

# A new time-dependent analytic compact model for radiation-induced photocurrent in epitaxial structures

Jason C. Verley, Eric R. Keiter,  
Charles E. Hembree, Carl L. Axness (ret.)  
Sandia National Laboratories  
Albuquerque, New Mexico, USA

Bert Kerr  
Mathematics Department  
New Mexico Institute of Mining and Technology  
Socorro, New Mexico, USA

## Introduction

Photocurrent generated by ionizing radiation represents a threat to microelectronics in radiation environments. Circuit simulation tools that employ compact models for individual electrical components (SPICE and Xyce [1], e.g.) are often used to analyze these threats. Historically, many photocurrent compact models ([2], [3], e.g.) have suffered from accuracy issues due to the use of empirical assumptions, or physical approximations with limited validity. In this work, an analytic model is developed for epitaxial diode structures that have a heavily-doped sub-collector.

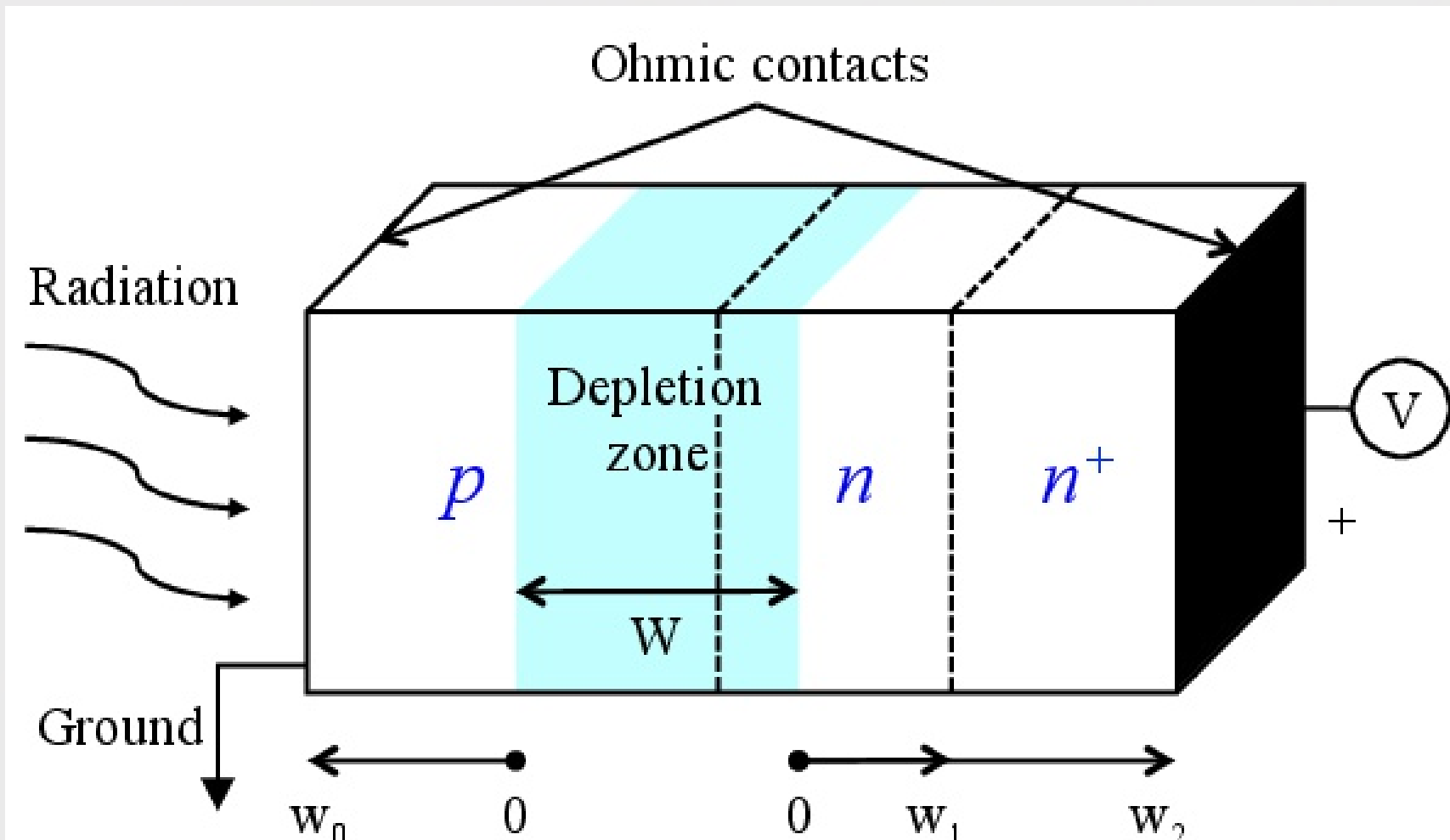


Fig. 1. Reverse-biased 1D abrupt junction pnn+ diode under irradiation

## Mathematics

An improved photocurrent model for pn-type structures was developed in [4], and we extend that here to include highly doped sub-collectors. Since the drift-diffusion equations are not amenable to analytic techniques, we model the excess carrier behavior using the ambipolar diffusion equation:

$$\frac{\partial u}{\partial t} = D_a \nabla^2 u - \mu_a \mathbf{E} \cdot \nabla u - \frac{u}{\tau} + g$$

where  $g$  is the generation rate. Assuming 1D and  $E=0$ :

$$u_t = D_i u_{xx} - \frac{1}{\tau_i} u + g(t)$$

where  $i=1$  in the n-type region, and  $i=2$  in the sub-collector ( $n^+$ ) region.  $D_1$  and  $D_2$  are the ambipolar diffusion coefficients, and  $\tau_1$  and  $\tau_2$  are the carrier lifetimes in those regions. Internal BC1, continuity of current through the  $nn^+$  junction:

$$D_1 \frac{\partial u}{\partial x} \Big|_{x=w_1^-} = D_2 \frac{\partial u}{\partial x} \Big|_{x=w_1^+}$$

Internal BC2, discontinuity due to doping:

$$N_1 u(w_1^-, t) = N_2 u(w_1^+, t)$$

External boundary conditions:  $u(0, t) = u(w_2, t) = 0$

The boundary value problem can be solved using the finite Fourier transform method [4], [5]. Assuming  $u(x, 0) = 0$ , the solution is given by:

$$u(x, t) = \sum_{n=1}^{\infty} w_n \int_0^t g(v) e^{-\lambda_n(t-v)} dv \frac{X_n(x)}{\|X_n\|^2}$$

where  $X_n$  are the eigenfunctions,  $\lambda_n$  are the eigenvalues, and  $w_n = \langle 1, X_n(x) \rangle$ .

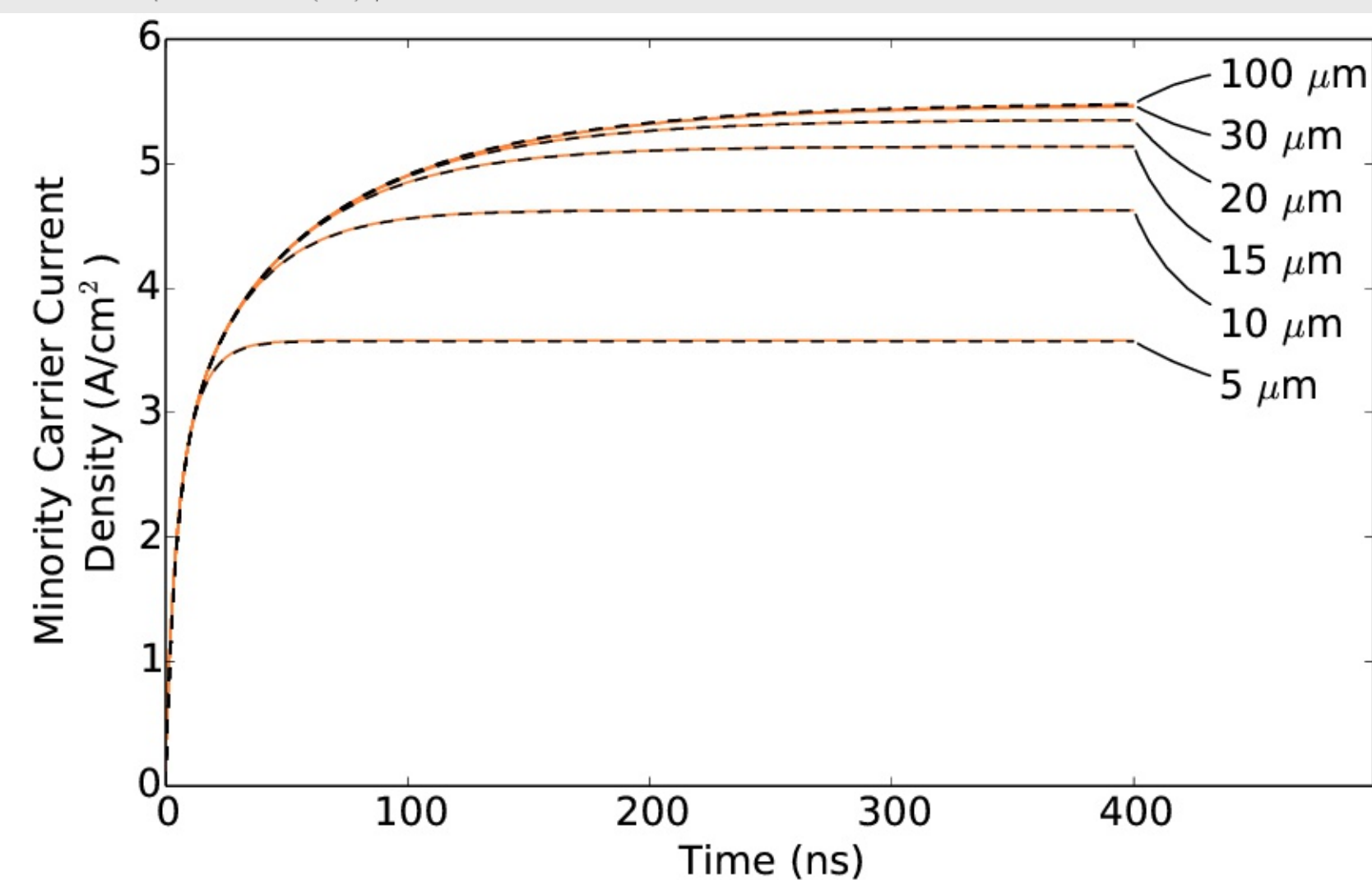


Fig. 3. Transient, normalized analytic and TCAD photocurrent densities for the  $nn^+$  region as a function of the sub-collector thickness, for a long  $10^9$  rad(Si)/s irradiation. The TCAD results are plotted as dashed lines.

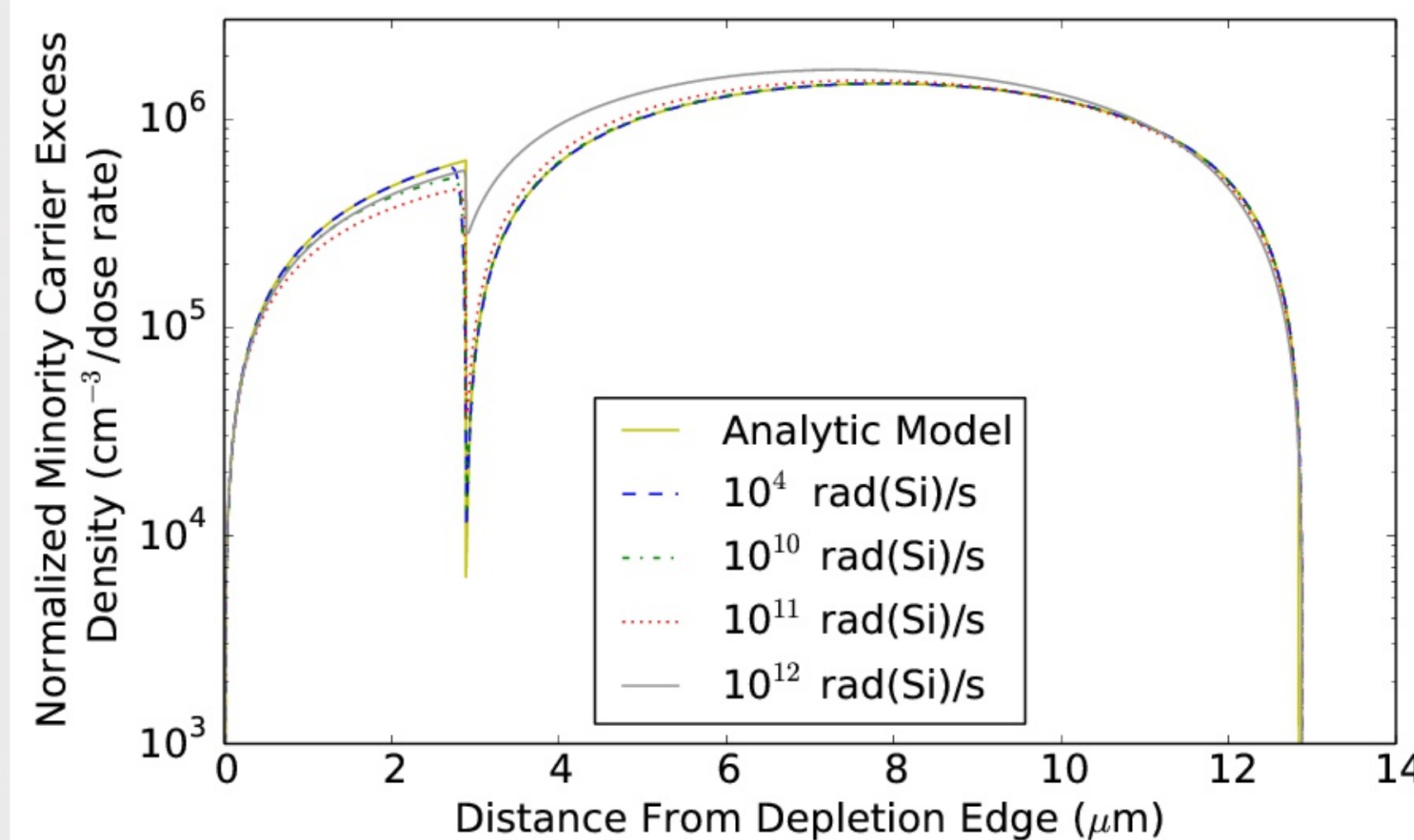


Fig. 2. Normalized (with respect to dose rate) steady-state excess minority carrier density for an irradiated  $nn^+$  region as a function of position. The right depletion edge in the  $pn^+$  diode (Figure 1) would correspond to  $x=0$  in the figure. The densities with dose rate labels are simulated with TCAD.

## Code comparison

Comparing to previous work, we assumed the same boundary conditions as in [3], but they took the sub-collector to be infinite; and they solved the problem analytically only for steady-state conditions. The photocurrent compact model by Fjeldly, et al. [2] treats the sub-collector by assuming that photocurrent collection from the sub-collector is limited to one diffusion length from the boundary.

Figure 2 compares a density calculation from the analytic model to TCAD simulations. The (quasi-)discontinuity at the  $nn^+$  interface is due to the boundary conditions in the analytic calculation, but is a natural consequence of the doping profile in the TCAD simulations. The curves are in close agreement for dose rates less than  $10^{10}$  rad(Si)/s, illustrating the effectiveness of the compact model. It also supports the choice of interface boundary conditions.

Figure 3 illustrates the photocurrent contribution from the sub-collector. For the parameters and doping levels used in these simulations, a significant amount of charge is collected from deep within the sub-collector (in the range of  $20 \mu\text{m}$ – $30 \mu\text{m}$ ). The diffusion length in this case is approximately  $5 \mu\text{m}$ , which indicates that the Fjeldly assumption does not account for all of the delayed photocurrent contributions. Also, the infinite sub-collector assumption in [3] is valid only for sub-collectors longer than  $\sim 30 \mu\text{m}$ .

A comparison of the analytic model to the Fjeldly model and TCAD simulations is shown in Figure 4 for a hypothetical npnn+ BJT. A sawtooth waveform was chosen to represent the radiation pulse to provide a good exercise for the model. The waveform is shown in the inset of Figure 4a.

The total current in each sub-figure is the sum of the photocurrent going through the  $50 \Omega$  resistor from the individual regions [4] plus the photocurrent from the  $nn^+$  region, which is given by

$$J_{pp}(t) = qD_1 \frac{\partial u}{\partial x} \Big|_{x=0} = qD_1 \sum_{n=1}^{\infty} w_n \int_0^t g(v) e^{-\lambda_n(t-v)} dv \frac{X'_n(0)}{\|X_n\|^2}$$

where  $q$  is the elementary charge.

Figure 4a compares the Fjeldly model calculation to the analytic model. The Fjeldly model requires calibration, and was calibrated on the same time scale as in Figure 4a. The two model calculations agree well with the TCAD calculation on this time scale.

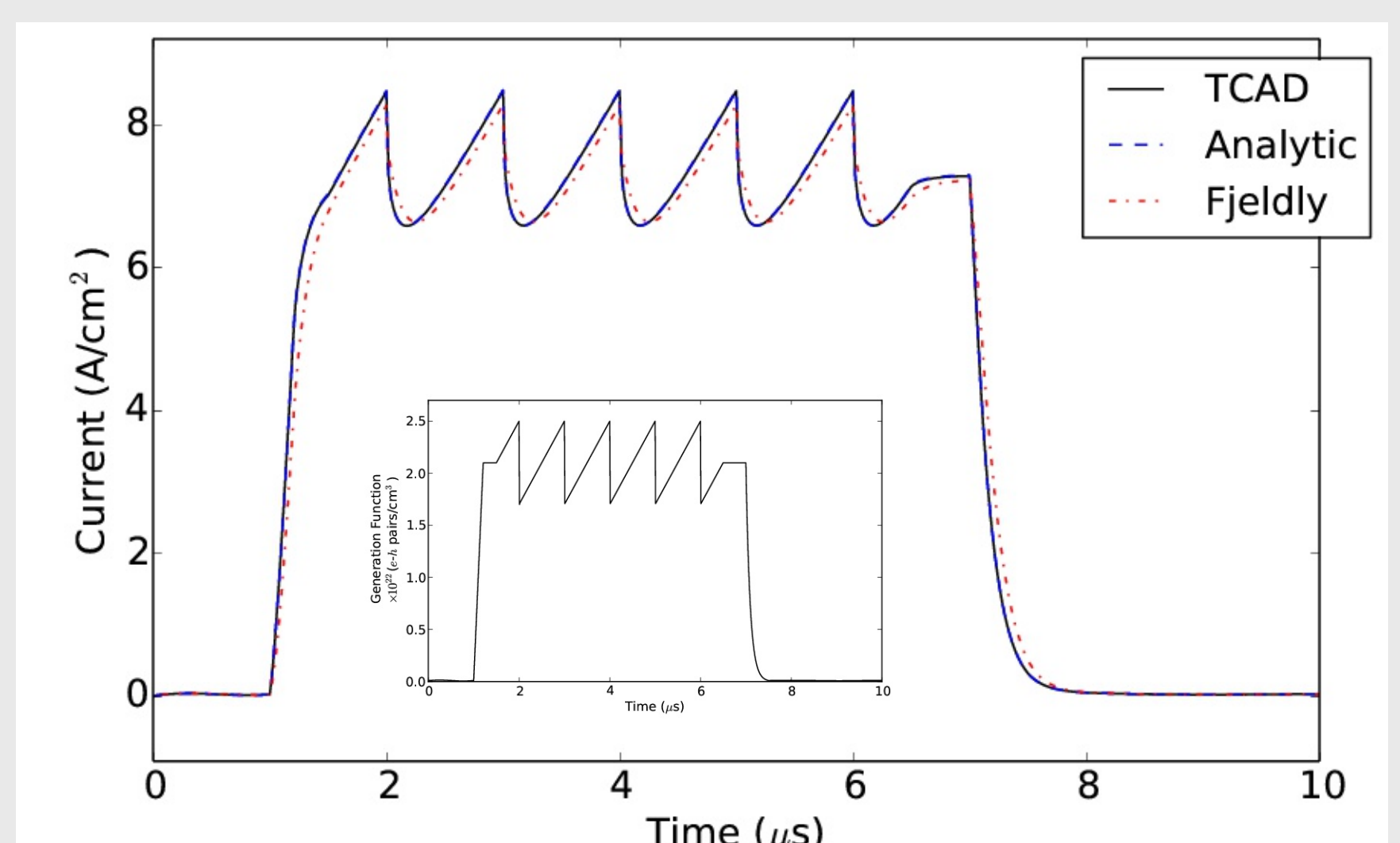
The limitations of the Fjeldly model become apparent in Figure 4b, where the time scale is compressed by a factor of 100. The analytic model still has excellent agreement with TCAD, however.

When the time scale is shortened to a pulse of a few nanoseconds of duration (Figure 4c), agreement of the analytic model with the TCAD simulation begins to diverge. The excessive sharpness of the analytic solution compared to the TCAD simulation indicates that some of the disagreement is due to the simple method for computing the photocurrent from the depletion region, which assumes an infinite drift velocity.

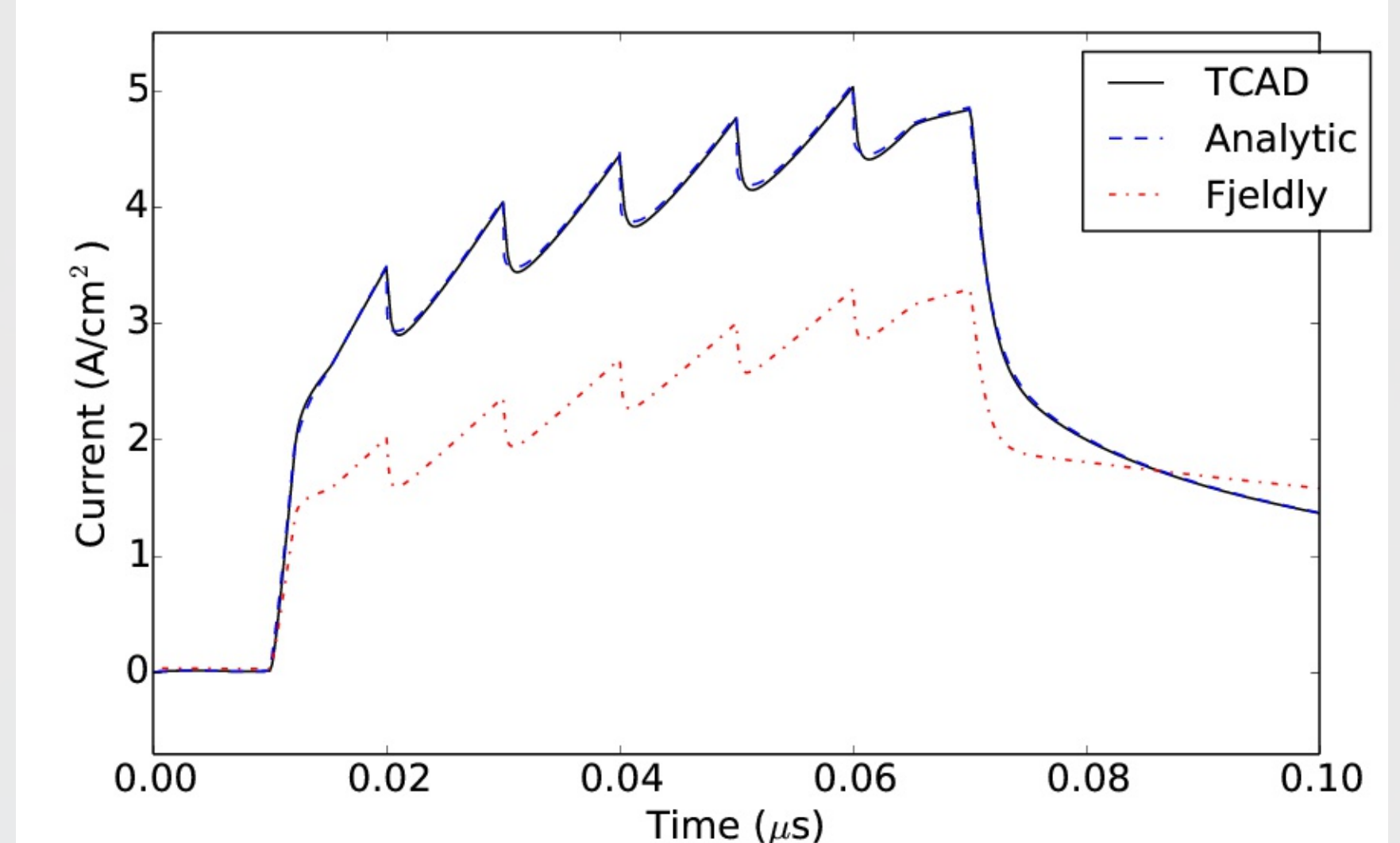
## Conclusions

We presented a new analytic solution that determines the current density coming from an irradiated finite 1D reverse-biased  $pn^+$  abrupt junction epitaxial diode. The solution uses the correct  $nn^+$  boundary conditions, as found in [3]. It also improves on the solution in [3] by solving the problem for a finite diode, and taking into account an arbitrary time-dependent radiation generation density. We also developed the analytic solution for a piecewise linear generation function, so that it may be used to analyze realistic pulses—including those based on experimental data [5]. The analytic results compare favorably to TCAD simulations, and represent a substantial improvement over the Fjeldly model [2].

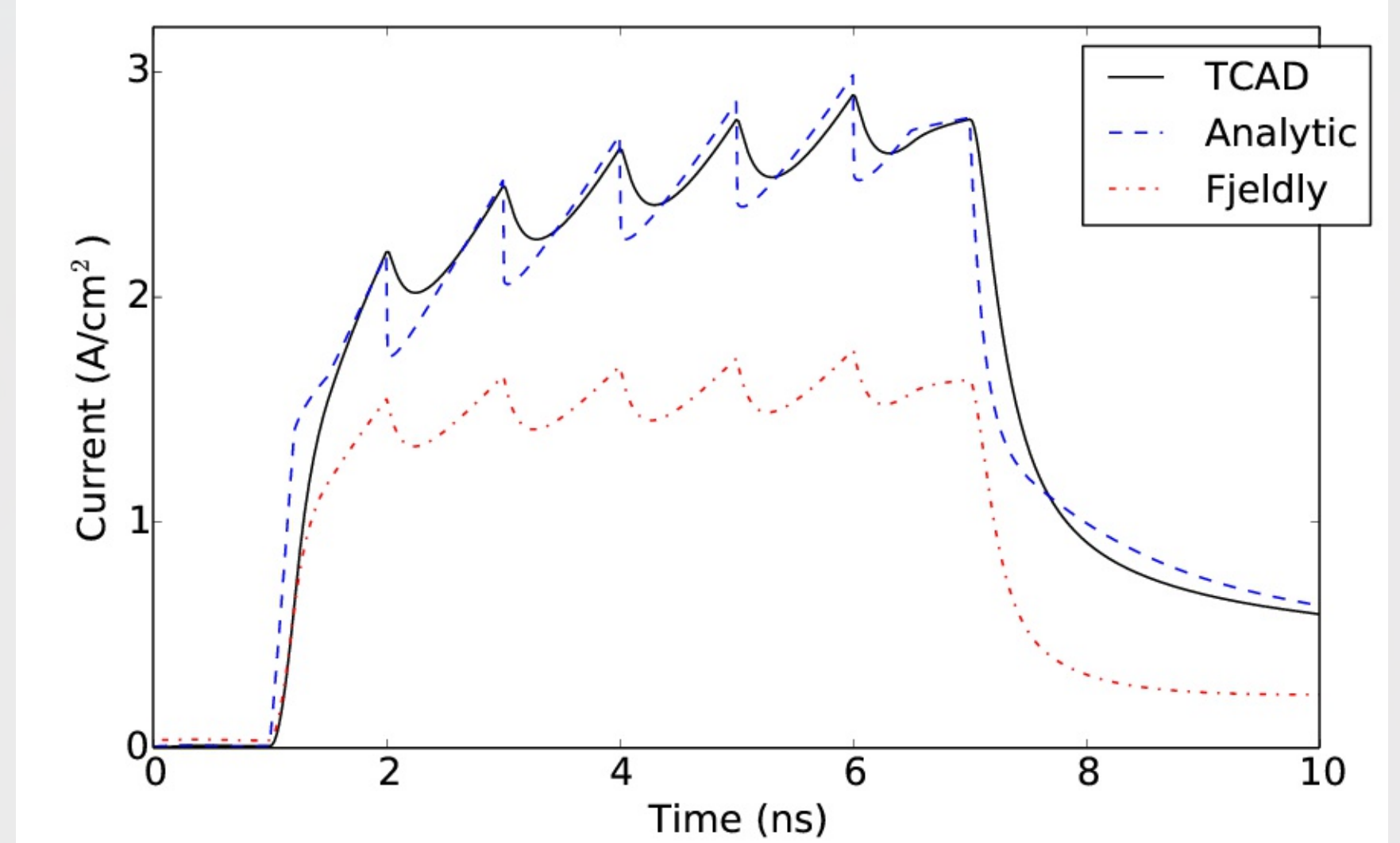
Finally, we note that the analytic model presented here may also be readily adapted for other applications involving the behavior of excess carriers in undepleted device regions. This includes devices where the generation function has a dependence on both position and time. Examples include optical sensors, photovoltaics and power devices.



(a) Comparison at the calibrated time scale



(b) Time-compressed by a factor of 100



(c) Time-compressed by a factor of 1000

Fig. 4. Hypothetical BJT photocurrent comparison, as computed with TCAD, the analytic model, and the Fjeldly model. The transistor was simulated as part of a standard test circuit, where the base and emitter are shorted together, and attached to ground via a  $50 \Omega$  resistor; the collector is attached to a  $5 \text{ V}$  bias.

## References

- [1] E. R. Keiter, et al., Sandia National Laboratories, Albuquerque, NM, Tech. Rep. SAND2012-4085, 2012. Also see <http://xyce.sandia.gov> (QR code below).
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