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Electromagnetic XFEM with Weak Discontinuities

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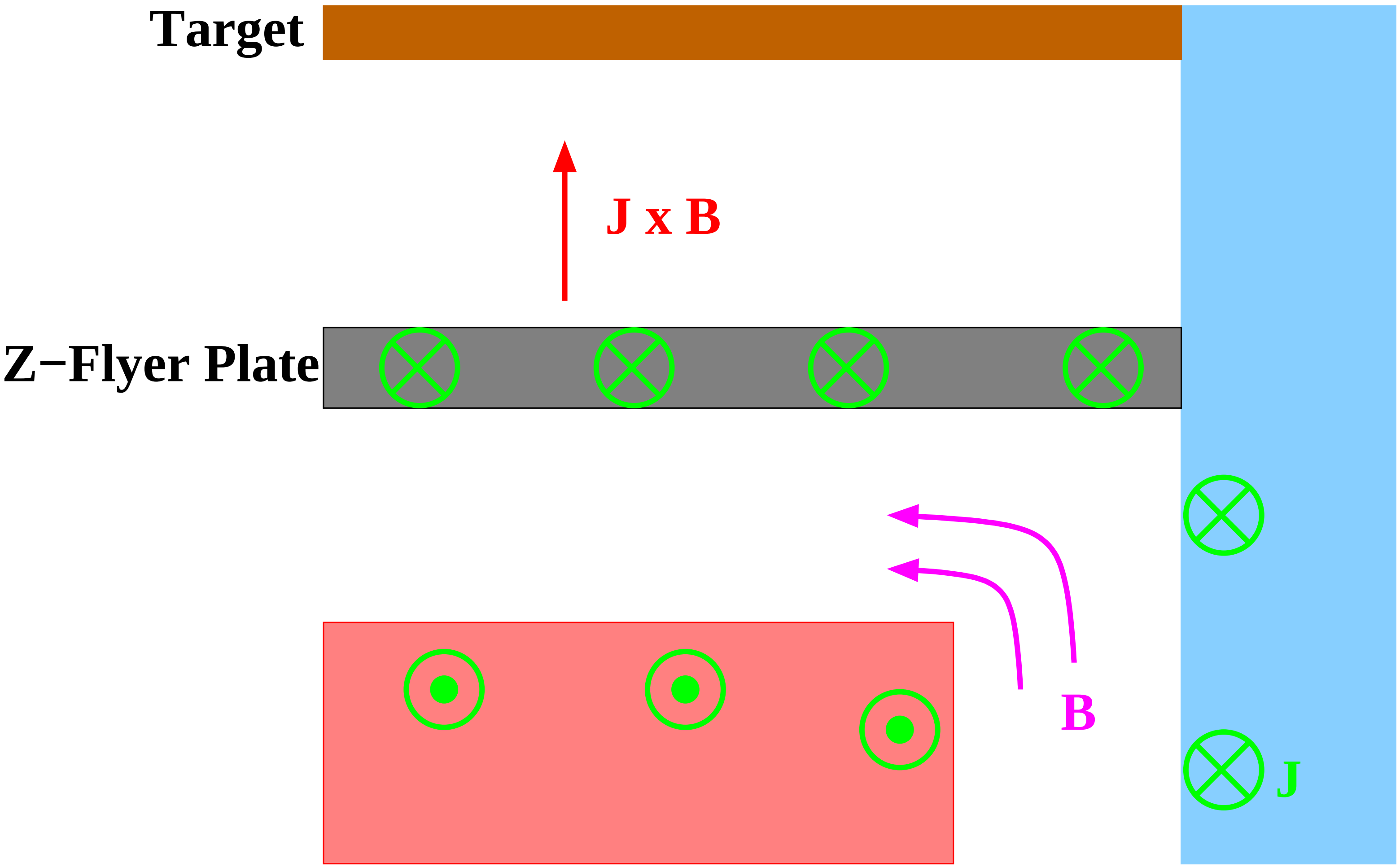
Outline

- Introduction
- Tied Heaviside XFEM
- XFEM with Algebraic Constraints
- Results & Conclusions

Z-Flyer Plate Application



How Can We Handle This?

- Eddy current Maxwell's equations:

$$\nabla \times \mu^{-1} \nabla \times \mathbf{E} + \sigma \frac{\partial \mathbf{E}}{\partial t} = \mathbf{f}$$

- Current concentrates on surfaces.
  - Interfaces are important.
  - Material/void interfaces “count” for EM.
- Treatment Options
  - Lagrangian Mesh: Not for large-scale deformation.
  - Eulerian Mixture Models: OK for hydro, poor for EM.
  - “XFEM”: Interface tracking + local “mesh” refinement.

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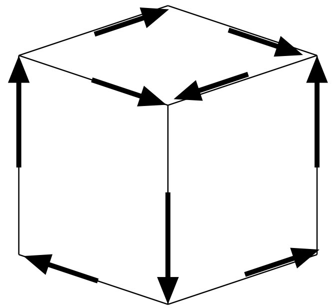
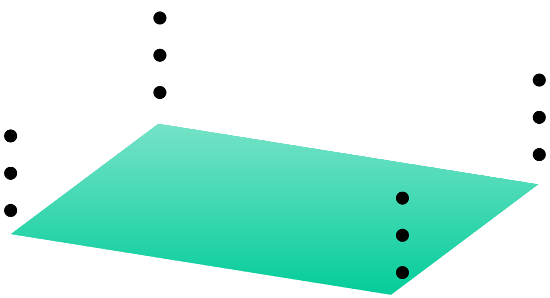
Edge Elements

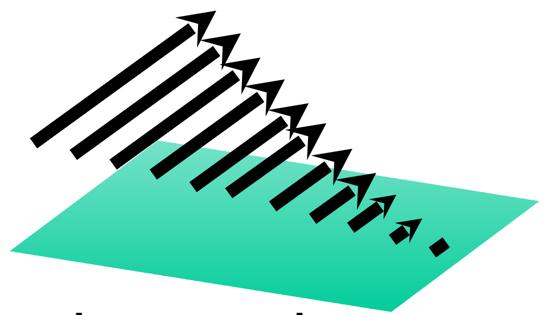
$$\nabla \times \mu^{-1} \nabla \times \mathbf{E} + \sigma \frac{\partial \mathbf{E}}{\partial t} = \mathbf{f}$$



- Scalar coefficients for vector basis functions.
- $\nabla \times \nabla \phi = 0$ preserved discretely.



Nodal Basis



Edge Basis

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XFEM 101

- Adds intra-element discontinuities w/o changing mesh topology
  - Strong (e.g. cracks; displacement)
  - Weak (e.g. bonded materials)
- Uses a Partition of Unity to preserve convergence

$$u^h(x) = \sum_A \sum_I N_I(x) F_A(x) u_{I,A}$$

where the constant is in the span of the F_A's.

Note: XFEM is very similar to local mesh refinement.

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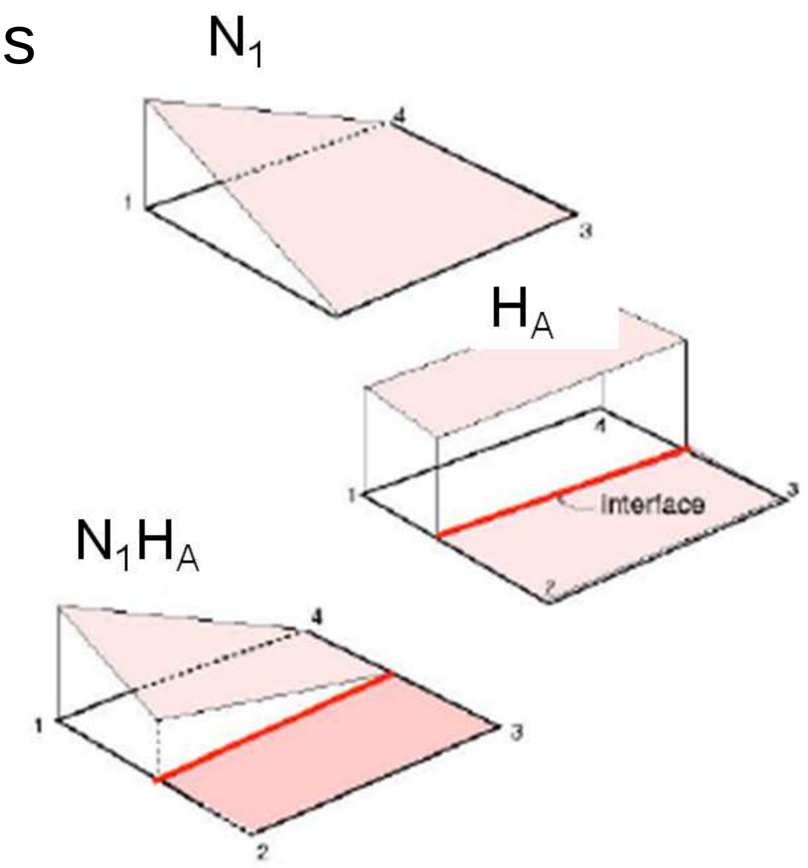
Tied Heaviside Methods

- Approach: Use Heaviside functions for enrichment.
- How to enforce continuity?
  - Standard Approach: Lagrange multiplier basis functions along cut(s) [1,2].
  - Our Approach: Use *virtual*, *algebraic* Lagrange multipliers.



[1] *Möes et al., 2006.*, [2] *Hautefeuille et al., 2012.*

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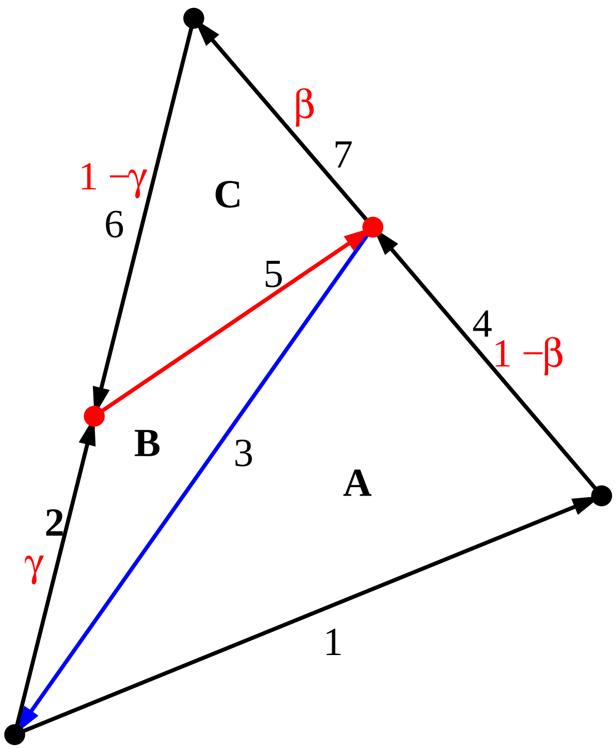
Algebraic Constraints



- Idea #1: Decompose everything into triangles.
- Note: Lowest order basis tangentially constant on cut.
- Idea #2: One pointwise constraint ties a constant function everywhere.

Why This Works

- This yields the system:

$$\begin{bmatrix} A_X & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

- These specific constraints also give a matrix Π s.t.

$$A_F = \Pi^T A_X \Pi$$

  where A_F is the body-fit FEM matrix.

- Moreover, $C\Pi = 0$.
- Proves equivalence between XFEM & local mesh refinement.

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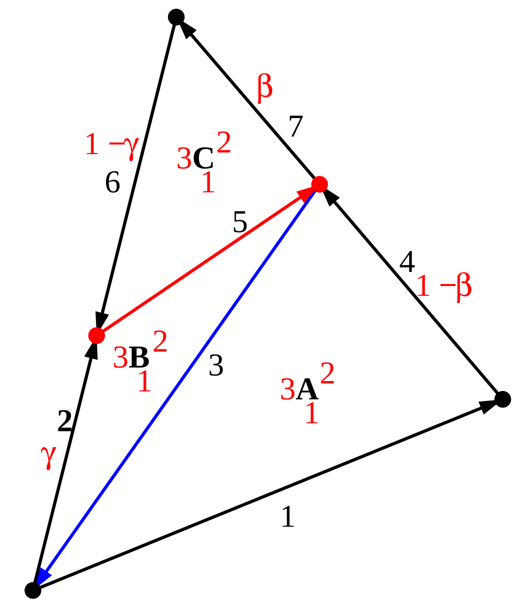
Change of Basis



- We can write the FEM-to-XFEM change of basis, Π, as follows:

$$
\begin{array}{c c}
 & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ B_1 \\ B_2 \\ B_3 \\ C_1 \\ C_2 \\ C_3 \end{array} &
\left(\begin{array}{ccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{1-\beta} & 0 & 0 & 0 \\
\frac{\beta}{1-\beta} & 0 & \frac{1}{1-\beta} & 0 & 0 & 0 & 0 \\
0 & \frac{\beta-1}{\gamma\,\beta} & -\frac{1}{\beta} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\gamma-1}{\gamma\,\beta} & 0 & -\frac{1}{\gamma\,\beta} & 0 & 0 \\
0 & -\frac{1}{\gamma} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{(1-\gamma)\beta} & \frac{\beta-1}{(\gamma-1)\beta} & \frac{\gamma}{(1-\gamma)\beta} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\beta} \\
0 & 0 & 0 & 0 & 0 & \frac{1}{1-\gamma} & 0
\end{array}\right)
\end{array}
$$

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Steady-State Test Problem

$$\mathbf{B}_z(x,y) = \begin{cases} -x^2 + y^2 + 4x + y - \frac{5}{7}, & x \le \frac{3}{7} \\ -x^2 + y^2 + x + y + 2, & x > \frac{3}{7} \end{cases}$$

$$\mathbf{E}(x,y) = \begin{cases} \begin{bmatrix} 2y+1 \\ 2x-4 \end{bmatrix}, & x \le \frac{3}{7} \\ \begin{bmatrix} 44y+22 \\ 44x-22 \end{bmatrix}, & x > \frac{3}{7} \end{cases}$$

with $\mu = 1$, $\sigma_L = 1$ and $\sigma_R = \frac{1}{22}$.

- Has tangential, but not normal, continuity across $x = \frac{3}{7}$.

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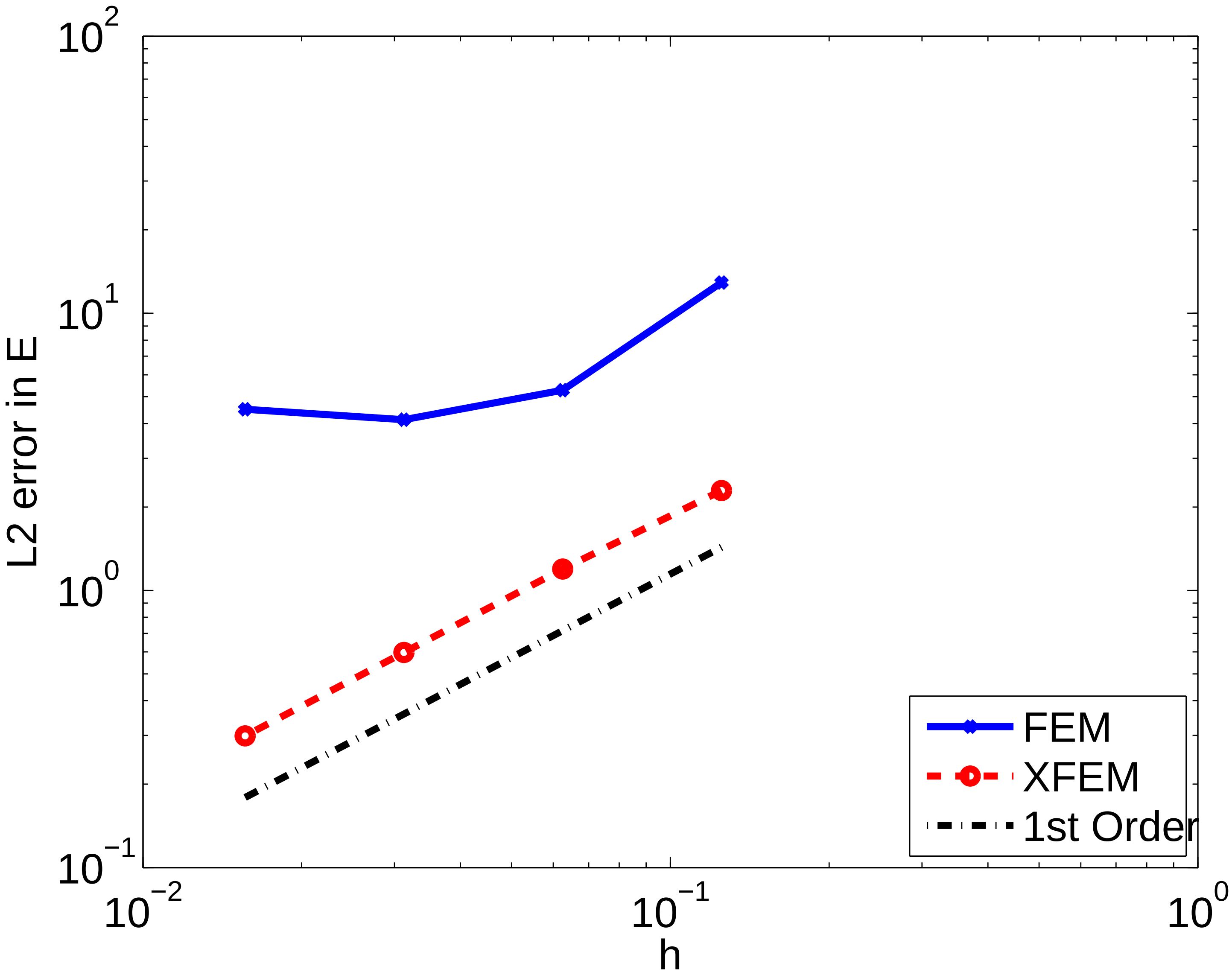
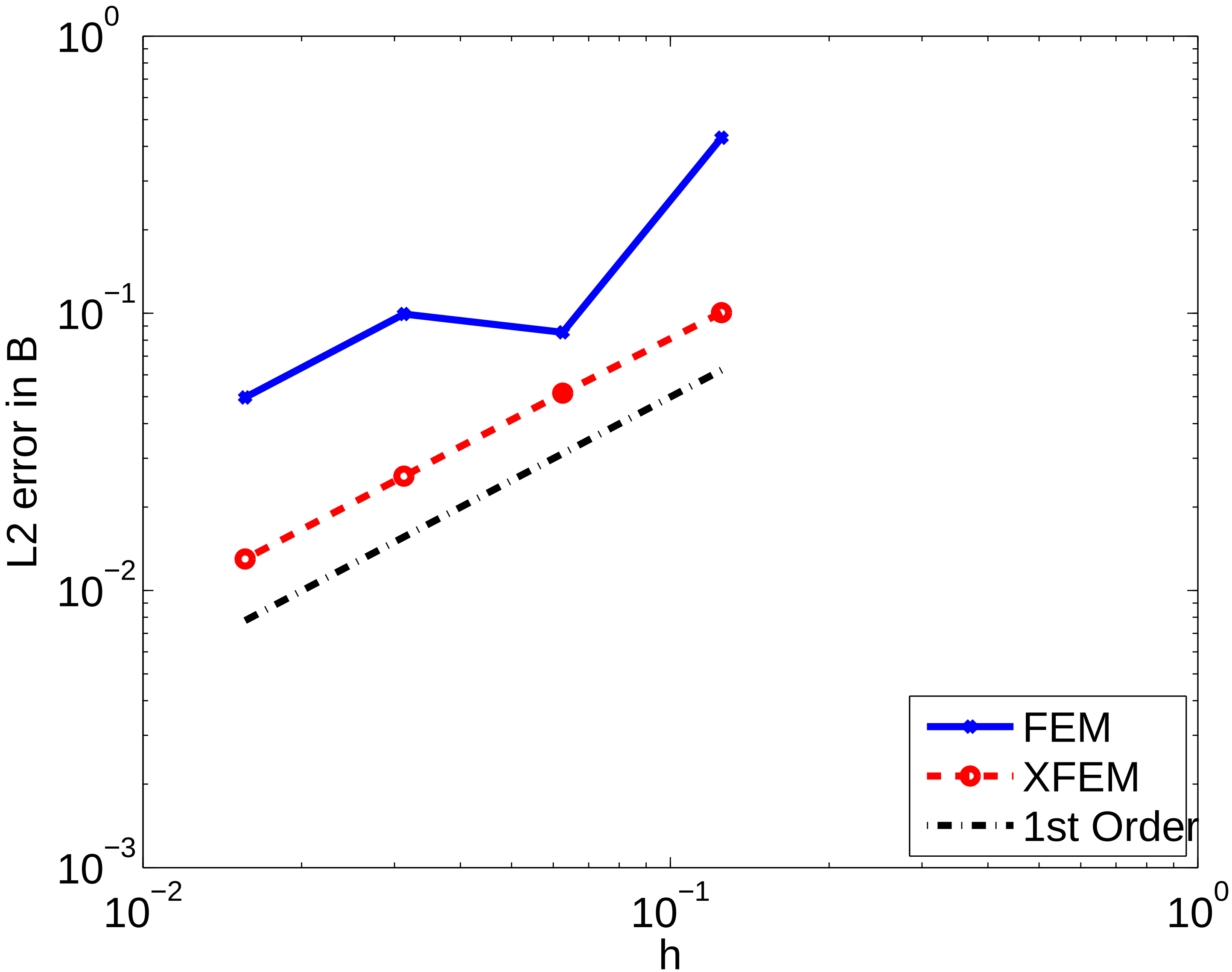
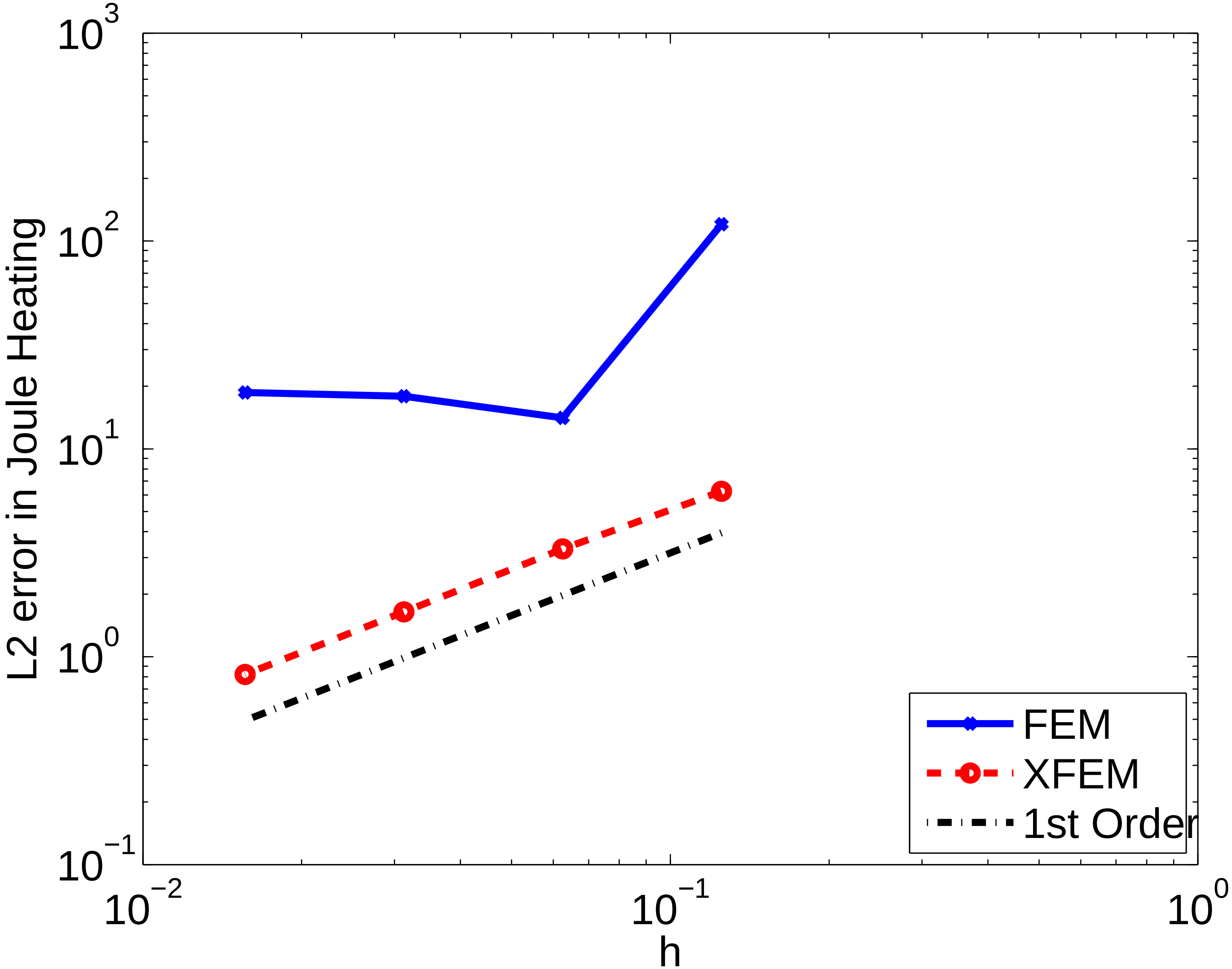
E Field



B Field



Joule Power ($\sigma \mathbf{E} \cdot \mathbf{E}$)



Conclusions

- XFEM-AC offers substantially better accuracy than mixed-cell models.
- Equivalence to body-fit problem offers provable convergence.
- Current / future work
  - 3D edge elements.
  - Solving the resulting linear system.