



Polymer Field Theories and Classical DFT: Applications to Polymer Brushes and Nanocomposites

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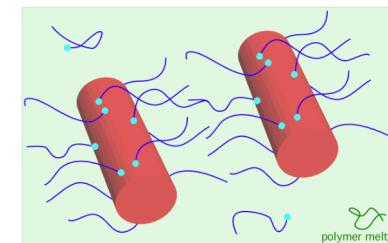
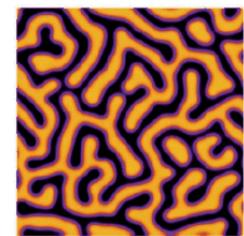
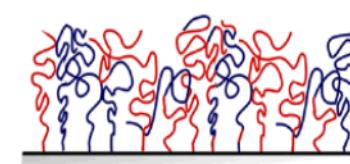
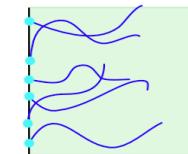
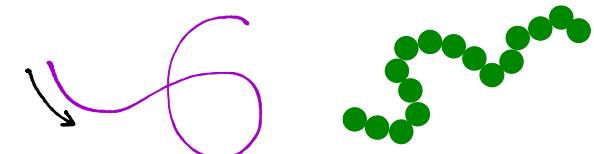
*Exceptional
service
in the
national
interest*



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Outline of the talk

- polymer theories
 - self-consistent field theory (SCFT)
 - classical density functional theory (DFT)
- intro to end-grafted polymers
- pattern formation in mixed brushes
 - SCFT calculations
 - comparison to experiment
- polymer brushes on nanoparticles
 - DFT and SCFT calculations
 - comparison to experiment

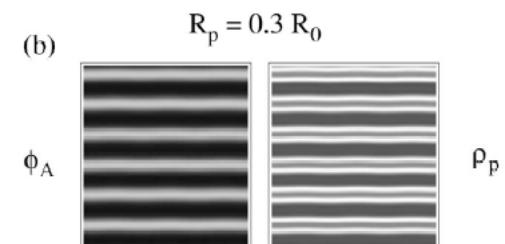
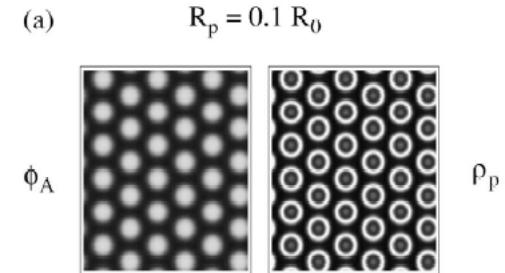


Polymer Theories

- treat larger length/time scales than simulation
- direct access to free energy
- exploration of phase space
- *but* approximate

bulk

- equations of state
- PRISM



inhomogeneous

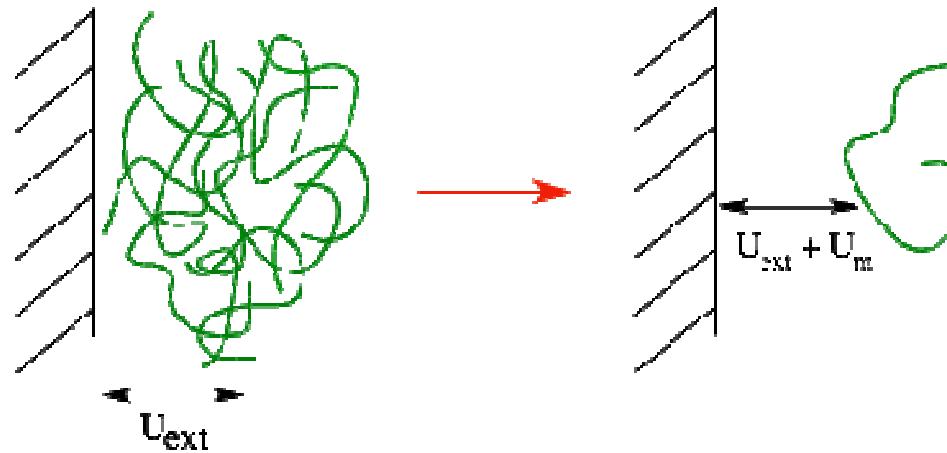
- self-consistent field theory (SCF)
- classical density functional theory (DFT)
- hybrid SCF/DFT

Balazs, Curr. Opin. Solid St. & Mat. Sci., 2003

Inhomogeneous Theories

SCFT and DFT

basic idea: replace many chain problem with single chain in a field



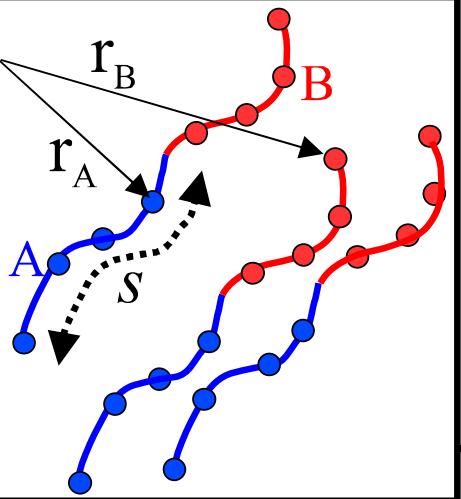
input: a model for the system
chain type
interactions

output: minimized free energy
density profiles

Self-Consistent Field Theory

From Particles to Fields

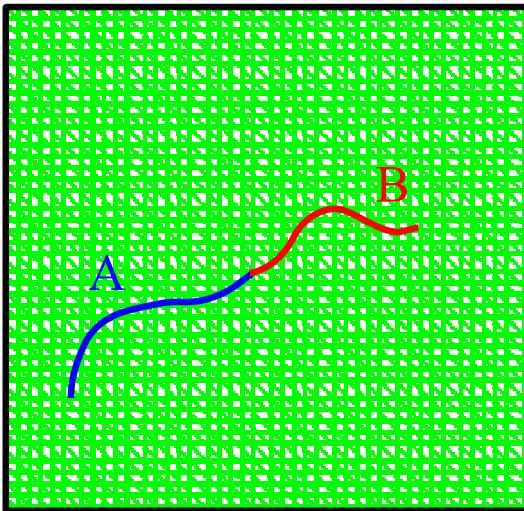
Example: diblock copolymers



$$Z = \int \mathcal{D}\mathbf{r}_A \mathcal{D}\mathbf{r}_B e^{-\beta H[\mathbf{r}_A, \mathbf{r}_B]}$$

$$\beta H = \frac{3}{2b^2} \sum_{\alpha=1}^n \int_0^N ds \left(\frac{d\mathbf{r}_\alpha(s)}{ds} \right)^2 + \frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}' \hat{\rho}_K(\mathbf{r}) v_{KL}(\mathbf{r} - \mathbf{r}') \hat{\rho}_L(\mathbf{r}')$$

Hubbard-Stratonovich transformation



$$Z = \int \mathcal{D}w_A \mathcal{D}w_B e^{-\beta H[w_A, w_B]}$$

$$\beta H = \int d\mathbf{r} h(w_A(\mathbf{r}), w_B(\mathbf{r})) - n \ln Q[iw_A, iw_B]$$

Q = *single-chain partition function*

G. H. Fredrickson

Mean-Field Approximation: SCFT

- SCFT is derived by a *saddle point* approximation to the FT:

$$e^{-F} \equiv Z = \int \mathcal{D}[w] e^{-H[w]} \sim e^{-H[w^*]}$$

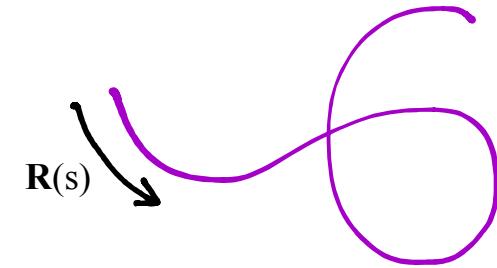
- The approximation is asymptotic for $C \rightarrow \infty$
- We can simulate the field theory at two levels:
 - “Mean-field” approximation (SCFT): $F \approx H[w^*]$
 - Full stochastic sampling of the complex field theory: “Field-theoretic simulations” (FTS)

Typical Features of SCFTs

typically (but not always):

- use Gaussian thread model of chain
 - leads to PDE for chain propagators

$$U_0[\mathbf{R}(s)] = \frac{3}{2a^2} \int_0^N ds \left| \frac{\partial \mathbf{R}(s)}{\partial s} \right|^2$$



- use Flory-Huggins free energy for interactions

$$U[\mathbf{R}_A(s), \mathbf{R}_B(s)] = v_0 \int d\mathbf{r} \chi_{AB} \phi_A(\mathbf{r}) \phi_B(\mathbf{r})$$

- incompressible (or nearly incompressible)
- many numerical solution methods (nonlinear equations)
 - real space
 - Fourier space

- model polymers on the length scale of the chain (R_g)
- great for phase behavior, structure

To learn more about SCFT

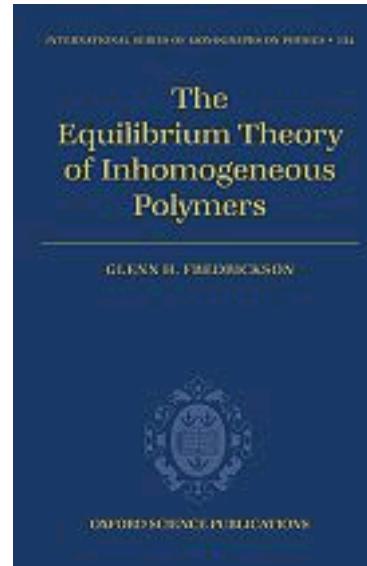


Read Glenn Fredrickson's book!

The Equilibrium Theory of
Inhomogeneous Polymers

Glenn H. Fredrickson

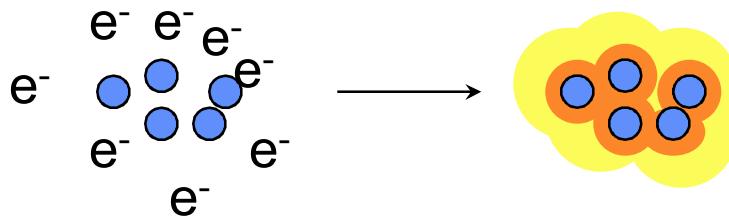
Oxford Univ. Press, 2006



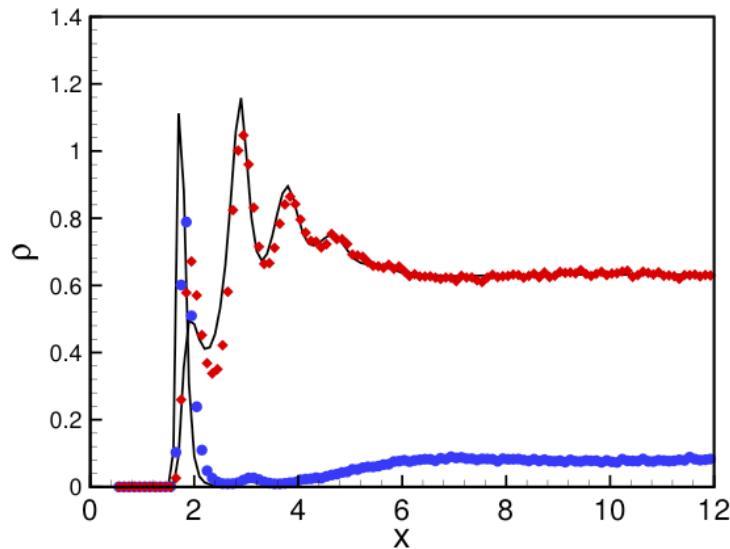
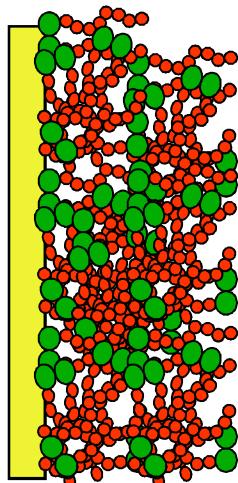
Density Functional Theory (DFT)

$$\Omega[\rho(r)] : V(r) \rightarrow \rho(r)$$

External field Density profile

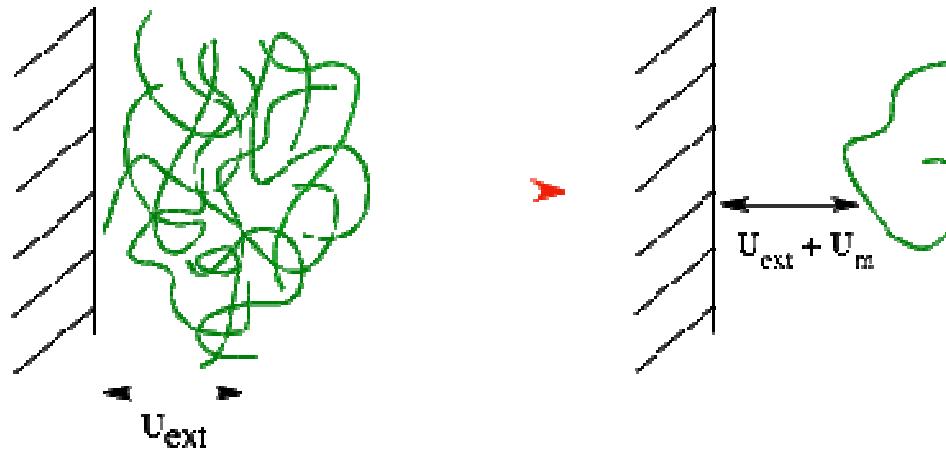


Electronic Structure
Minimizing quantum Hamiltonian



Fluid Structure
Minimizing free energy
(Often open system
with fixed chemical
potential)

Structure of Fluids DFT



$$\Omega[\rho_\alpha(\mathbf{r})] = F[\rho_\alpha(\mathbf{r})] + \sum_\alpha \int d\mathbf{r} \rho_\alpha(\mathbf{r}) [V_\alpha(\mathbf{r}) - \mu_\alpha]$$

Helmholtz free energy F : ideal gas, hard sphere, attractions, bonding, ...

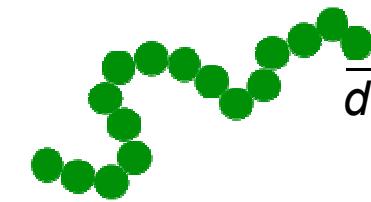
minimize free energy

$$\frac{\delta \Omega}{\delta \rho(\mathbf{r})} = 0 \longrightarrow \text{equations to solve for } \langle \rangle(\mathbf{r})$$

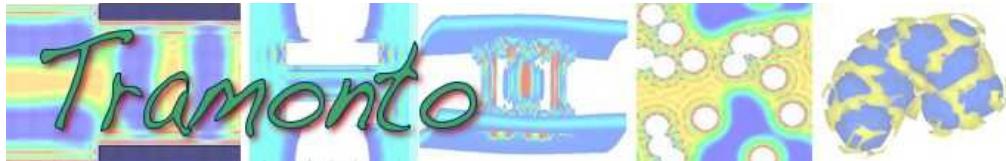
Typical Features of Polymer DFTs

- use freely-jointed chain of hard spheres for model
 - includes segment length scale d
 - leads to integral equations for chain propagators
- use hard sphere repulsions + LJ attractions for interactions
 - directly comparable to MD, MC simulations
- compressible
- different types of classical DFTs:
 - CMS-DFT (Chandler, McCoy, Singer)
 - based on 2nd order expansion of free energy
 - weighted DFTs (esp. Wu, Chapman)
 - based on perturbations to hard sphere reference fluid

- model packing effects + chain length scale
- great for comparison to simulation
- local structure, phase behavior, mixtures with particles, etc.



Tramonto: Sandia's DFT Code



<http://software.sandia.gov/tramonto>

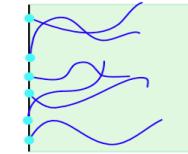
F-DFTs = nonlinear integral equations

- solve in 3D, Cartesian grid
- modified Newton's method, Picard solver
- parallel
- sophisticated linear solver algorithms
- arc-length continuation algorithms
- hard spheres
- polymers
 - CMS-DFT
 - modified iSAFT
- mean-field attractions
- charged systems
 - includes Poisson solver

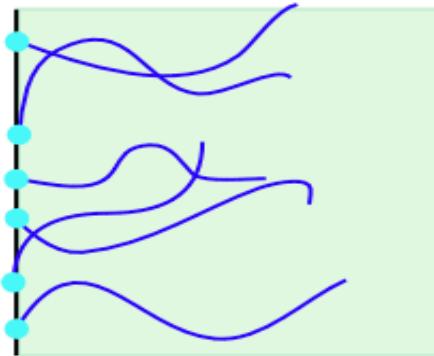
The Trilinos Project

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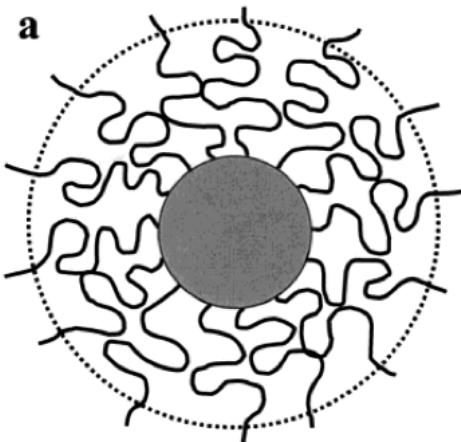


Polymer Brushes



- graft or adsorb polymer to surface
- if dense enough, chains stretch away from surface

height of brush:
balance stretching energy penalty vs. interactions



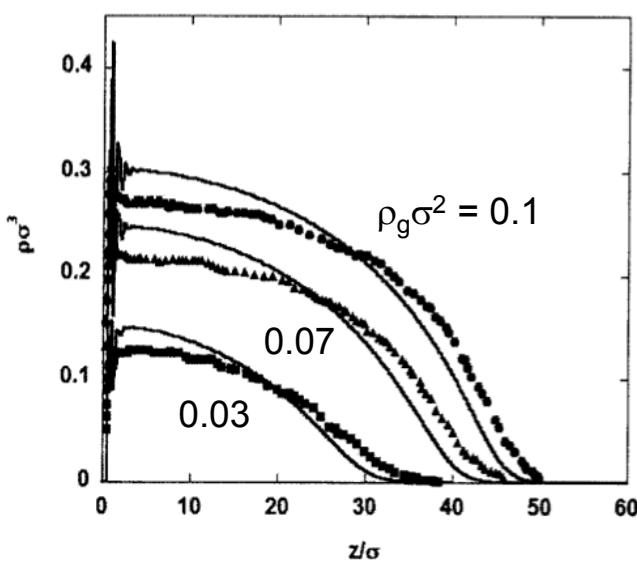
curved brushes

- more volume than on flat surface at same grafting density
- chains splay out more
- brush is less extended

Theory for Polymer Brushes

DFT and SCFT capture basic physics

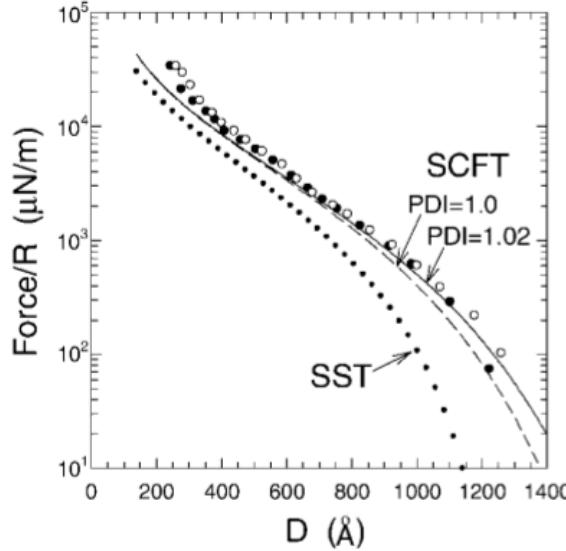
DFT vs. MD simulation



brush in vacuum
 $N = 100$

Jain et al., J. Chem. Phys. 128, 154910 (2008); (simulations from Grest and Murat, 1989)

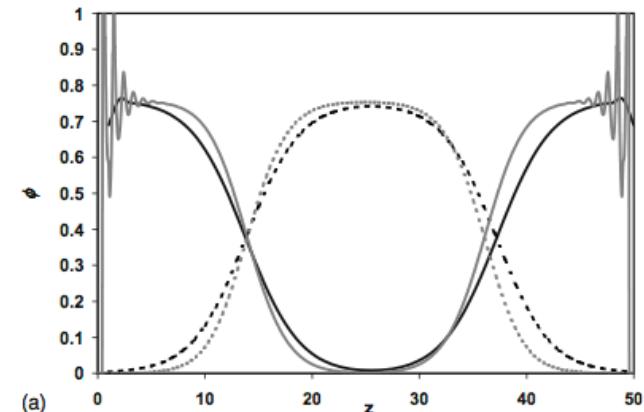
SCFT vs. Experiment



force between brushes
in solvent

Kim and Matsen, Macromolecules 42, 3430 (2009); (exp. from Taunton et al, 1988)

DFT vs. SCFT



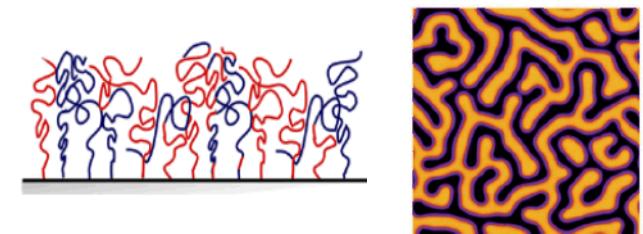
(a)

brushes in polymer melt
 $N = P = 100, \rho_g = 0.1$

Jain et al., J Chem Phys 131, 044908 (2009)

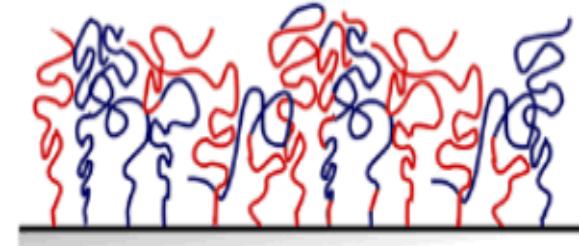
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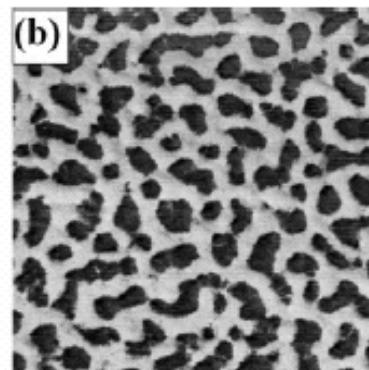


Mixed Polymer Brushes

- A mixture of two polymers in which one end of each polymer chain is tethered to the substrate
- Phase separate in a manner similar to block copolymer thin films

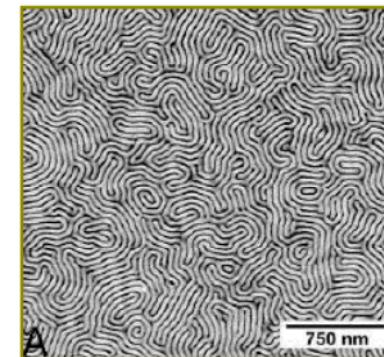


“Ripple” phase of symmetric mixed brush (PS – PMMA) under non-selective solvent



Usov et al., *Macromolecules*, 40, 8774 (2007)

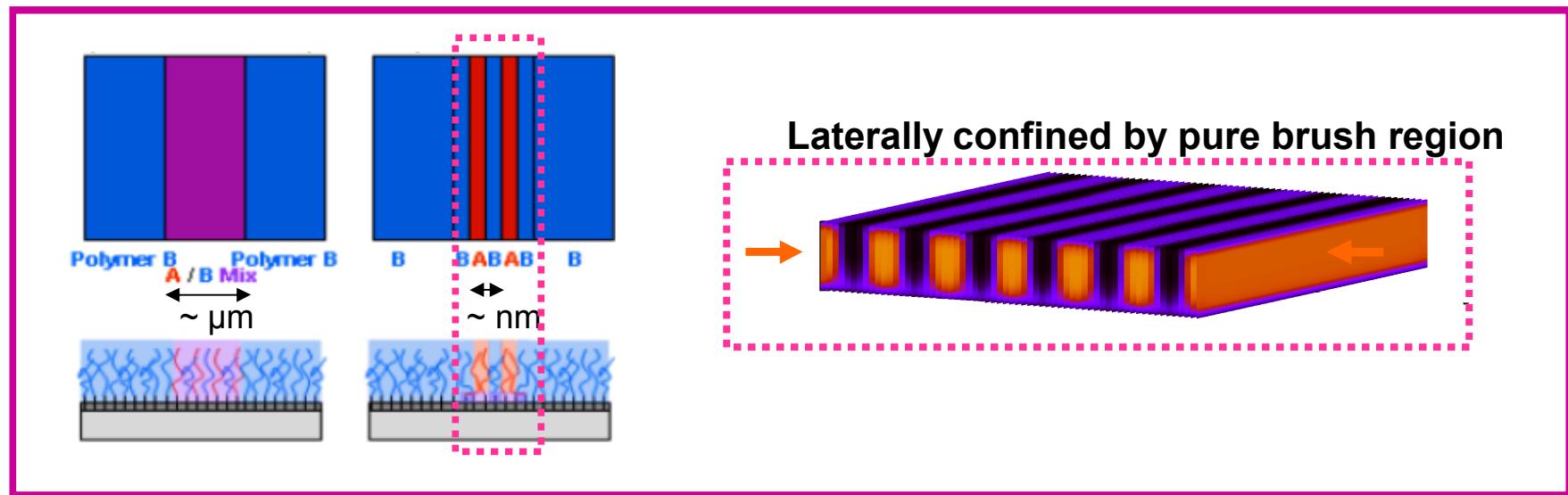
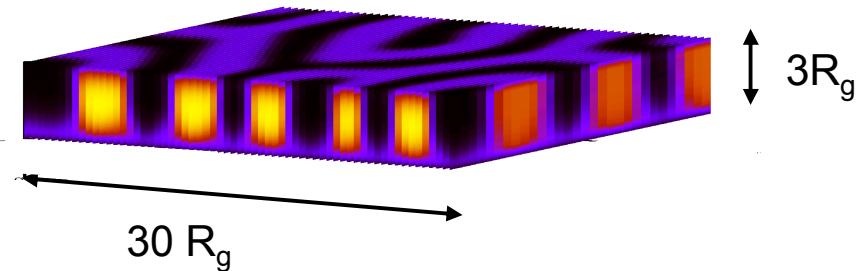
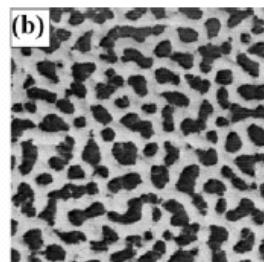
Perpendicular lamella of PS-b-PMMA block copolymer thin film



Daniel J.C. Herr, *Future Fab. Intl. Sec.5.* Issue 18 (2005) ¹⁷

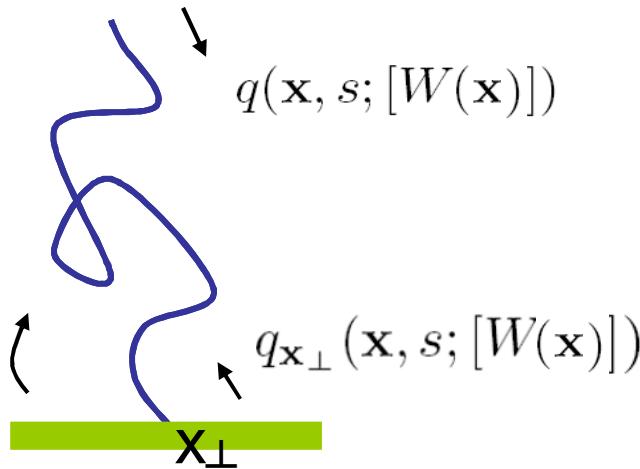
Directed Assembly?

- New graphoepitaxy-type technique
- Mixed polymer brushes laterally confined by pure brush region



SCFT of Melt Mixed Brush

- melt with high grafting density
- brush chains A, B: Flory interaction energy χ
- large polymer/air surface tension so flat top surface
- “walls” in z-direction (substrate + top surface)
- periodic boundaries in (x,y)



Free-end: uniform initial condition

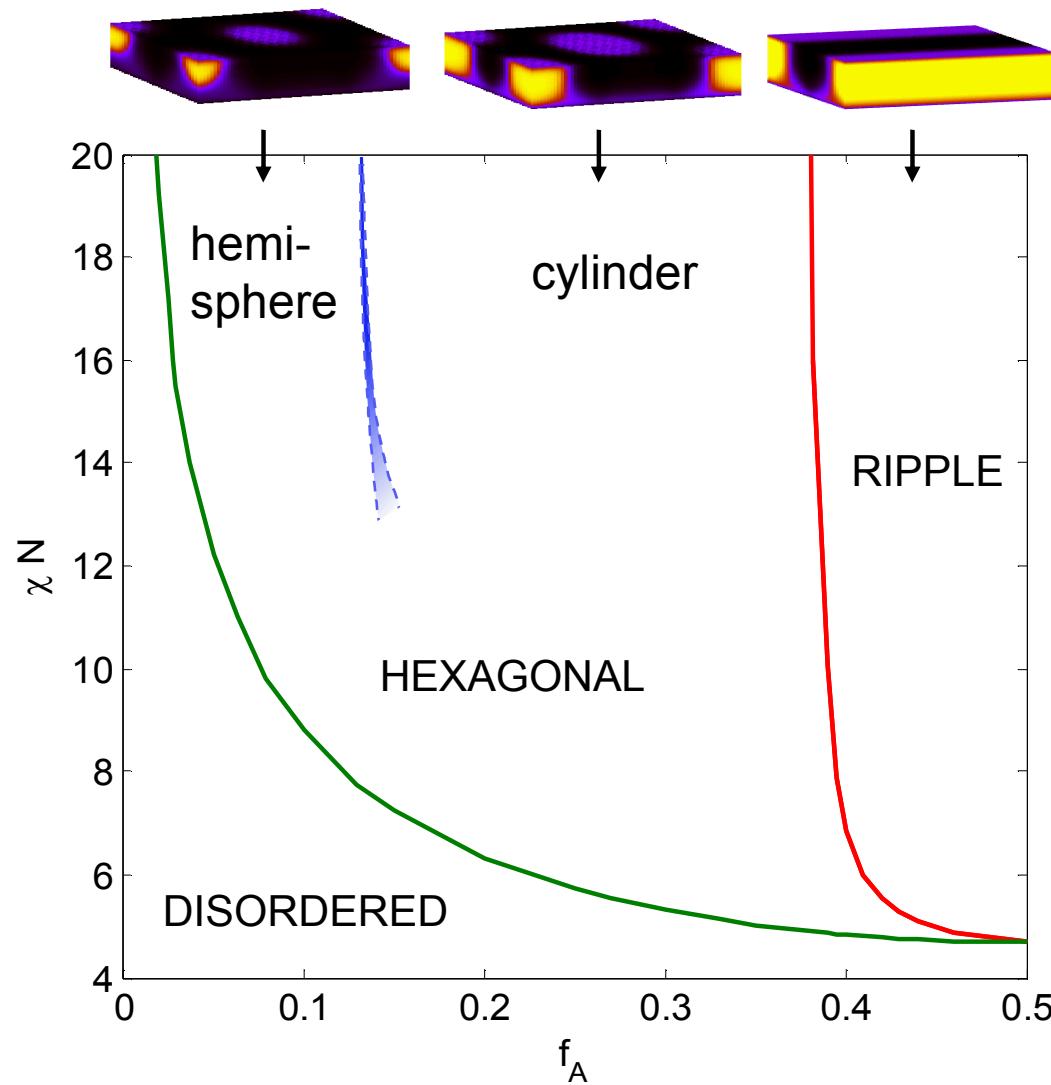
Tethered-end: initialized with a delta function

$$q_{\mathbf{x}_\perp}(\mathbf{x}, 0; [W]) = \delta(\mathbf{x} - \mathbf{x}_\perp)$$

Generalize to arbitrary grafting density distribution

$$g_{A,B}(\mathbf{x}_\perp, z)$$

SCFT Phase Diagram

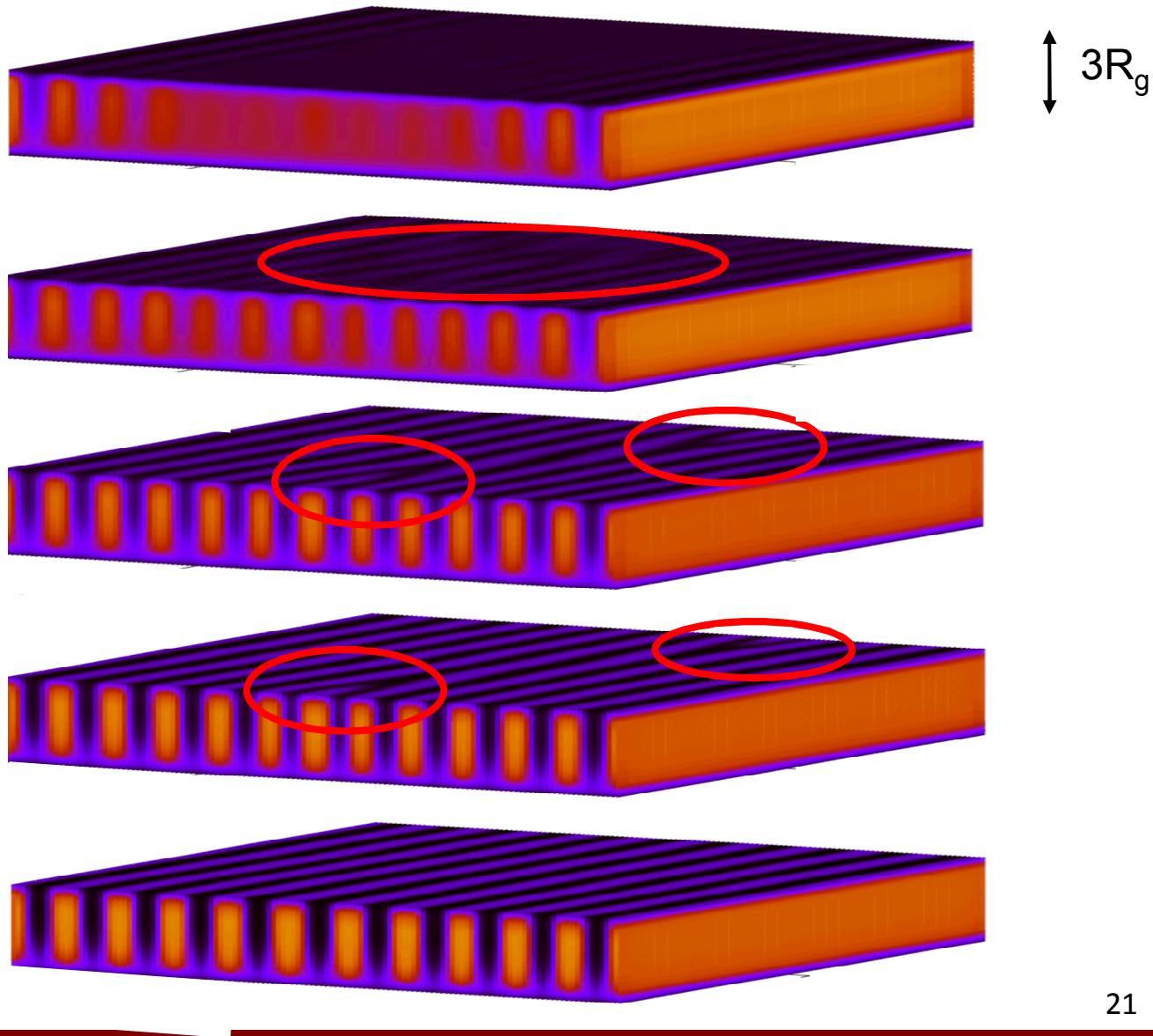


Directed Assembly in SCFT

$f_A = 0.5$
system size
 $\sim 56 R_g$

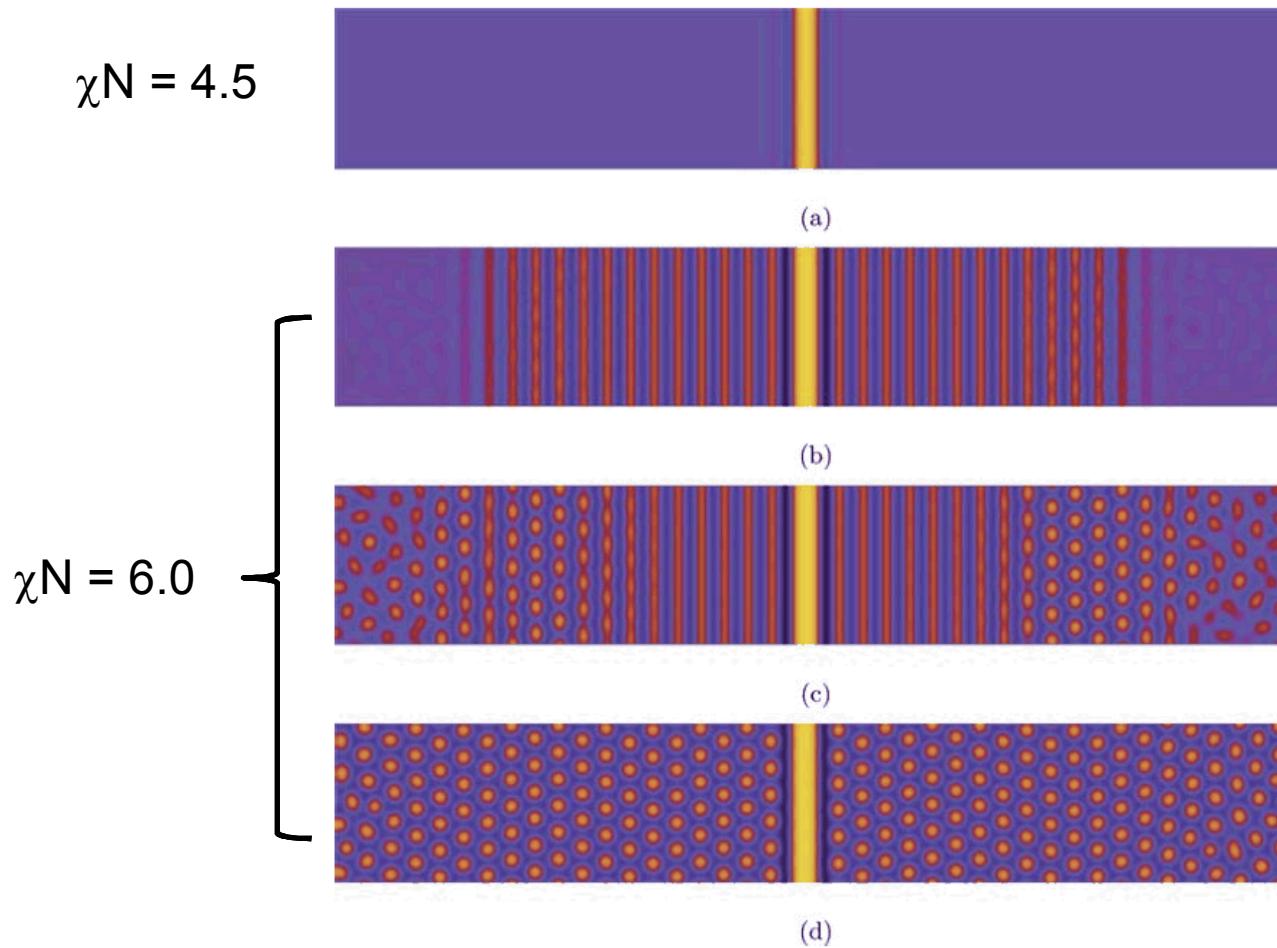
Evolution of
long-ranged
ordering

anneal χN slowly



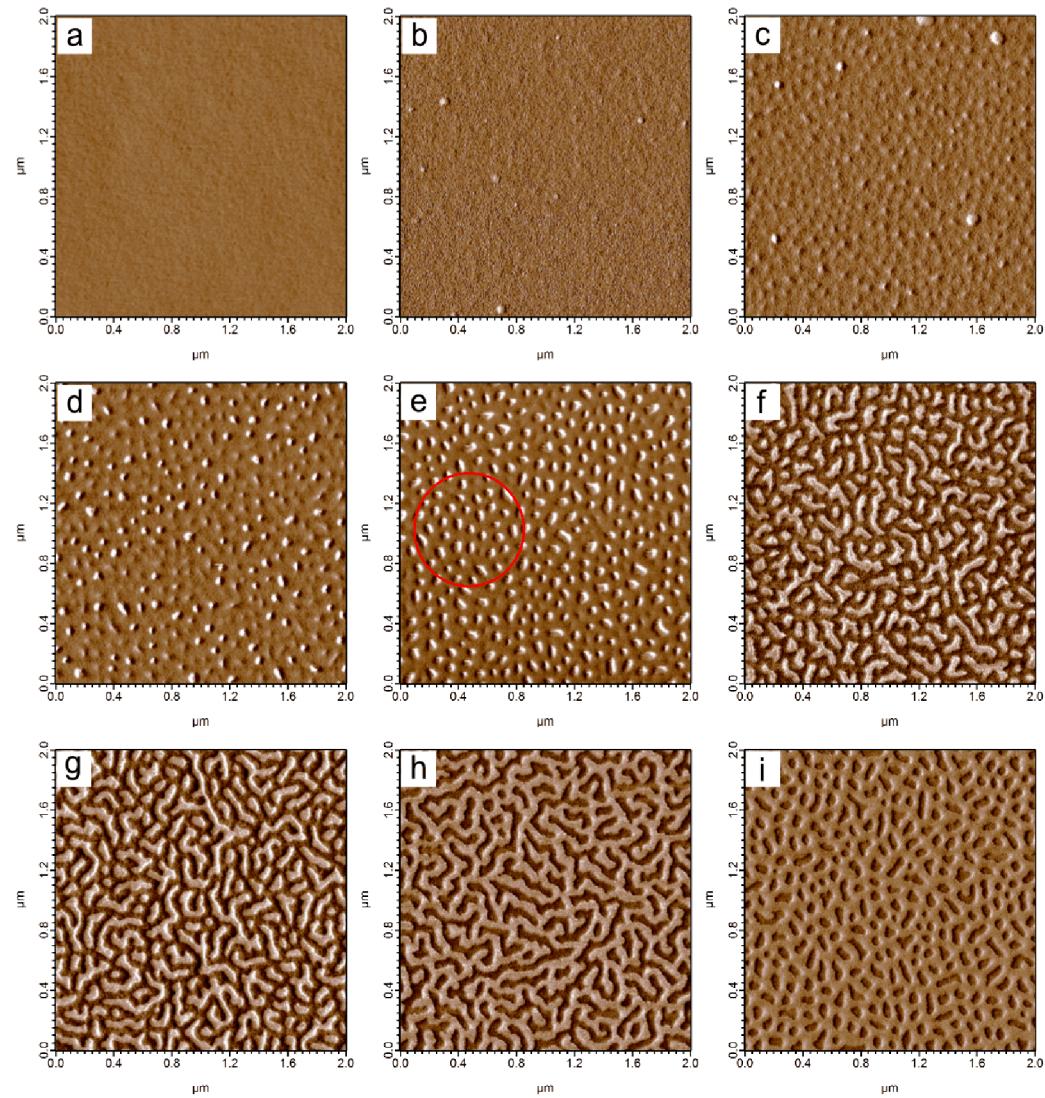
Directed Assembly of Cylinders

$f_A = 0.3$



Experimental Phases

- PS-PMMA mixed brushes
- solvent anneal
- PS volume fraction from 0.0 to 0.68
- AFM phase contrast images



Why the difference?

spatial variations in grafting density!

SCFT

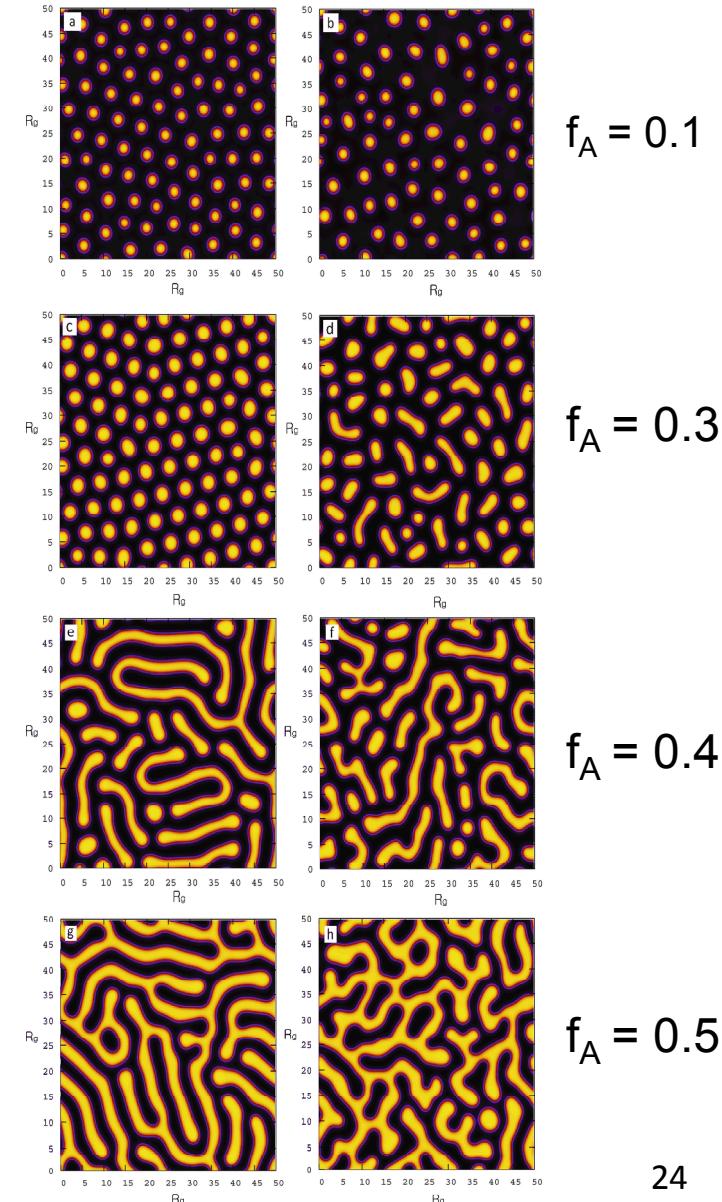
uniform grafting density (left)

$$g_A(\mathbf{x}_\perp) = f_A$$

Gaussian random distribution of grafting (right)

$$\langle (g_A(\mathbf{x}_\perp) - f_A)(g_A(\mathbf{x}'_\perp) - f_A) \rangle = \Lambda^2 \exp(-|\mathbf{x}_\perp - \mathbf{x}'_\perp|^2/2\sigma^2)$$

$$\sigma = 0.5R_g, \Lambda^2 = 0.02$$



Conclusions

- spatial variations in grafting destroy long-range order
- good qualitative agreement in phase diagram
- can direct self-assembly
 - with sufficiently uniform grafting density

Acknowledgments:



Su-Mi Hur Glenn Fredrickson
UC Santa Barbara



Dale Huber

Hur et al., *Soft Matter* **7**, 8776 (2011); Price et al.,
Macromolecules **45**, 510 (2012)

S-M. Hur, A. L. Frischknecht, D. L. Huber, and G. H. Fredrickson,
Soft Matter **9**, 5341 (2013).

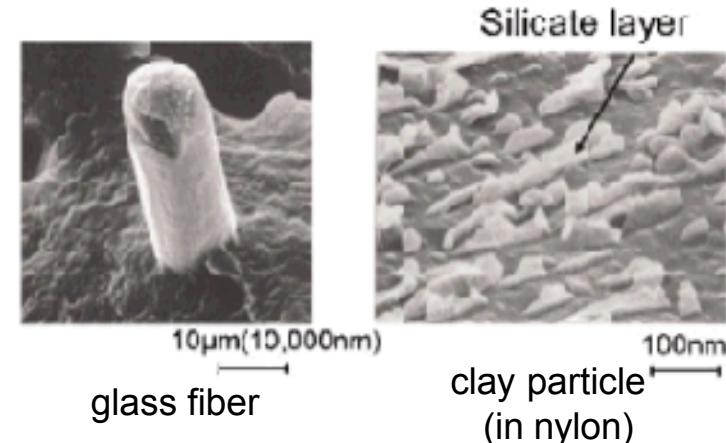


Polymer Nanocomposites

majority component: polymer

minority component: particle with dimensions < 100 nm

- improved material properties
- mechanical
- electrical
- optical
- need control over dispersion



Okada & Usuki, Macromol. Mater. Eng., 2006

- functional materials
- self-healing
- photovoltaics
- others...
- need control over interfaces

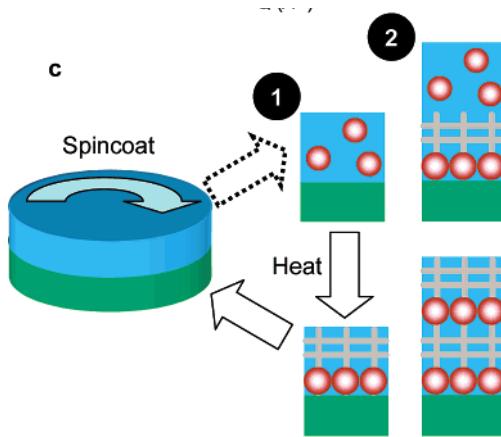


Cartoon of PCBM/P3HT solar cell

Kiel et al, Phys Rev Lett, 2010

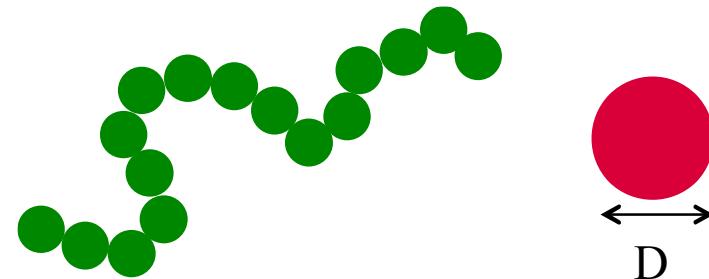
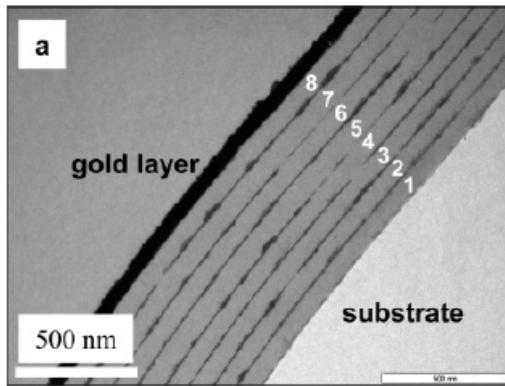
Digression: NPs in Polymer Films

nanoparticles tend to go to surfaces



athermal system: model as hard spheres
only interactions are entropic
polymer and particles are fluid components

ideal for DFT: capture packing interactions



Krishnan et al., *Nano Lett* (2007)

Weighted DFT: iSAFT



$$\Omega[\rho_\alpha(\mathbf{r})] = F[\rho_\alpha(\mathbf{r})] + \sum_\alpha \int d\mathbf{r} \rho_\alpha(\mathbf{r}) [V_\alpha(\mathbf{r}) - \mu_\alpha]$$

$$F = F^{id} + F^{hs} + F^{ch}$$

ideal gas part: $F^{id}[\rho_\alpha(\mathbf{r})] = kT \sum_\alpha \int d\mathbf{r} \rho_\alpha(\mathbf{r}) [\ln \rho_\alpha(\mathbf{r}) - 1]$

hard sphere functional: $F^{hs}[\rho_\alpha(\mathbf{r})] = kT \int d\mathbf{r} \Phi[n_\gamma(\mathbf{r})]$ ("White Bear" FMT, Roth et al., 2002)

chain bonding contribution:

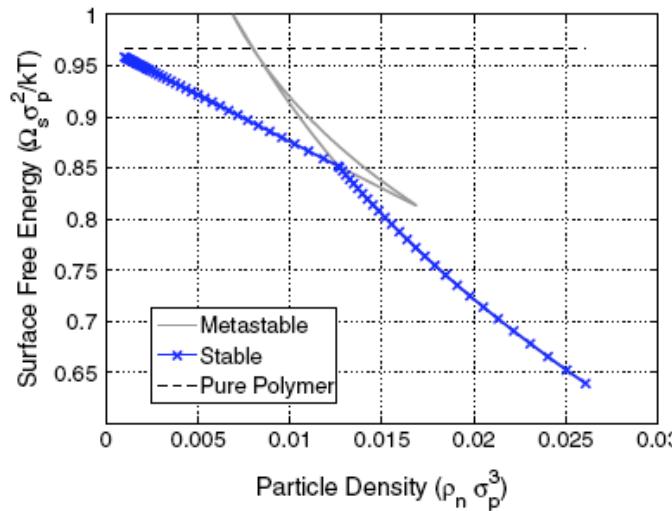
$$F^{ch}[\rho_\alpha(\vec{r})] = kT \int d\vec{r}' \sum_{\alpha=1}^m \rho_\alpha^{\text{seg}}(\vec{r}') \sum_{\alpha'}^{\{\alpha'\}} \left(-\frac{1}{2} \ln \int d\vec{r}'' \frac{\delta(|\vec{r}' - \vec{r}''| - \sigma^{\alpha\alpha'})}{4\pi(\sigma^{\alpha\alpha'})^2} y^{\alpha\alpha'}(\vec{r}', \vec{r}'') \rho_{\alpha'}^{\text{seg}}(\vec{r}'') + \frac{1}{2} \right)$$

minimize free energy

$$\frac{\delta \Omega}{\delta \rho(\mathbf{r})} = 0 \longrightarrow \text{equations to solve for } \langle \rangle(\mathbf{r})$$

S. Tripathi and W.G. Chapman, *Phys. Rev. Lett.* **94**, 087081 (2005); *J. Chem. Phys.* **122**, 094506 (2005); S. Jain et al., *J. Chem. Phys.* **127**, 244904 (2007)

A Layering Phase Transition

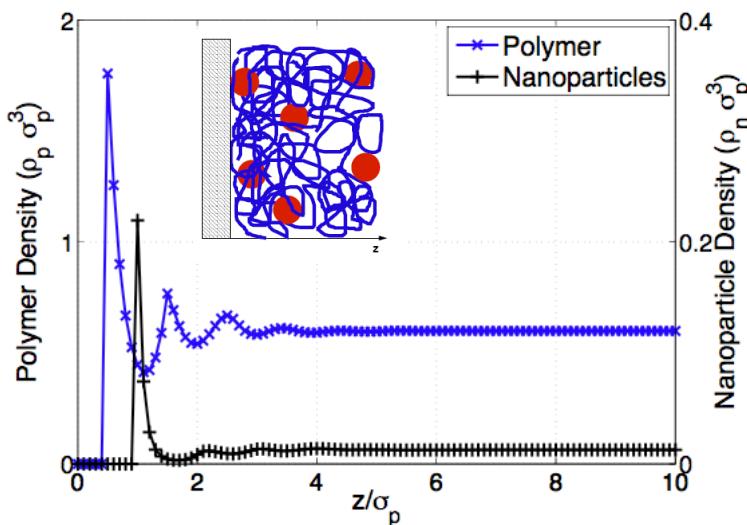


$N = 40, D=2 \int H3\text{nm}$

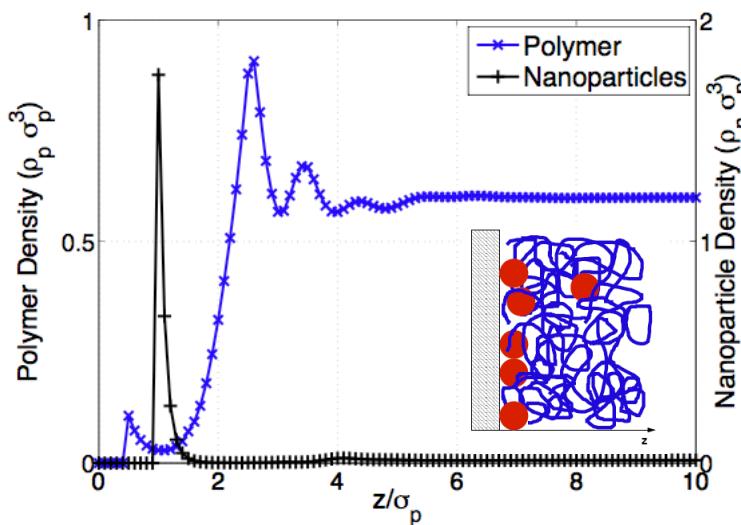
- fixed total packing fraction

$$\rho_p \sigma^3 + \rho_{np} \sigma_{np}^3 = 0.79$$

- first-order transition
- entropy-driven



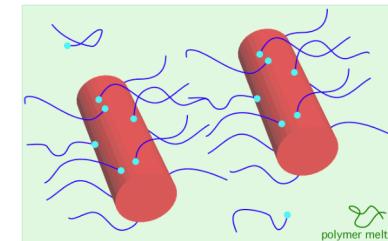
typical fluid structure



NP layer, areal coverage 0.82

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Polymer-Grafted Nanorods

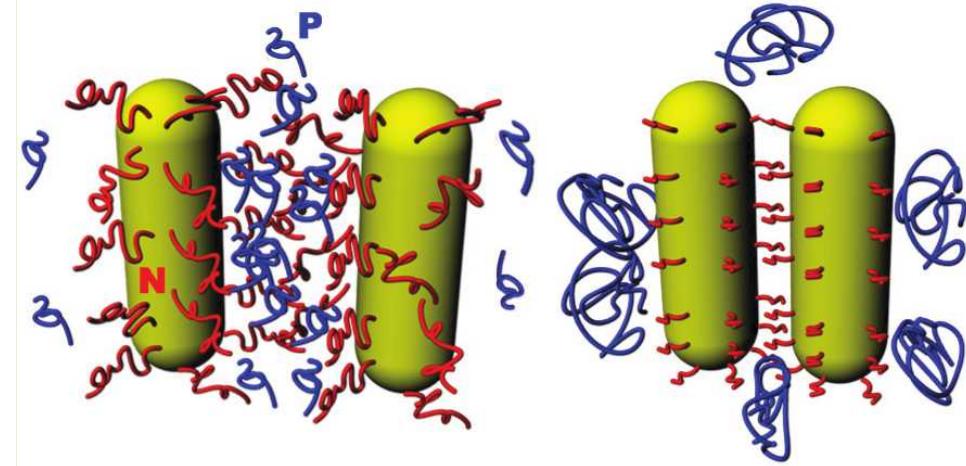
What controls nanorod dispersion/aggregation?

- gold nanorods
- polymer brush coating
- 5% rods in polymer thin film
 - rods confined in the plane of the film

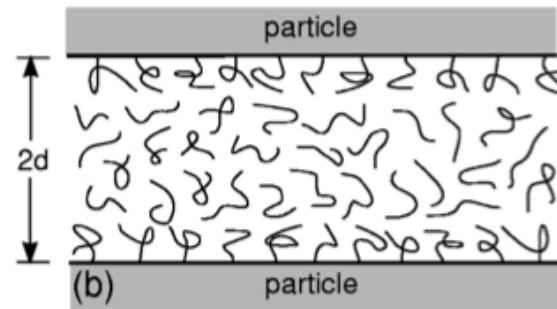
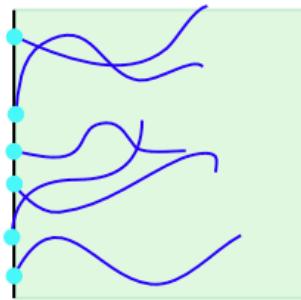
athermal systems:

PS brush in PS

PEO brush in PEO

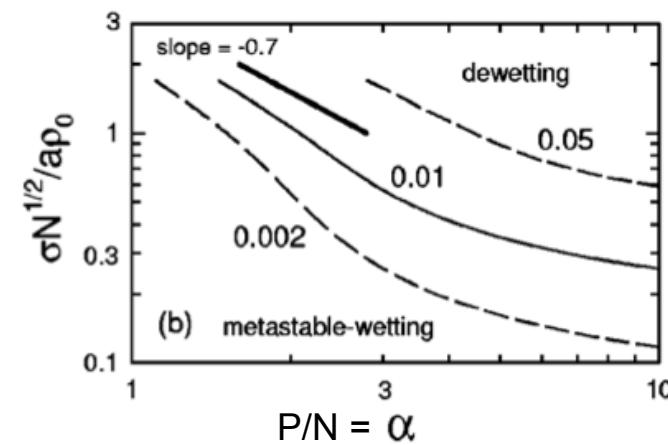
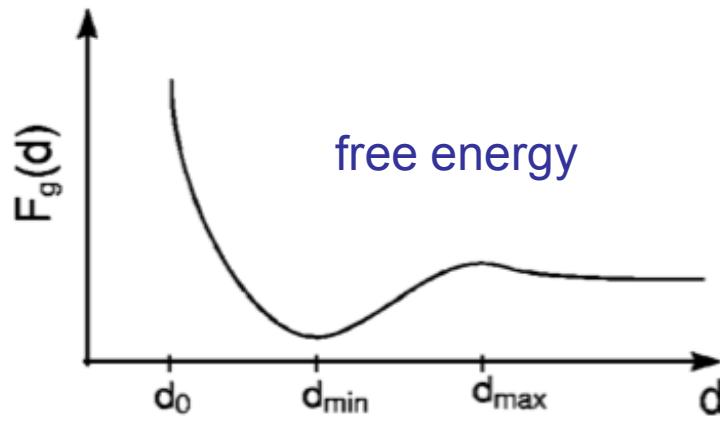


Brush-Brush Interactions

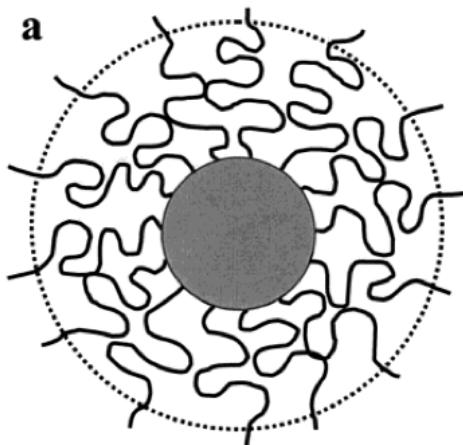


autophobic dewetting:

- entropy cost for matrix chains to enter brush
- leads to positive brush-matrix surface tension
- brushes are attracted



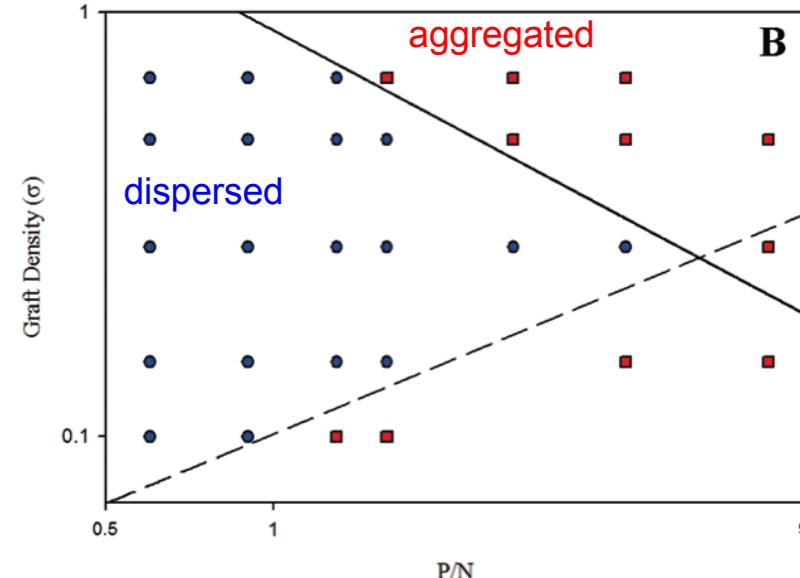
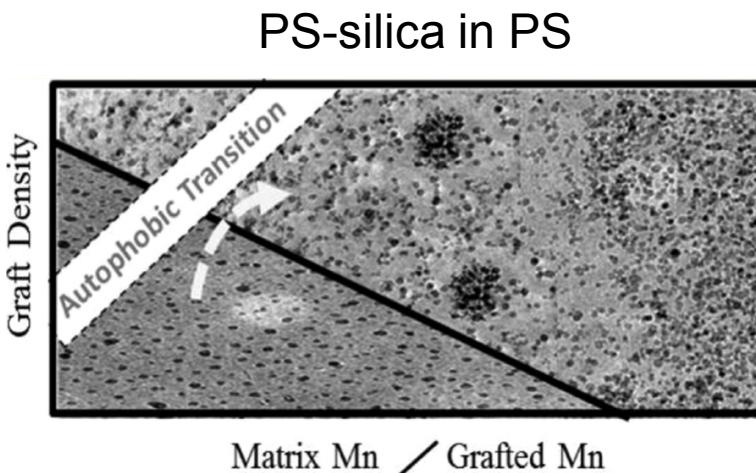
Brushes on Curved Surfaces



- more volume than on flat surface at same grafting density
- chains splay out more
- brush is less extended

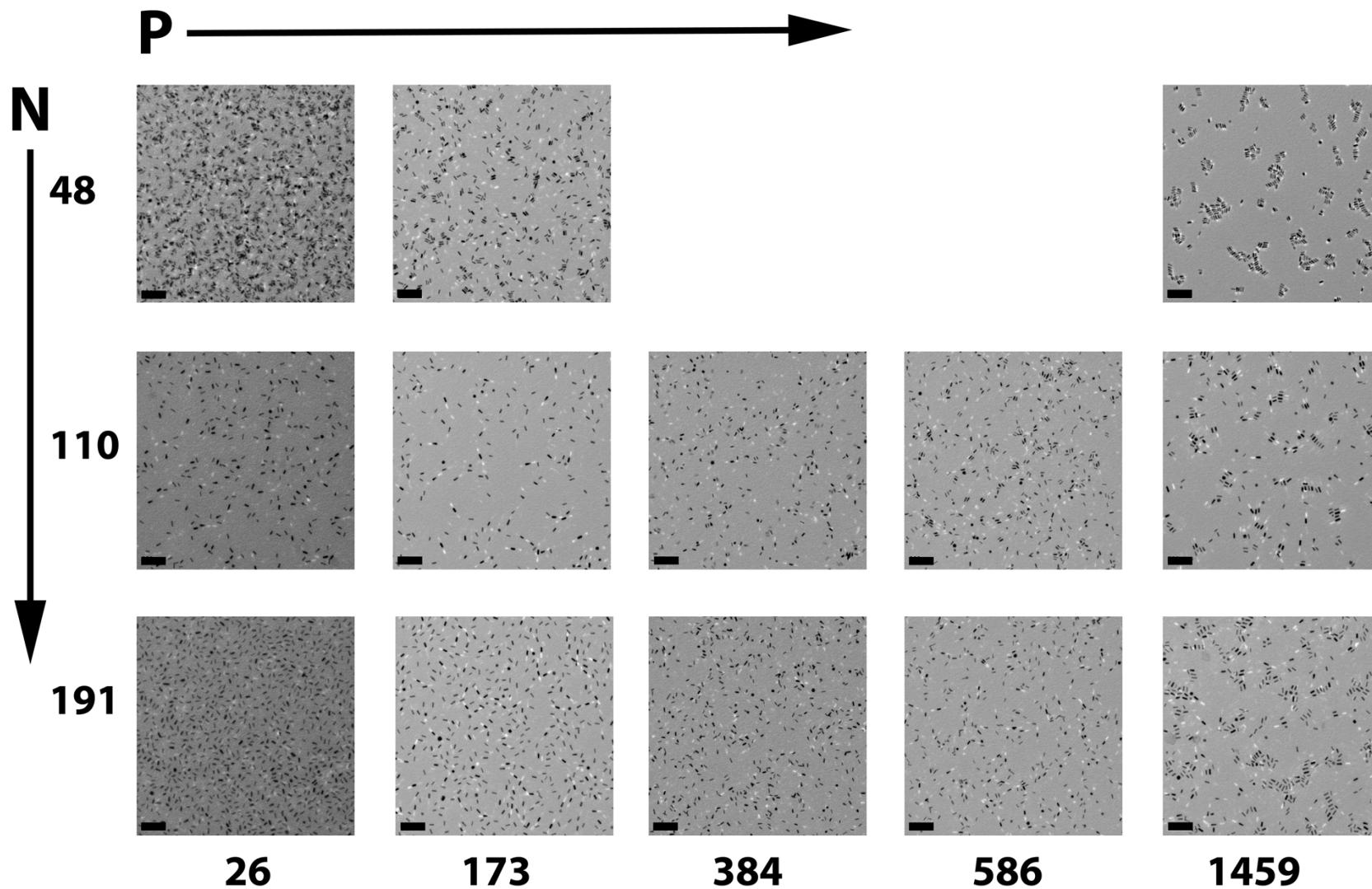
expect wet to dry transition to occur for

- larger grafting densities
- larger P/N

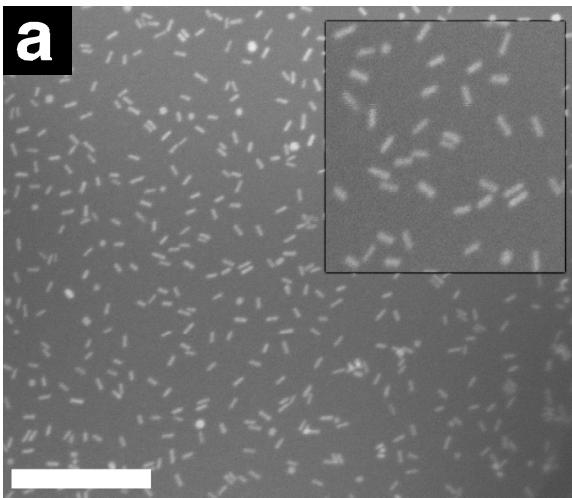


Sunday, D., Ilavsky, J., & Green, D. L. (2012).
Macromolecules, 45, 4007–4011.

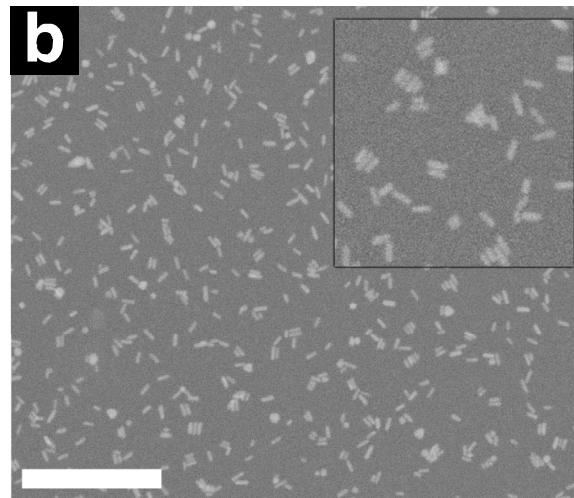
Nanorods: PS-Au(*N*):PS(*P*)



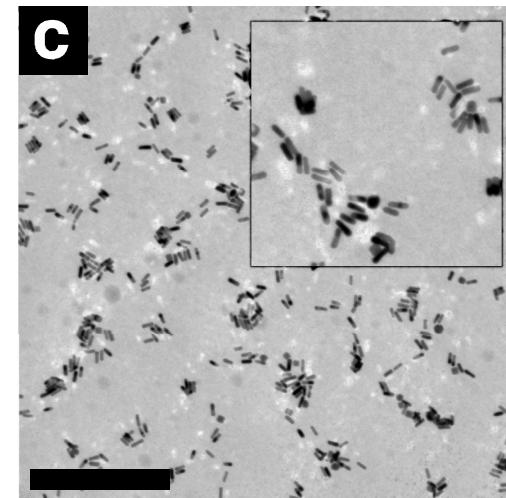
PEO-Au(N):PEO(P)



P/N = 0.43

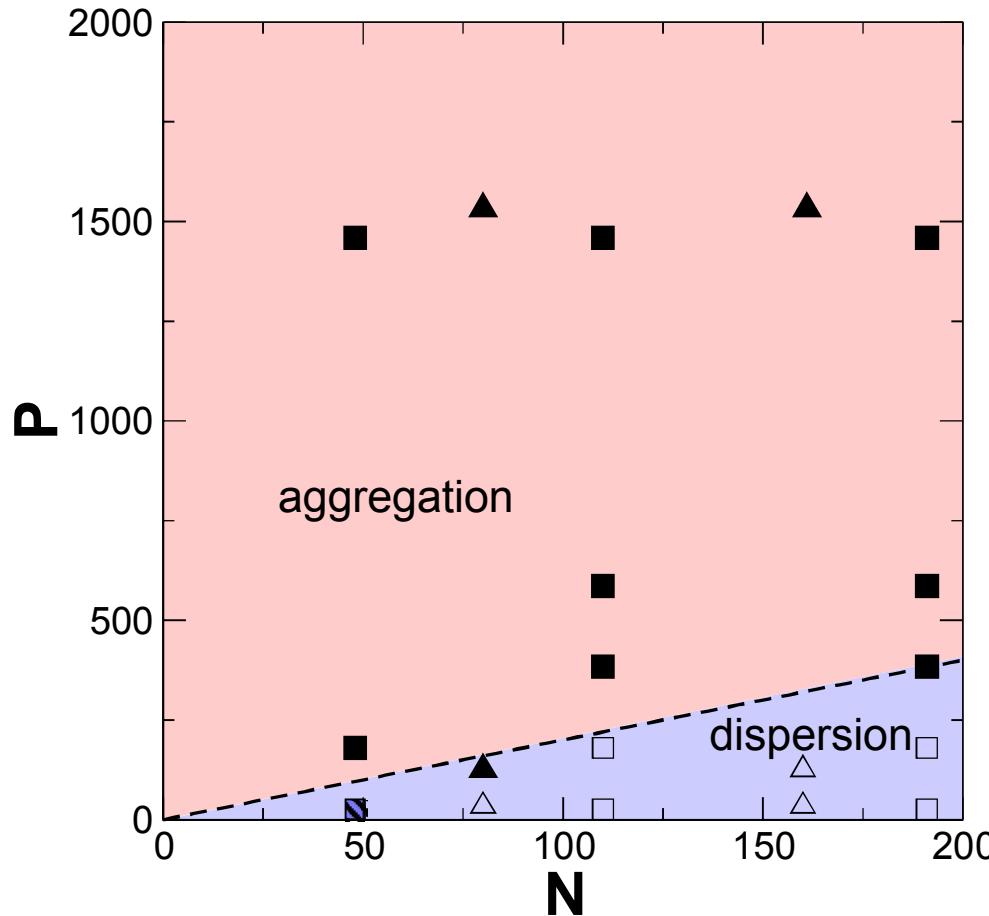


P/N = 1.58



P/N = 19.2

Dispersion “Map” for NRs



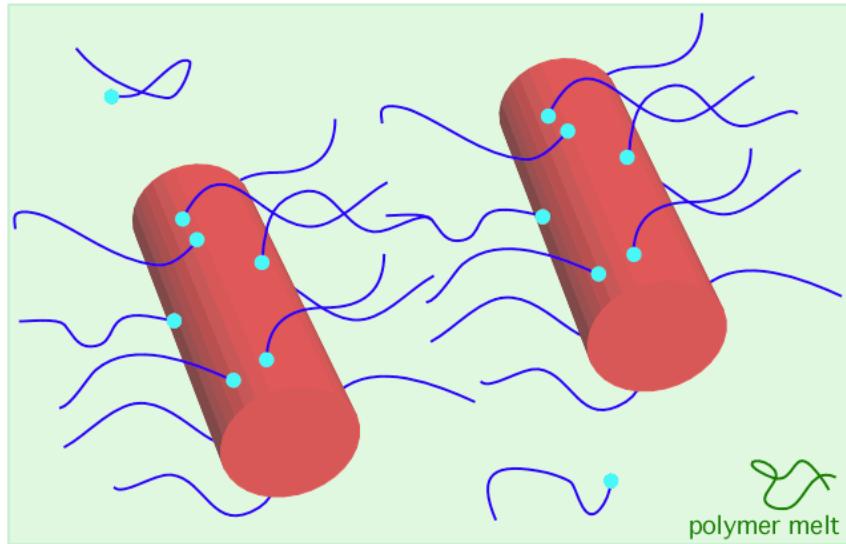
dispersion for $P < 2N$

ignores possible effects of:

- rod curvature
- grafting density
- rod length

squares: PS
triangles: PEO

Modeling



classical DFT and SCFT

- brush chains length N
- matrix chains length P
- athermal ($\chi = 0$)
- nanorods with radius R_{rod}
- grafting density σ

note nanorods exclude chains from interior
are not part of the fluid in the theory

goal: calculate polymer-mediated interaction free energy

variables: $\alpha = P/N$

$$R_{rod}/R_g$$

$$\sigma^* \equiv \frac{\sqrt{6}\sigma N^{1/2}}{a\rho_0}$$

experimental ranges:

$$\alpha = P/N = 0.15 - 30$$

$$R_{rod}/R_g = 1.5-3.2$$

$$\sigma^* = 0.95-2.38$$

earlier work: Frischknecht, J. Chem. Phys., **128**, 224902 (2008)

DFT approach: CMS-DFT

- chains are flexible
- 2nd order density expansion

Chandler, McCoy, Singer (1986);
McCoy et al. (1990s)

$$\rho_\alpha(r) = \frac{\rho_\alpha^b}{N_\alpha} \sum_{s=1}^{N_\alpha} \frac{G_s(r)G_s^i(r)}{e^{-\beta U_\alpha(r)}}$$

Chain density distribution

$$U_\alpha(r) = V_{ext}(r) - \sum_\gamma \int c_{\alpha\gamma}(r - r') [\rho_\gamma(r') - \rho_\gamma^b] dr'$$

$$c(r) = c_{rep}(r) - u_{att}(r)$$

Unknown field
PRISM
RPM

Theory
Approx

$$G_s(r) = e^{-\beta U_{\alpha,s}} \int w(r - r') G_{s-1}(r') dr'$$

Chain Architecture
(freely-jointed chains)

$$G_s^i(r) = e^{-\beta U_{\alpha,s}} \int w(r - r') G_{s+1}^i(r') dr'$$

$$G_1 = G_N^i = e^{-\beta U(r)} \quad w(r) = \frac{1}{4\pi\sigma^2} \delta(|r| - \sigma)$$

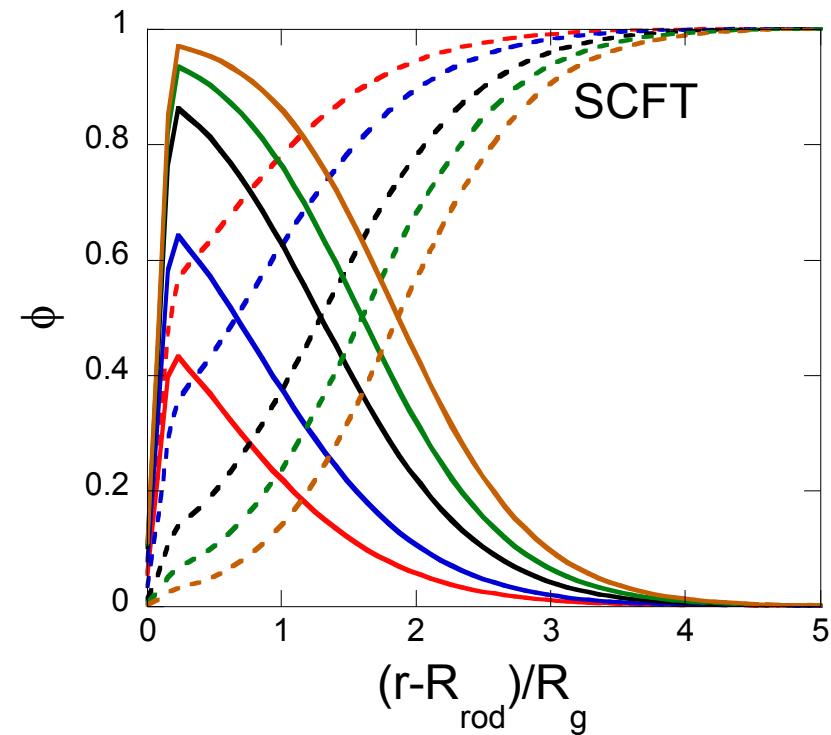
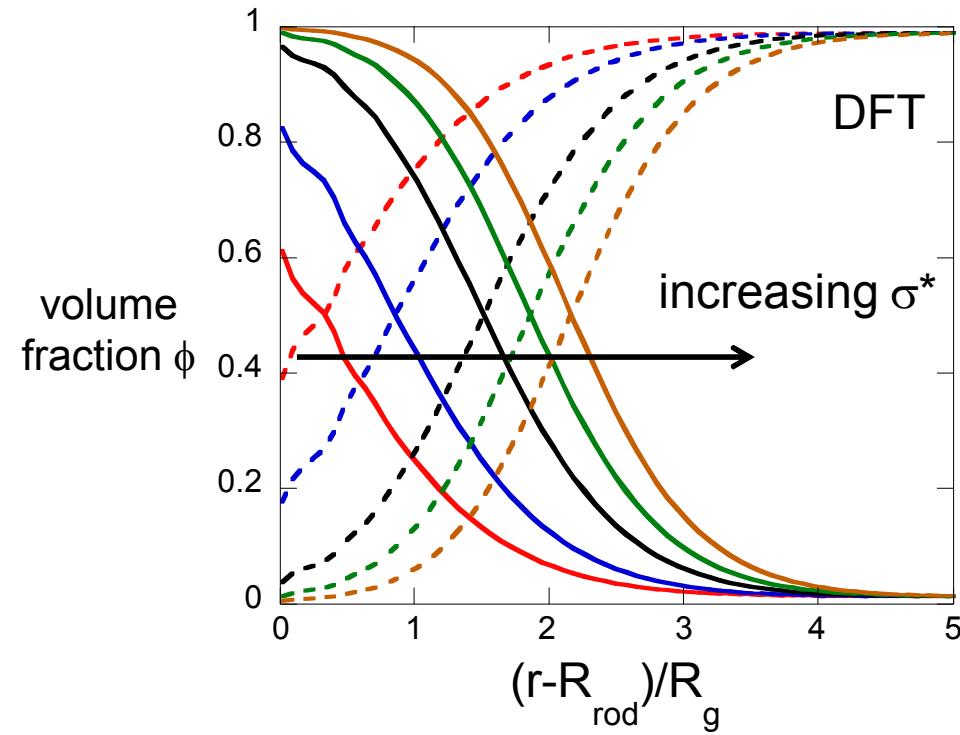
free energy

$$\Delta\Omega = -\frac{1}{N_L} \sum_\alpha \int d\mathbf{r} (\rho_\alpha(\mathbf{r}) - \rho_{b,\alpha}) +$$

$$\frac{1}{2} \sum_{\alpha\beta} \int \int d\mathbf{r} d\mathbf{r}' c_{\alpha\beta}(\mathbf{r} - \mathbf{r}') [\rho_\alpha(\mathbf{r})\rho_\beta(\mathbf{r}') - \rho_{b,\alpha}\rho_{b,\beta}]$$

Single Nanorod: Brush Profiles

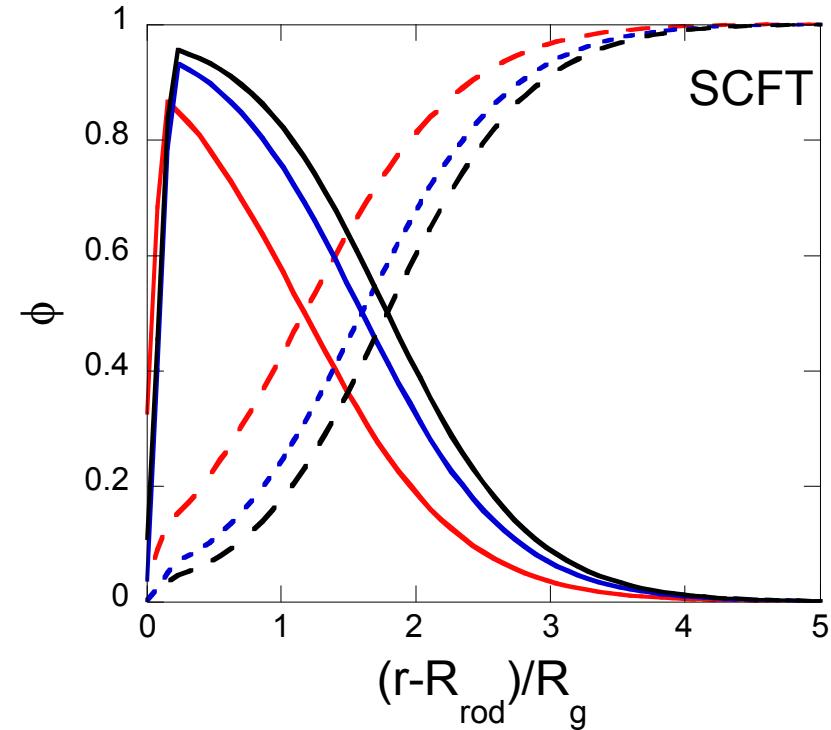
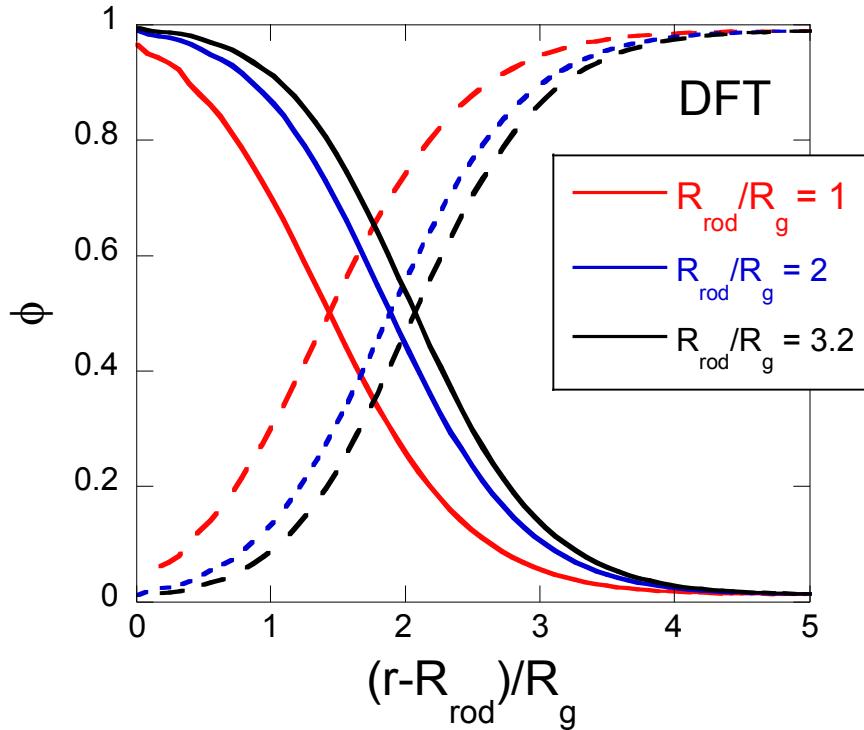
$$R_{\text{rod}}/R_g = 3.2, P/N = \alpha = 3$$



grafting density important

Single Nanorod: Brush Profiles

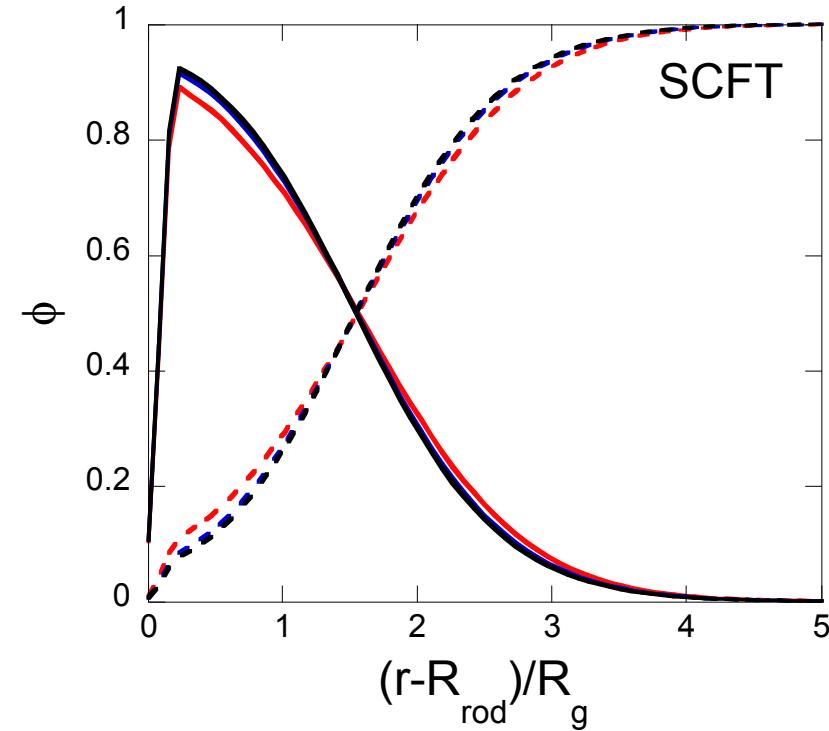
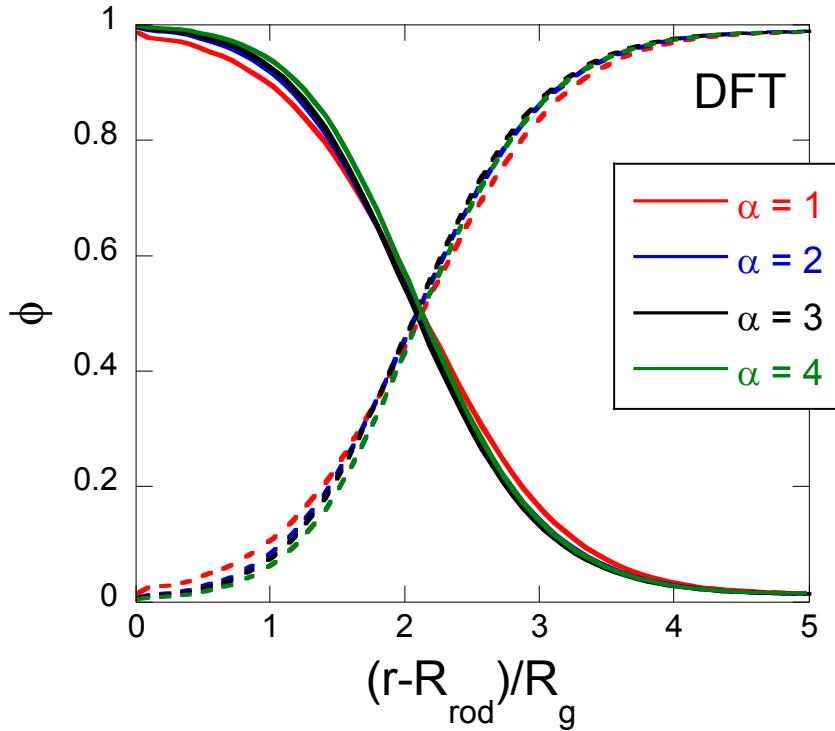
$$\alpha = 2, \sigma^* = 2.38$$



nanorod curvature important

Single Nanorod: Brush Profiles

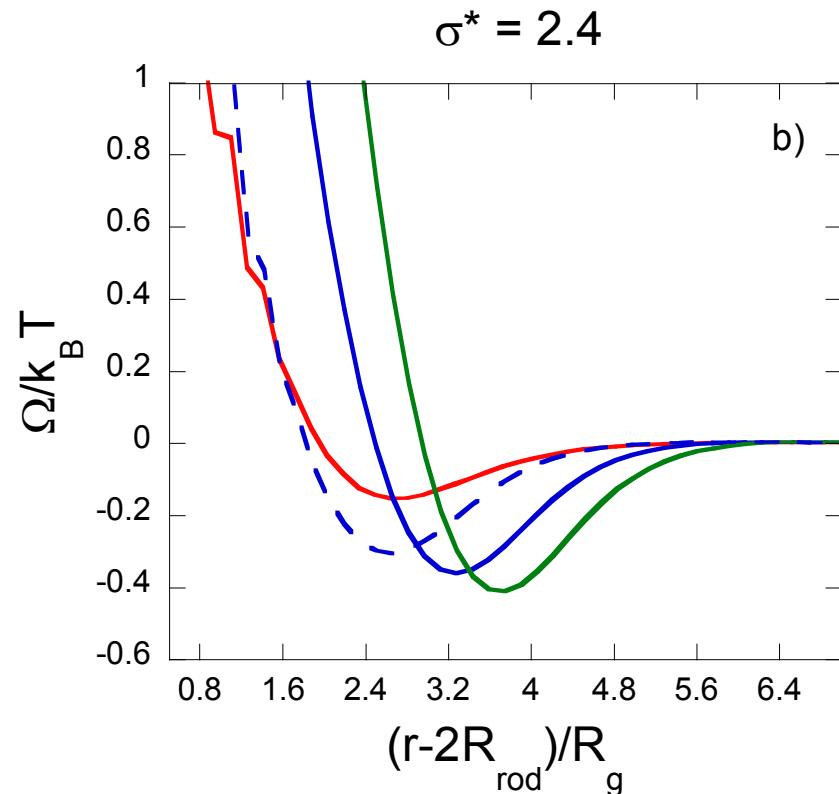
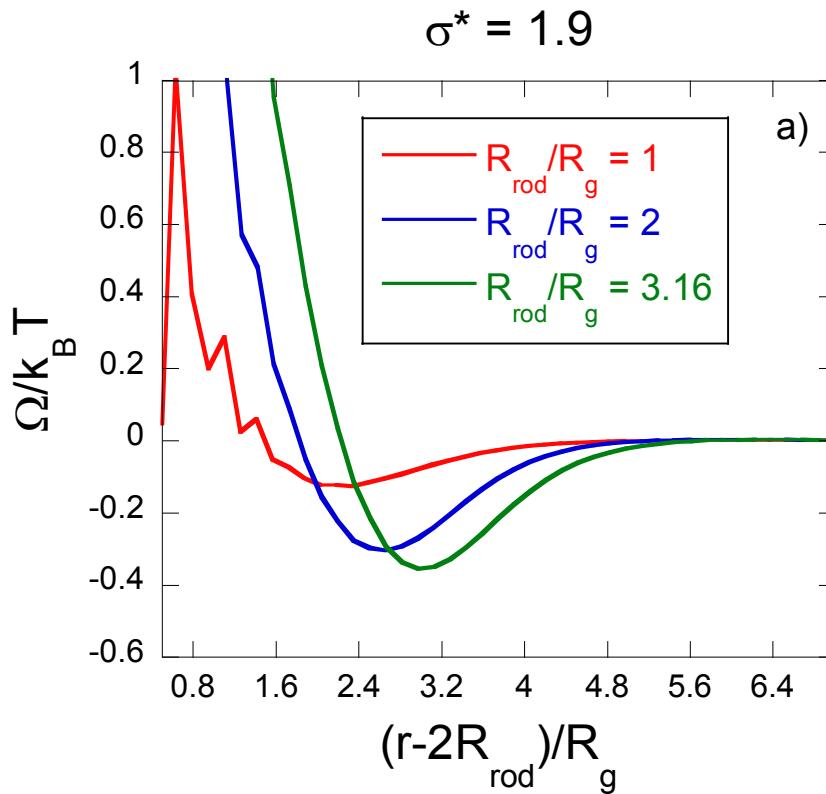
$$R_{\text{rod}}/R_g = 3.2, \sigma^* = 2.38$$



matrix chain length doesn't affect brush profiles much

Rod-rod interaction energy

P/N = α = 3

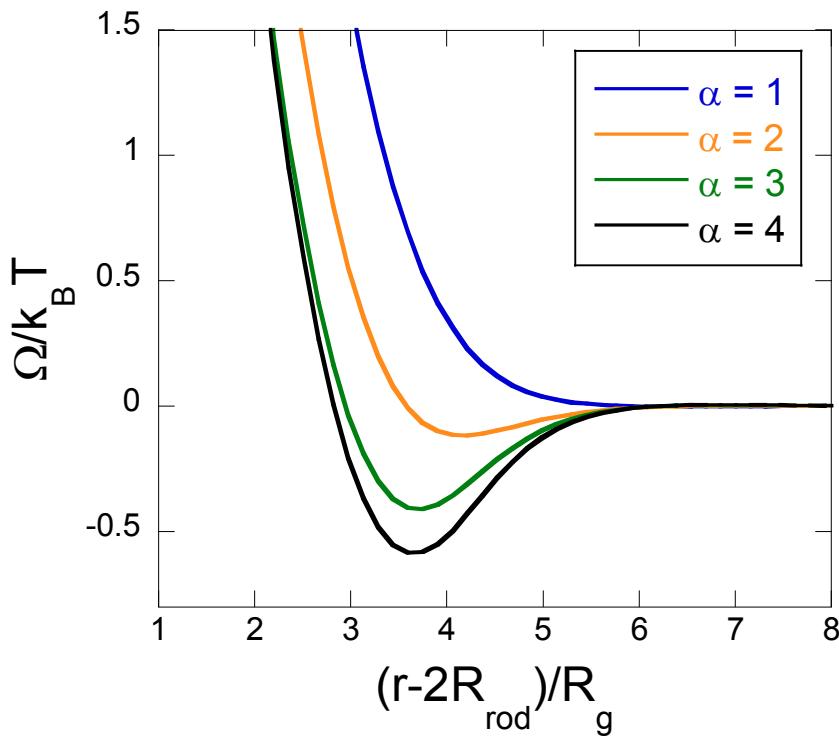


deeper attractive well as:
grafting density increases
 R_{rod} increases

Rod-rod interaction energy

$$R_{\text{rod}}/R_g = 3.2, \sigma^* = 2.4$$

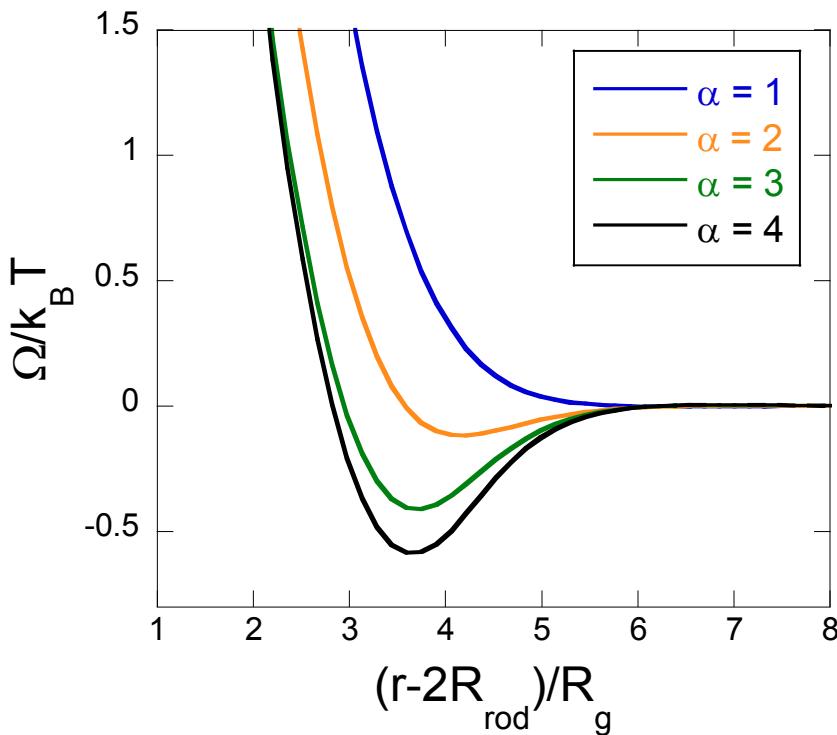
matrix chain length crucial!



Rod-rod interaction energy

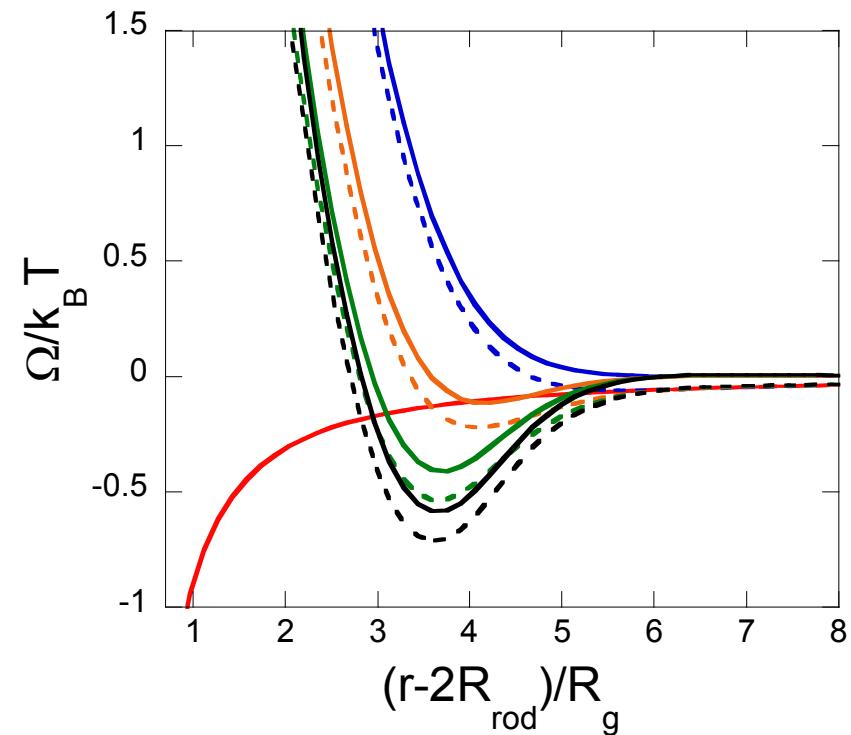
$$R_{\text{rod}}/R_g = 3.2, \sigma^* = 2.4$$

matrix chain length crucial!



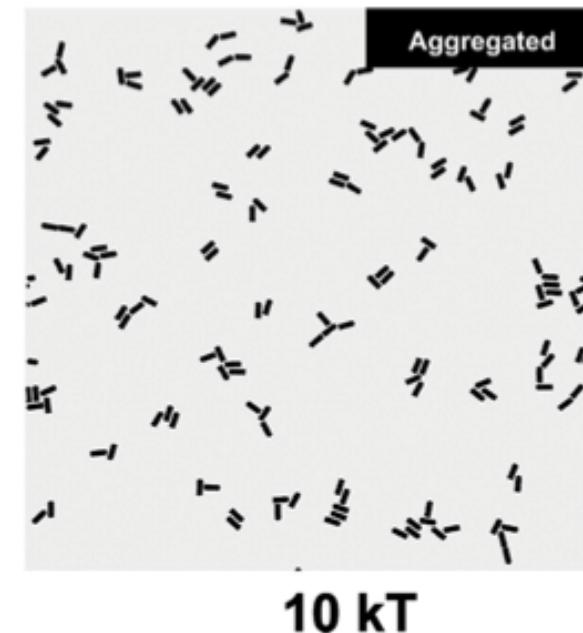
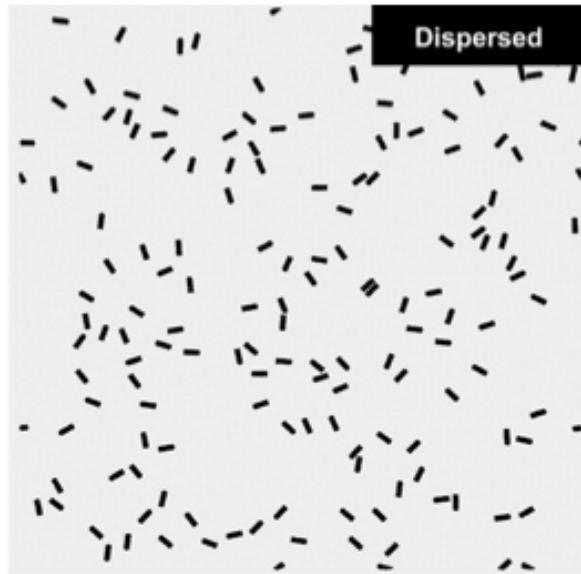
include van der Waals
between Au rods:

$$W = -\frac{ALR_{\text{rod}}^{1/2}}{24H^{3/2}}$$



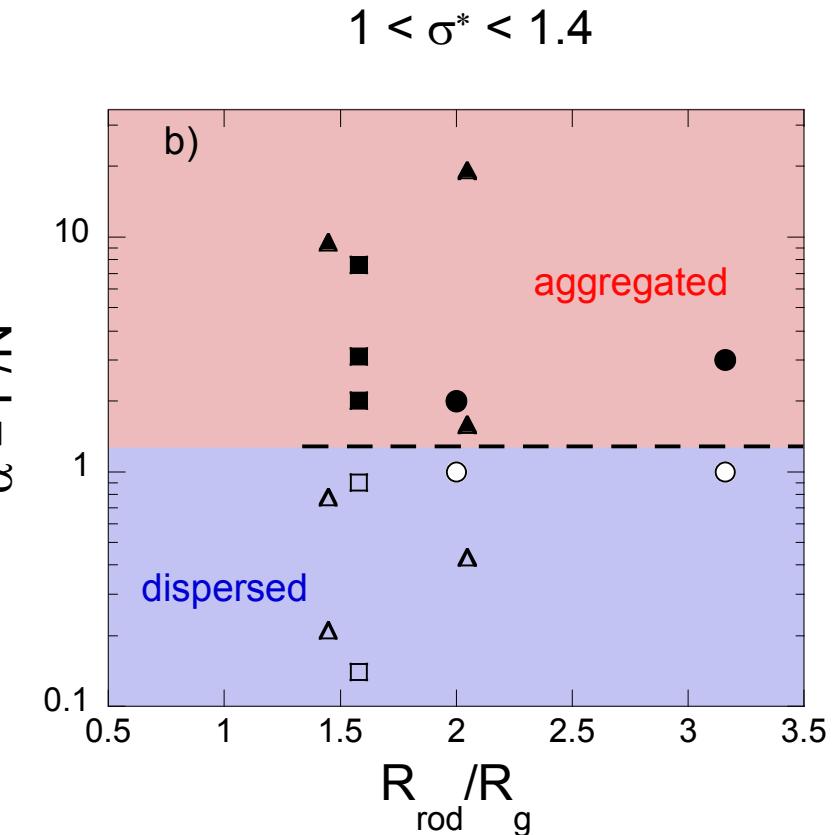
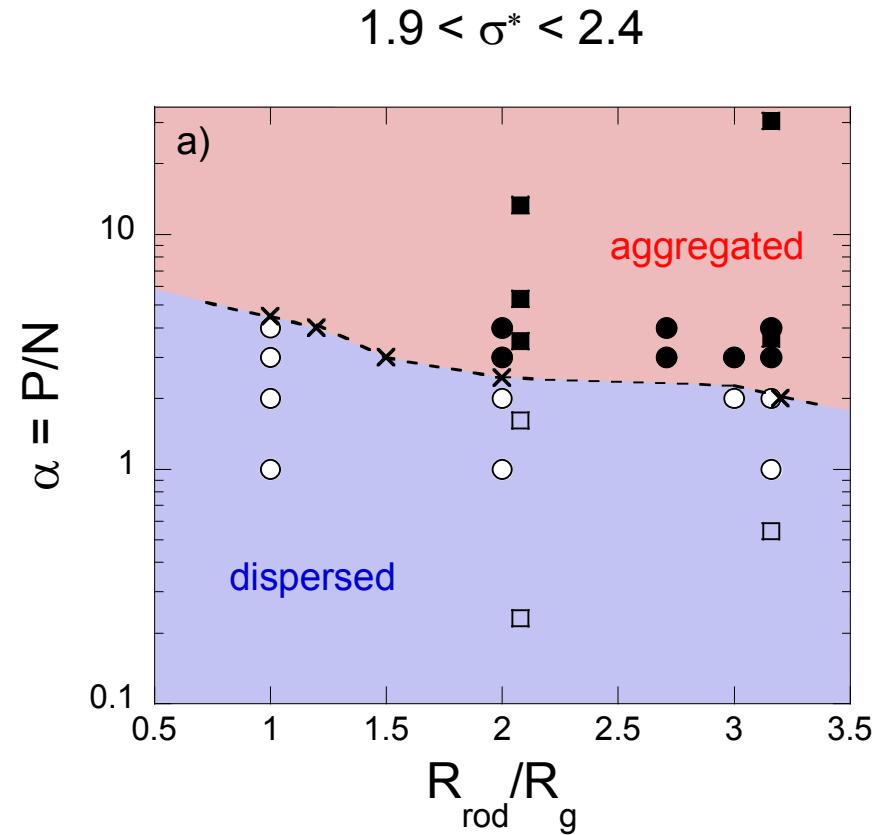
Criterion for Aggregation?

Monte Carlo simulations
square-well potential



assume aggregation for $E_{\text{tot}} > 5 \text{ kT}$

Comparison with experiment

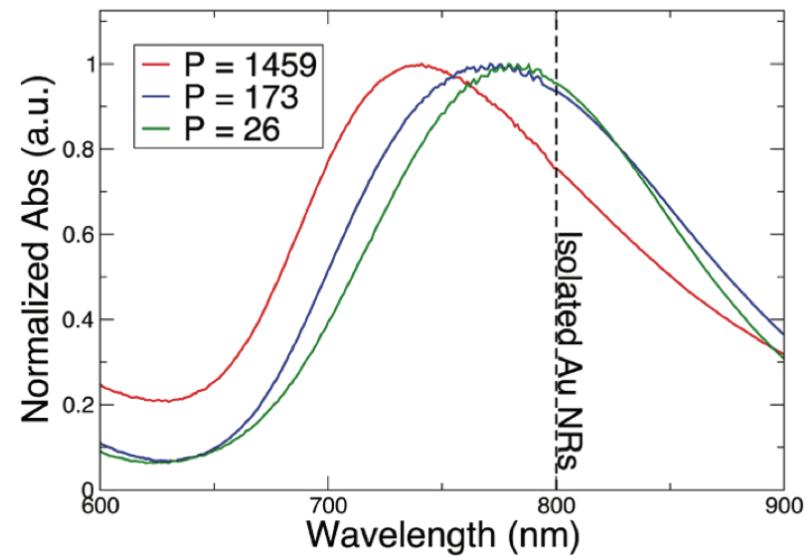
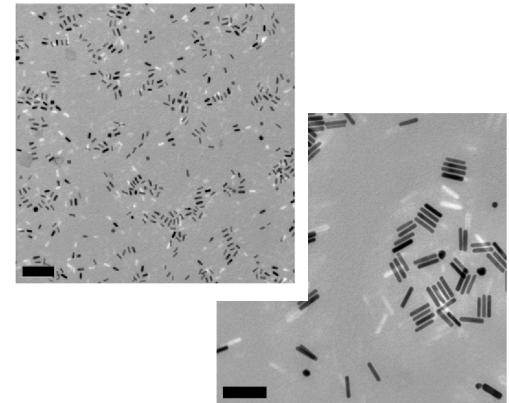


circles = DFT
squares = PS/PS
triangles = PEO/PEO

good overall agreement

Conclusions

- brush profiles remarkably insensitive to α
- interaction energy sensitive to σ^* , R_{rod} , and α
- DFT captures correct trends
 - transition to aggregation near $P/N = 2$
- design: promote dispersion for
 - small σ^* , R_{rod} , or α
 - can potentially control rod spacing in aggregates
 - to control optical properties



Acknowledgments



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Michael Mackay



University of Delaware



Su-Mi Hur



Glenn Fredrickson



Dale Huber

UC Santa Barbara

Funding

- CINT User Program
- LDRD



CINT

The Center for Integrated Nanotechnologies

Nanomaterials

Integration

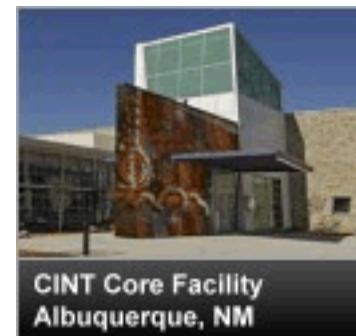
Home

*A US DOE Office of Science User Facility
and Nanoscale Science Research Center*

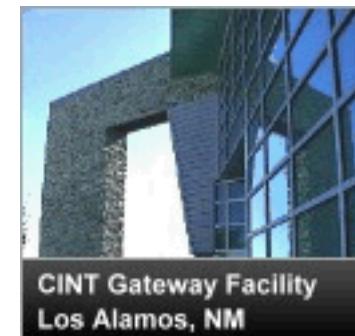
<http://cint.lanl.gov>



- Focus on nanoscale integration
- User facility with a diverse portfolio of customers (academia, labs, industry)

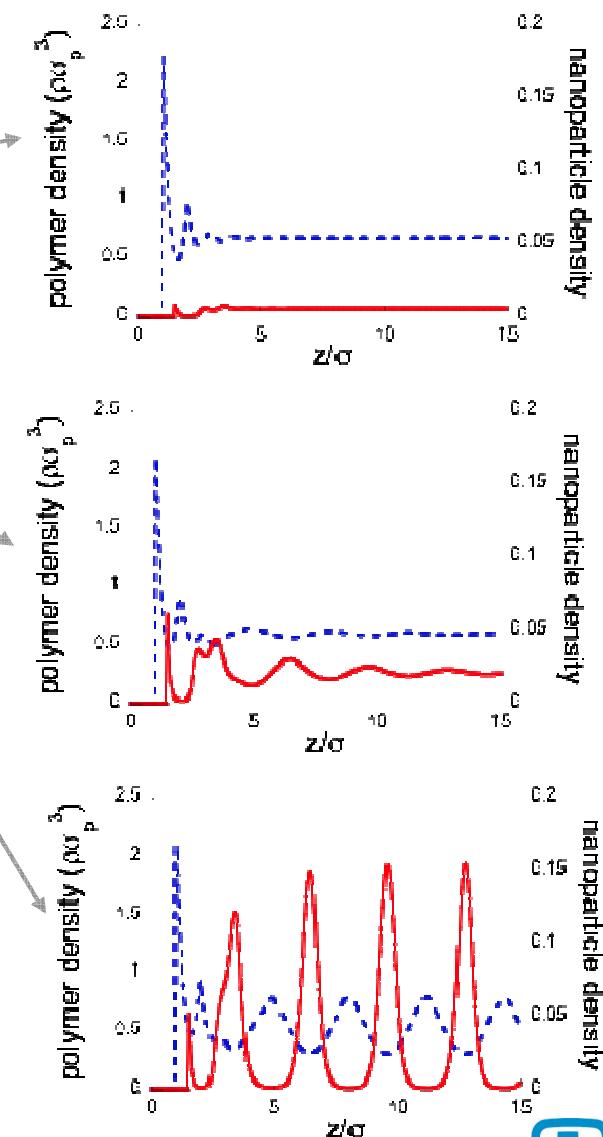
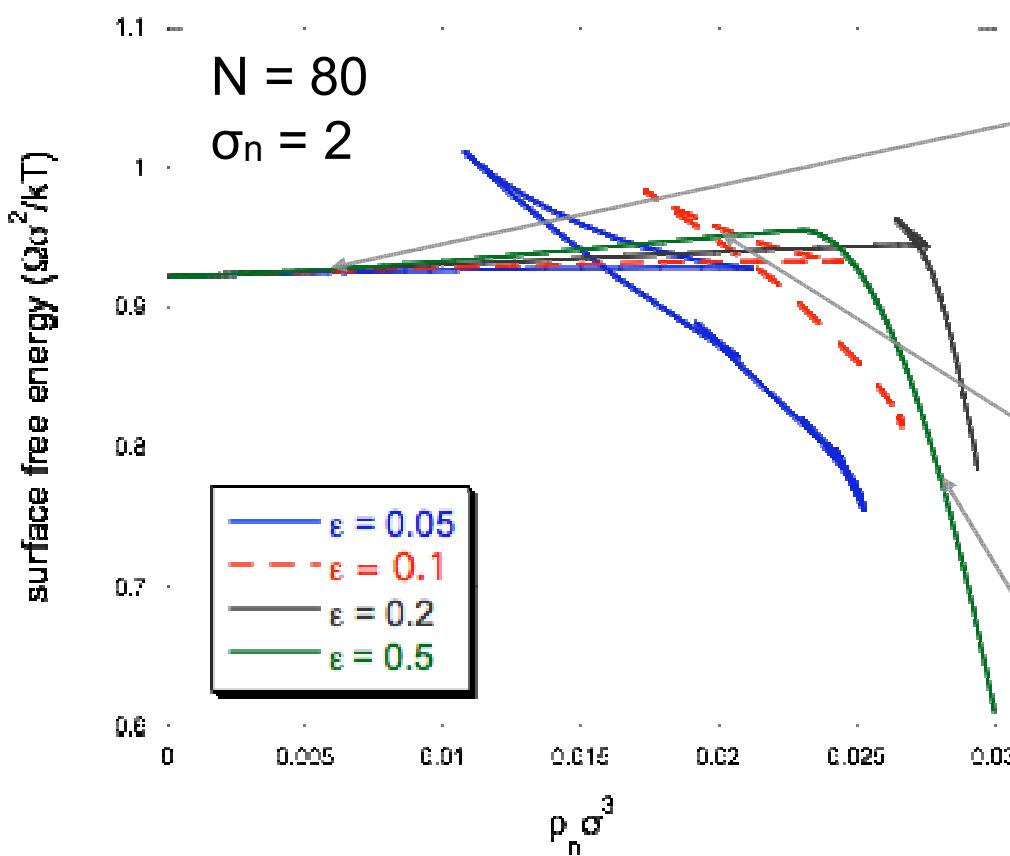


CINT Core Facility
Albuquerque, NM



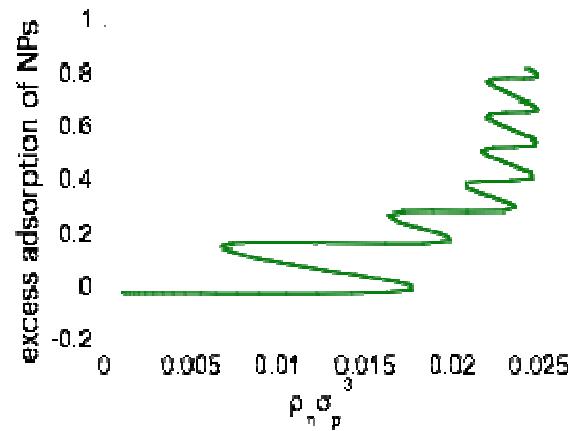
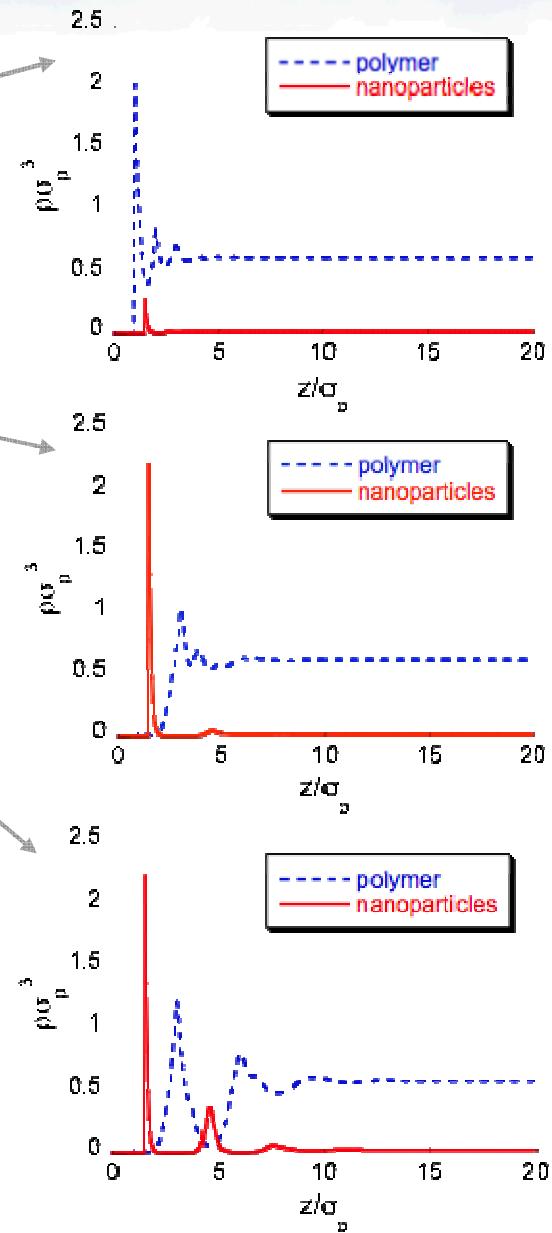
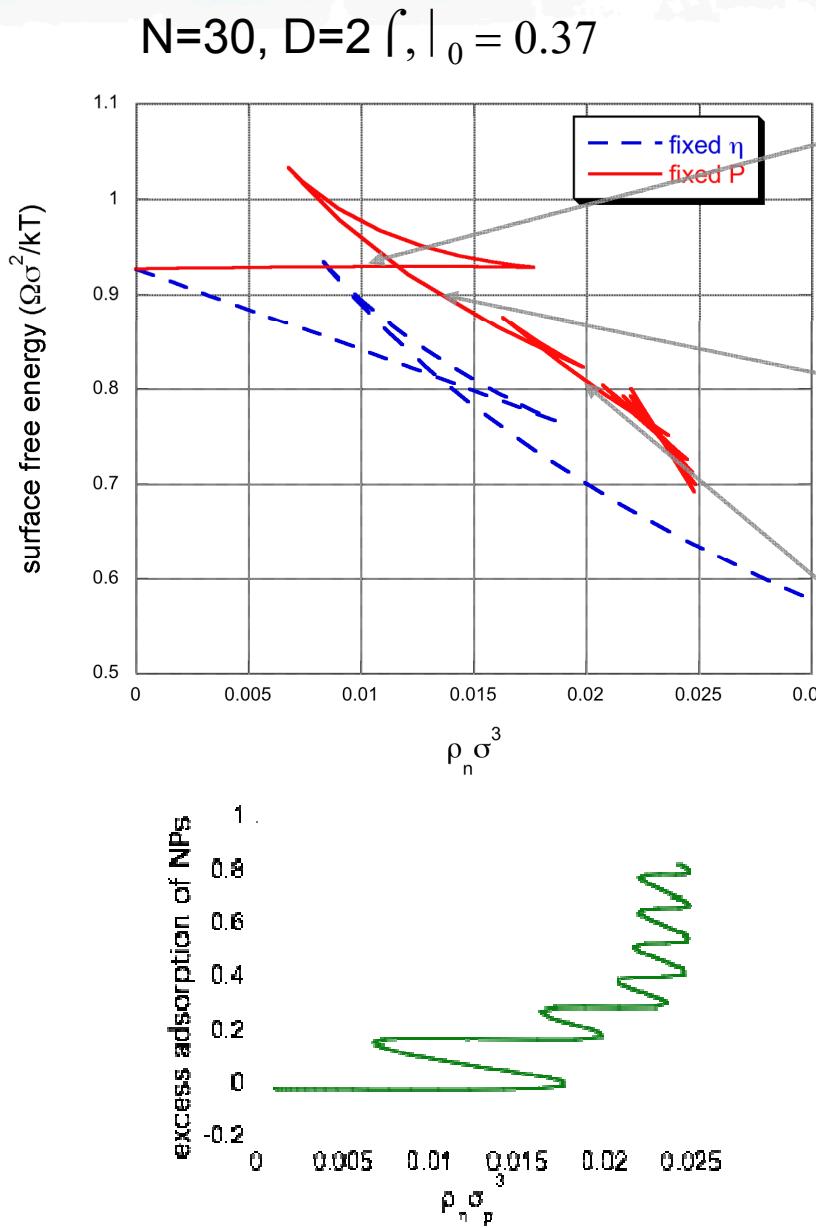
CINT Gateway Facility
Los Alamos, NM

Effect of Attraction Strength



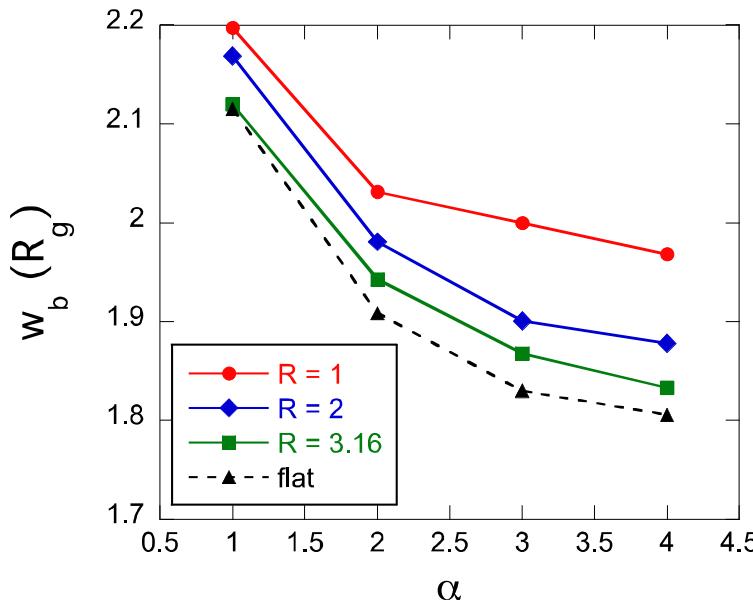
- van der Waals loop vanishes at high ϵ
- transition becomes continuous

Constant pressure: still a transition

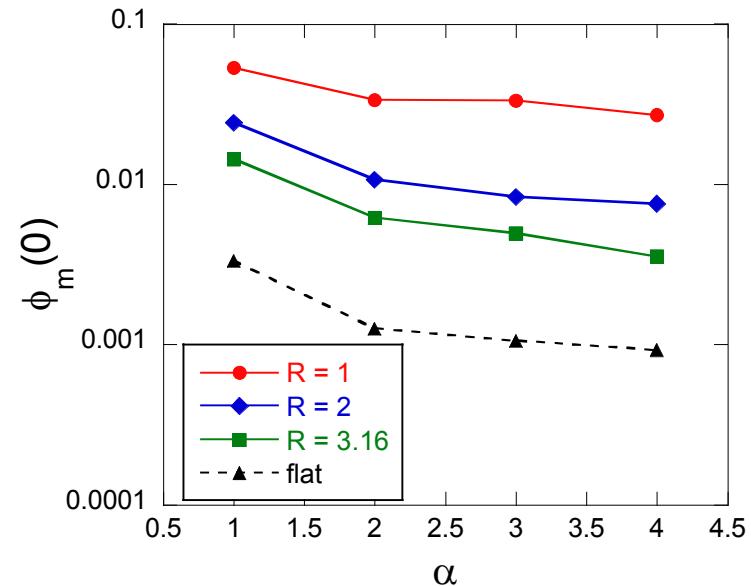


Brush Characteristics vs α

brush width



volume fraction of matrix chains
at NR surface



- brush height insensitive to α
- details of interface matter for interactions

Brush Characteristics vs σ^*

height h_b
(nearly linear)

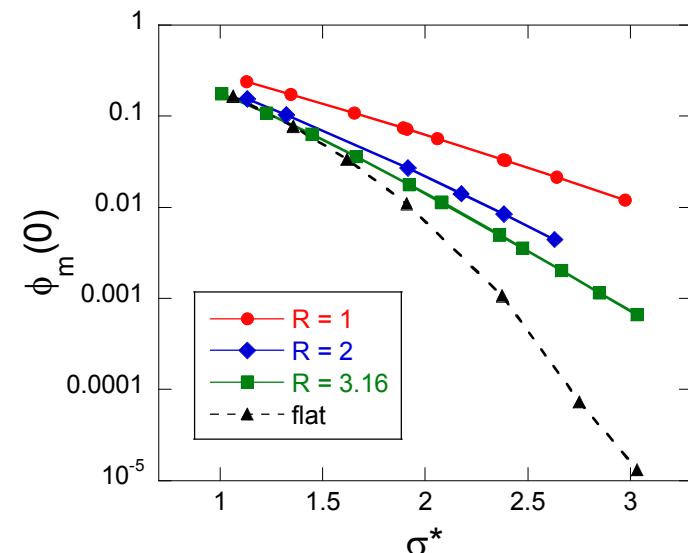
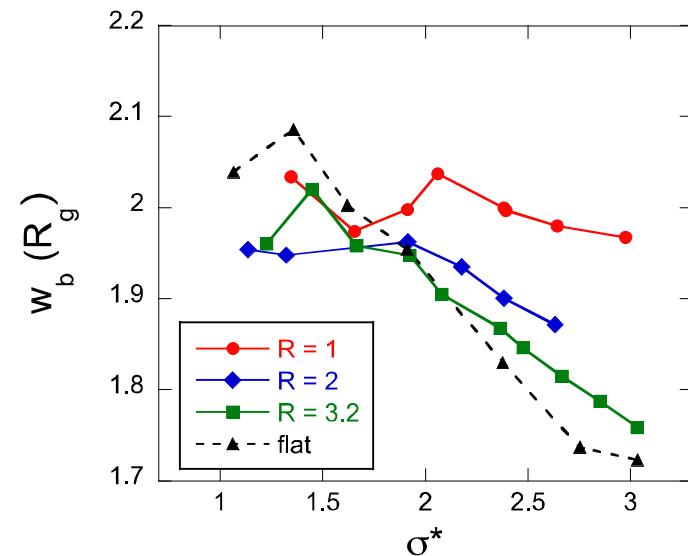
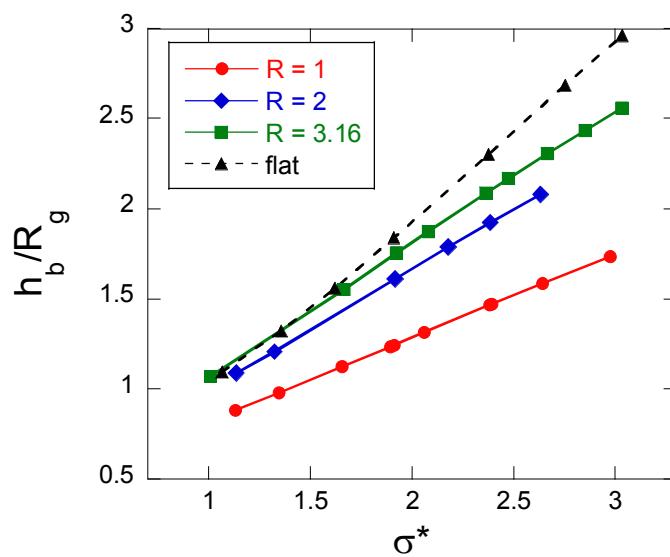
$$\phi_b(h_b) = \frac{1}{2}\phi_b(0)$$

width w_b

$$w_b = \frac{\phi_b(0)}{|\phi'_b(h)|}$$

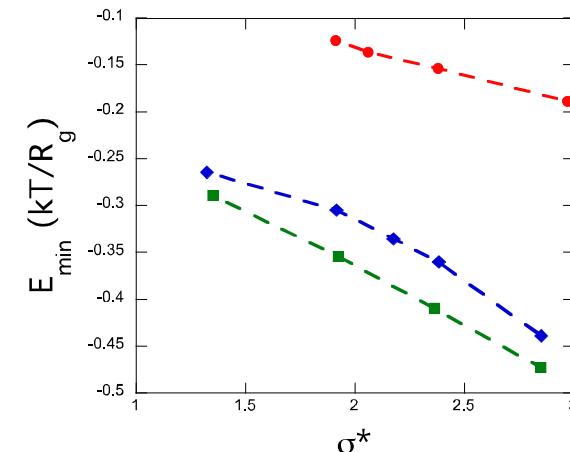
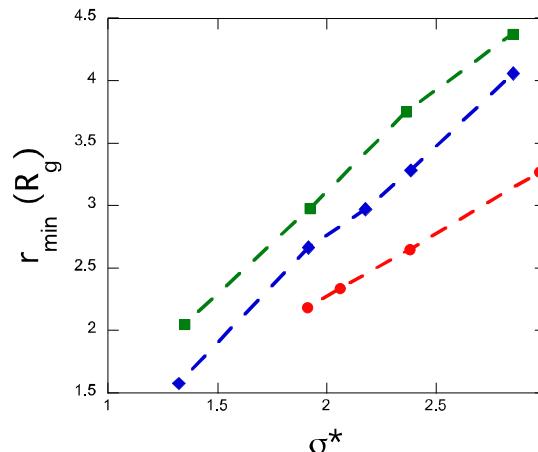
matrix fraction
at rod surface

$$\phi_m(0)$$

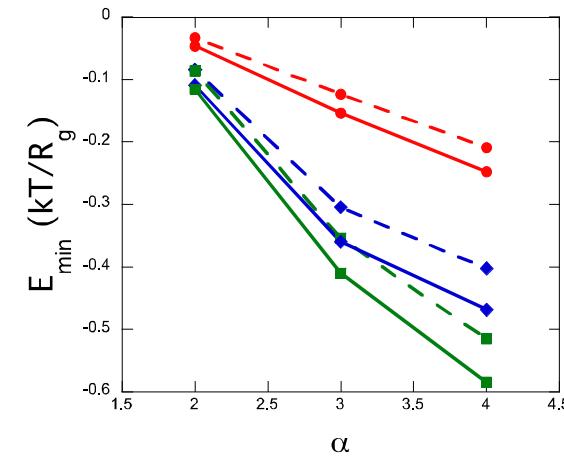
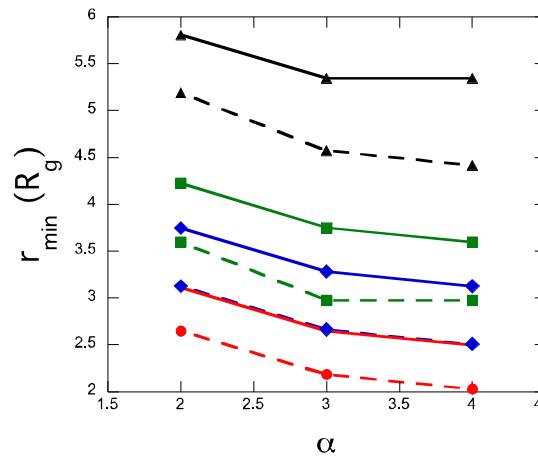


Interactions vs σ^* and α

grafting density

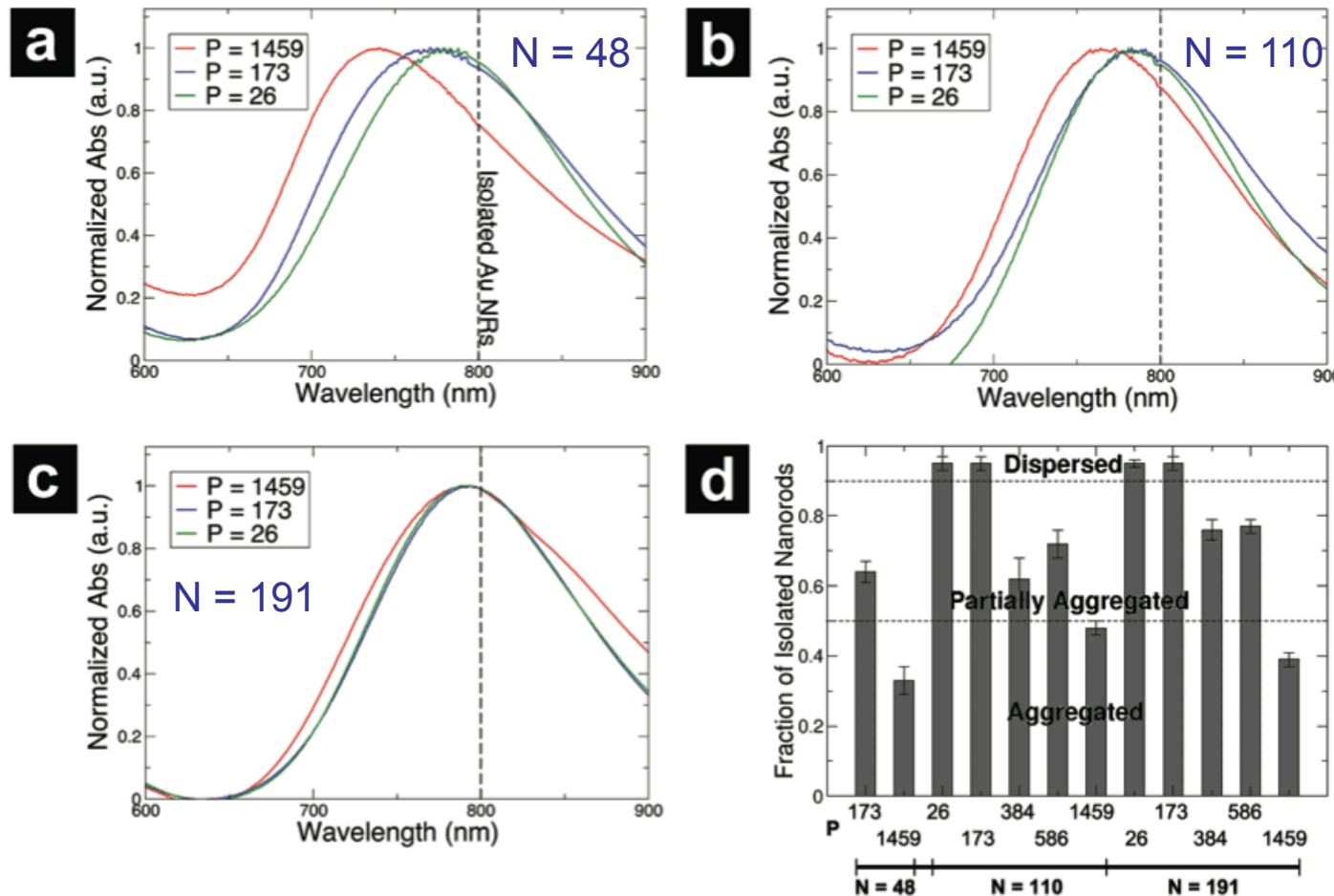


chain ratio



- depth of well decreases with both σ^* , α
- r_{\min} increases with increasing σ^* , decreases with increasing α

Surface Plasmon Resonance



Fluids-DFT Implementation

solve nonlinear integral eqtns

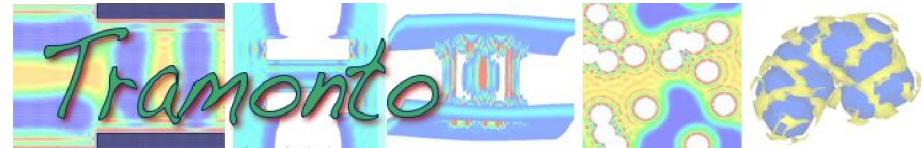
- in 3D, Cartesian grid
- modified Newton solver
- parallel

• **inputs**

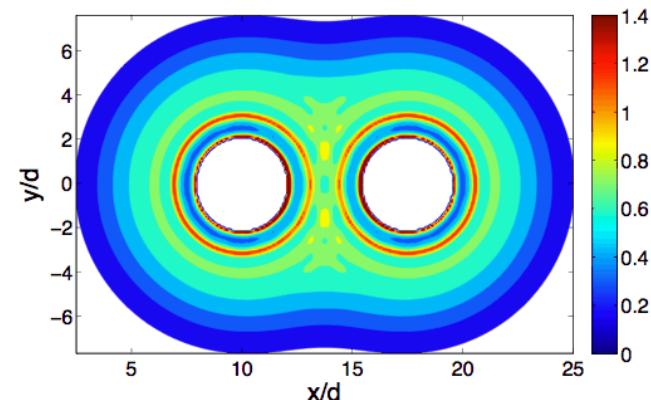
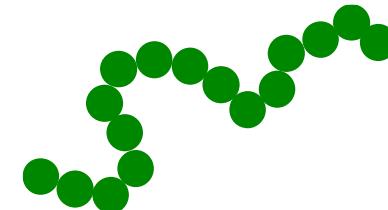
- model of the fluid
 - freely-jointed tangent chains
 - repulsive LJ interactions
- bulk fluid densities (chemical potentials)
- surface geometry
 - NR exclude polymer
 - sticky ends attracted with LJ energy

• **outputs**

- fluid density profiles
- equilibrium free energy
 - phase diagrams
- adsorption, stress profiles, ...



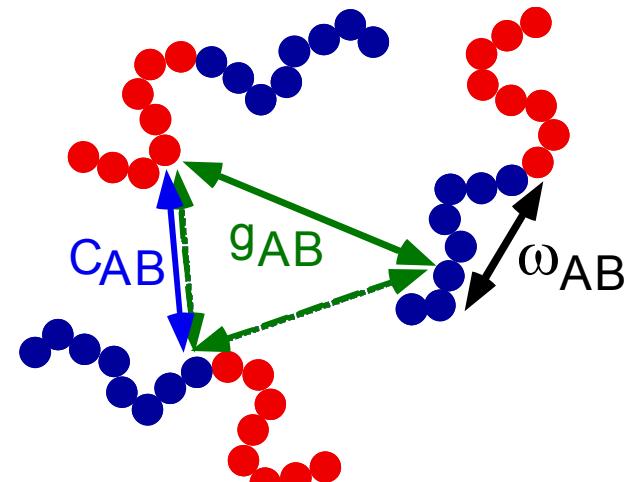
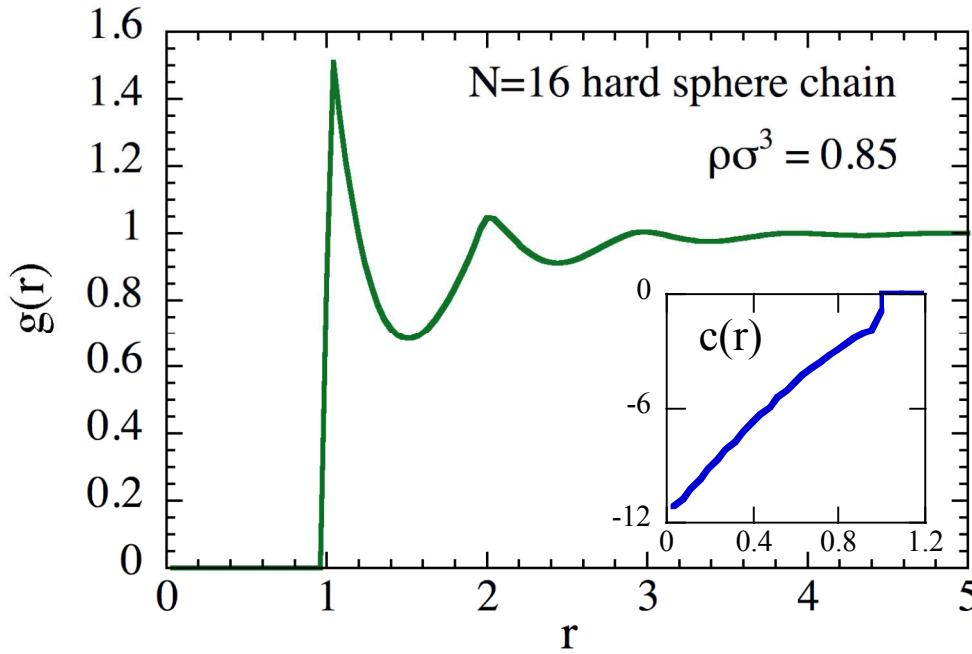
<http://software.sandia.gov/tramonto>



Input to CMS-DFT: PRISM Theory

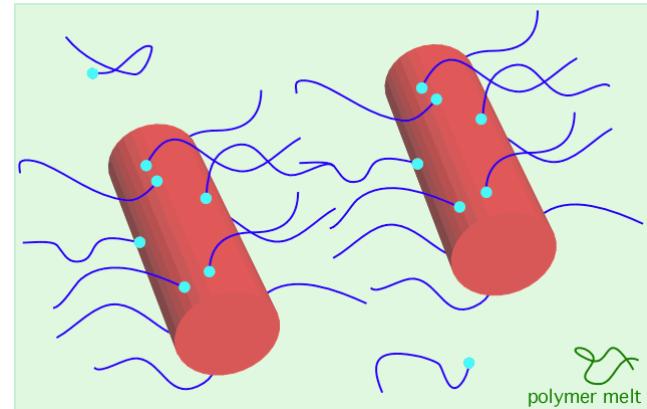
Curro and Schweizer

- Liquid state theory for homogeneous polymer fluids
 - intramolecular correlations ω_{AB}
 - intermolecular correlations $g_{AB}(r)$, $c_{AB}(r)$
- Excellent for repulsive interactions



Calculation Details

- parallel cylinders
- athermal (repulsive interactions)
- adsorbed chains
 - $N = 40$
 - $\rho_b a^3 = 0.01$
- matrix chains
 - $P = 40, 80, 120, 160$
 - $\rho_b a^3 = 0.85$



surface interactions:

- repulsive for matrix chains, all except end on adsorbing chains
- attractive to one end of adsorbing chains, depth ε_e