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Probabilistic Analysis Across Multiple Scales

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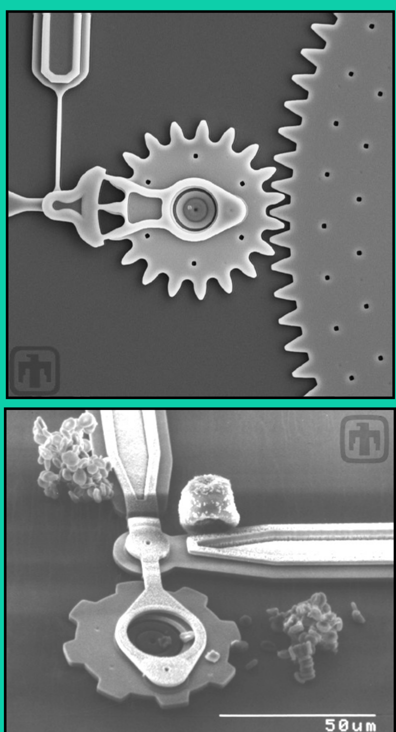
Outline

- Probabilistic Analysis
  - Model uncertainty
  - Assess design and performance margins
- The “Very Small”
  - Micro-Electro-Mechanical Systems (MEMS)
  - Objective is robust design
  - Simple models, very customer focused
- The “Very Large”
  - Earth climate
  - Objective is to assess “margin”
  - Research focus
- In between
  - Aerospace and automotive vehicles, civil structures, etc.

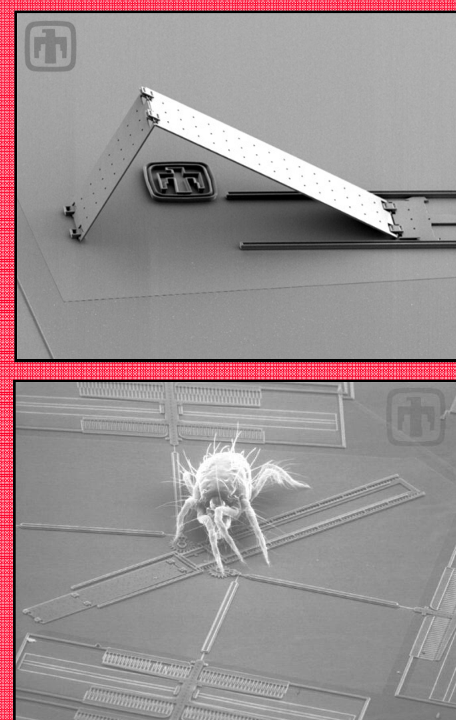
MEMS at Sandia

- Applications include microengines, mirror assemblies, and inertial actuators

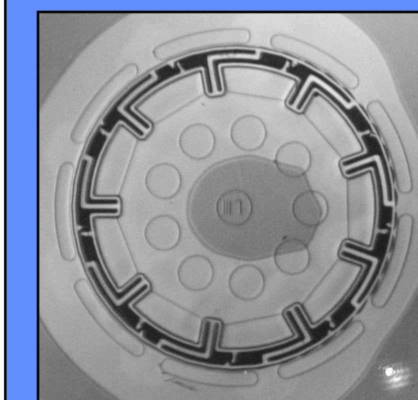
Microengine drive gear



Silicon mirror and drive assembly



Inertial switch



Significant unit-to-unit variability introduced by fabrication process – this affects design performance

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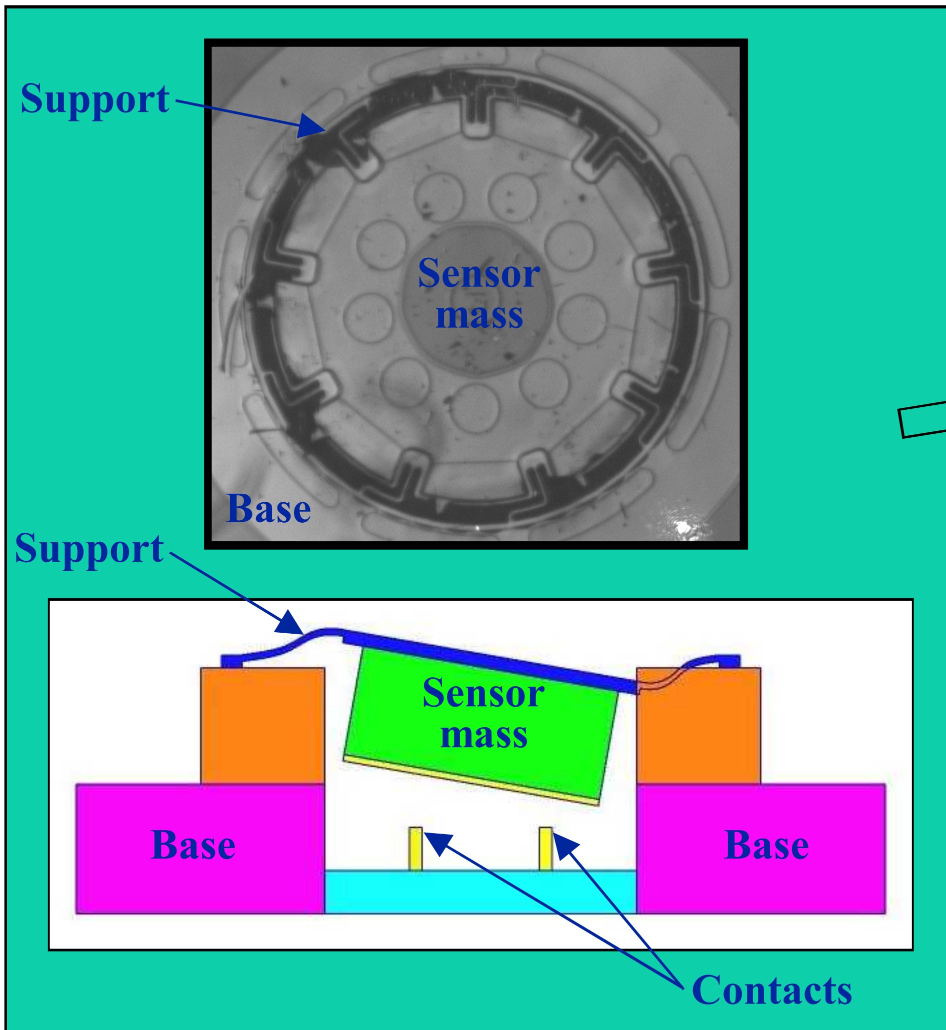
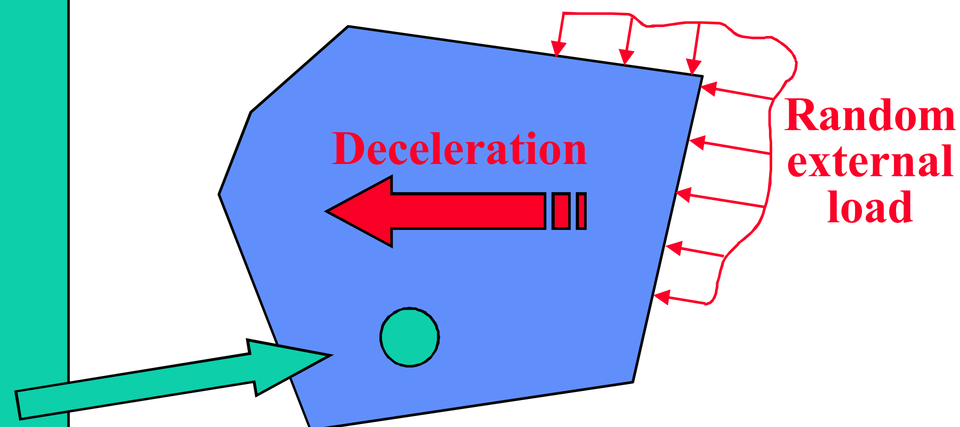
Problem definition and objective

MEMS switch



Parent structure



Objective: Design MEMS switch to close, and remain closed, once a critical level of deceleration of parent structure is achieved; otherwise, switch should remain open

Some complicating factors

- Parent structure is excited by fluctuating external load in time and space
  - Modeled as stochastic process

- MEMS switch is subject to repetitive mechanical contact
  - Nonlinear vibro-impact model used for analysis

- Significant unit-to-unit variability observed during testing due to the fabrication process
  - Some parameters modeled as (correlated) random variables

Nonlinear dynamic system with random parameters driven by stochastic forcing function

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Model for structure and impact

- SDOF nonlinear vibro-impact system

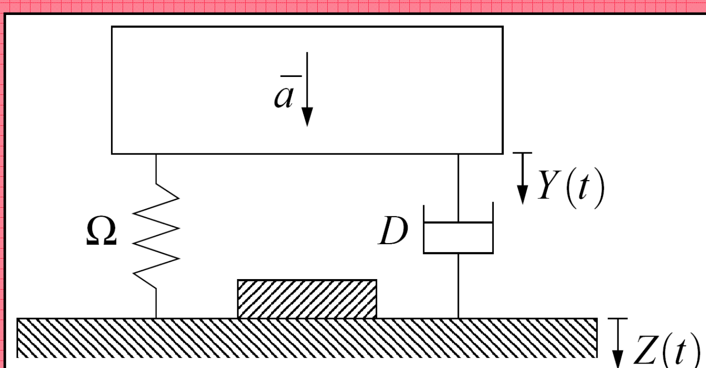
$$\ddot{W}(t) + 2D\Omega\dot{W}(t) + \Omega^2 W(t) = \bar{a} - \ddot{Z}(t)$$
$$W(0) = \min\{\bar{a}/\Omega^2, X\},\ \dot{W}(0) = 0$$

Equation of motion

Instantaneous impact with coefficient of restitution, η

$$\dot{W}(t^+) = \begin{cases} \dot{W}(t^-) & \text{if } W(t^-) < X, \\ -\eta\dot{W}(t^-) & \text{if } W(t^-) = X, \end{cases}$$

Equation of impact



- Y, Z = Random disp. of mass and base
  - $\ddot{Z}(t)$ denotes stochastic forcing function
- W = Y – Z is random relative disp. of mass
- X = Random travel distance for closure
  - switch is open if W < X; closed if W = X
- -, D = Random parameters
  - natural frequency, damping ratio of supports
- $\bar{a}$ = Static deceleration of parent

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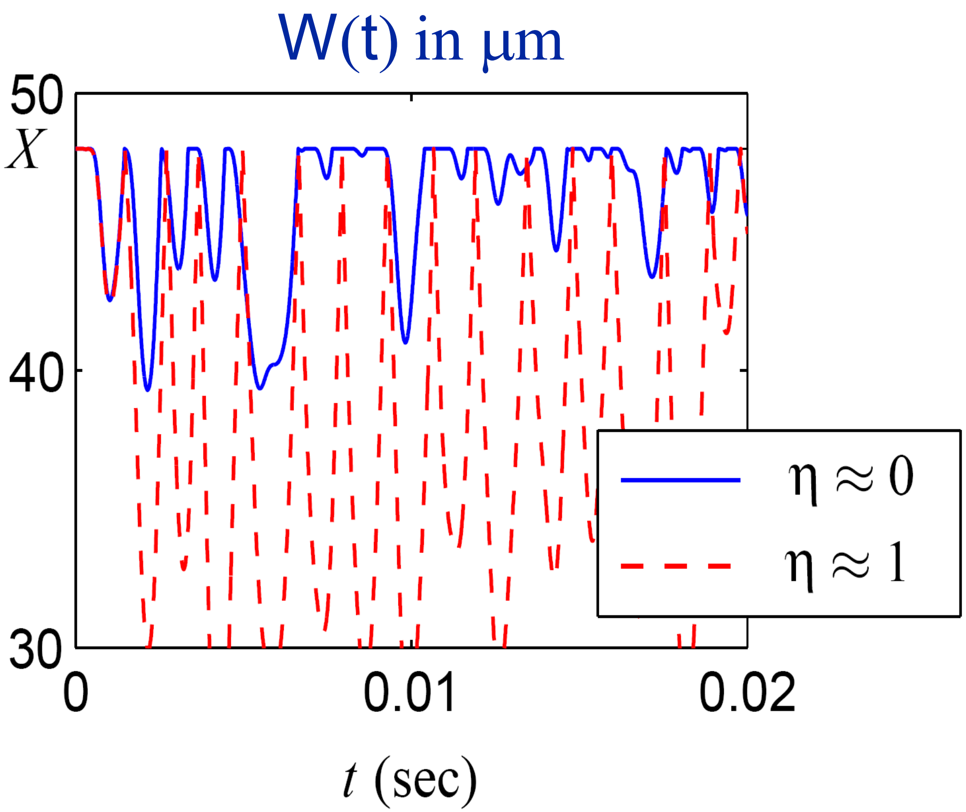
Solution is sensitive to value for ´



- Parameters
  - - = 2,000 rad/s
  - D = 2%
  - X = 40 μm
  - $\bar{a}$ = 25 g
  - Switch is closed at t = 0
- Two values for ´ shown
  - Perfectly inelastic (´ = 0)
  - Perfectly elastic (´ = 1)
  - **Calibration needed!**

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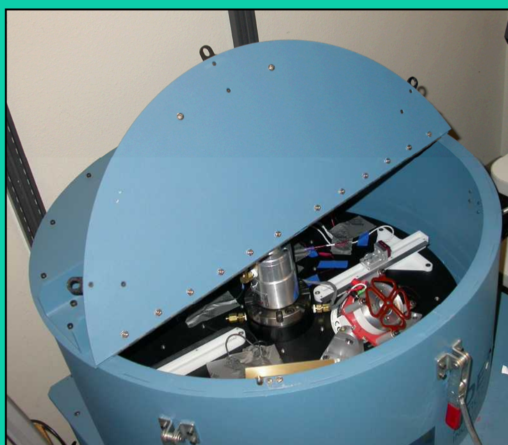
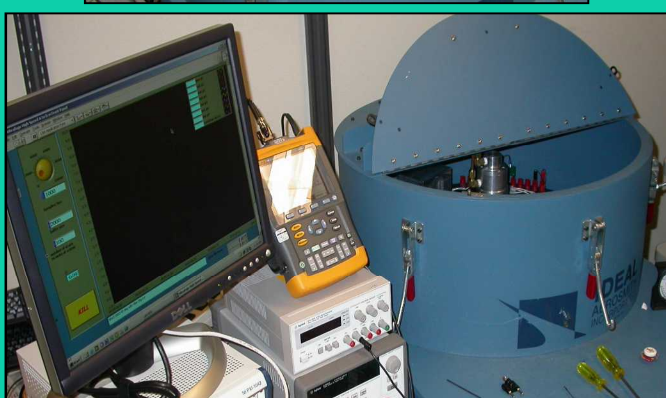
Calibration of impact model

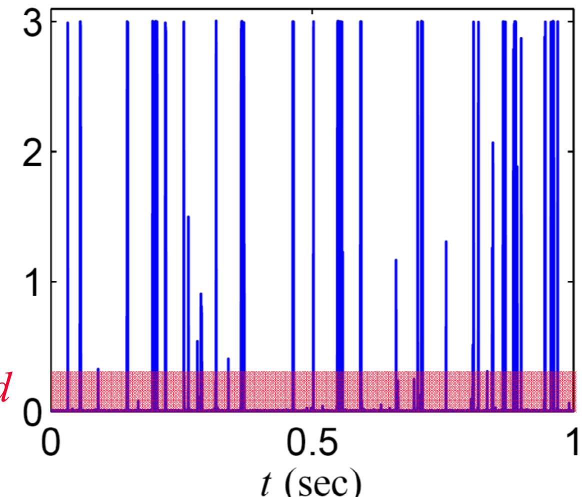
Centrifuge with internal shaker for transient dynamic testing





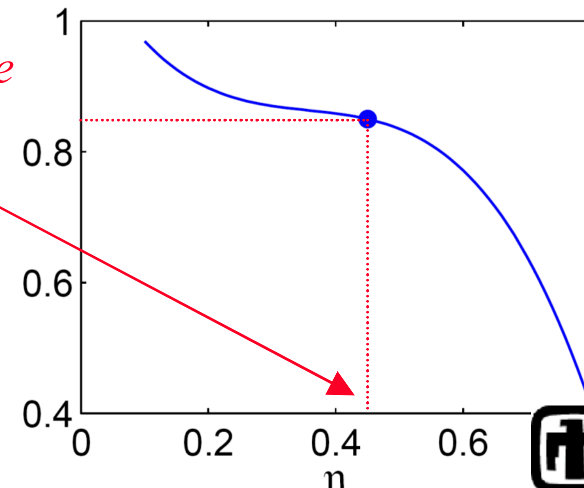
Applied load: static acceleration with superimposed random vibration

Data: voltage *vs.* time



Switch closed

Model: mean closure time *vs.* ´



Calibrated value
´ = 0.45

Thanks to D. Graf (SNL) for experimental results

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Model for environment and variability

- Model for environment: stationary stochastic process
  - Zero-mean, Gaussian, band-limited white noise
- Model for variability
  - Natural frequency and damping ratio (-, D) modeled as correlated random vector with prescribed marginal PDFs

$$\boxed{f_\Omega(\omega;\lambda,\xi) = \frac{1}{\sqrt{2\pi}\xi\omega}\exp\left[-\frac{1}{2}\left(\frac{\ln\omega-\lambda}{\xi}\right)^2\right],\ \ \omega>0}$$

Lognormal

$$\boxed{f_D(d;a,b,q,r) = \frac{\Gamma(q+r)}{\Gamma(q)\Gamma(r)}\frac{(d-a)^{q-1}(b-d)^{r-1}}{(b-a)^{q+r-1}},\ \ a\le d\le b}$$

Beta

  - Travel distance, X, modeled as uniform RV based on expert opinion
  - **Model for (-, D) can be calibrated to data**

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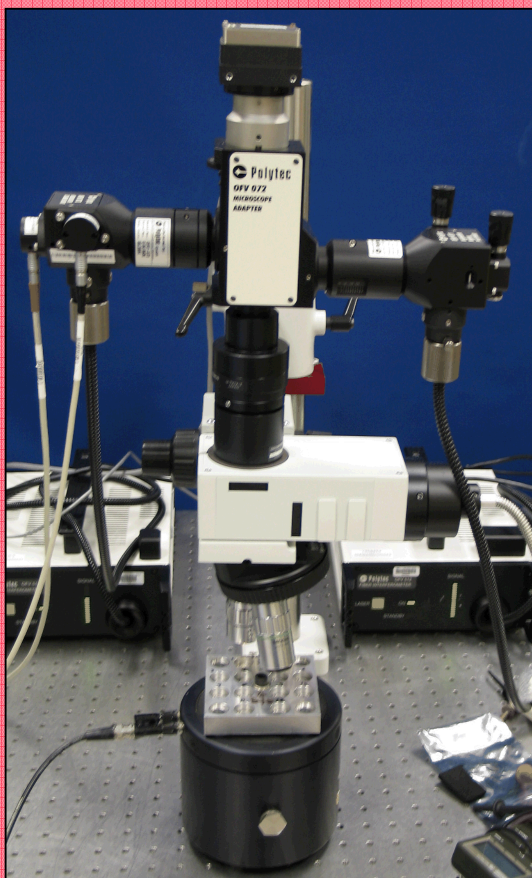
Calibration of variability models

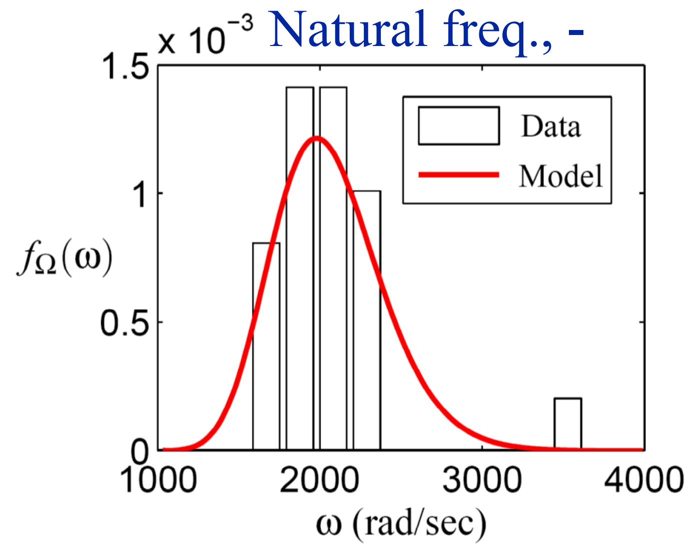
Microscope vibrometer with 10 lb. shaker for modal tests

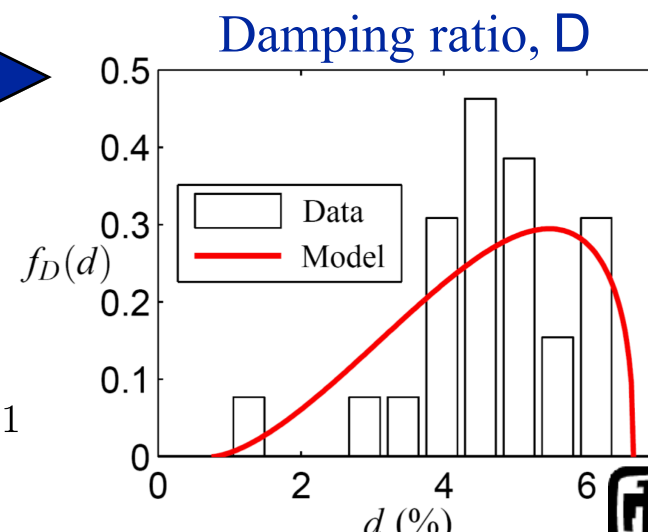


Measurements of - *and* D *from 24 nominally-identical MEMS switches*

Correlation

$$\frac{\mathrm{E}[\Omega D]}{\mathrm{E}[\Omega^2]\mathrm{E}[D^2]} = -0.81$$





Thanks to D. Epp (SNL) for experimental results

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Design requirements

- Metric: proportion of time switch is closed
  - Approximate by occupation time

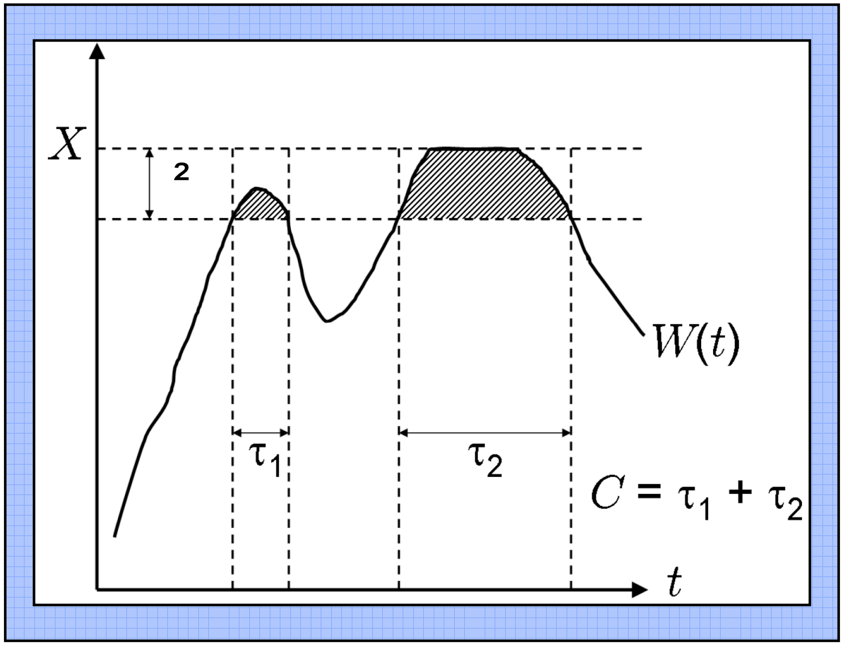
$$\boxed{C = \int_0^t 1\left(X - W(s) < \epsilon\right)\ \mathrm{d}s}$$

  - $\bar{C} = \frac{1}{t} C$ is a random variable
  - Case I: $\bar{a} = 2$ g; Case II: $\bar{a} = 25$ g
- Requirements

$$\boxed{\begin{aligned} P(\bar{C}_{\mathrm{I}} \leq 0.03) &\geq 0.95 \\ P(\bar{C}_{\mathrm{II}} > 0.50) &\geq 0.95 \end{aligned}}$$

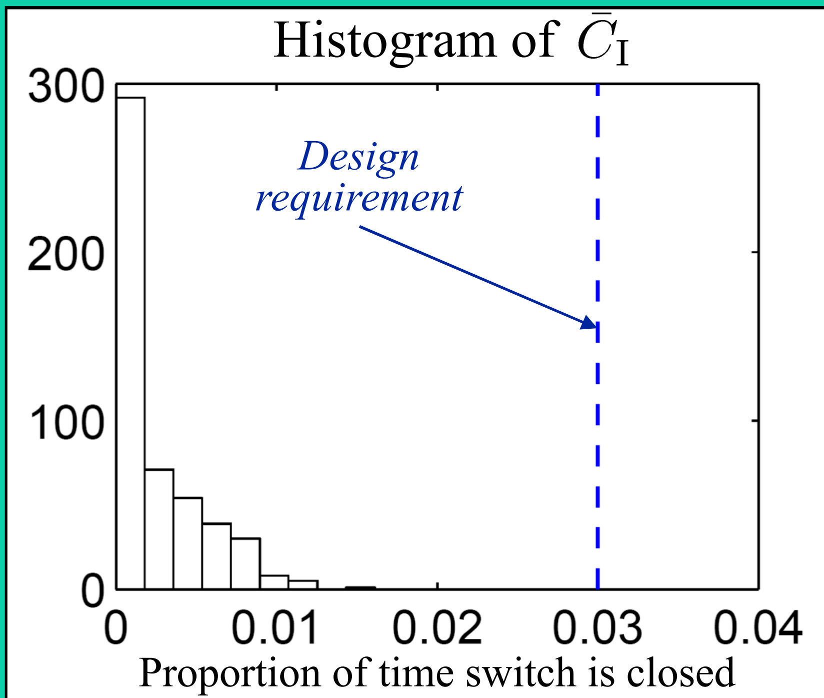
  - **Cannot be computed exactly**

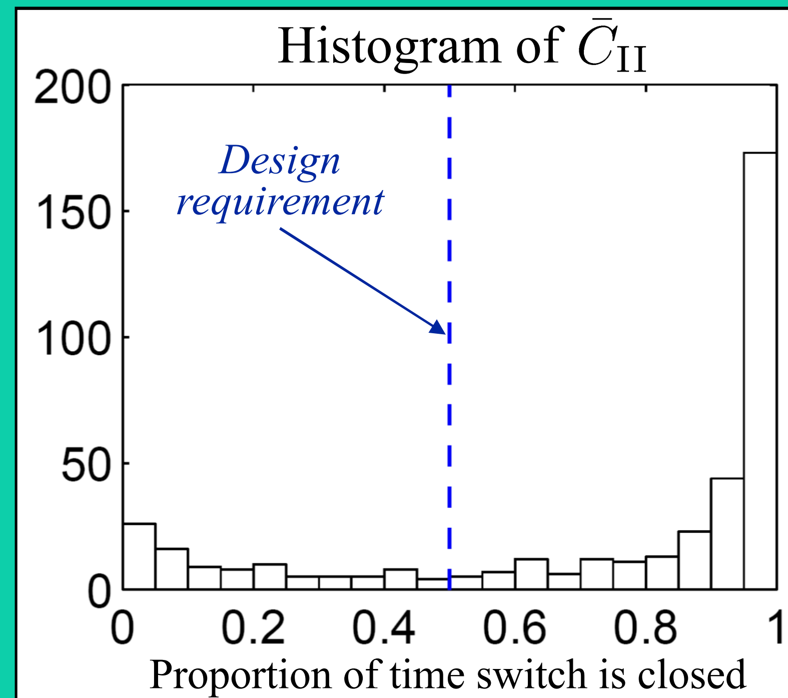
Occupation time of W(t) within ² of X



Note: ² = *0.5 μm for calculations*

Performance of initial design





- Case I: $P(\bar{C}_{\mathrm{I}} \leq 0.03) \approx 1$
- Case II: $P(\bar{C}_{\mathrm{II}} > 0.5) \approx 0.76$
  - **Design modifications needed!**

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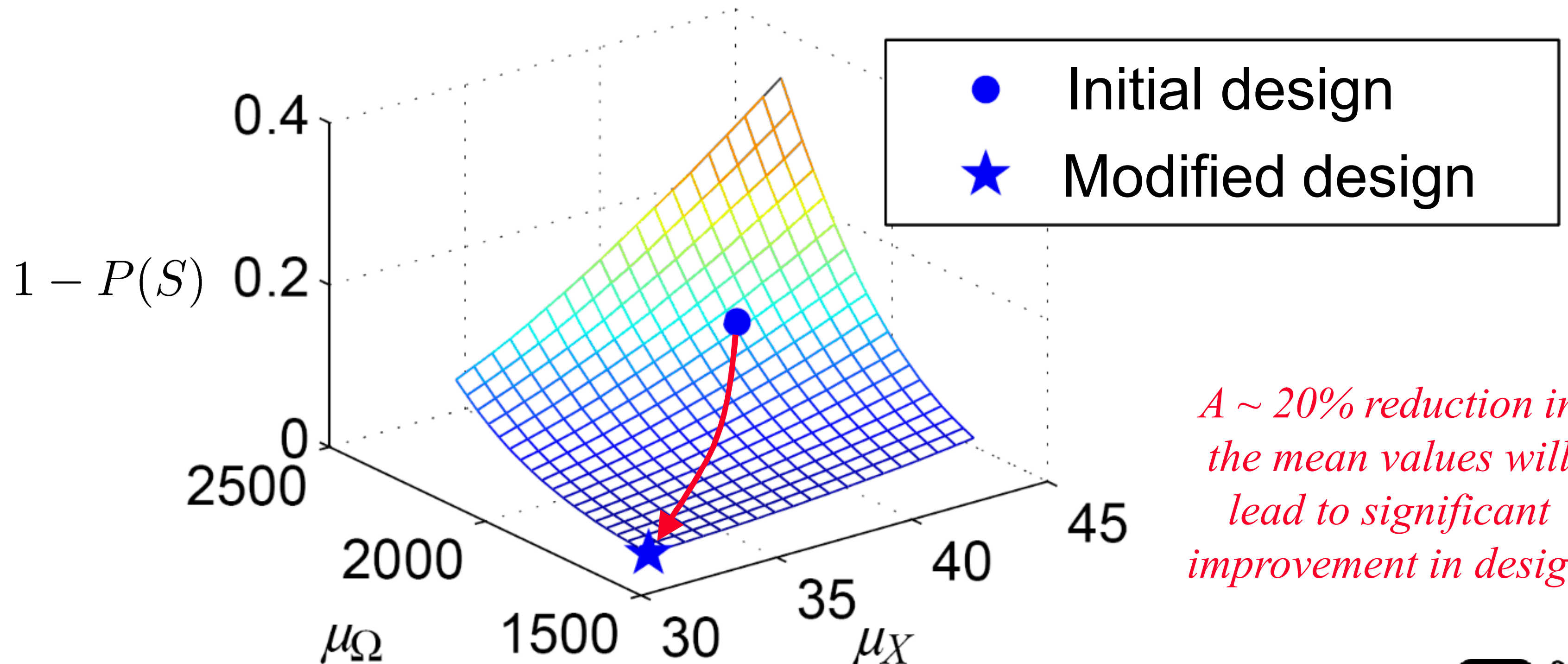
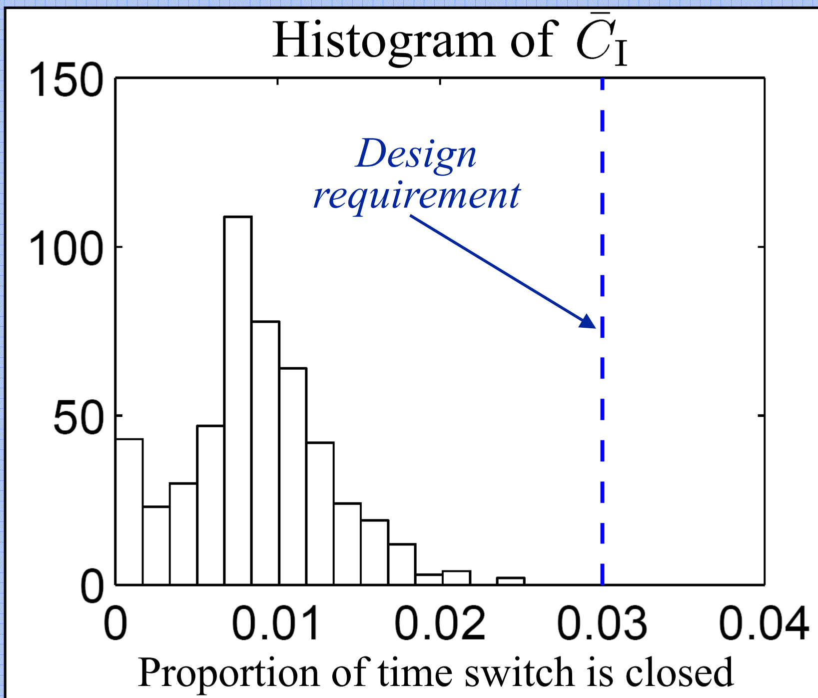
Design study

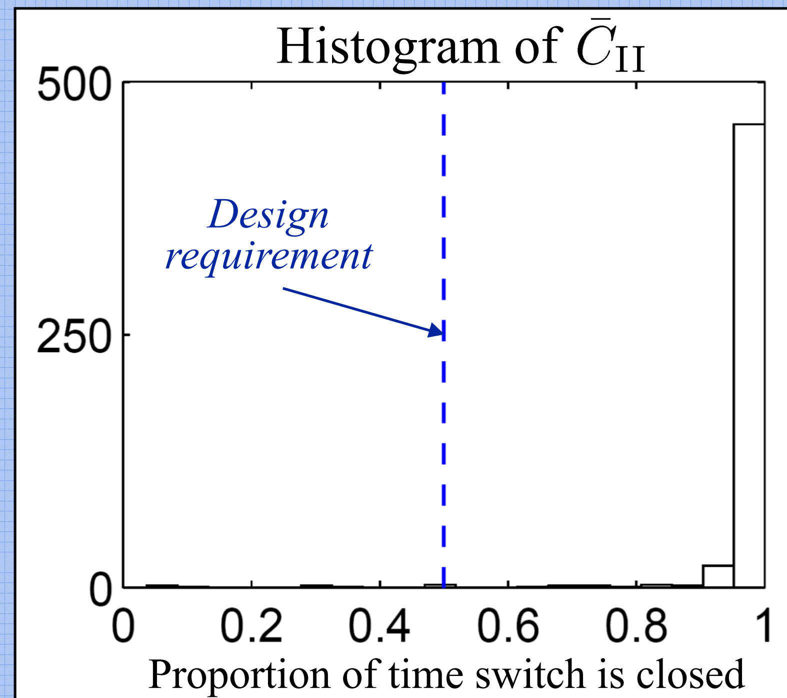
- Assumptions
  - Values for $\mu_X = \mathrm{E}[X]$ and $\mu_\Omega = \mathrm{E}[\Omega]$ can be prescribed
  - PDFs for X and - remain otherwise unchanged



A ~ 20% reduction in the mean values will lead to significant improvement in design

Performance of modified design





- Both design requirements are satisfied
  - Case I: $P(\bar{C}_{\mathrm{I}} \leq 0.03) \approx 1$
  - Case II: $P(\bar{C}_{\mathrm{II}} > 0.5) \approx 0.98$

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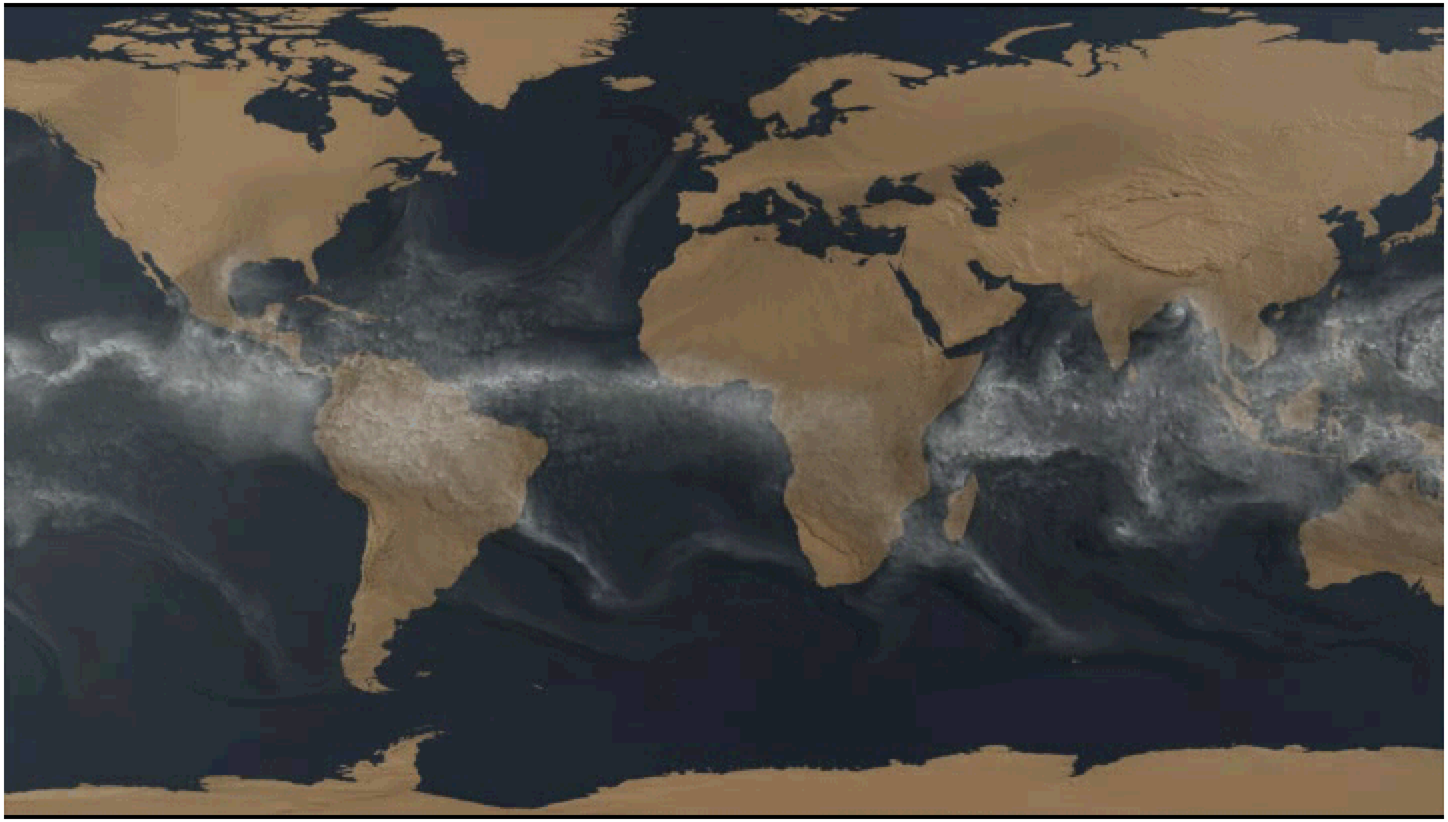
The “Very Large”

Climate Modeling at Sandia



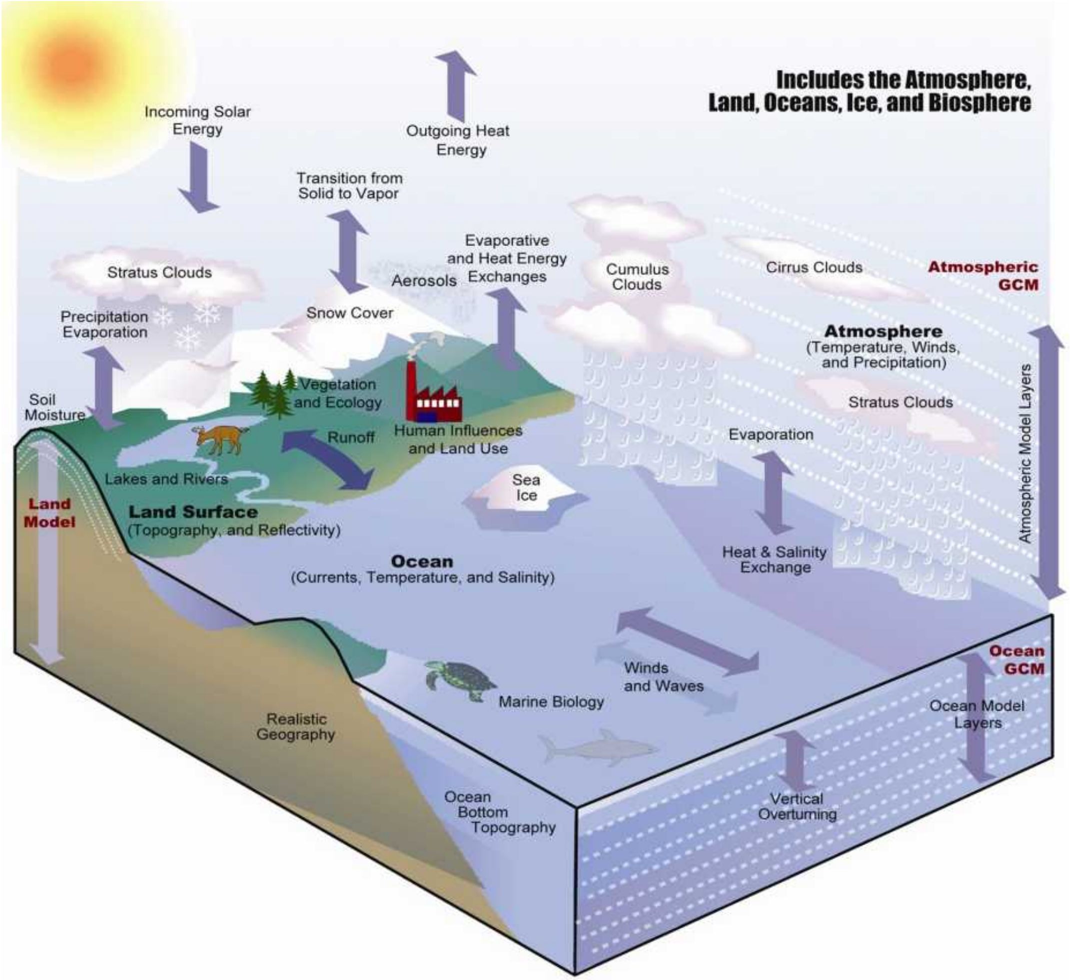
- Funded by DOE Office of Science & LDRD
- Sandia focus: Large-Scale Computing & Uncertainty Quantification
- Animation: 1/8 degree (12.5 km) atmosphere time slice simulation
  - 66,816 cores of ORNL's JaguarPF

Outline

- Climate physics
  - Nonlinear, multi-physics, multi-scale in time and space
  - Components of the climate and their interactions
  - Forcings, feedback, and sensitivity
- Climate models
  - Community System Climate Model (CCSM)
- Probabilistic analysis of climate models
  - Objective: Quantify the margin of high-consequence climate change
  - Bring the philosophy of probabilistic mechanics and risk analysis to the climate community

Components of Earth Climate

1. Atmosphere
   – Composition, temperature, circulation, precipitation
   – Very active; first to be modeled
2. Hydrosphere (Oceans)
   – Temperature, salinity, circulation, structure (layers)
   – Upper layer has strong interactions with atmosphere
3. Cryosphere (Frozen water)
   – Sea ice, glaciers, ice caps, ice sheets, frozen ground
   – Plays major role in heat balance due to large albedo (ratio of reflected to incoming radiation)
4. Biosphere (Life)
   – All living organisms in atmosphere, land, oceans
5. Land surface



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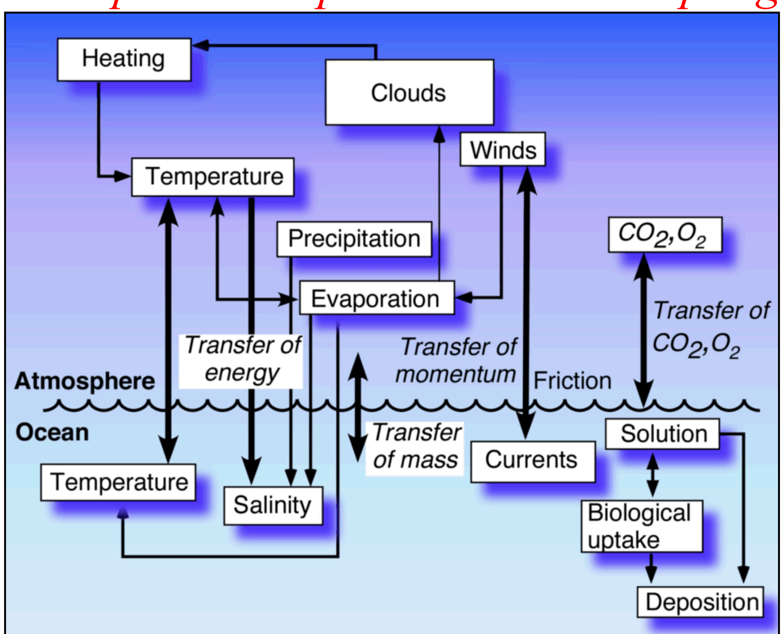
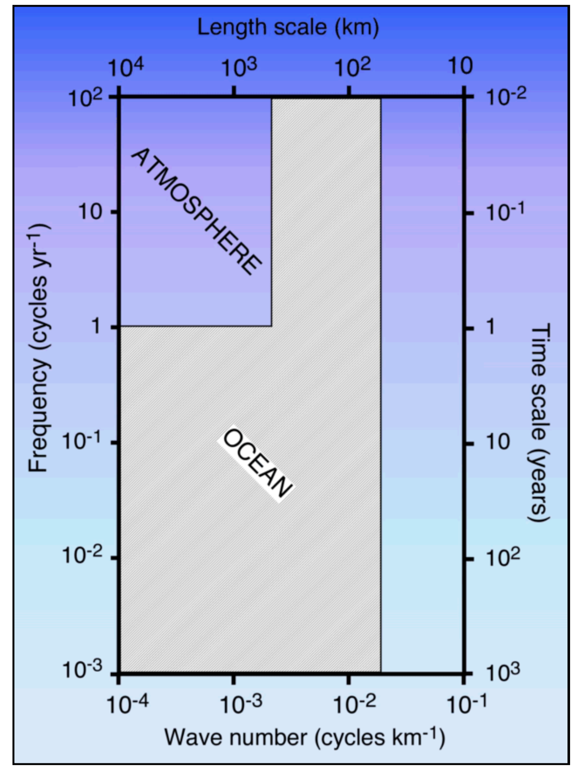
Component Coupling

- Interconnection of energy, momentum, and mass fluxes between components
- Time / space dependent

Example: Atmosphere / Ocean Coupling





*Coupling occurs on "scales of motion ranging from molecular to planetary, and from time scales of nano-seconds to geological eras"**

*K. McGuffie and A. Henderson-Sellers, *A Climate Modelling Primer*, Wiley, 2005.

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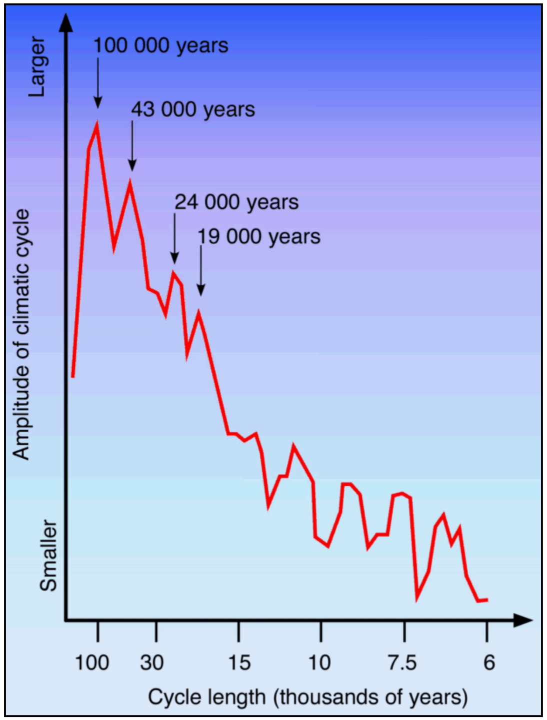
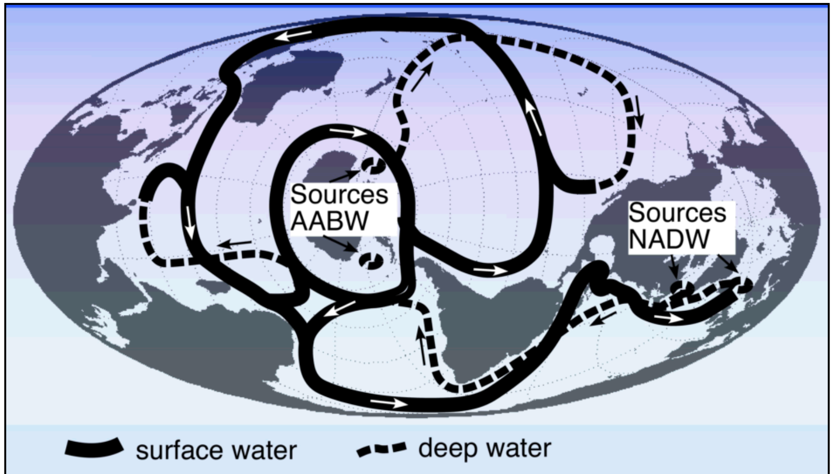
Climate Forcings, Feedbacks, and Equilibrium Sensitivity

- Forcing: *A change imposed on the planetary energy balance that causes a change in global mean temperature*
  - External
    - Earth's orbit around the sun; Solar activity
  - Internal
    - Natural: Volcanic eruptions; Ocean circulation
    - Human-induced: Greenhouse gas emissions; Aerosols; Ozone; Land surface changes

Spectrum of climatic variations over the last 500,000 years



Thermohaline circulation of the ocean illustrating the North Atlantic Deep Water and Antarctic Bottom Water sources



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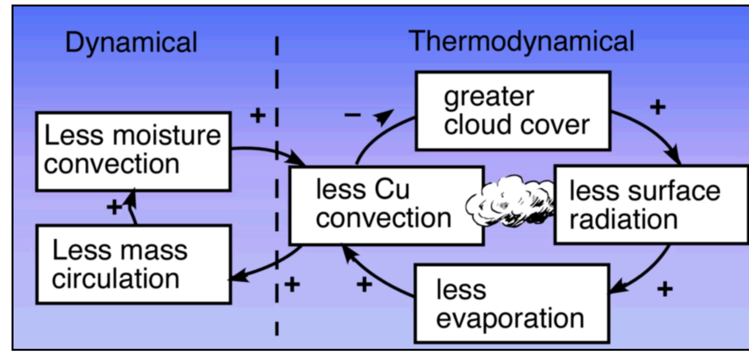
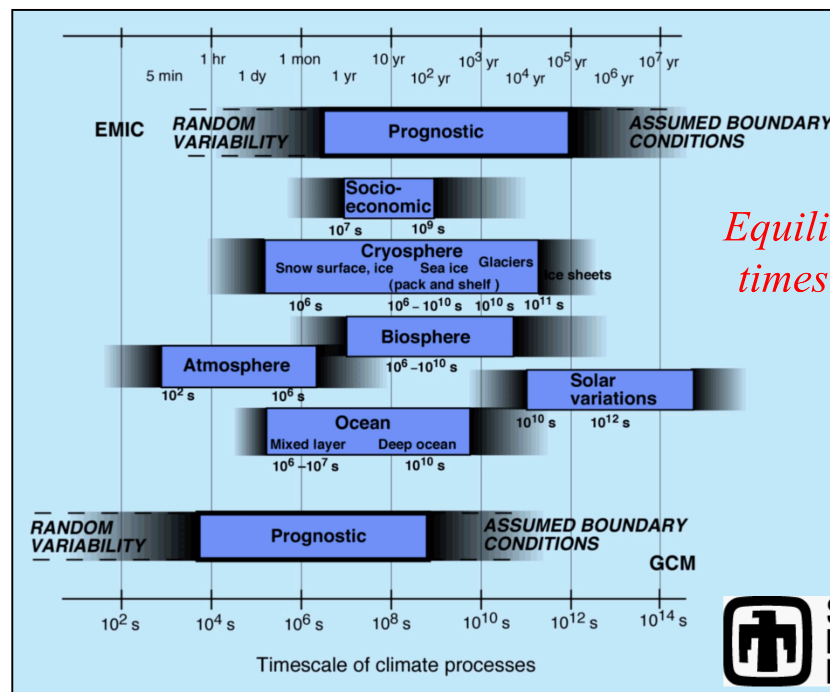
Climate Forcings, Feedbacks, and Equilibrium Sensitivity

- Feedback: *Internal mechanism that amplifies (positive) or reduces (negative) temperature change due to a forcing*
  – Examples: Ice - albedo mechanism; Cloud feedback
- Sensitivity: *Change in the global mean surface temperature after the climate system has reached a new equilibrium in response to a doubling of the* CO_2 *concentration in the atmosphere*

Cloud feedback





Equilibrium timescales

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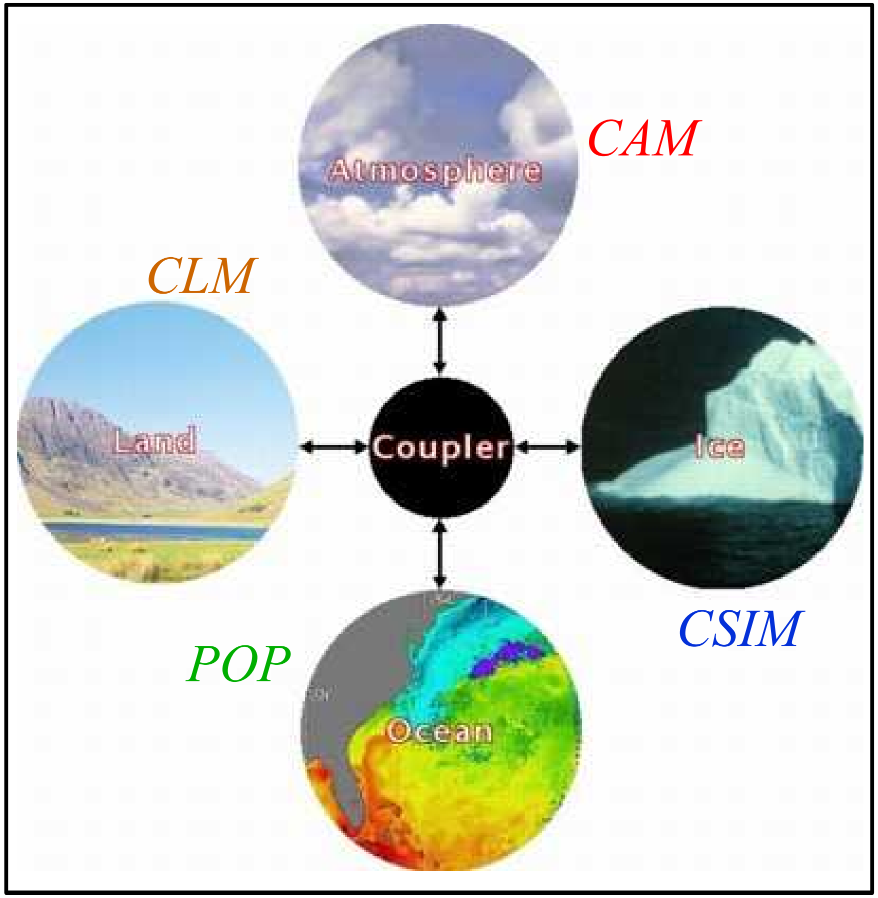
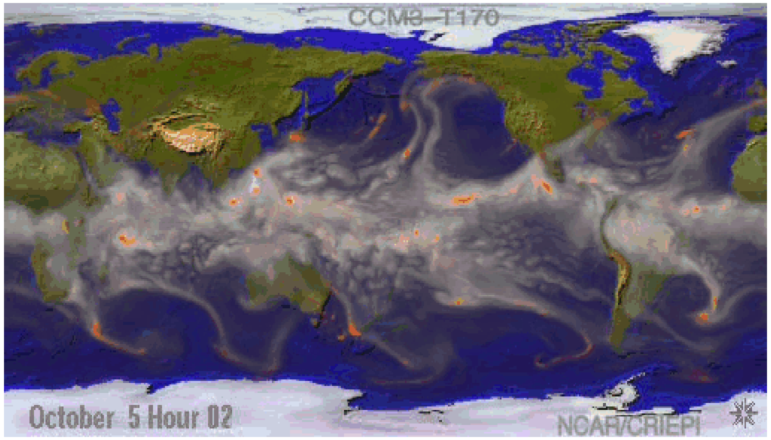
The Community Climate System Model (CCSM)

- A fully-coupled, global climate model that provides state-of-the-art computer simulations of the Earth's past, present, and future climate states
  - Cooperative effort among US climate researchers
  - Supported by NSF and centered at NCAR; collaborations with DOE and NASA
  - Originally created by NCAR in 1983
- Some run statistics:
  - 104 CPUs on “Bluesky” can simulate 7.5 years in 1 day

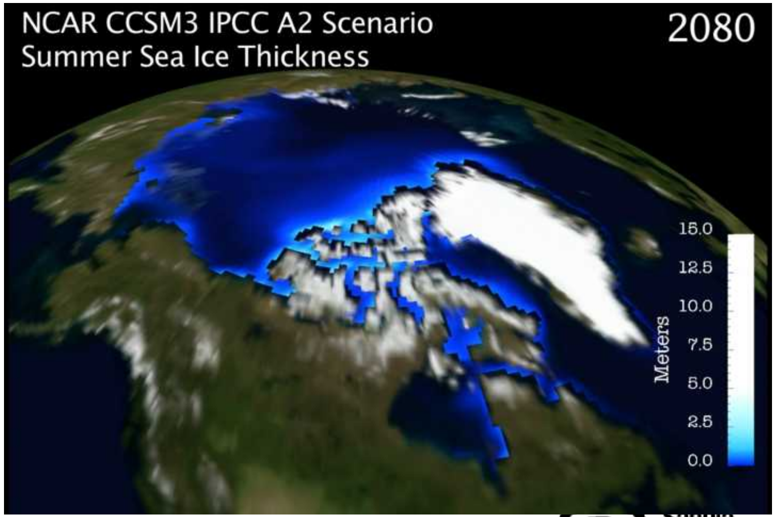


Some Details of the CCSM

- Atmosphere
  - Euler equations with hydrostatic and shallow atmosphere approximations, ideal gas
  - Includes effects of water vapor, CO_2, ozone and clouds
  - Models predict cloud type & fraction but not individual clouds; parameterize cumulus convection and boundary layer mixing, gravity wave drag
- Ocean
  - Euler equations with hydrostatic & Boussinesq approximations; empirical equation of state
  - Parameterizations: Ocean mixing near the surface, vertical mixing, mesoscale eddies



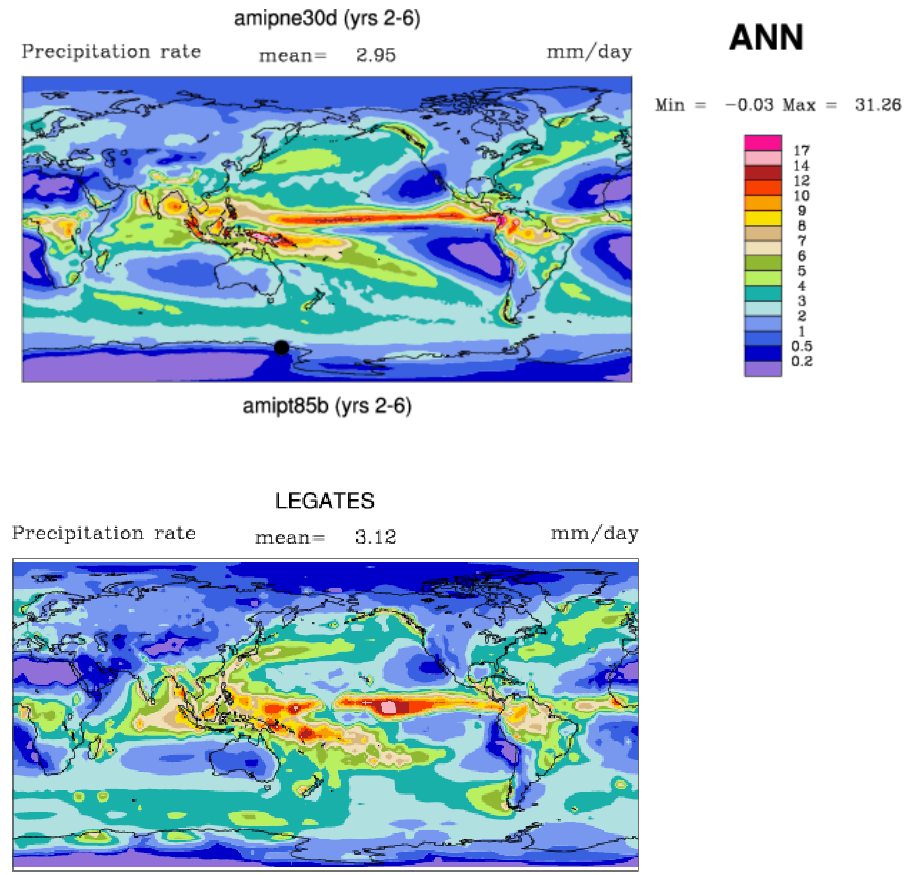
Precipitable H_20 in the atmospheric component of CCSM



Sea Ice thickness as forecast by CCSM

Climate Model V&V

- Verification
  - PDE component of atmospheric model has undergone extensive verification: mesh convergence studies for idealized problems for both short-term forecasts and long term statistics.
- Validation
  - Forecast community uses very similar global atmospheric models. Forecast skill monotonically improving since the 70's.
  - Atmospheric Model Inter-comparison Project (AMIP): Atmospheric component model compared with observations.
  - Data is sparse (100 years)
  - Ensembles of runs, Hindcasting?



Model is forced with observed ocean/ice conditions for a 25 year period

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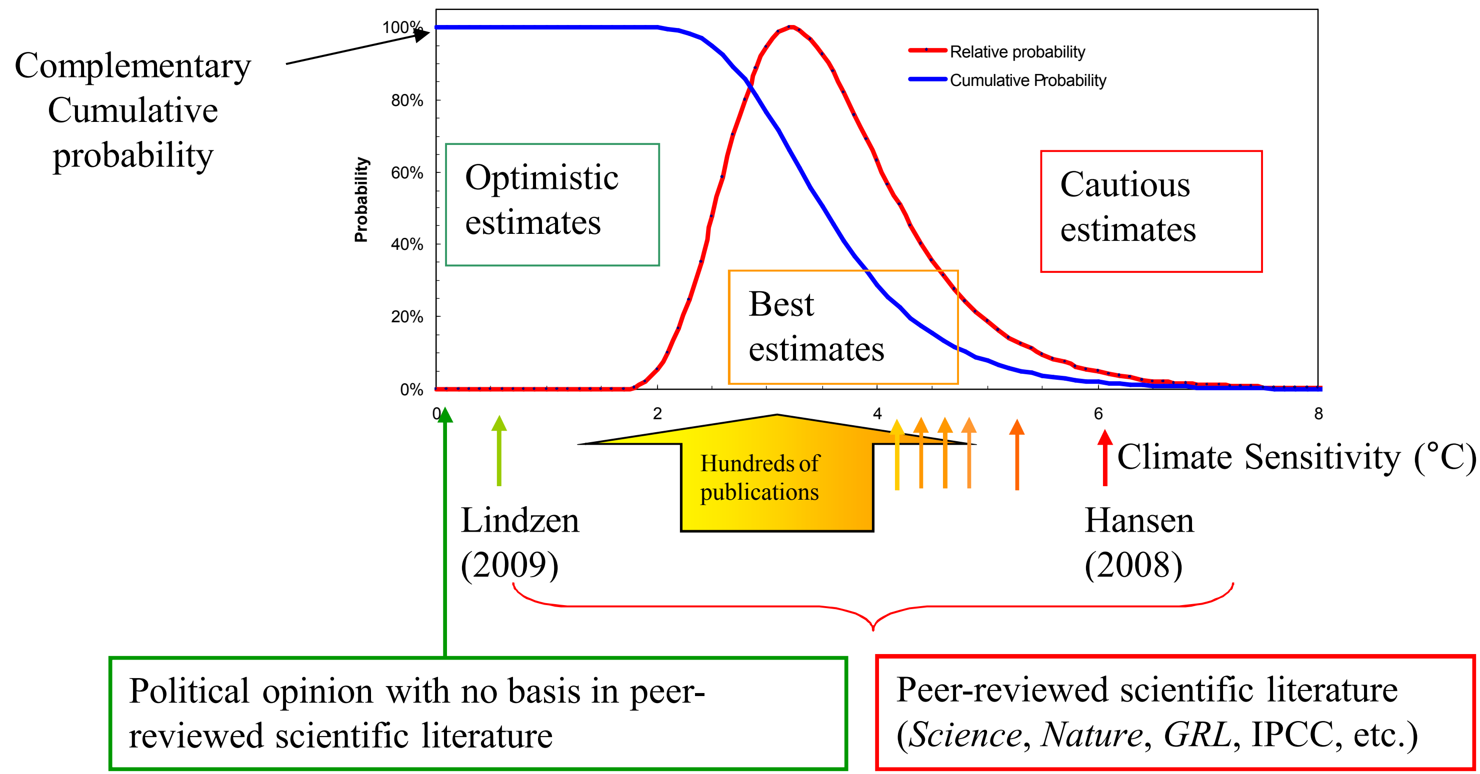
Uncertainty in Climate Modeling



- Tail of model predictions is often ignored by climate scientists
  - A "best estimate"
  - This is precisely the QMU problem

Uncertainty in Climate Modeling

- Climate models depend on LOTS of parameters
  - Many are non-physical and/or unmeasurable; treated as random
  - Equations are very sensitive to initial conditions, making ensemble of model runs necessary
  - Some physical processes are unknown / misunderstood
- Our approach
  - Model climate variables as random fields (a SROM)
  - Introduce tools from structural reliability / random vibration to make risk assessments
  - Use Bayesian approach to quantify confidence in these assessments

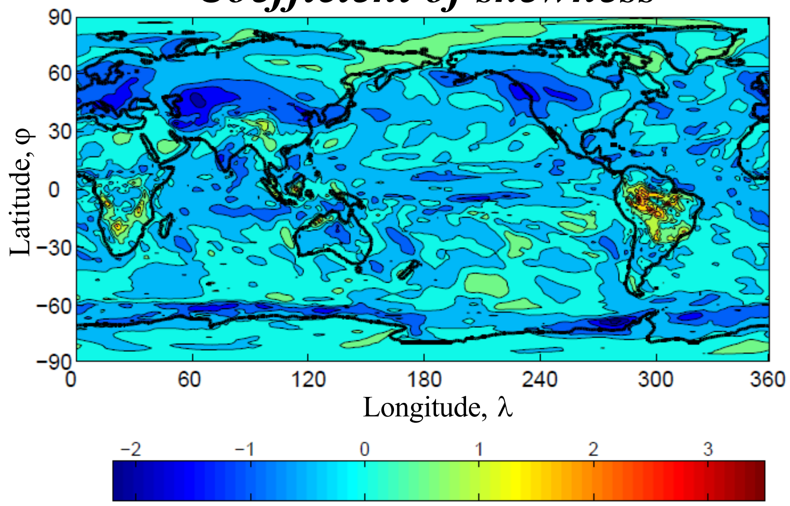
A Translation Random Field Model for Climate Variables

- Translation random field – a stochastic reduced-order model for climate variables
  - Examples: Temperature, precipitation flux, total cloud fraction
- Objective: utilize the random field to make predictions about the probability of certain climate events of interest
  - Calibrate random field to available CCSM output data
  - Match covariance function, any higher-order statistics (e.g., skewness) and marginal distribution
  - Bayesian framework facilitates quantitative measure of confidence on model predictions
- Approach
  - Karhunen-Loéve representations for second-moment properties
  - Monte Carlo simulation on reduced-order model for prediction and confidence

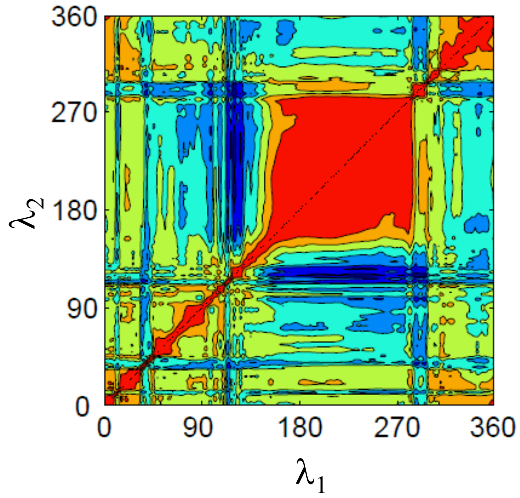
Calibration to CCSM 3.0 Surface Temperature Predictions

- Observe non-zero skewness and long-range dependence

Coefficient of skewness



Covariance function at equator



- Any time-dependence has thus far been ignored
- Power-law distributions ("heavy tails")

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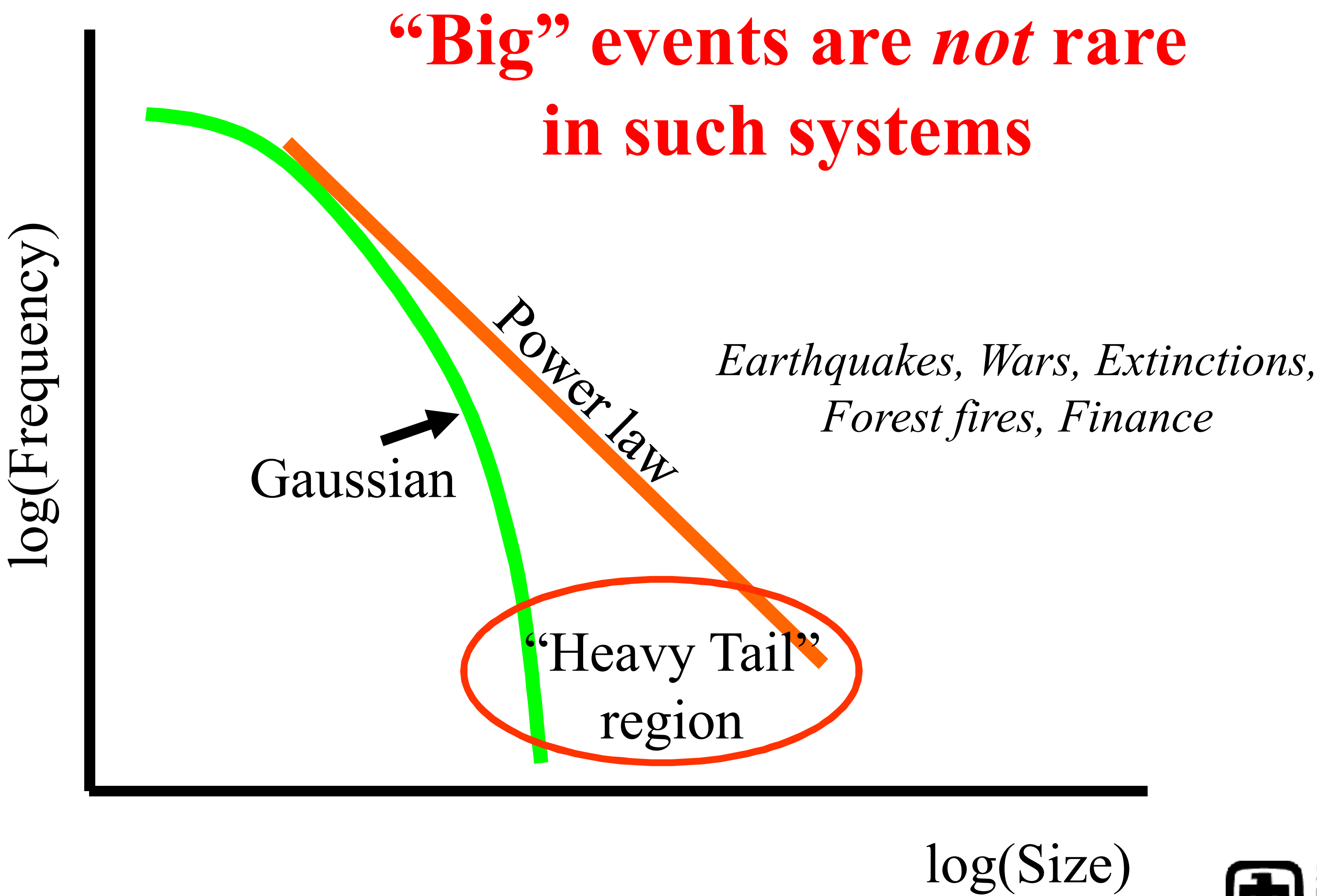
More on “Heavy Tails”



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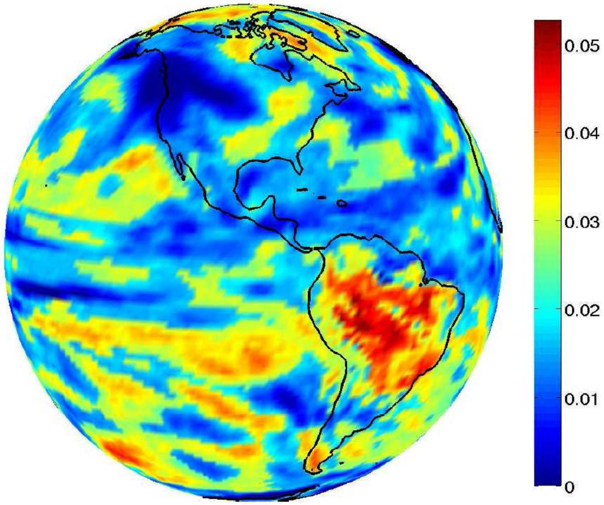
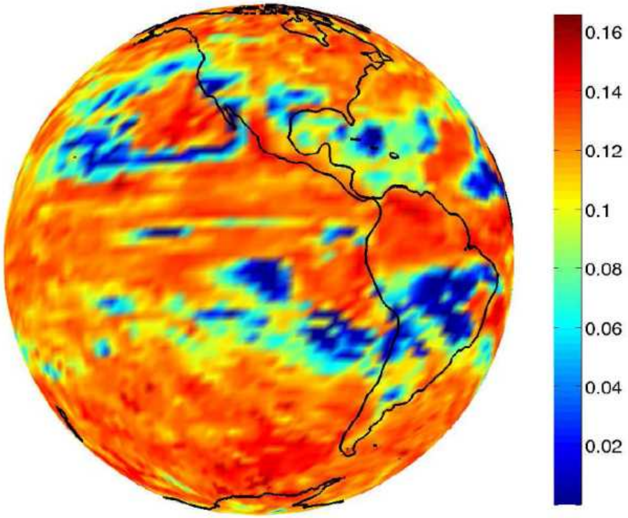
Use of Calibrated Random Field Model for Prediction

$$p(\boldsymbol{\theta}) = \Pr\left(h(A(\mathbf{u})) \in S \mid \boldsymbol{\theta}\right) = \int_S h(a)\, f_A(a|\boldsymbol{\theta})\, \mathrm{d}a$$

Probability of exceeding "2-σ" level of surface air temperature



Probability of not meeting "1-σ" level of precipitation rate



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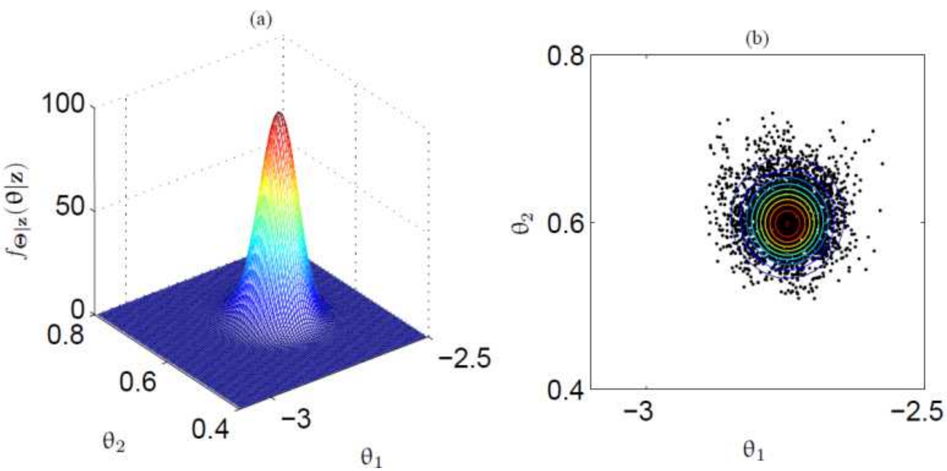
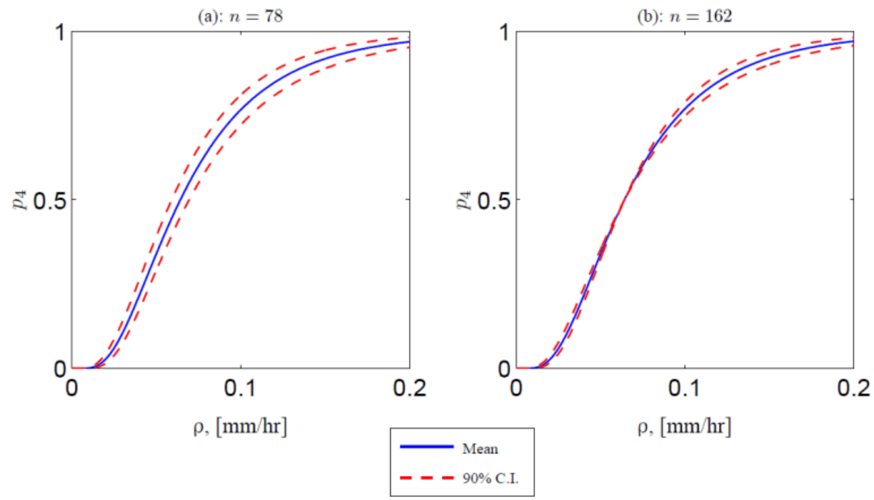
Quantify Prediction Confidence Using Bayesian Credible Sets

$$1 - \alpha \leq \int_{\{\boldsymbol{\theta} \in D_{\boldsymbol{\Theta}} \,:\, p(\boldsymbol{\theta}) \in C\}} p(\boldsymbol{\theta})\, f_{\boldsymbol{\Theta}|\mathbf{z}}(\boldsymbol{\theta}|\mathbf{z}) \mathrm{d}\boldsymbol{\theta}$$

Bayesian prior-posterior analysis of model parameters



Probability law of precipitation rate near Albuquerque with 90% Bayesian credible intervals



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Summary

- Work at Sandia can be very diverse
- Application #1: MEMS Dynamics
  - The "Very Small"
  - Mix of computer modeling and experimental work
  - Robust design problem with real customer focus
- Application #2: Climate Modeling
  - The "Very Large"
  - Research focus