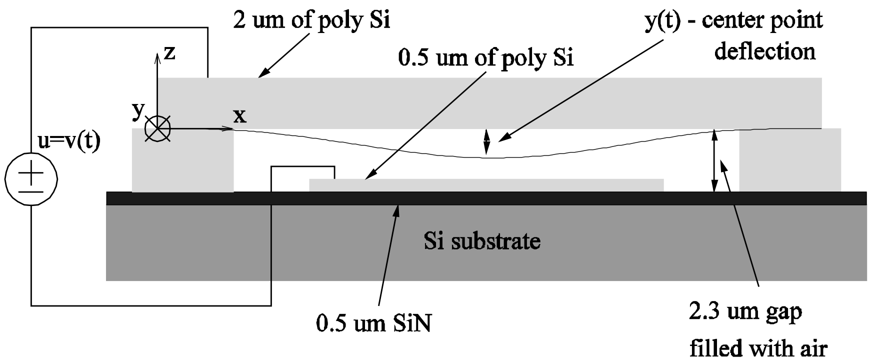
Identify model

$$\hat{E}\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$$

Continuous Time, Linear Model



Example: Traditional NLMOR Application - MEMS



- 400th order original model
- 4th order CT compact model
- 7th degree polynomial
- 52 coefficients
- 40X speedup for Transient

$$\dot{x} = f(x,u)$$

Simulate original system

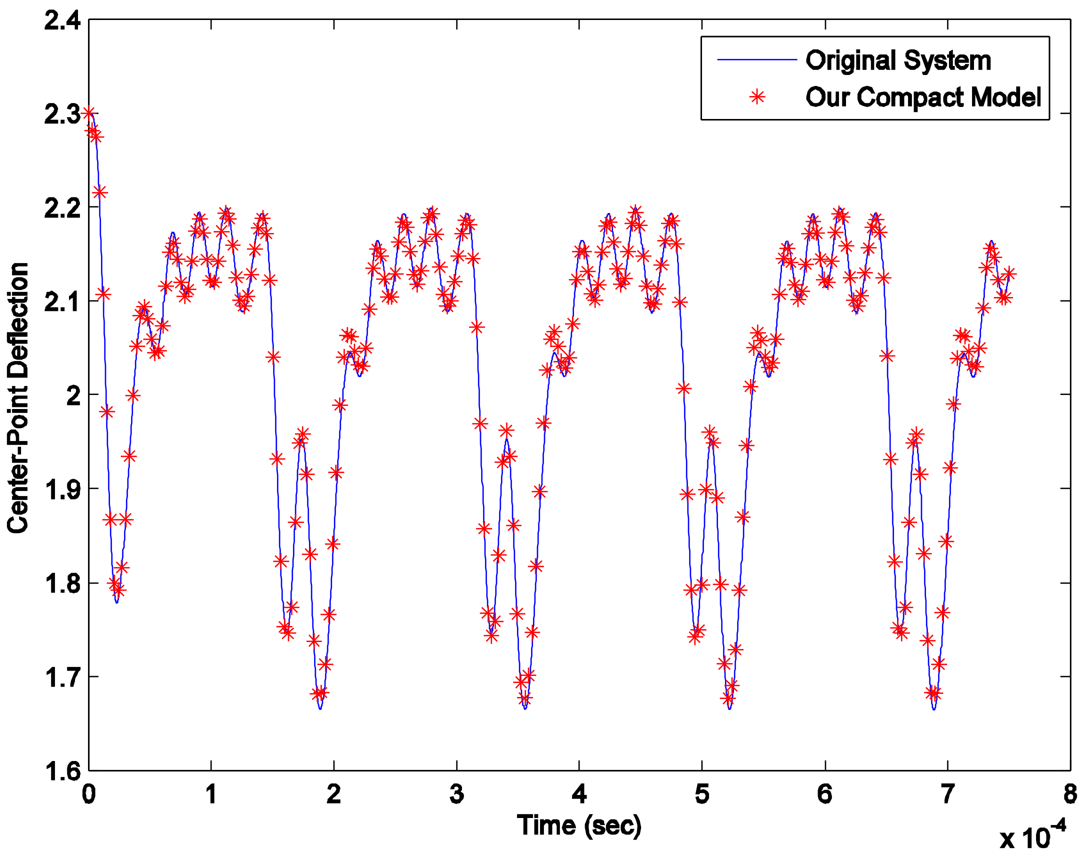
$$\{x,u\}$$

Project data

$$\{\hat{x},u\}$$

Identify model

$$\dot{\hat{x}} = \hat{f}(\hat{x},u)$$



Conclusion

- Accelerated simulation using compact models
- Need techniques guaranteeing robust models
- Encounter all types of systems in analog applications
  - Unstructured Linear, Nonlinear, Data samples
- SYSID approach based on semidefinite optimization

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Papers, codes, slides