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# Hybrid X-pinch with the SPHINX Accelerator

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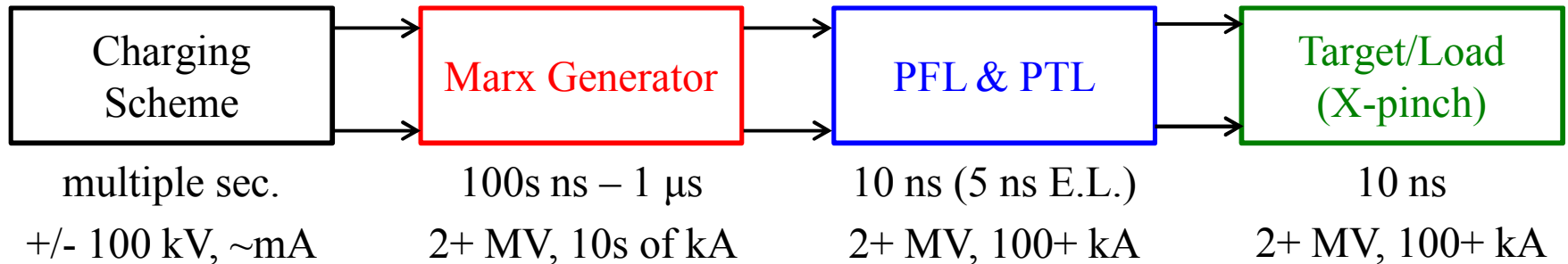
# Project Tasks

- Use the SPHINX system to generate x-rays using an x-pinch radiation source
  - Primarily intent is for micro-channel plate (MCP), and other x-ray diagnostic calibration

# Accomplishments

- Developed a model for SPHINX completed initial SPHINX characterization (First shot 07/29/2013)
- Completed initial calculations to determine the viability of an X-pinch (XP) or Hybrid X-pinch (HXP) on SPHINX
- Worked with Andy Biller developing a complete design for the XP or HXP to interface with SPHINX
- Using ALEGRA, performed initial 2D cylindrical radiative MHD simulations of the HXP

# SPHINX Pulsed Power

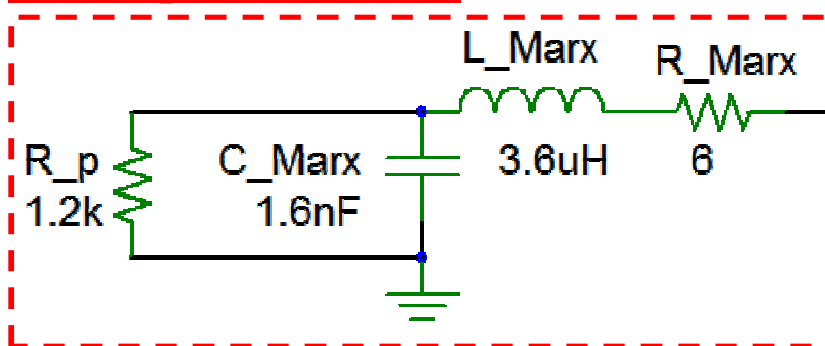


## ■ Marx

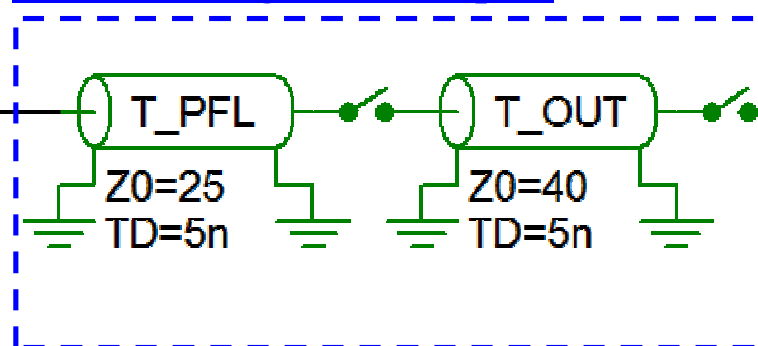
- 24 stages
- 100 kV/stage
- 40 nF/stage

- Pulse Forming Line (PFL) and Pulse Transport Line (PTL)
  - Transformer oil based line
  - 25 Ohm PFL, 40 Ohm PTL

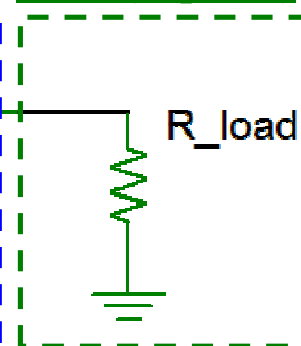
### Marx Equivalent Circuit



### Pulse Forming and Transport

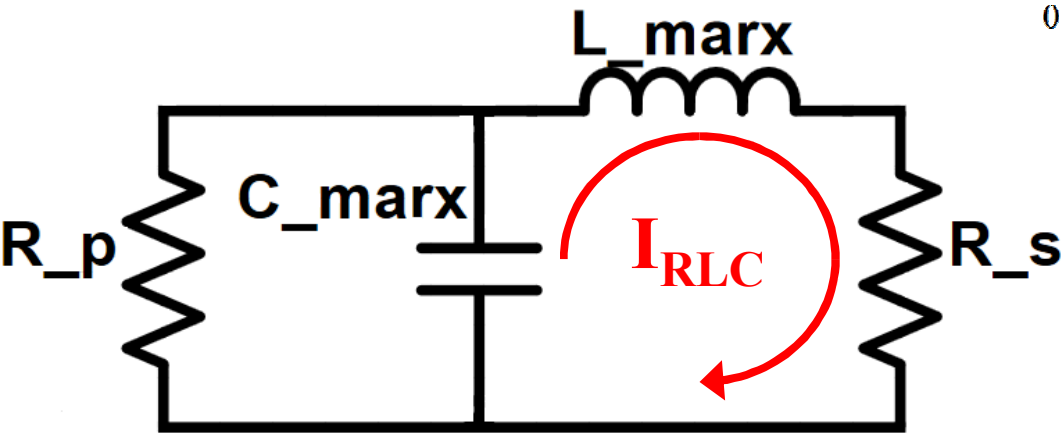
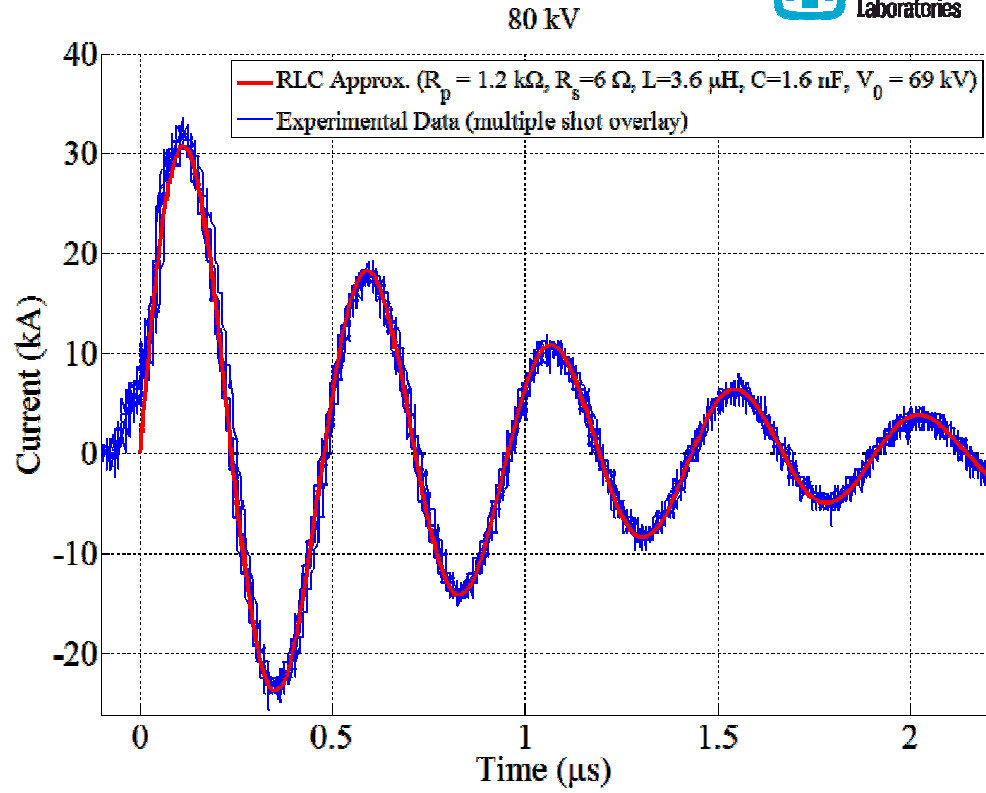


### Load (X-pinch)



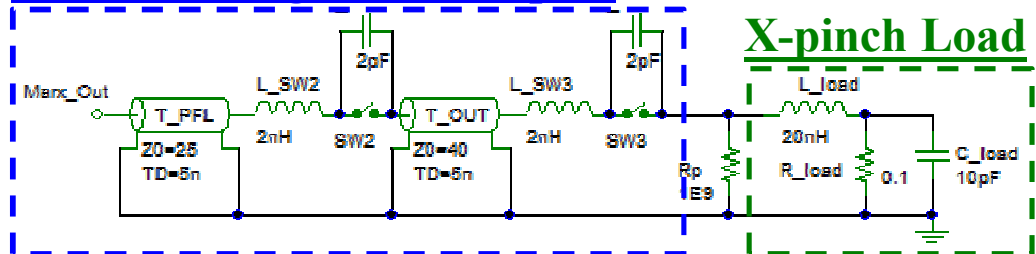
# SPHINX Marx

- Marx Generator
  - 24 stages
  - 100 kV/stage
  - 40 nF/stage
- Voltage adder
  - $V_{\text{out}} = N \cdot V_{\text{charge}}$  ( $N$  stages)
- Approximated by an RLC circuit

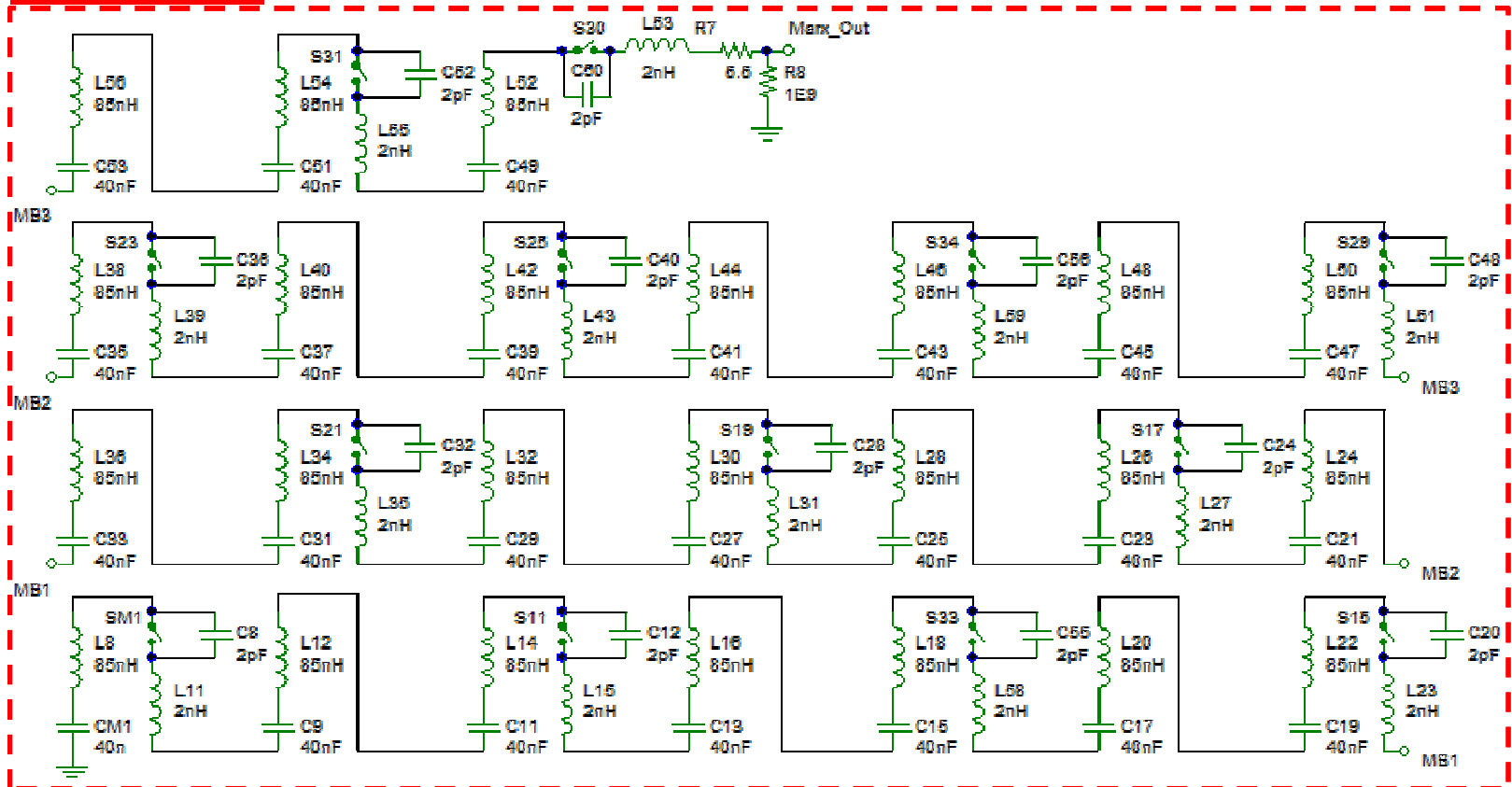


# SPHINX Schematic with Parasitics

## Pulse Forming and Transport



## Marx Circuit



# SPHINX Modeling

## Matched load

- Correctly calculates 60 kA, 10 ns FWHM

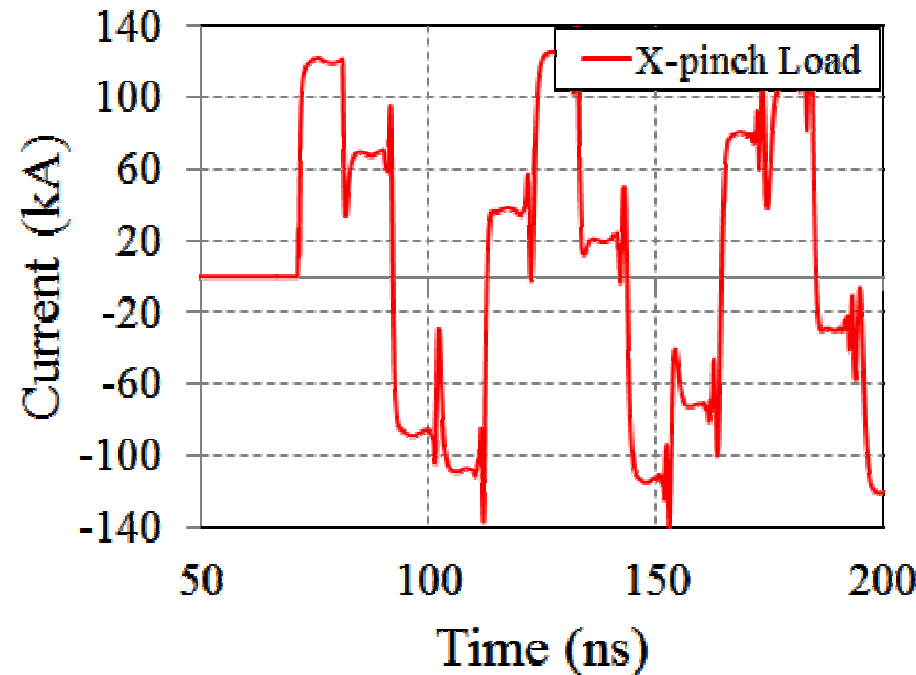
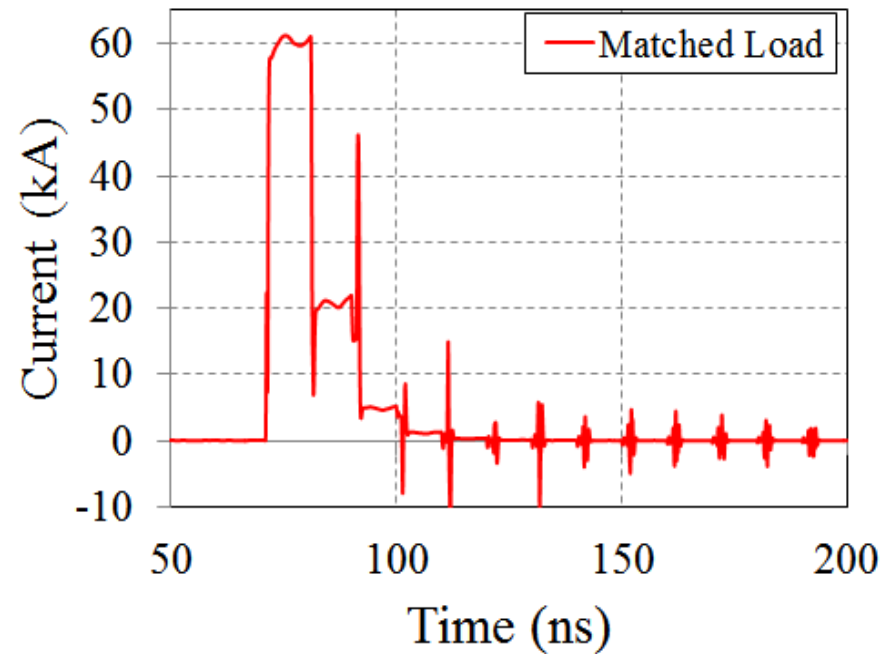
$$I_{\max(\text{matched})} = \frac{V_{T\text{-line}}}{Z_{T\text{-line}}} = 60\text{kA}$$

## X-pinch load

- Predicts 120 kA pulse, with 10 ns FWHM

$$I_{\max(\text{short})} = \frac{2 \cdot V_{T\text{-line}}}{Z_{T\text{-line}}} = 120\text{kA}$$

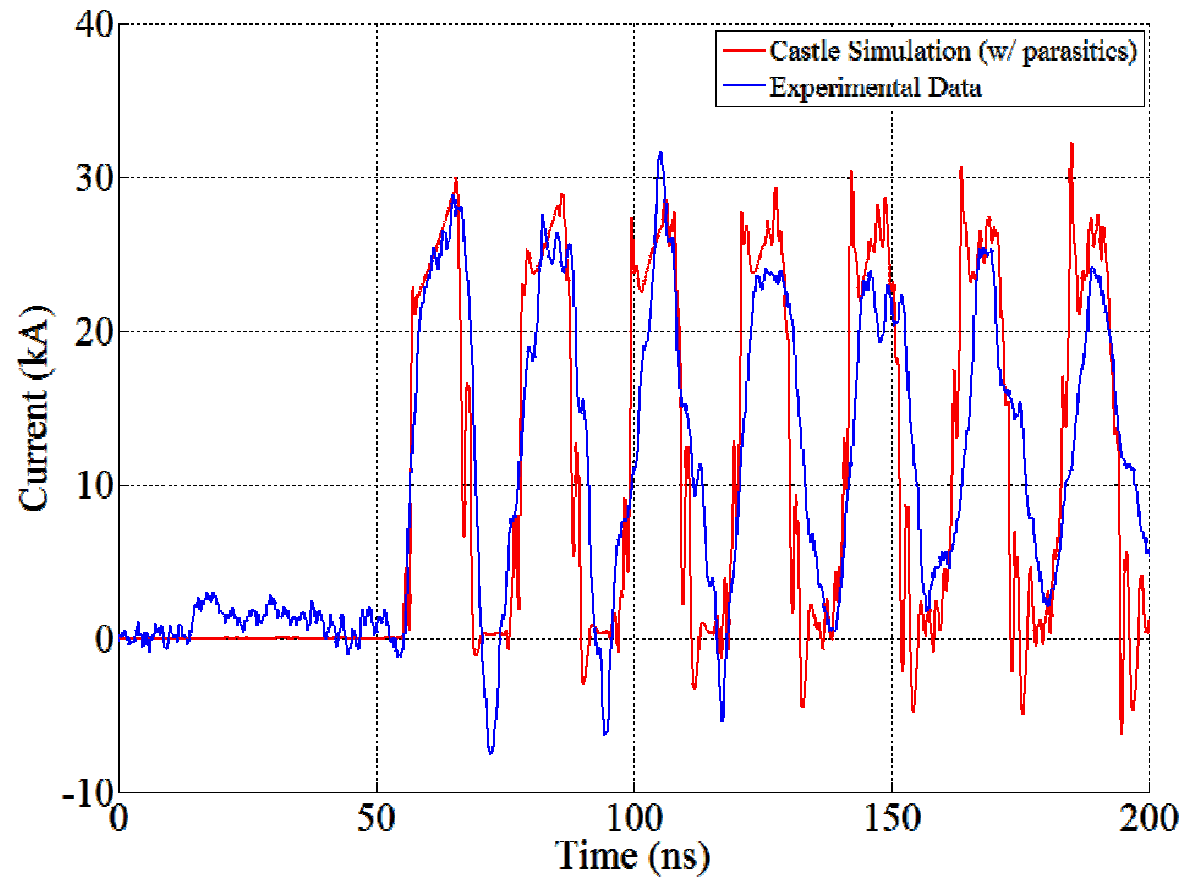
- Significant reflections predicted
  - Expected in simulation
  - Should be less pronounced in the experiments
  - Some ringing in marx capacitors



# SPHINX Modeling

## Complete System Shots

- +/- 60 kV marx charge
- Fired into a shorted 60 Ohm Vacuum Transmission Line (~MITL)
- PFL gap and pulse sharpening switch were set at 1.1 MV and 70 kV respectively (DC)
- Results demonstrate reasonable agreement with Castle circuit simulation software predictions.



# SPHINX X-pinch Preliminary Evaluation

- Current action at x-ray burst

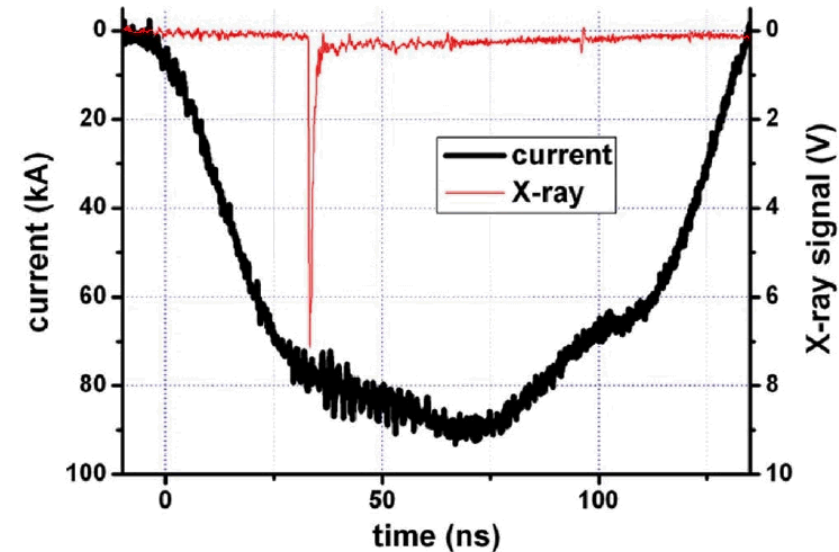
$$h_{x-ray} = \frac{1}{A_0^2} \int_0^{t_{x-ray}} I^2(t) dt$$

- Magnetic pressure

$$P_B \approx \frac{\mu_0 I^2}{8\pi^2 r^2}$$

- Current rise rate

- 25-100 kA/ns
- Above the empirical 1 kA/ns requirement [1]



Typical current and x-ray response [3]

Table 1: Comparison of proposed system with existing x-pinch systems (approximated from published data).

	Wire diameter	$h_{x-ray}$ (A <sup>2</sup> ·sec/m <sup>4</sup> )	$P_B$ (N/m <sup>2</sup> )
Kalantar & Hammer [2]	50 μm (Al)	$1.32 \cdot 10^{20}$	$1.95 \cdot 10^{11}$
R. Zhang et al. [3]	13 μm (Mo)	$4.24 \cdot 10^{21}$	$7.63 \cdot 10^{11}$
D. Haas et al. [4]	13 μm (Mo)	$1.77 \cdot 10^{22}$	$6.03 \cdot 10^{11}$
SPHINX	13 μm	$6.87 \cdot 10^{21}$	$1.14 \cdot 10^{12}$

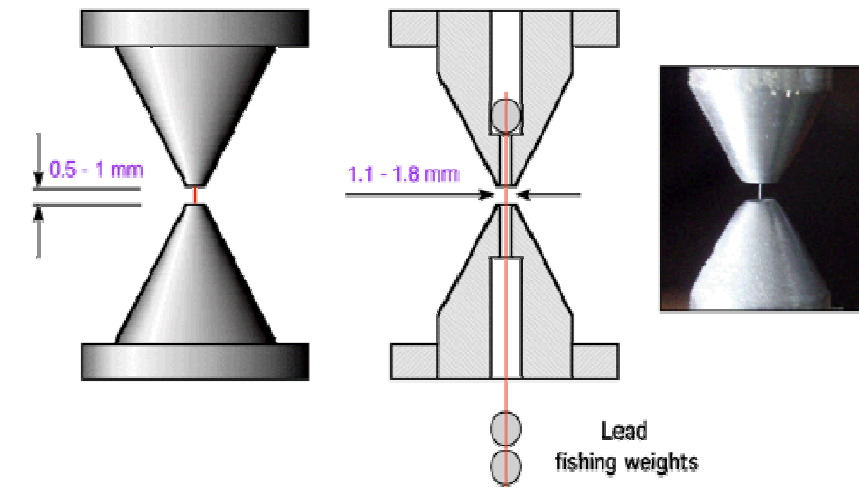
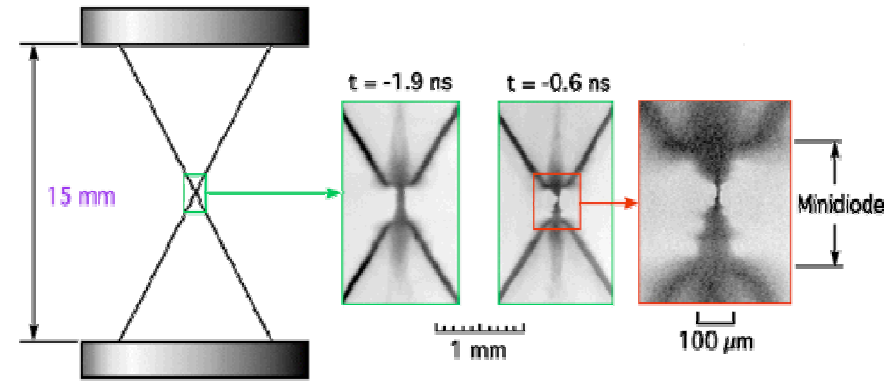
# Hybrid X-pinch

- Shelkovenko [5] and Chittenden [6] show that radiation is produced by spot formations in a “micro-z” region at the cross point.
- Recent experiments show that similar performance can be obtained from the “Hybrid X-pinch” geometry [7].
  - Increase load manufacturing reproducibility
  - Simplifies geometry for 2D axisymmetric simulation

## Similarity Scaling [8-10]

$$\Pi = \frac{\mu_0 I^2 \tau^2}{4\pi^2 \rho r^4}$$

- Suggests the use of small wires (5 μm or less)
- Shelkovenko recommends we look at 12.7 μm or larger [11]



Hybrid X-pinch geometry [7]

# MHD and ALEGRA

- Magnetohydrodynamics (MHD)
  - A combination of Navier-Stokes, Maxwell's Equations, Joule heating, Thermal conduction, Radiation etc.
- Using ALEGRA 2-D, cylindrical axisymmetric (r-z) simulations ( $\partial/\partial\phi=0$ ) conducted

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0$$

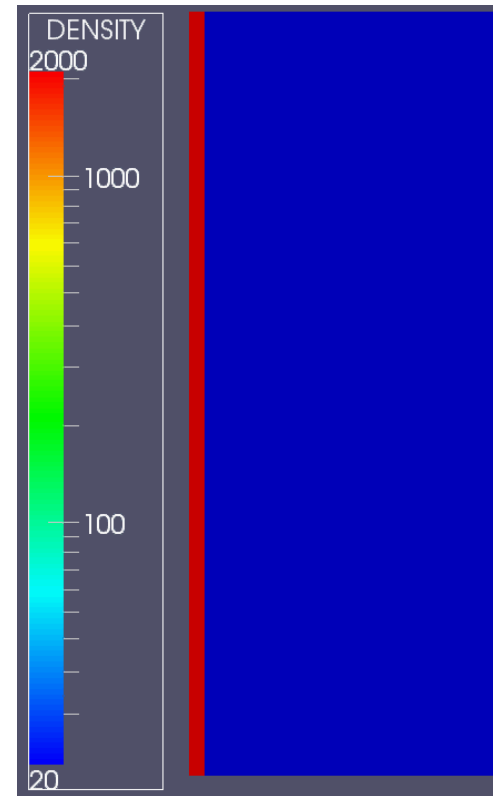
$$\rho \left[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = -\nabla p + \vec{J} \times \vec{B}$$

$$\frac{\partial}{\partial t} (\rho \varepsilon) = -\nabla \cdot (\rho \varepsilon \vec{v}) - p \nabla \cdot \vec{v} + \nabla \cdot \vec{q} + \frac{\vec{J} \cdot \vec{J}}{\sigma}$$

- $\vec{J} \times \vec{B}$  forces (Lorentz forces)
  - Assuming complete magnetic diffusion

$$\vec{J} = \frac{I}{\pi r^2} (\hat{e}_z) \quad \vec{B} = \frac{\mu_0 I}{2\pi r} (\hat{e}_\phi)$$

$$\vec{J} \times \vec{B} = \frac{\mu_0 I^2}{2\pi^2 r^3} (-\hat{e}_r)$$



# Instability and Hot Spot Formation

- Radius perturbation

$$r = r_0 + \xi(r, \varphi, z)$$

- Where  $\xi$  describes perturbations, or inhomogeneities in the wire radius.

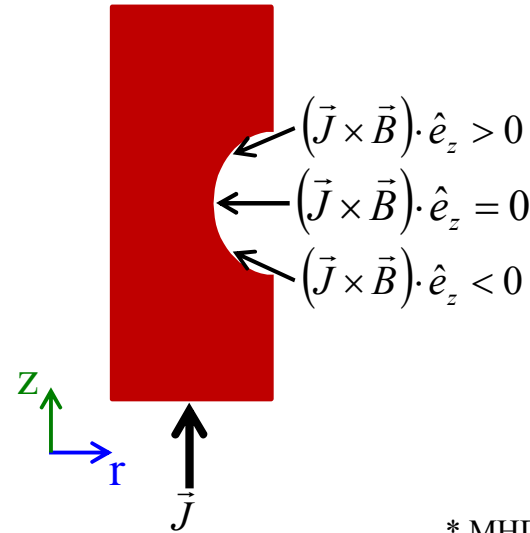
$$\xi = \xi(r) e^{im\varphi + ikz}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} = \left( -\frac{\partial B_\varphi}{\partial z} \hat{e}_r + \frac{1}{r} \frac{\partial(r B_\varphi)}{\partial z} \hat{e}_z \right)$$

$$\vec{J} \times \vec{B} = (-J_z B_\varphi) \hat{e}_r + (J_r B_\varphi) \hat{e}_z$$

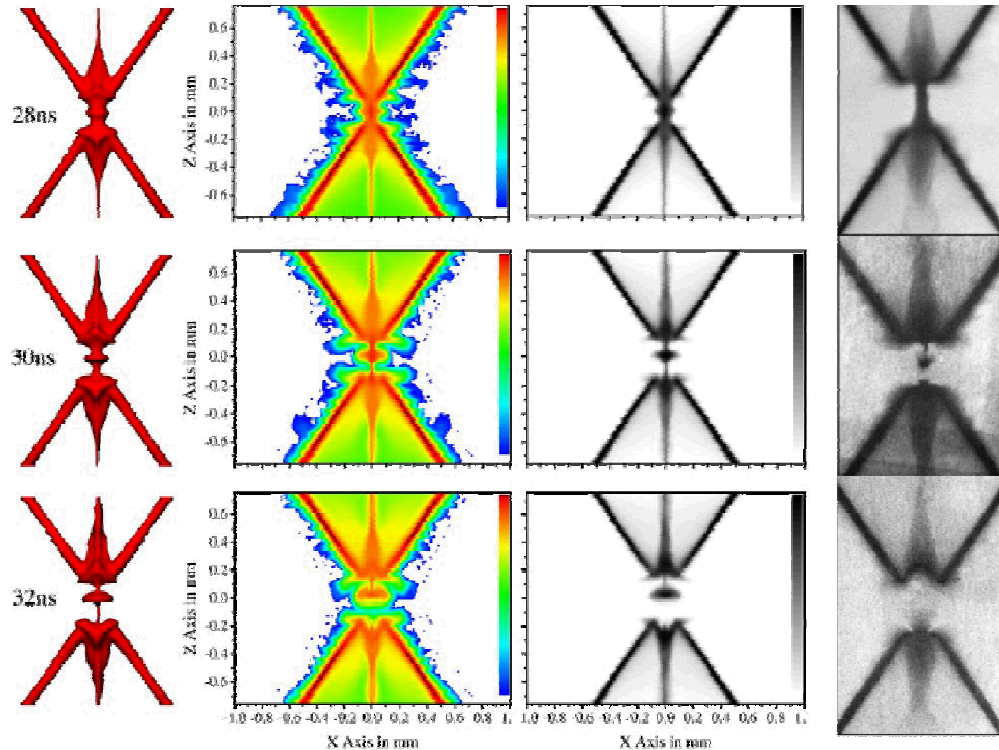
- Axial Mass Transport (see Chittenden et al. [6])

$$(\vec{J} \times \vec{B}) \cdot \hat{e}_z \propto J_r B_\varphi$$



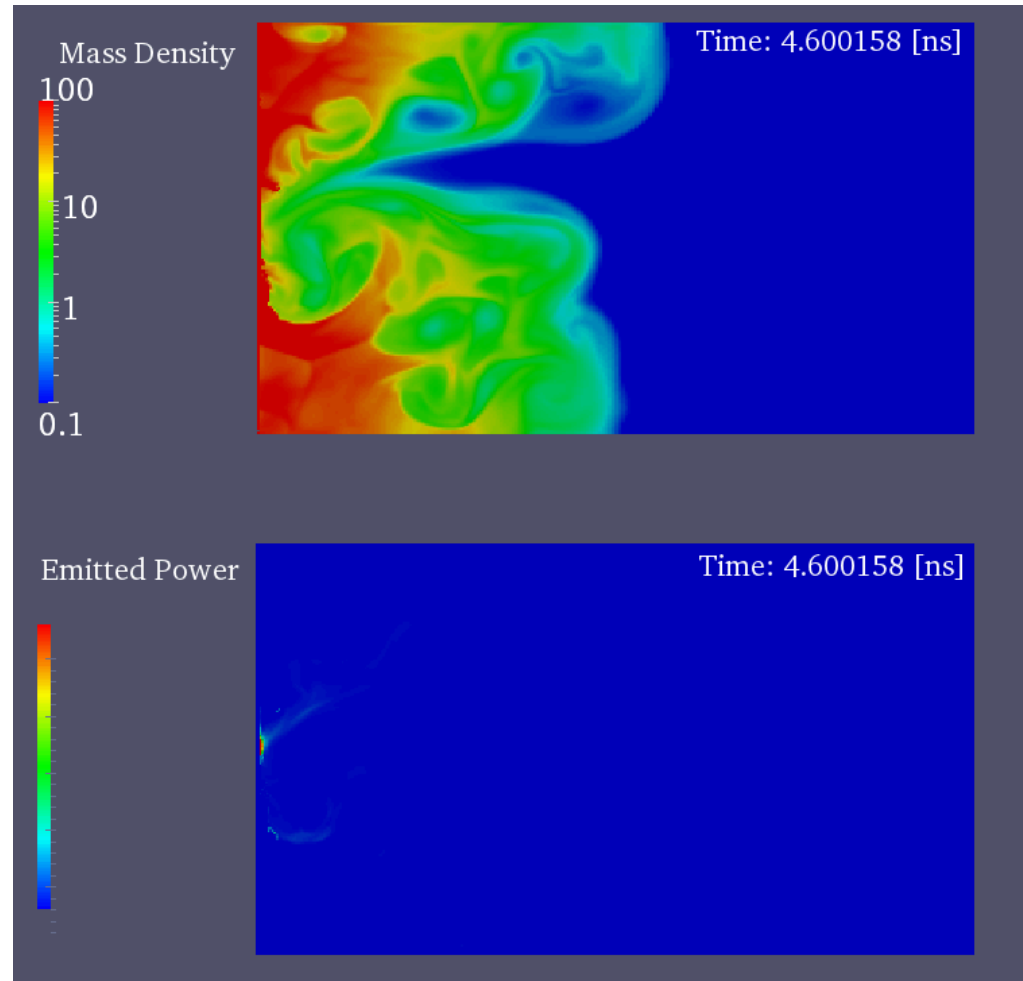
\* MHD simulation Chittenden et al. [6]

\*\* Experimental radiographs Shelkovenko et al. [5]



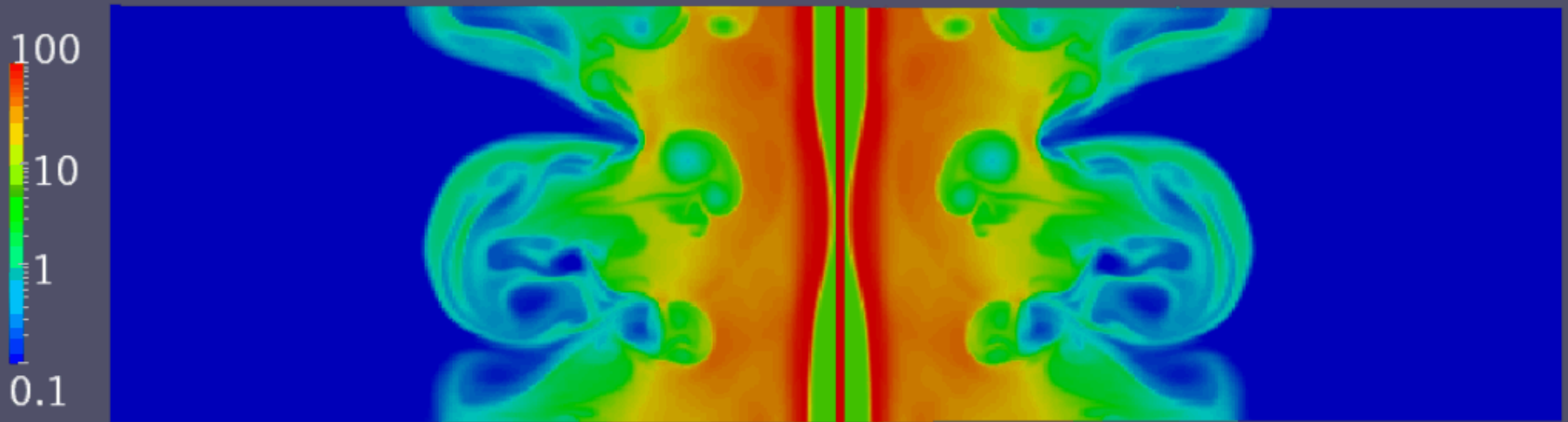
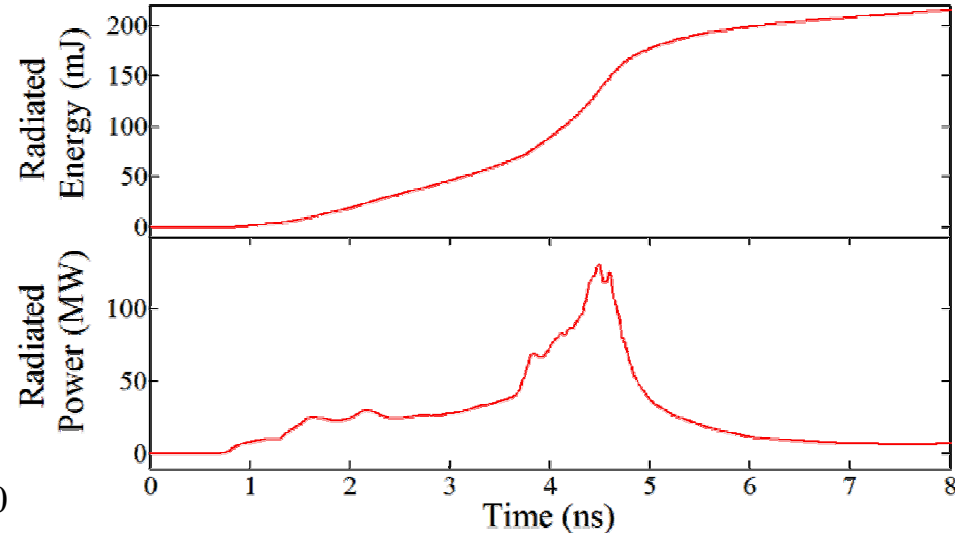
# ALEGRA Simulations

- “Engineered Solutions”
  - Several ad hoc mechanisms in place in the code for stability (density floors etc.)
    - ALEGRA has known complications with fine meshes at material-vacuum boundaries.
  - Kramer’s opacity is used, with very little justification for the input parameters
    - Difficult to obtain opacity data for high  $Z$  ( $Z > 30$ ) materials
    - Simulation is sensitive to Kramer’s parameters
    - Kramer’s is an extremely crude oversimplification
- Not a sufficient predictive model, yet.



# ALEGRA Simulations

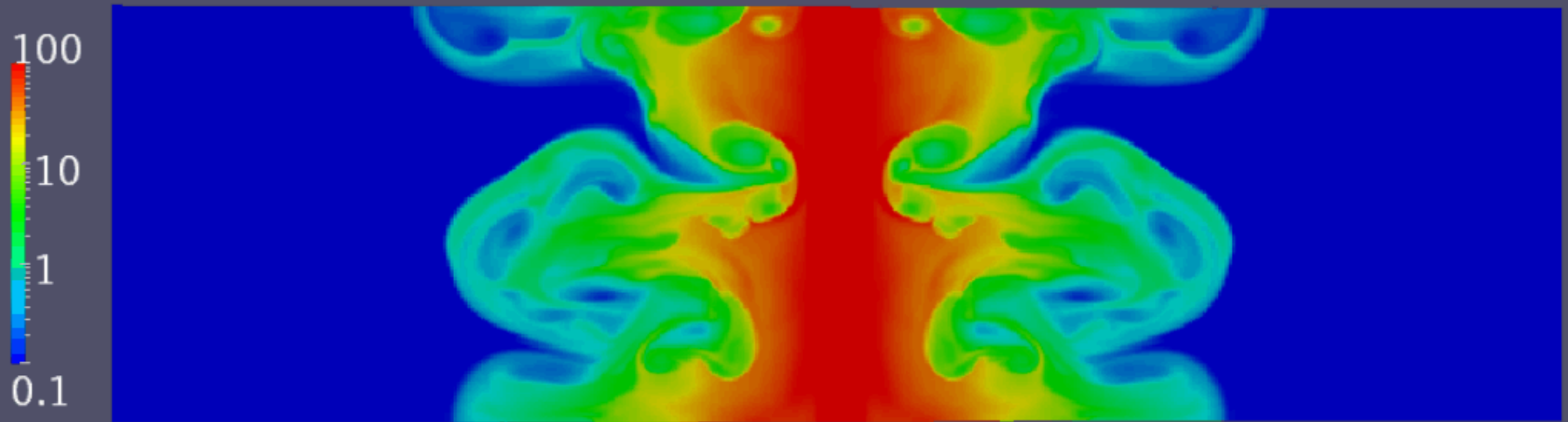
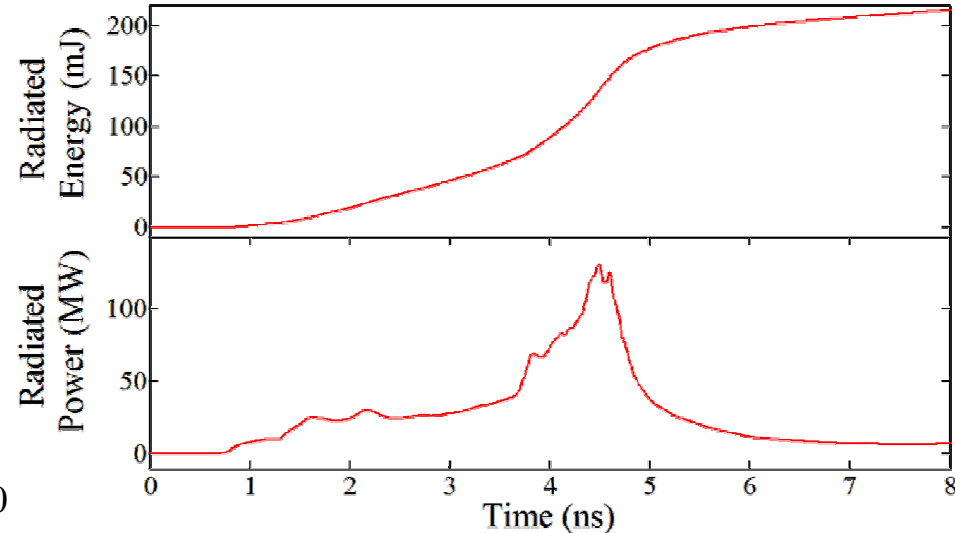
- 7  $\mu\text{m}$  OD tungsten wire
  - Energy and power yields consistent with experimental results from systems of comparable energy [10]
  - Radiation source;  $<5 \mu\text{m}$ , 500eV - 1 keV
- **LIMITED ACCURACY**
  - Mass on axis remains unrealistically high
    - Partially a limitation of 2D treatment
    - Extreme impact on simulation behavior
  - Instabilities are not seeded
  - Kramer's opacity is not accurate here
  - SPHINX may not generate the 90 kA, 2 ns risetime, 10 ns FWHM pulse (high inductance load)



Time: 3.500373 [ns]

# ALEGRA Simulations

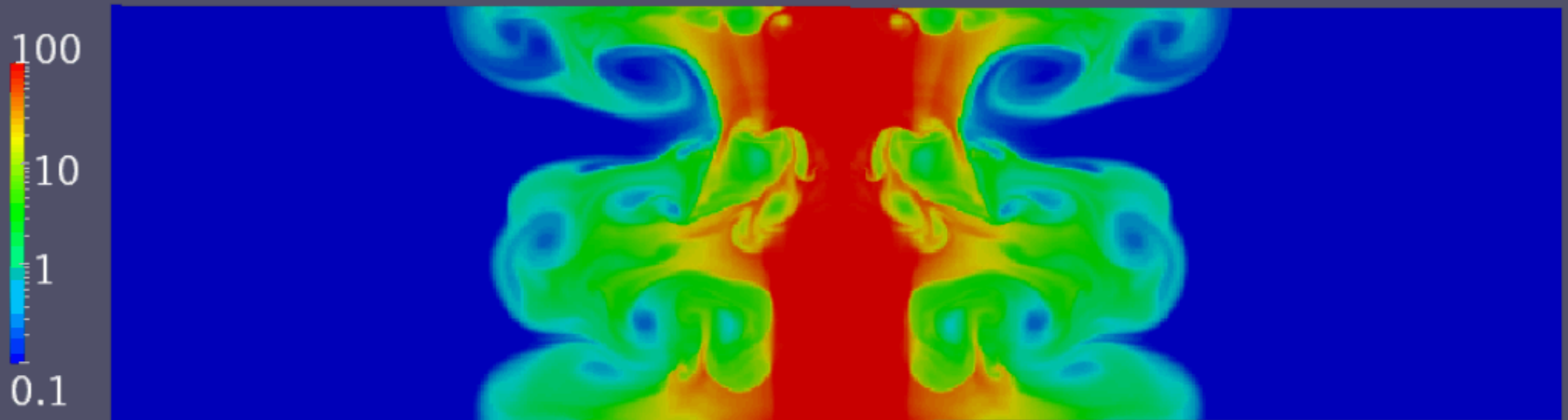
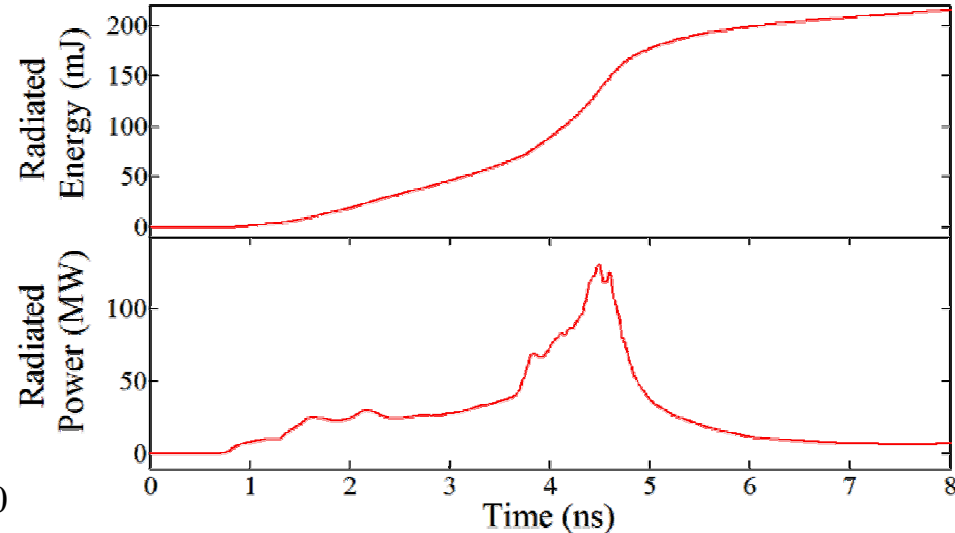
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Time: 3.750424 [ns]

# ALEGRA Simulations

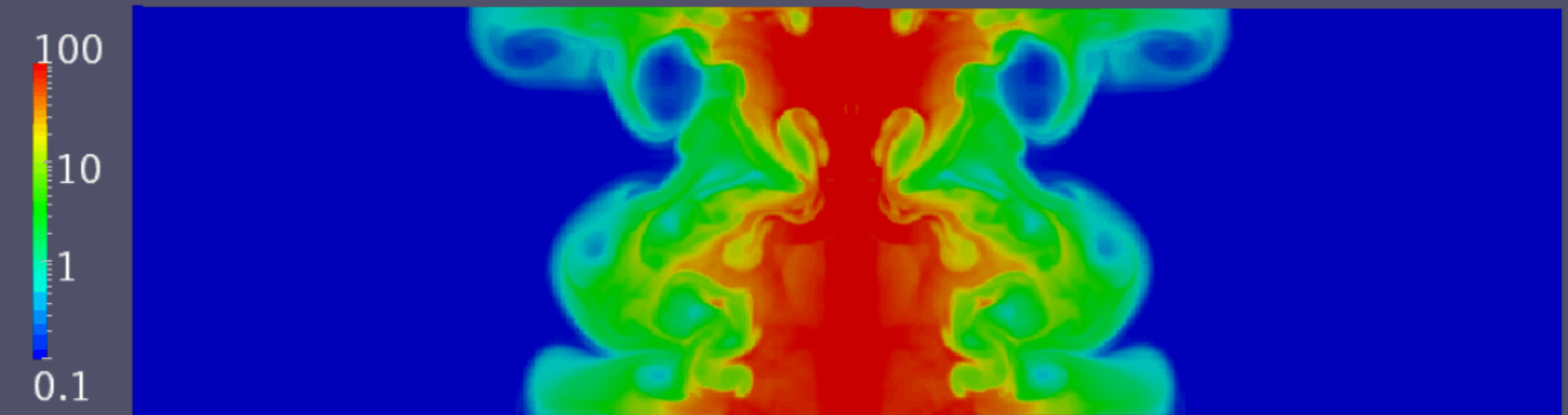
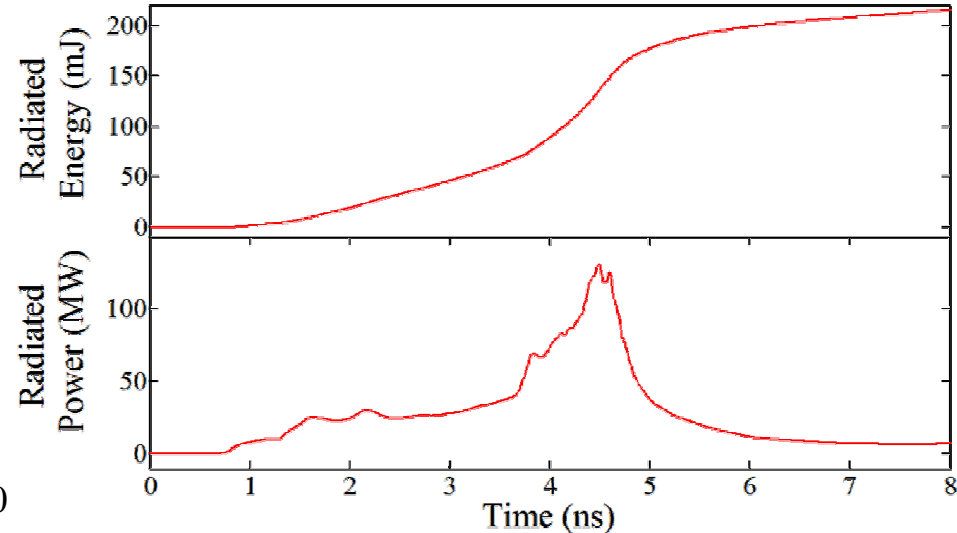
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Time: 4.000154 [ns]

# ALEGRA Simulations

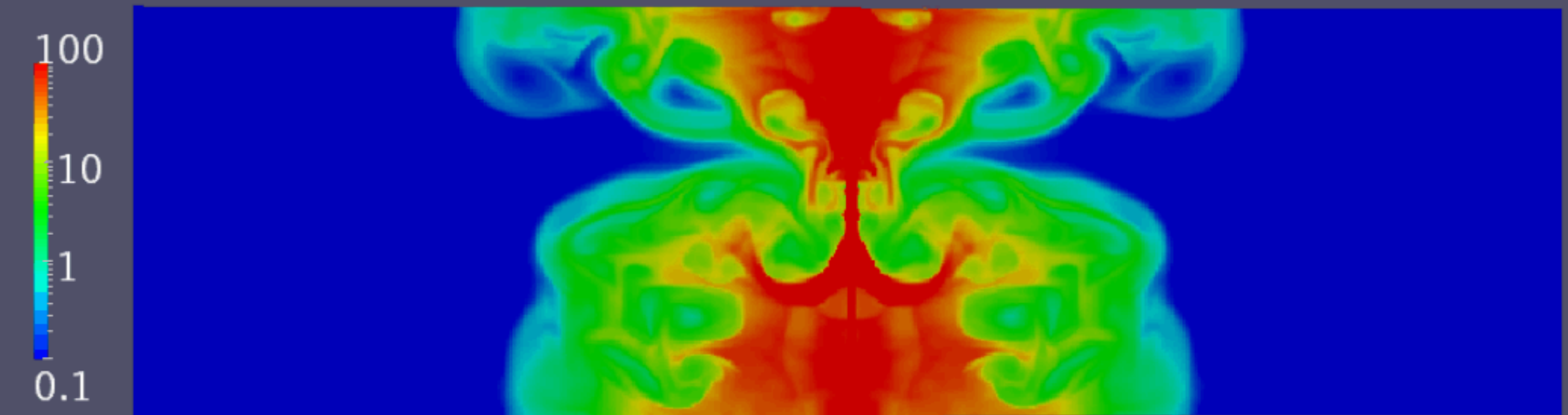
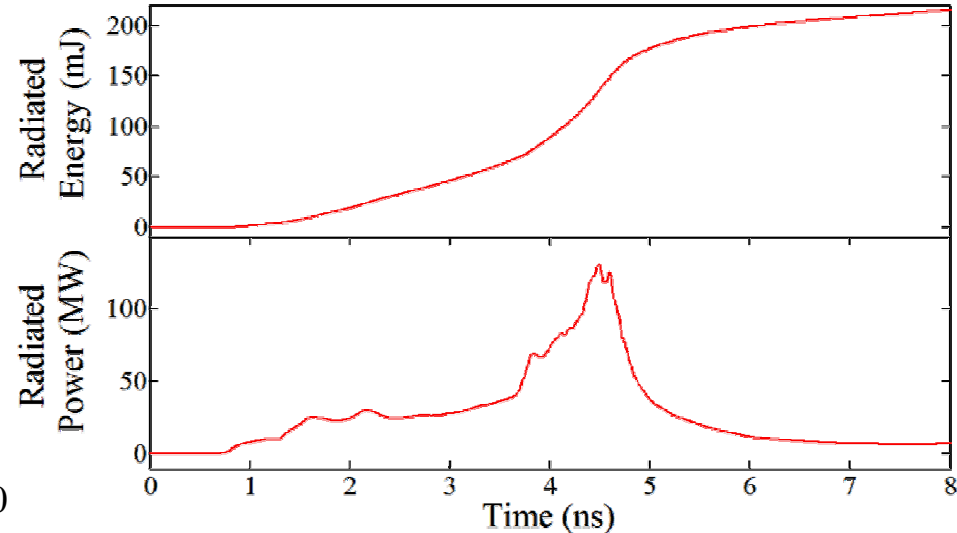
- 7  $\mu\text{m}$  OD tungsten wire
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  - Radiation source;  $<5 \mu\text{m}$ , 500eV - 1 keV
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Time: 4.250260 [ns]

# ALEGRA Simulations

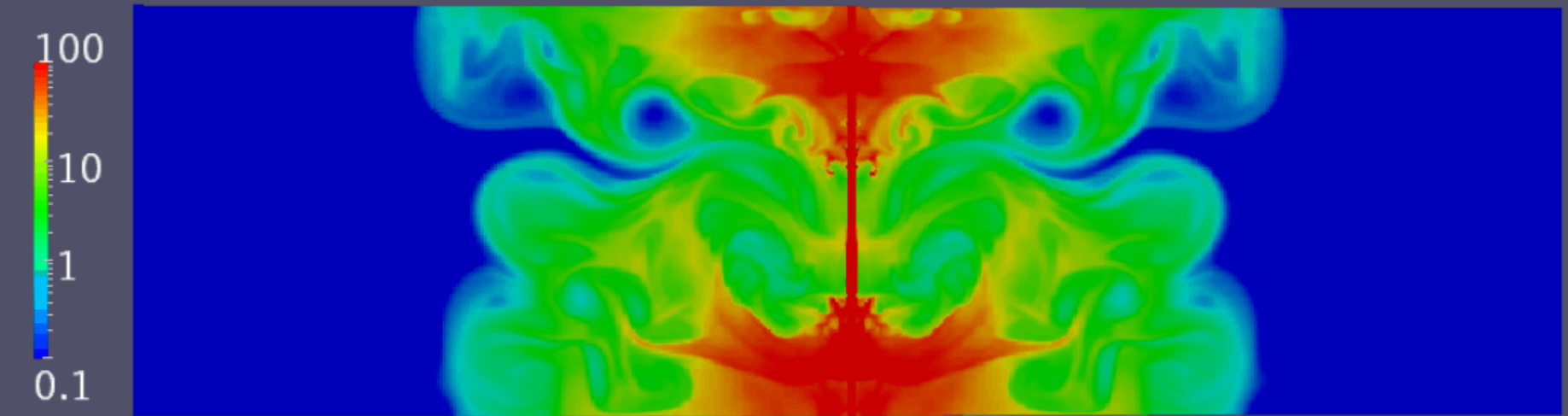
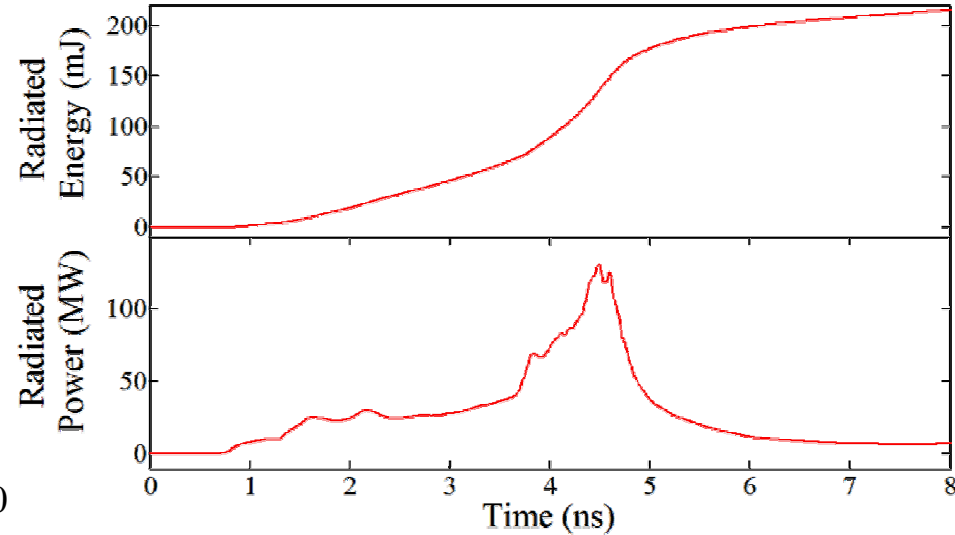
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Time: 4.500101 [ns]

# ALEGRA Simulations

- 7  $\mu\text{m}$  OD tungsten wire
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Time: 4.750249 [ns]

# References

- [1] T.A. Shelkovenko, et al. IEEE Trans. on Plasma Sci. 34, 2336 (2006).
- [2] D.H. Kalantar, D.A. Hammer, Rev. Sci. Instrum. 66, 779 (1995).
- [3] R. Zhang, et al. IEEE Trans. Plasma Science, Vol. 40, No. 12, Dec. (2012).
- [4] D. Haas, et al. Plasma Phys. Control. Fusion, 49 1151-1162 (2007).
- [5] T.A. Shelkovenko, et al. Phys. Plasmas, 8, 4, 2001.
- [6] J.P. Chittenden et al. Phys. Rev. Lett. 98, 2007.
- [7] T.A. Shelkovenko et al. Phys. Plasmas, 17, 112707, 2010.
- [8] D.D. Ryutov, M.S. Derzon, M.K. Matzen, Rev. Mod. Phys. 71, 1, 2000.
- [9] D.B. Sinars, et al. “6 MA X-pinch Scaling Experiments”, SAND-2009-3065C, 2009.
- [10] G.A. Mesyats et al. Journal of Experimental and Theoretical Physics, 111, 3, pp 363-370, 2010.

# Timescale of interest in HXPs

- System parameters
  - $j_{\max} \approx 5.9 \cdot 10^9 \text{ A/cm}^2 - 6.1 \cdot 10^{11} \text{ A/cm}^2$ 
    - Indicates instability formation through ablative processes (MRT? or high Alfvén velocity  $\rightarrow m=0$ )

Table 2: Characteristic timescales for phenomenon with tungsten (W) wire

wire diameter, tungsten (W)	magnetic diffusion time* $\tau_{B\text{-diff}} = \mu_0 \sigma r^2$	$m=0$ development time** $\tau_{MHD(m=0)} = \frac{r}{c_A} = \frac{2}{j_{\max}} \sqrt{\frac{\rho}{\mu_0}}$
5 $\mu\text{m}$	0.14 ns	0.041 ns
12.7 $\mu\text{m}$	0.96 ns	0.26 ns
25.4 $\mu\text{m}$	3.84 ns	1.05 ns
50.8 $\mu\text{m}$	15.4 ns	4.18 ns

\* H. Knoepfel, *Magnetic Fields, A Comprehensive Theoretical Treatise for Practical Use*, 2000.

\*\* V.I. Oreshkin, “Thermal instability during an electrical wire explosion”, *Phys. Plasmas* **15**, 092103 (2008).