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# Characterization of Forward and Backward Flux Correlations in 1D Stochastic Media

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# Overview

- Used SCEPTE\* on Pegasus and Redsky to generate realizations of stochastic meshes
- Mined data at points throughout the mesh for forward and backward flux
- Discovered five points of interest that define extreme correlation cases in a given stochastic mesh
  - Points 1-4: Generated from maximum possible absorption and scattering in regions
  - Point 5: Generated as average path length decreases and mesh approaches homogeneity

\*Sandia's Computational Engine for Particle Transport for Radiation Effects

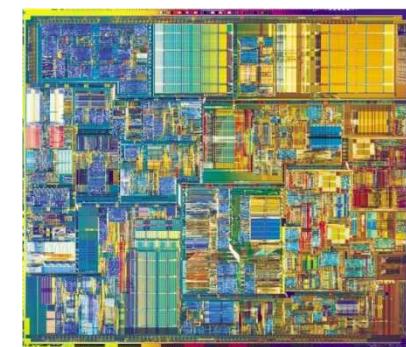
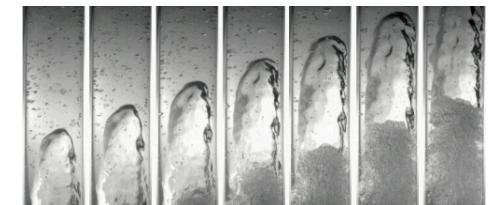
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# Introduction – Stochastic Media

- Stochastic Media – Two or more materials mixed in a way that cannot easily be exactly modeled

- Compound mixtures
- Two-phase flow
- Small repetitive systems



- Meshes built using
  - average path lengths ( $\lambda$ )
  - and a probability distribution

$$\lambda_0 = 99/10$$

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Examples

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$$\lambda_1 = 11/10$$

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Examples

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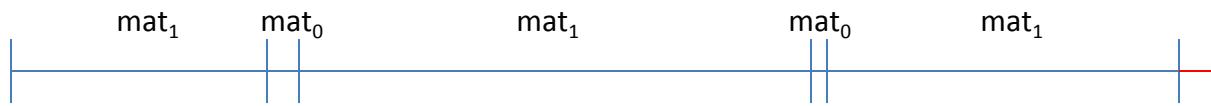
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# Introduction – Building Stochastic Meshes

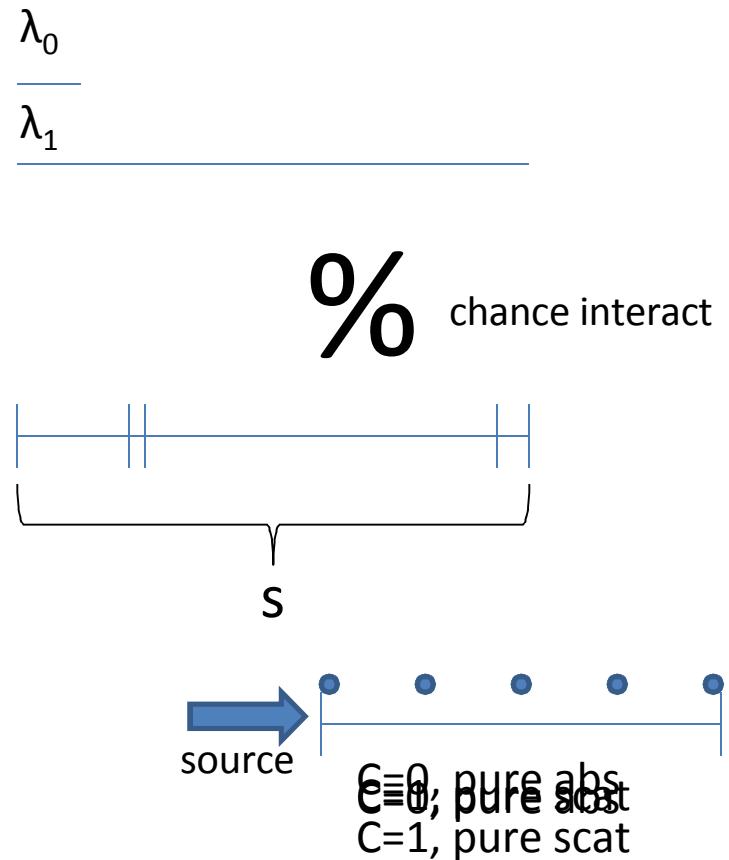
$\lambda_0=99/10$  —

$\lambda_1=11/10$  —



# Introduction – Stochastic Mesh Parameters

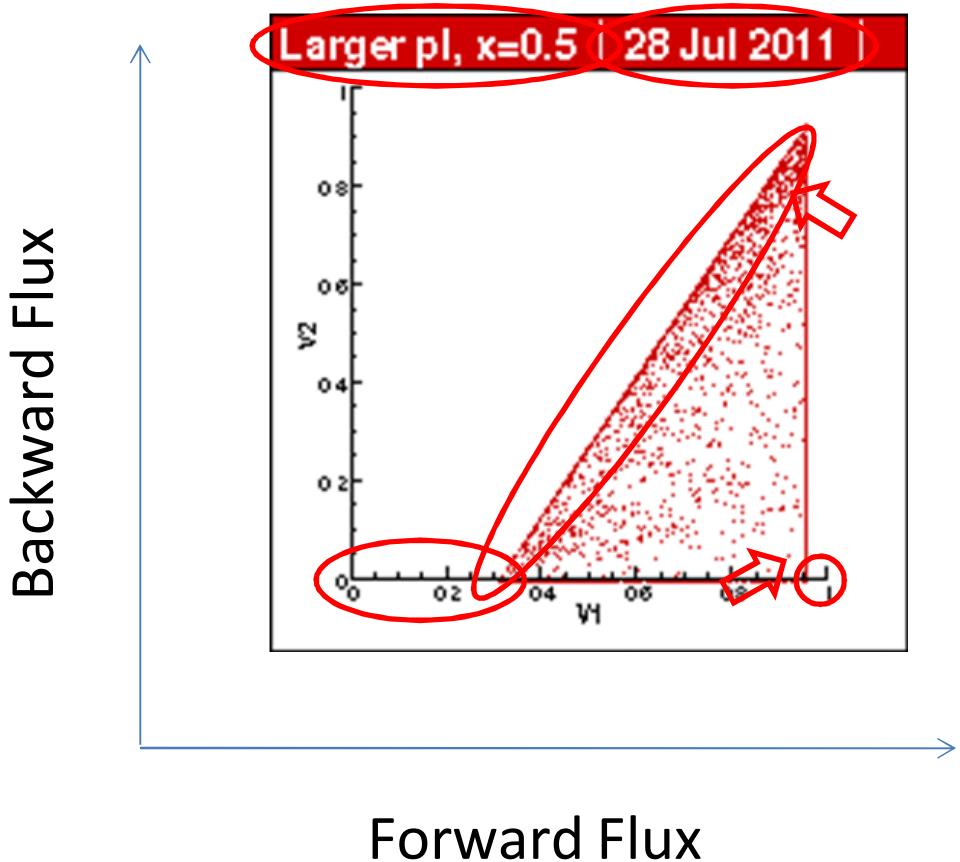
Stochastic Parameters Reference	
$\lambda$	<b>Average path length</b> (of material segment)
$\sigma$	<b>Cross-section</b> (of interaction)
$s$	<b>Total length</b> (of mesh)
$c$	<b>Scattering ratio</b> ( $\sigma_s / \sigma_t$ , 0=pure abs, 1=pure scat)
$x$	<b>Position in mesh</b> (relative to left boundary)



# Introduction – Correlation Plots

- Smaller  $\lambda$  later slides
- At  $x=0.5$  (of  $s=10$ )

- Date I created this plot on



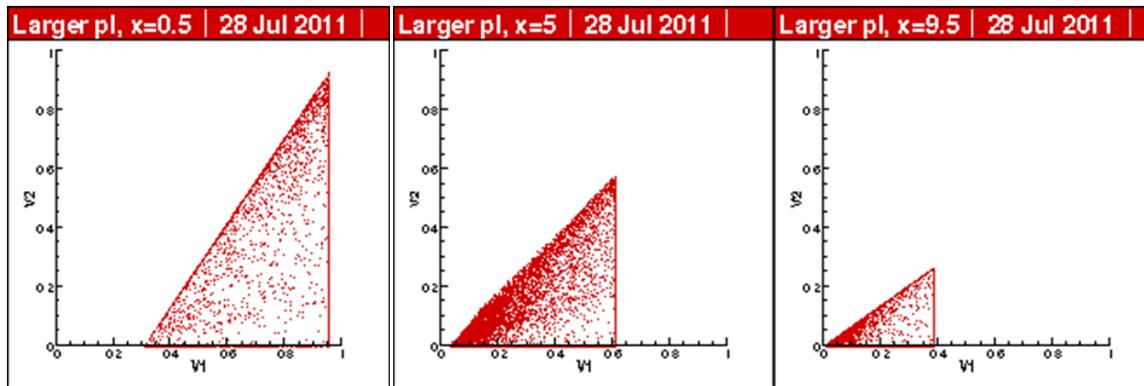
- Each dot represents the fluxes with a given mesh
- Example 1: Lots of absorbing
- Example 2: Lots of scattering past  $x=0.5$
- At least 30% always gets here
- About 5% never gets here
- Must have forward to have backward



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# Results & Analysis – Material Depth



## Mesh Parameters

$s = 10$

mat0

$\lambda_0 = 99/10$

$\sigma_0 = 10/99$

$c_0 = 0 \text{ (abs)}$

mat1

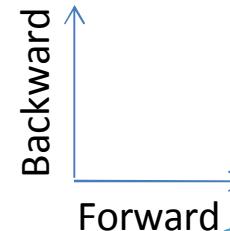
$\lambda_1 = 11/10$

$\sigma_1 = 100/11$

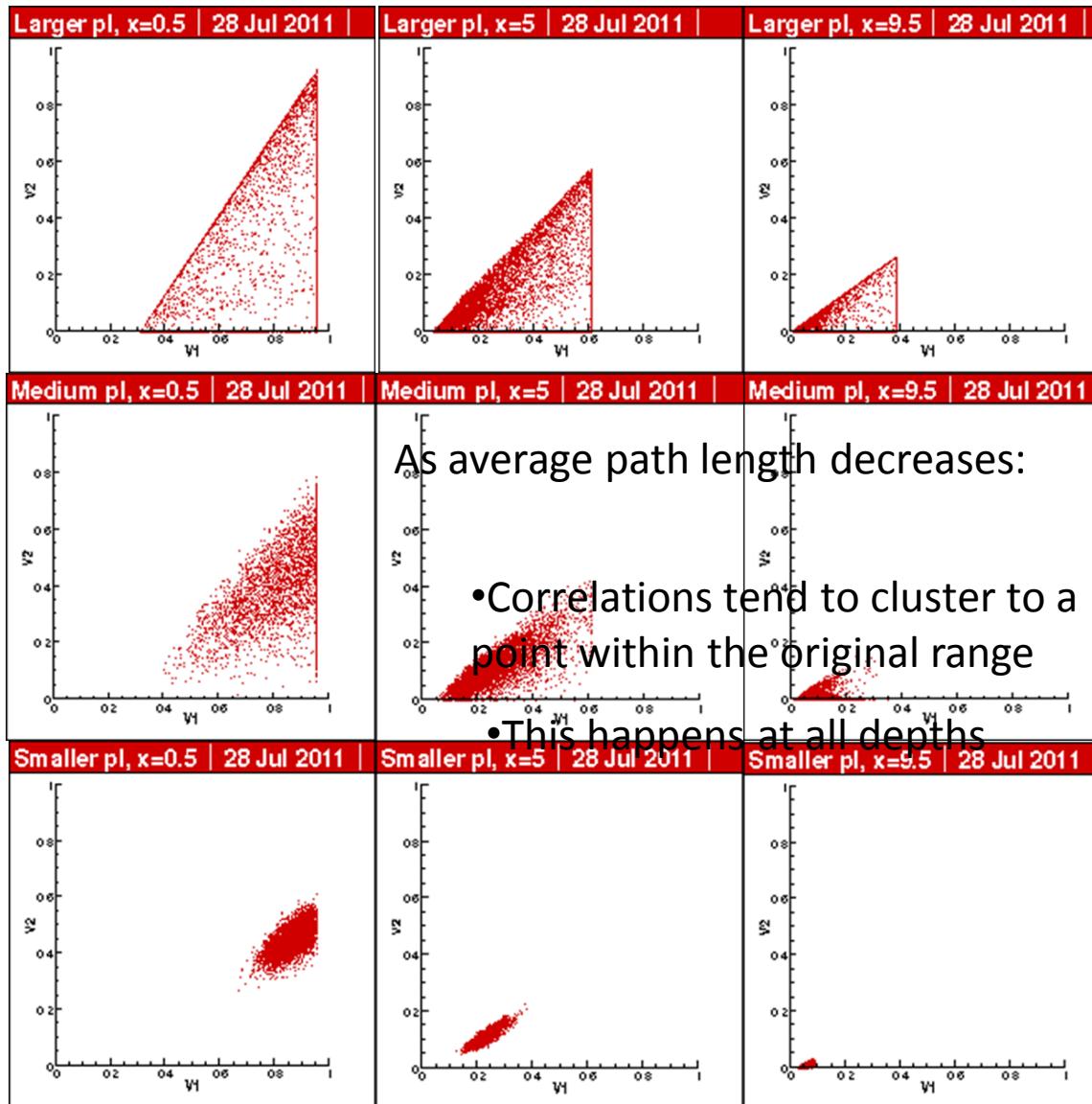
$c_1 = 1 \text{ (scat)}$

As data is taken further through the mesh:

- there is a lower flux
- the backwards flux dies more than the forward flux

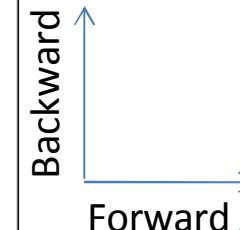


# Results & Analysis – Average Path Length



## Mesh Parameters

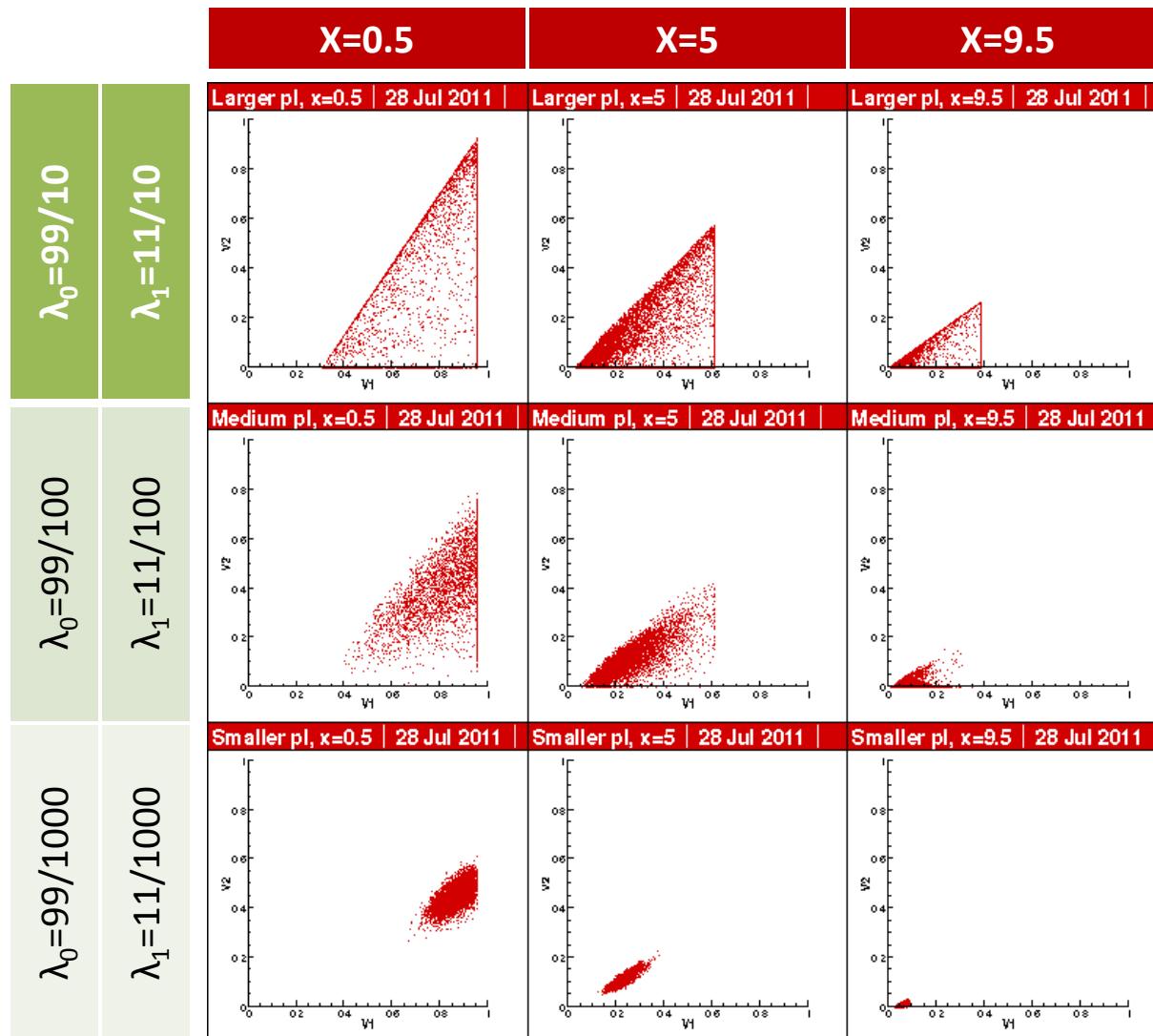
$s =$	10
$mat0$	
$\lambda_0 =$	varies
$\sigma_0 =$	10/99
$c_0 =$	0 (abs)
$mat1$	
$\lambda_1 =$	varies
$\sigma_1 =$	100/11
$c_1 =$	1 (scat)



# Results & Analysis – Material Depth and Length Effects Summarized

## Depth in Mesh

### Average Path Length



### Mesh Parameters

$s=$	10
$mat0$	
$\lambda_0=$	varies
$\sigma_0=$	10/99
$c_0=$	0 (abs)
$mat1$	
$\lambda_1=$	varies
$\sigma_1=$	100/11
$c_1=$	1 (scat)

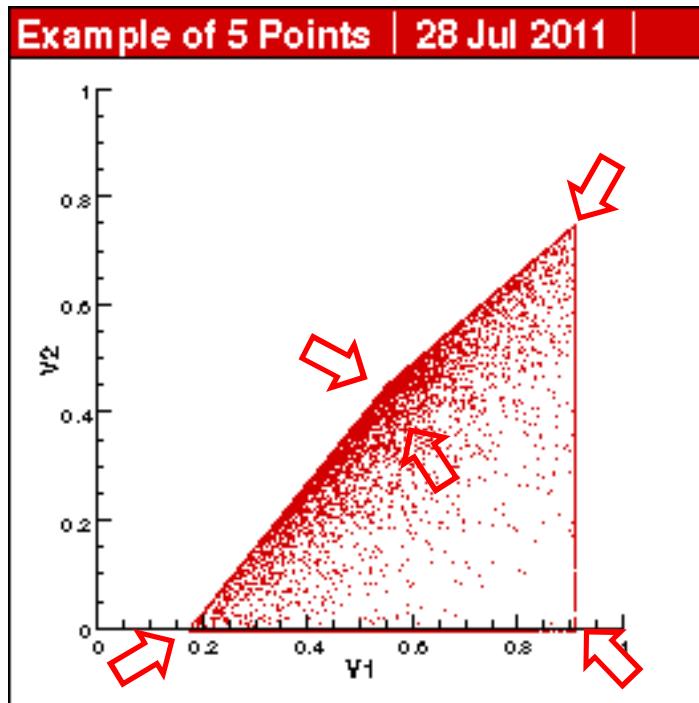
Backward  
Forward

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# Results & Analysis – Boundary Points

- What phenomenon caused these points?
- What can that tell us about the lines between them and where the points lie?



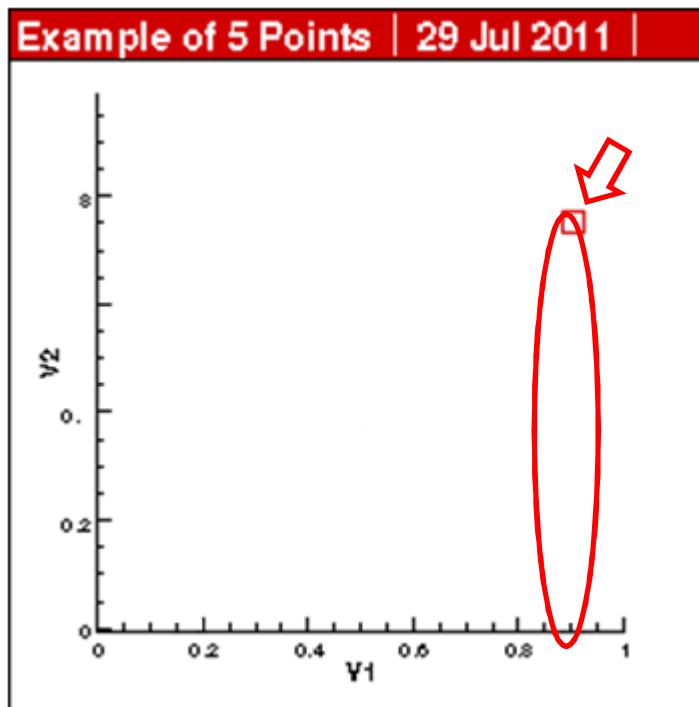
Mesh Parameters	
$s =$	10
$\lambda_0 =$	$101/20$
$abs_0 =$	$2/101$
$scat_0 =$	$0 (abs)$
mat1	
scat	$\lambda_1 = abs$
abs	$101/20$
$\sigma_1 = scat$	$200/101$
$c_1 =$	$1 (scat)$

Backward

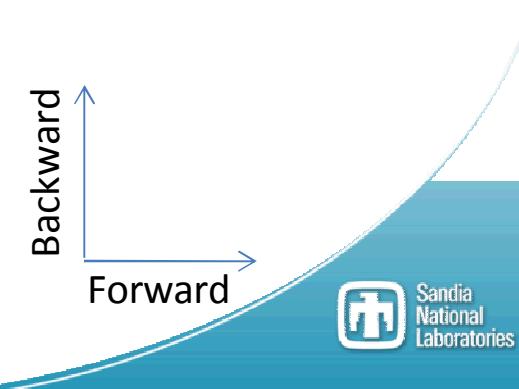
Forward

# Results & Analysis – Boundary Points Identified

- In this mesh:
  - Scattering blocks more than absorbing

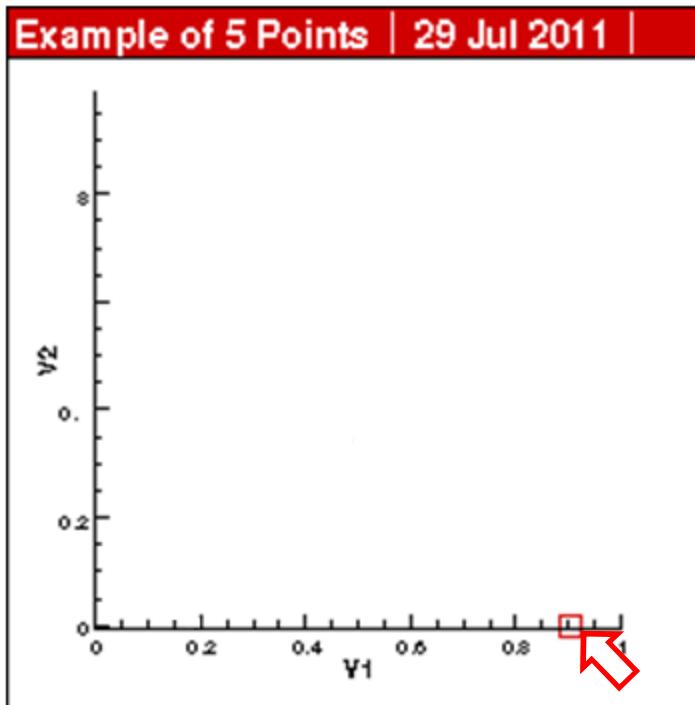


- Absorption lets the most through
  - Highest Forward Flux
- Scattering after reflects the most back
  - Highest Backward Flux

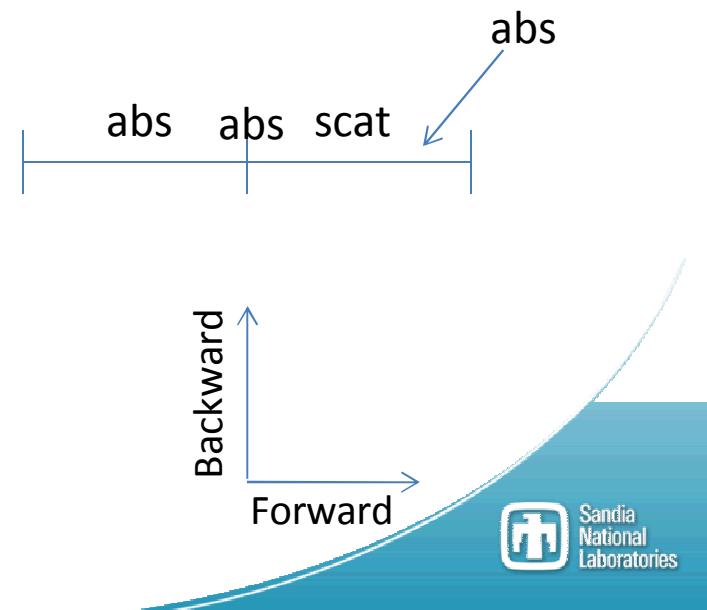


# Results & Analysis – Boundary Points Identified

- In this mesh:
  - Scattering blocks more than absorbing



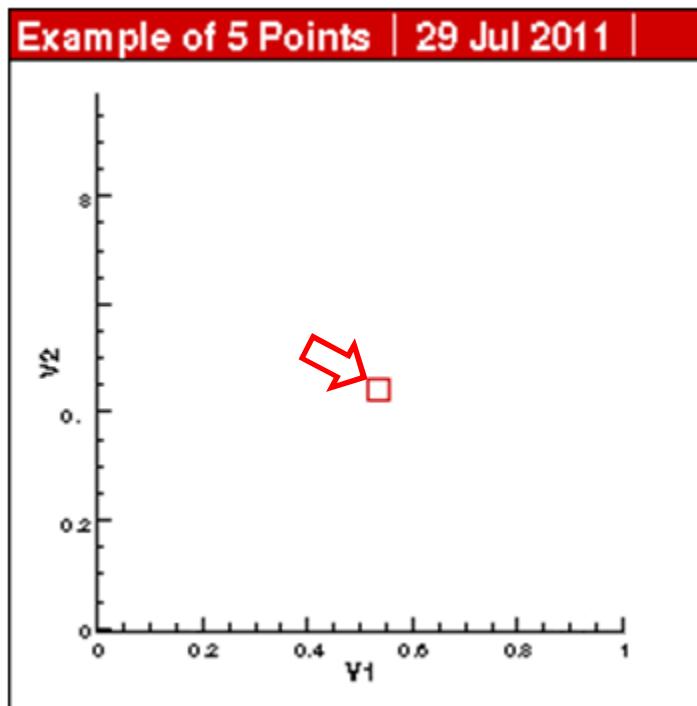
- If you add absorption to the scattering side...
- Forward Flux will stay the same, Backward flux will go to zero



# Results & Analysis – Boundary Points Identified

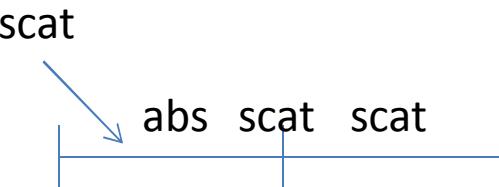
- In this mesh:

- Scattering blocks more than absorbing



- Going back to the first extreme condition, if you add scattering to the absorption side...

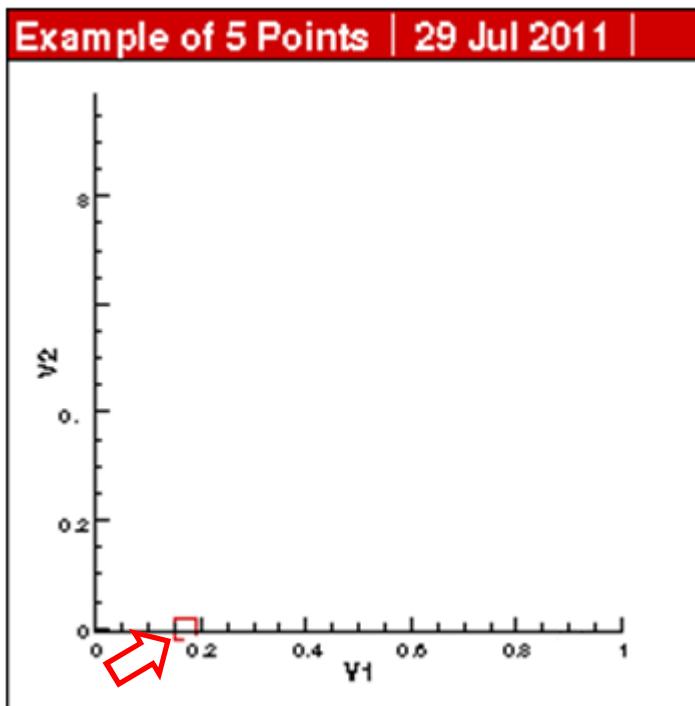
- Forward Flux will decrease some, Backward flux will as a result decrease some



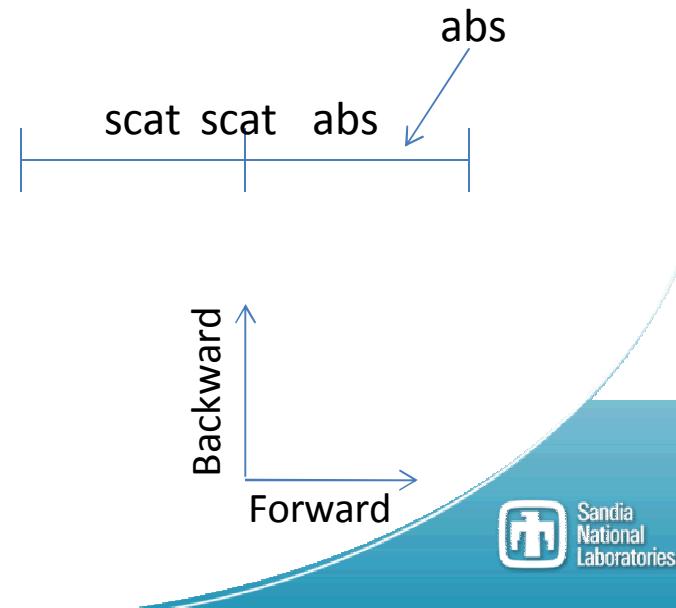
# Results & Analysis – Boundary Points Identified

- In this mesh:

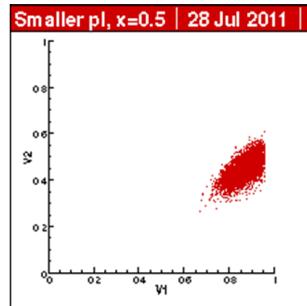
- Scattering blocks more than absorbing



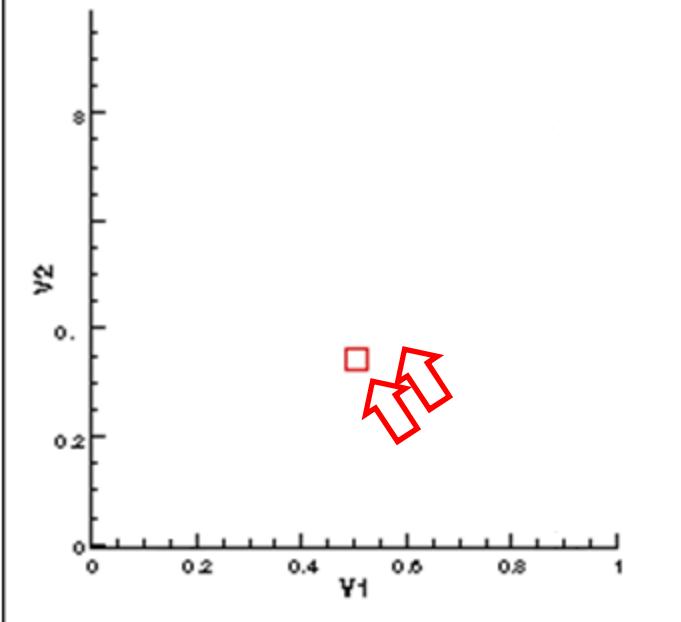
- Continuing from the previous, if we add absorption after the mining point...
- Forward Flux will decrease some, Backward flux will go to zero



# Results & Analysis – Boundary Points Identified



Example of 5 Points | 29 Jul 2011 |



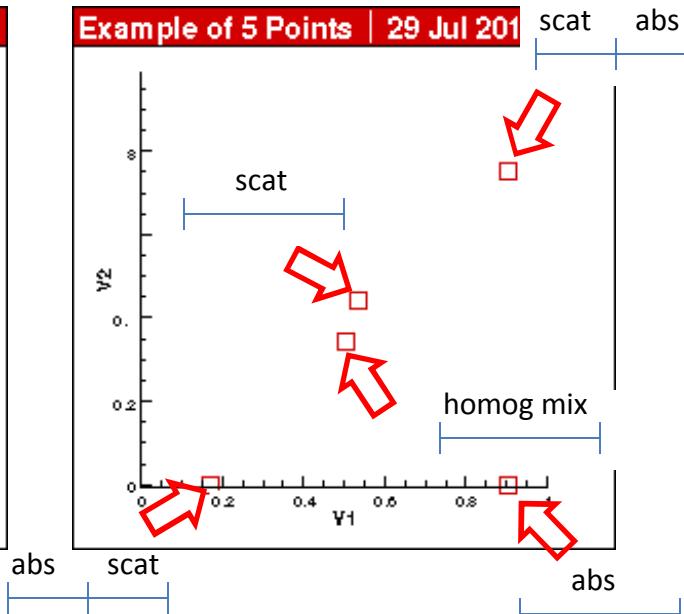
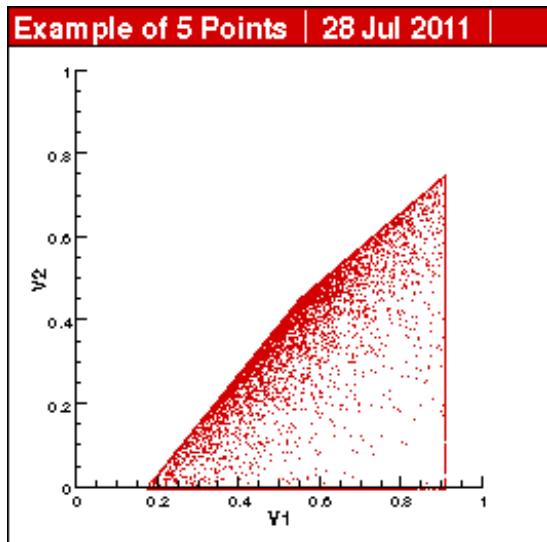
- The fifth Point of Interest is generated a little differently
- As the average path length ( $\lambda$ ) decreases, the mesh approaches homogeneity
- So the last point of interest is defined by a homogeneous mix

homogeneous mix

Backward  
Forward

# Results & Analysis – Boundary Points Summarized

In summary of these five points of interest:



Mesh Parameters	
$s =$	10
mat0	
$\lambda_0 =$	$101/20$
$\sigma_0 =$	$2/101$
$c_0 =$	0 (abs)
mat1	
$\lambda_1 =$	$101/20$
$\sigma_1 =$	$200/101$
$c_1 =$	1 (scat)

Backward  
Forward

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# Conclusions

- **With distance from the source flux decreases**
- **As path length decreases correlations approach homogeneity**
- **With large path lengths**
  - Four boundary points define the outer boundaries of flux relationships
  - Tend to cluster around the borders of the quadrilateral
- **With smaller path lengths**
  - One boundary point defines homogeneous mixture
  - Tend to cluster inside quadrilateral and around convergence point.

# Acknowledgments/References

A Thanks to:

Mentor – [Shawn Pautz](#)

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Technical/Coding Help – [Clif Drumm](#), [Mike Rigley](#), and [Peter Sabaiya](#)

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Papers referenced:

- M. L. Adams, E. W. Larsen, G. C. Pomraning, “Benchmark Results for Particle Transport in a Binary Markov Statistical Medium,” *J. Quant. Spectrosc. Radiat. Transfer*, 42, pp. 253-266 (1989).
- S. D. Pautz, B. C. Franke, “Generation of Accurate Benchmarks for Transport in Stochastic Media by Means of Dynamic Error Control,” *Proc. Int. Conf. on Mathematics and computational Methods Applied to Nuclear Science and Engineering*, Rio de Janeiro, Brazil (2011).
- B. C. Franke, A. K. Prinja, “Flux-Probability Distributions From the Master Equation for Radiation Transport in Stochastic Media,” *Proc. Int. Conf. on Mathematics and computational Methods Applied to Nuclear Science and Engineering*, Rio de Janeiro, Brazil (2011).

# Questions

