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Characterization of Forward and Backward Flux Correlations in 1D Stochastic Media

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Overview

- **Used SCEPTRE* on Pegasus and Redsky to generate realizations of stochastic meshes**
- **Mined data at points throughout the mesh for forward and backward flux**
- **Discovered five points of interest that define extreme correlation cases in a given stochastic mesh**
 - **Points 1-4: Generated from maximum possible absorption and scattering in regions**
 - **Point 5: Generated as average path length decreases and mesh approaches homogeneity**

*Sandia's Computational Engine for Particle Transport for Radiation Effects

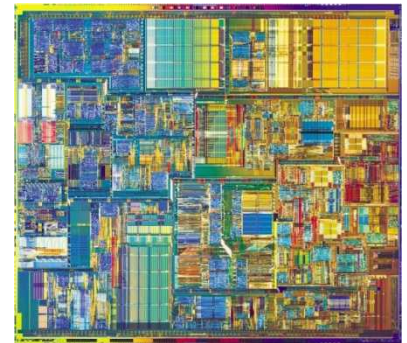
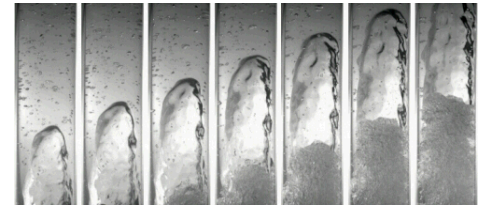
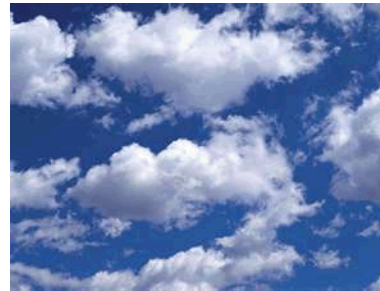
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Introduction – Stochastic Media

- Stochastic Media – Two or more materials mixed in a way that cannot easily be exactly modeled

- Compound mixtures
- Two-phase flow
- Small repetitive systems



- Meshes built using
 - average path lengths (λ)
 - and a probability distribution

$$\lambda_0 = 99/10$$

Examples

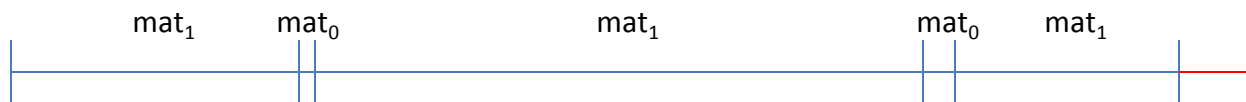
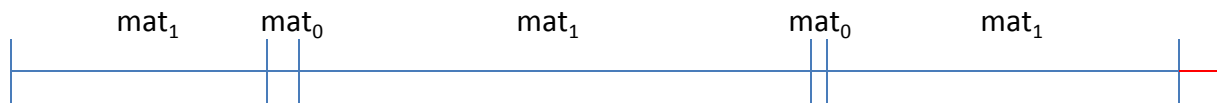
$$\lambda_1 = 11/10$$

Examples

Introduction – Building Stochastic Meshes

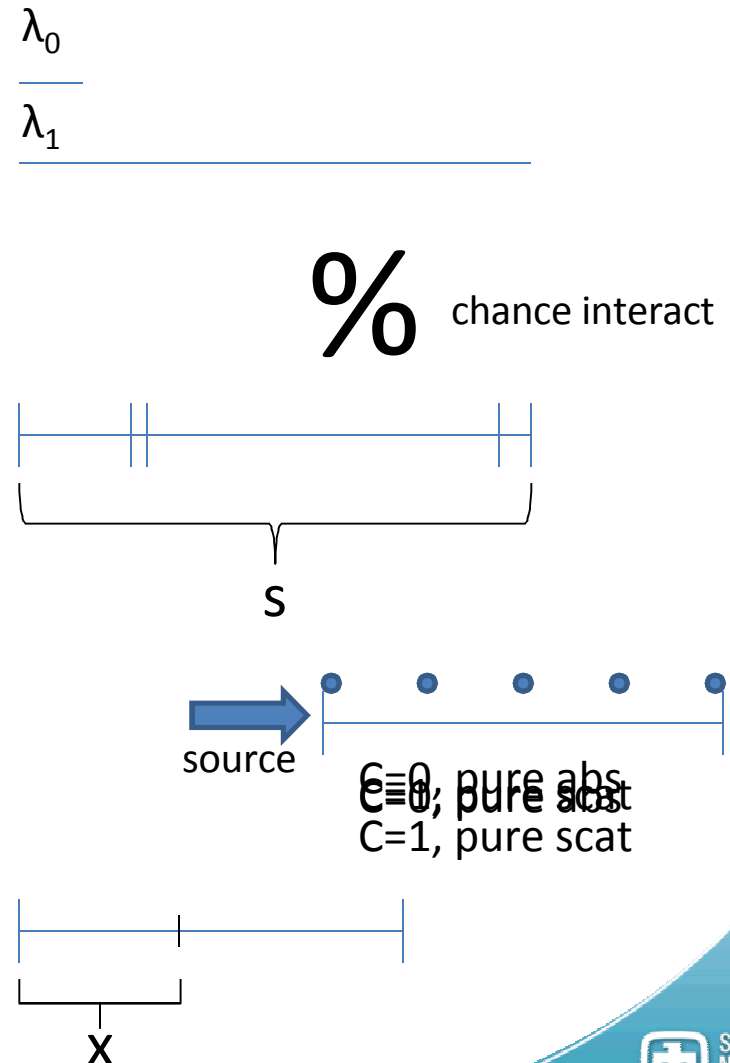
$$\lambda_0 = 99/10$$

$$\lambda_1 = 11/10$$



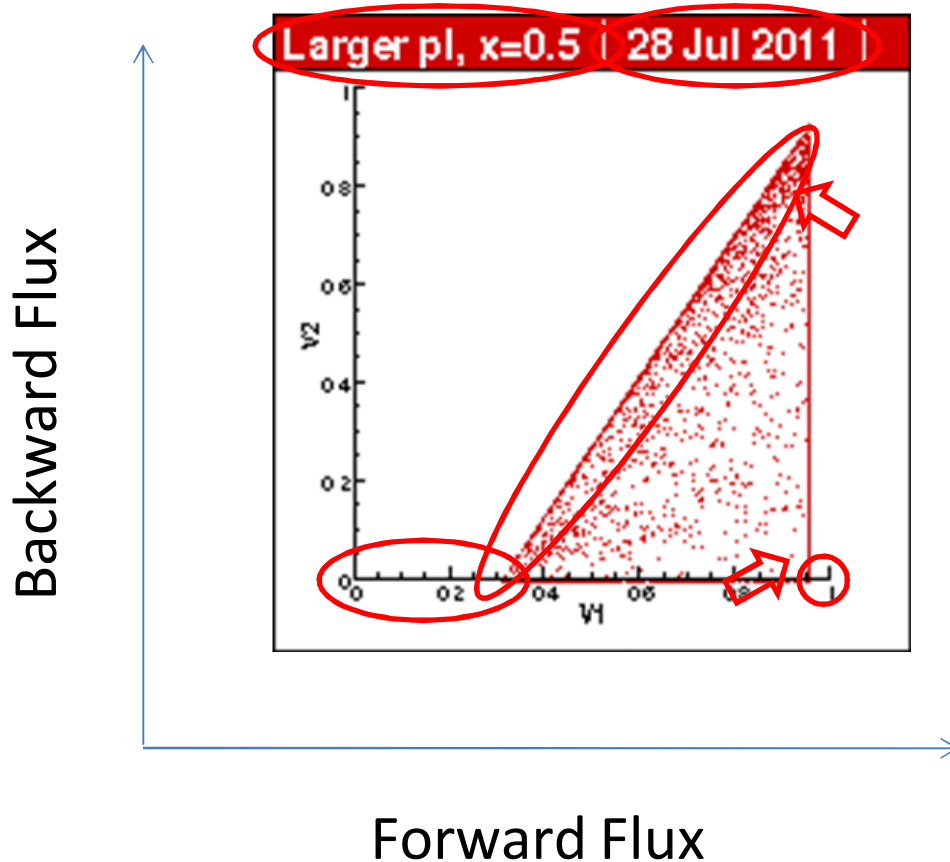
Introduction – Stochastic Mesh Parameters

Stochastic Parameters Reference	
λ	Average path length (of material segment)
σ	Cross-section (of interaction)
S	Total length (of mesh)
C	Scattering ratio (σ_s / σ_t , 0=pure abs, 1=pure scat)
X	Position in mesh (relative to left boundary)



Introduction – Correlation Plots

- Smaller λ later slides
- At $x=0.5$ (of $s=10$)
- Date I created this plot on



- Each dot represents the fluxes with a given mesh
- Example 1: Lots of absorbing
- Example 2: Lots of scattering past $x=0.5$
- At least 30% always gets here
- About 5% never gets here
- Must have forward to have backward

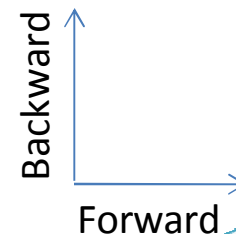
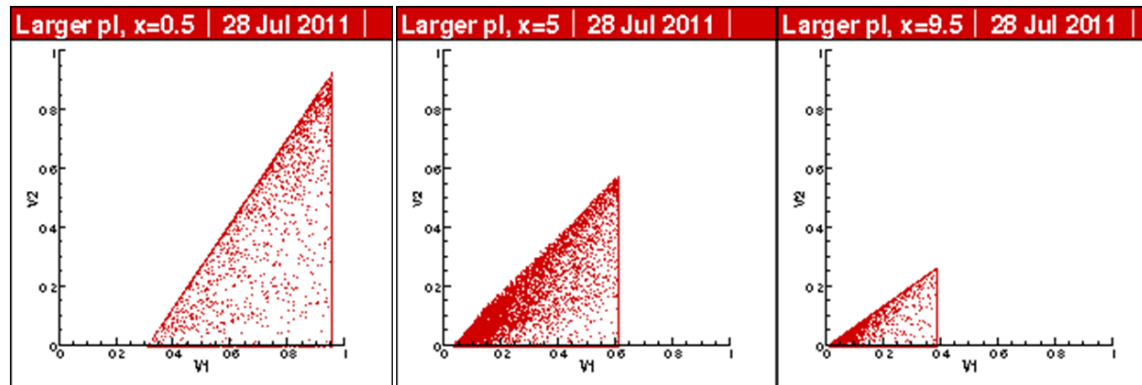


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Results & Analysis – Material Depth



As data is taken further through the mesh:

- there is a lower flux
- the backwards flux dies more than the forward flux

Mesh Parameters

$s = 10$

mat0

$\lambda_0 = 99/10$

$\sigma_0 = 10/99$

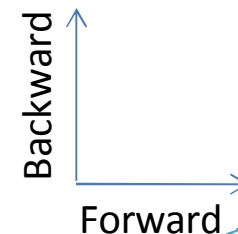
$c_0 = 0$ (abs)

mat1

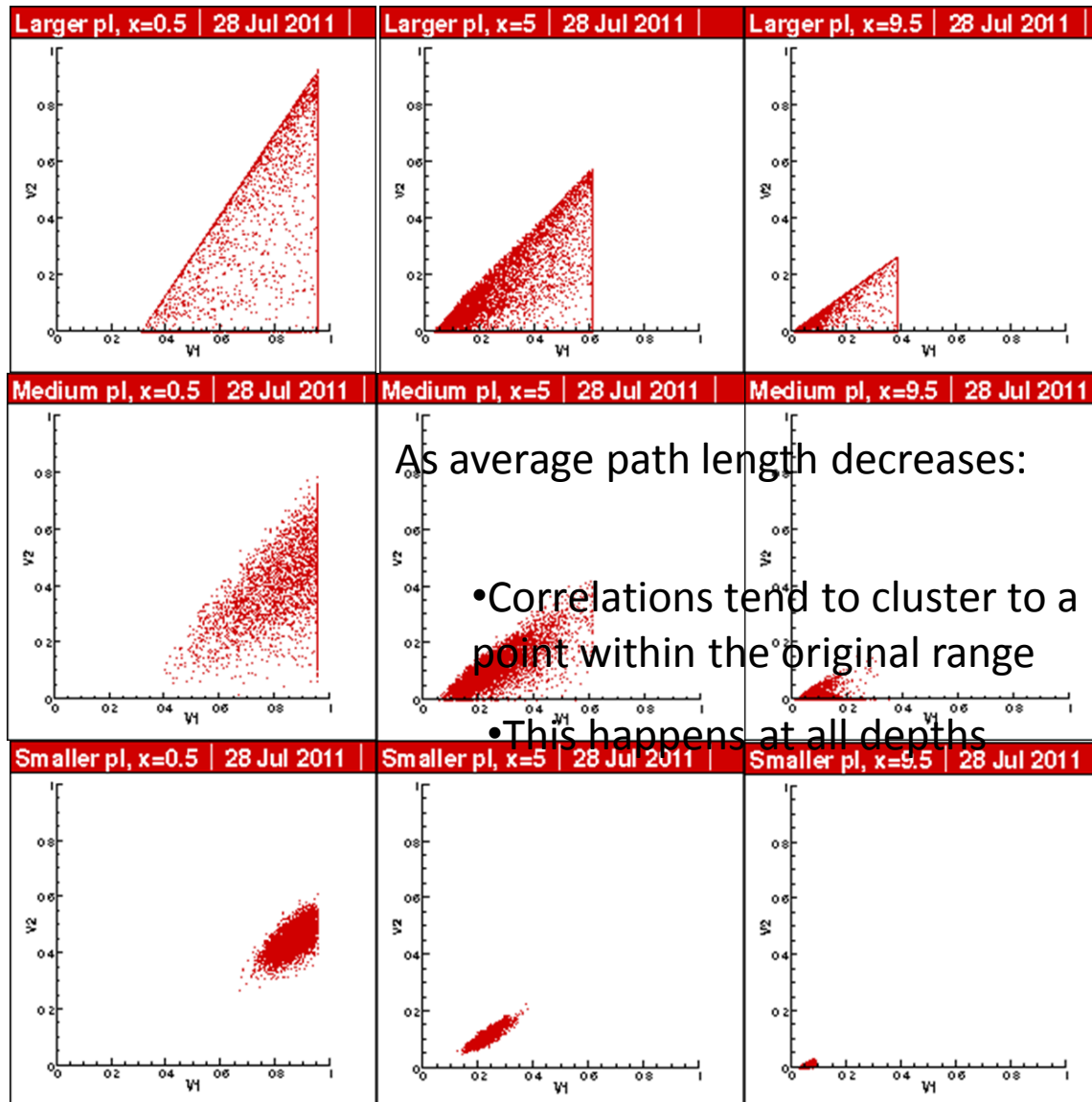
$\lambda_1 = 11/10$

$\sigma_1 = 100/11$

$c_1 = 1$ (scat)



Results & Analysis – Average Path Length



Mesh Parameters

$s = 10$

mat0

$\lambda_0 = \text{varies}$

$\sigma_0 = 10/99$

$c_0 = 0 \text{ (abs)}$

mat1

$\lambda_1 = \text{varies}$

$\sigma_1 = 100/11$

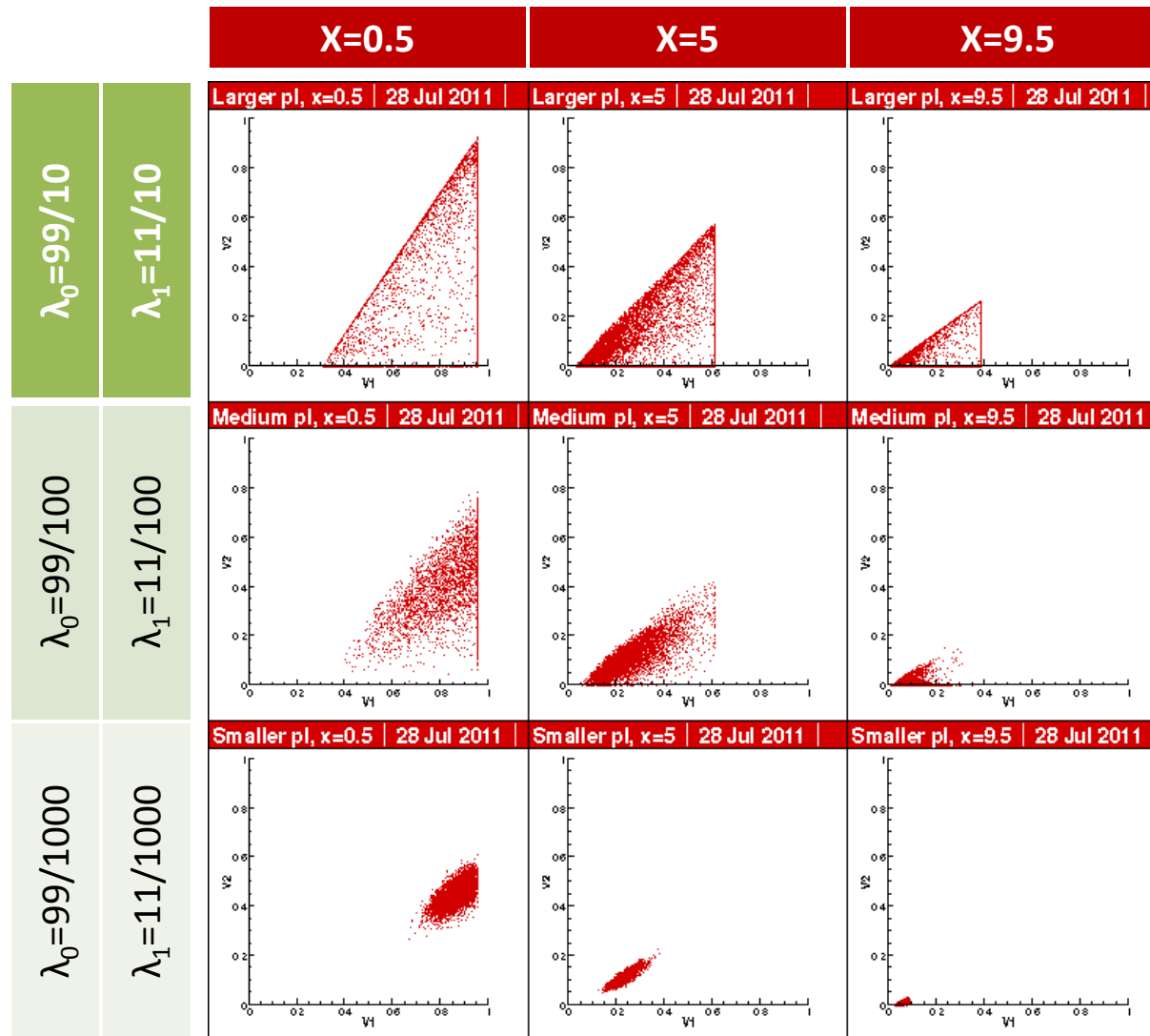
$c_1 = 1 \text{ (scat)}$

Backward
Forward

Results & Analysis – Material Depth and Length Effects Summarized

Depth in Mesh

Average Path Length



Mesh Parameters

$s=$ 10

mat0

$\lambda_0=$ varies

$\sigma_0=$ 10/99

$c_0=$ 0 (abs)

mat1

$\lambda_1=$ varies

$\sigma_1=$ 100/11

$c_1=$ 1 (scat)

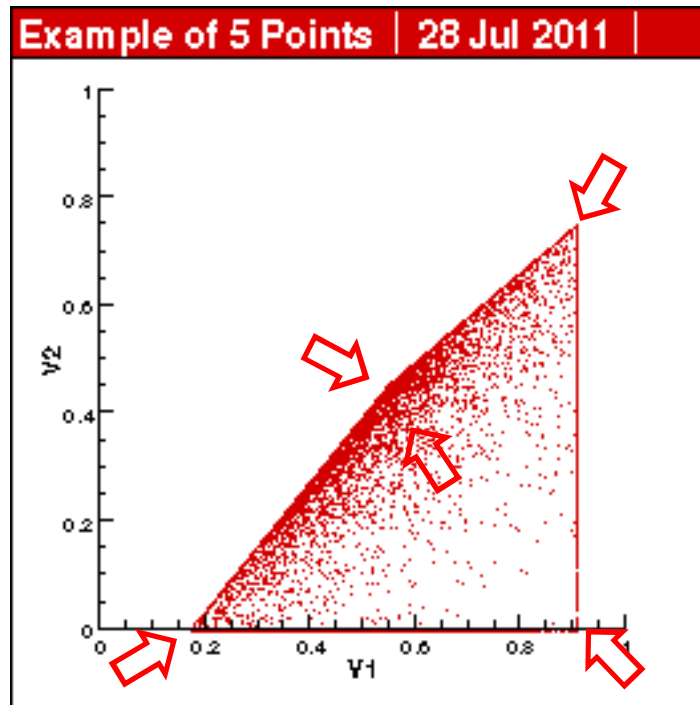
Backward
Forward

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Results & Analysis – Boundary Points

- What phenomenon caused these points?
- What can that tell us about the lines between them and where the points lie?



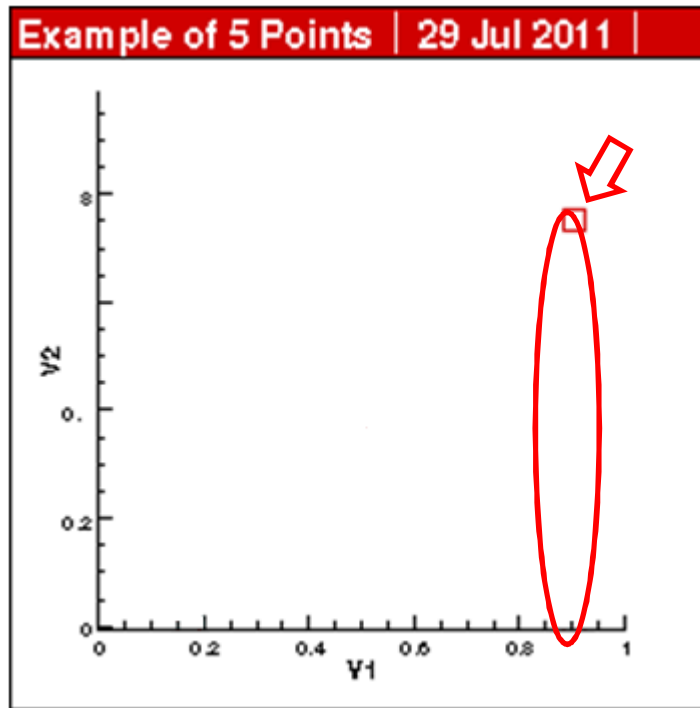
- Must examine extreme cases

Mesh Parameters	
s=	10
mat0	
$\lambda_0 =$	101/20
$\sigma_0 =$	2/101
$c_0 =$	0 (abs)
mat1	
scat $\lambda_1 =$ abs	101/20
abs $\sigma_1 =$ scat	200/101
$c_1 =$	1 (scat)

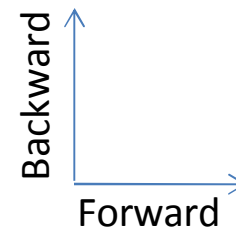
Backward ↑
Forward →

Results & Analysis – Boundary Points Identified

- In this mesh:
 - Scattering blocks more than absorbing

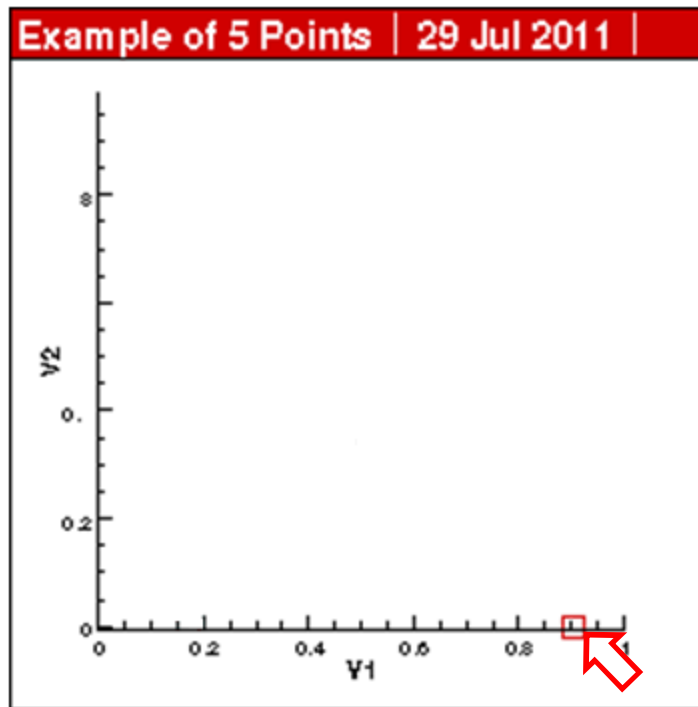


- Absorption lets the most through
 - Highest Forward Flux
- Scattering after reflects the most back
 - Highest Backward Flux



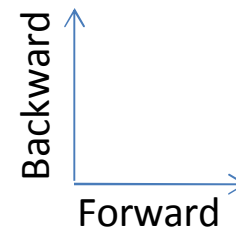
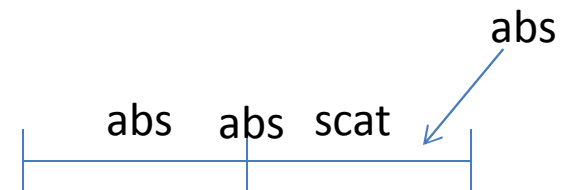
Results & Analysis – Boundary Points Identified

- In this mesh:
 - Scattering blocks more than absorbing



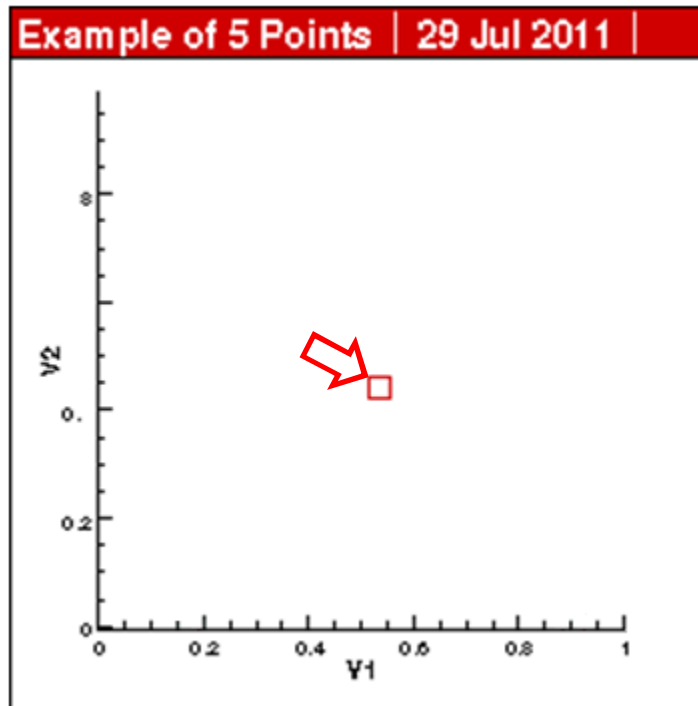
- If you add absorption to the scattering side...

- Forward Flux will stay the same, Backward flux will go to zero

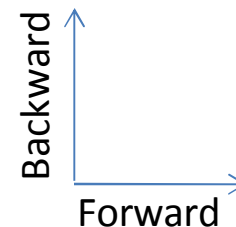
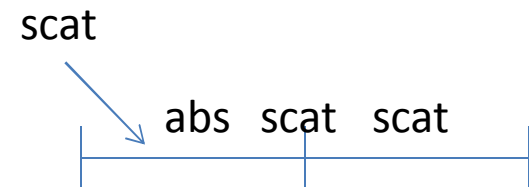


Results & Analysis – Boundary Points Identified

- In this mesh:
 - Scattering blocks more than absorbing

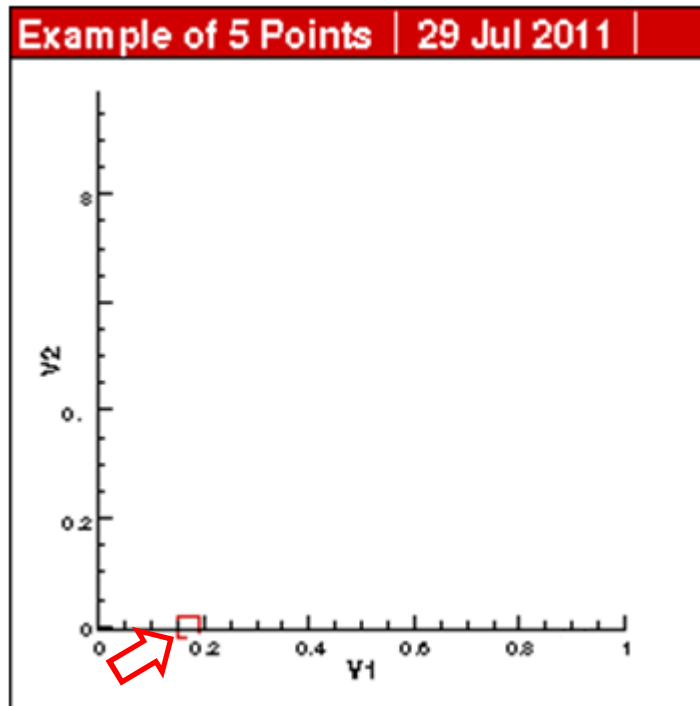


- Going back to the first extreme condition, if you add scattering to the absorption side...
- Forward Flux will decrease some, Backward flux will as a result decrease some



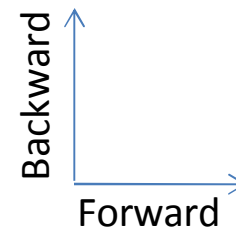
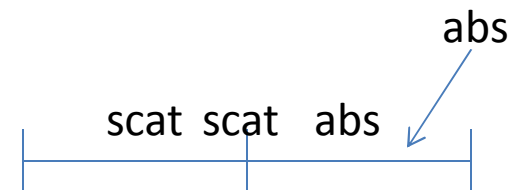
Results & Analysis – Boundary Points Identified

- In this mesh:
 - Scattering blocks more than absorbing

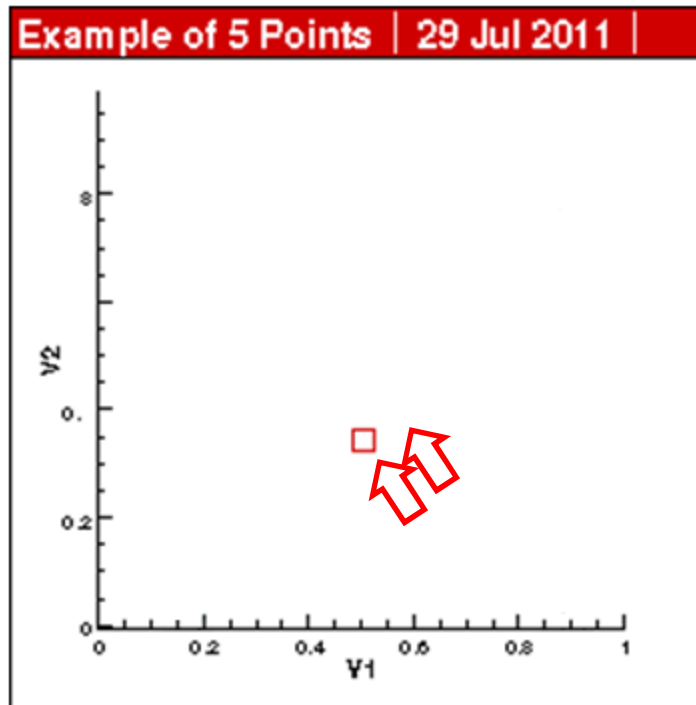
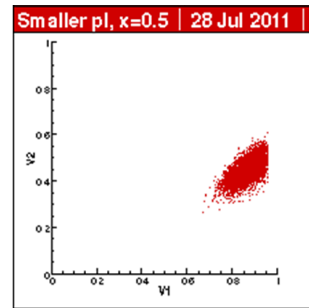
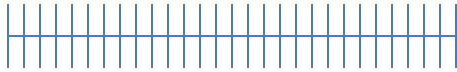


• Continuing from the previous, if we add absorption after the mining point...

• Forward Flux will decrease some, Backward flux will go to zero



Results & Analysis – Boundary Points Identified



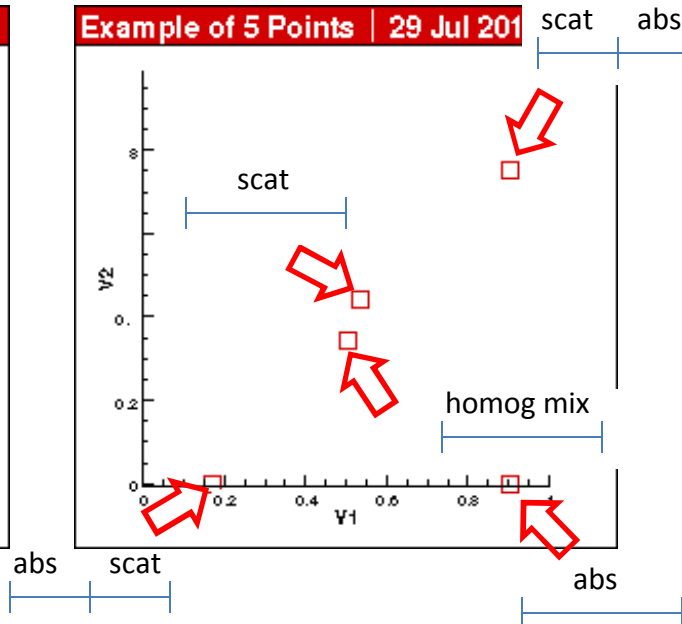
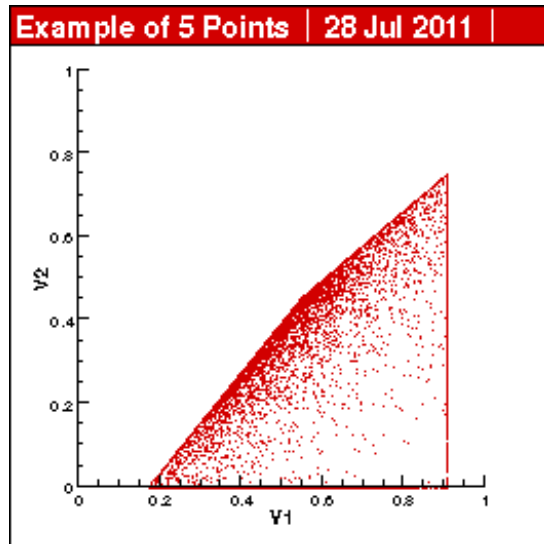
- The fifth Point of Interest is generated a little differently
- As the average path length (λ) decreases, the mesh approaches homogeneity
- So the last point of interest is defined by a homogeneous mix

homogeneous mix

Backward
Forward

Results & Analysis – Boundary Points Summarized

In summary of these five points of interest:



Mesh Parameters	
$s=$	10
mat0	
$\lambda_0=$	101/20
$\sigma_0=$	2/101
$c_0=$	0 (abs)
mat1	
$\lambda_1=$	101/20
$\sigma_1=$	200/101
$c_1=$	1 (scat)

Backward
Forward

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Conclusions

- **With distance from the source flux decreases**
- **As path length decreases correlations approach homogeneity**
- **With large path lengths**
 - Four boundary points define the outer boundaries of flux relationships
 - Tend to cluster around the borders of the quadrilateral
- **With smaller path lengths**
 - One boundary point defines homogeneous mixture
 - Tend to cluster inside quadrilateral and around convergence point.

Acknowledgments/References

A Thanks to:

Mentor – [Shawn](#) Pautz

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Technical/Coding Help – [Clif](#) Drumm, [Mike](#) Rigley, and [Peter](#) Sabaiya

Everything and anything else, our own wonderful – [Trish](#) St. John

Papers referenced:

- M. L. Adams, E. W. Larsen, G. C. Pomraning, "Benchmark Results for Particle Transport in a Binary Markov Statistical Medium," *J. Quant. Spectrosc. Radiat. Transfer*, 42, pp. 253-266 (1989).
- S. D. Pautz, B. C. Franke, "Generation of Accurate Benchmarks for Transport in Stochastic Media by Means of Dynamic Error Control," *Proc. Int. Conf. on Mathematics and computational Methods Applied to Nuclear Science and Engineering*, Rio de Janeiro, Brazil (2011).
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Questions

