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Block Preconditioning for Implicit Ocean Systems

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Collaborators

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- Andy Salinger
- Mark Taylor
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- Eric Cyr
- Henk Dijkstra
- Erik Bernsen
- Joe Fitzpatrick

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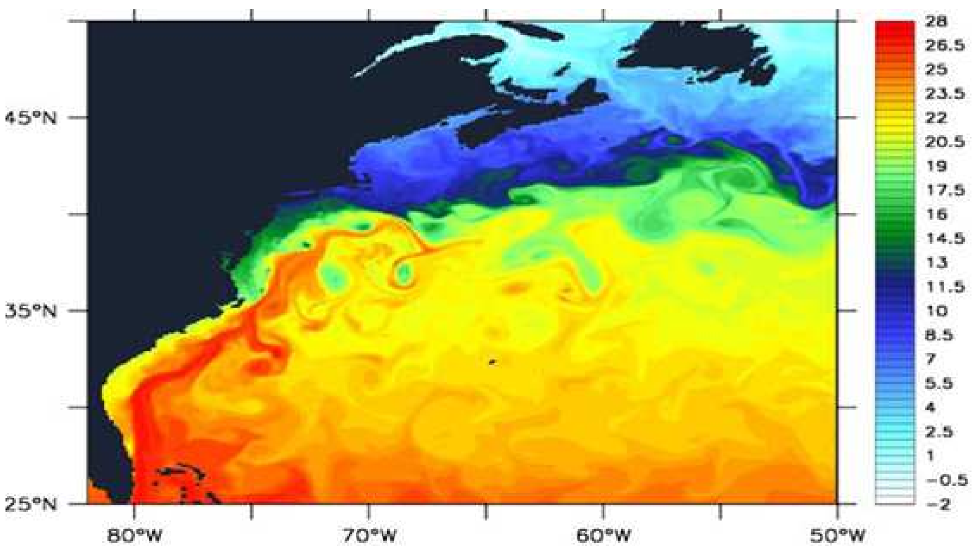
Outline

- Ocean Modeling with POP.
- The Coriolis Term.
- Pressure Coupling.
- Final Thoughts.

Ocean Modeling in POP

- Parallel Ocean Program (POP) is one of the models in the Community Climate System Model (CCSM).
- Physics of POP
  - Finite difference of thin stratified fluid equations w/ hydrostatic and Boussinesq approximations.
  - Coupled temperature & salinity advection-diffusion.



Left: THCM, J. Thies; Right: POP website

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POP Equations

$$\frac{\partial u}{\partial t} + \mathcal{C}_1(u) - \alpha_1 uv - \Omega v + \alpha_2 \frac{\partial \eta}{\partial \lambda} + \alpha_3 \frac{\partial p_{bc}(S,T)}{\partial \lambda} - D_1(u,v) = 0$$

$$\frac{\partial v}{\partial t} + \mathcal{C}_1(v) + \alpha_1 u^2 + \Omega u + \alpha_4 \frac{\partial \eta}{\partial \phi} + \alpha_5 \frac{\partial p_{bc}(S,T)}{\partial \phi} - D_2(u,v) = 0$$

$$\frac{\partial S}{\partial t} + \mathcal{C}_2(S) + \mathcal{C}_3(u,v,S) - D_3(S,T) = 0$$

$$\frac{\partial T}{\partial t} + \mathcal{C}_2(T) + \mathcal{C}_3(u,v,T) - D_3(S,T) = 0$$

$$\frac{\partial \eta}{\partial t} + \int_{-H}^{0} \left(\alpha_6 \frac{\partial u}{\partial \lambda} + \alpha_7 \frac{\partial v}{\partial \phi} \right) dz = 0$$

Diffusion
Convection
Coriolis
Coupling #1
Coupling #2

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POP Equations (Take 2)

$$\boxed{\frac{\partial u}{\partial t} + \mathcal{C}_1(u) - \alpha_1 uv - \Omega v} + \alpha_2 \frac{\partial \eta}{\partial \lambda} + \alpha_3 \frac{\partial p_{bc}(S,T)}{\partial \lambda} \boxed{- D_1(u,v)} = 0$$

$$\boxed{\frac{\partial v}{\partial t} + \mathcal{C}_1(v) + \alpha_1 u^2 + \Omega u} + \alpha_4 \frac{\partial \eta}{\partial \phi} + \alpha_5 \frac{\partial p_{bc}(S,T)}{\partial \phi} \boxed{- D_2(u,v)} = 0$$

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The Coriolis Term

- Consider the 2D Coriolis-Diffusion equation:

$$\begin{bmatrix} -\Delta & \Omega \\ -\Omega & -\Delta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}.$$

- (Periodic) Fourier analysis gives eigenvalues for Jacobi:

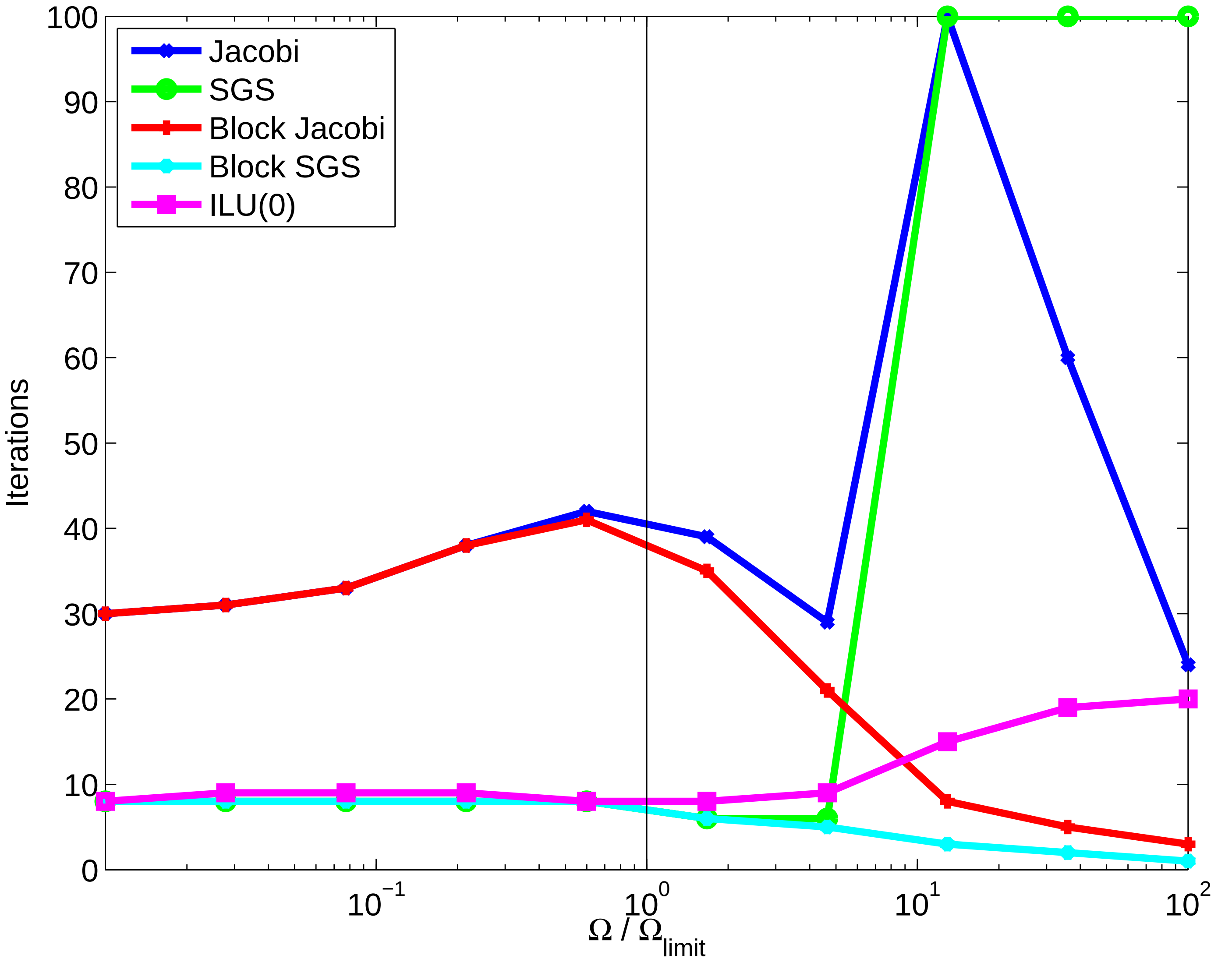
$$|\lambda_i|^2 = \frac{1}{4}(\cos\theta_s + \cos\theta_t)^2 + \frac{\Omega^2 h^4}{16}$$

- If $s, t > 0$, Jacobi diverges if

$$\Omega > \frac{4}{h^2}|\sin(2\pi h)|.$$

- Solution: Use block smoothers.

AMG on Coriolis-Diffusion



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POP Equations (Take 3)

$$\boxed{\frac{\partial u}{\partial t} + \mathcal{C}_1(u) - \alpha_1 uv - \Omega v + \alpha_2 \frac{\partial \eta}{\partial \lambda}} + \alpha_3 \frac{\partial p_{bc}(S,T)}{\partial \lambda} \boxed{- D_1(u,v)} = 0$$

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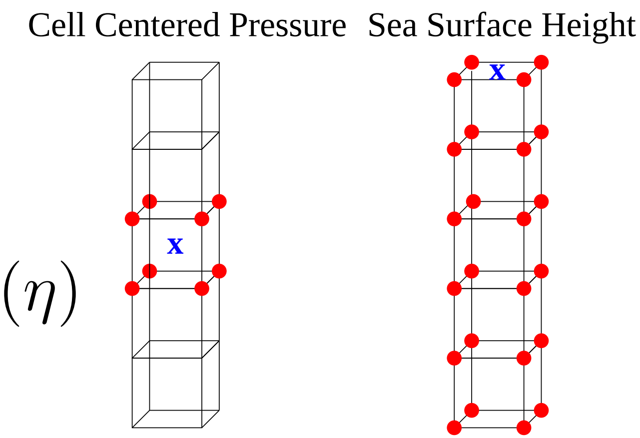
What POP does with Pressure

- Incompressible Stokes equations

$$\begin{aligned} \frac{\partial \mathbf{v}}{\partial t} - \mu \Delta \mathbf{v} + \frac{1}{\rho} \nabla p &= \mathbf{f} \\ \nabla \cdot \mathbf{v} &= 0 \end{aligned}$$

- POP discards pressure for *sea surface height* (η) which allows for a free ocean surface.
- Effect #1: 3D pressure field $\rightarrow$ 2D surface field.
- Effect #2: z-velocity term is removed.
- This leaves two questions:
  - What takes p's place in momentum equations?
  - How do we replace continuity equation?

Momentum Equations



- Baroclinic / Barotropic pressure split

$$\begin{aligned} p &= p_{bc}(z,T,S) + p_{bt}(\eta) \\ p_{bt} &= \rho g \eta \end{aligned}$$

- So

$$\frac{1}{\rho}\nabla p = g\nabla\eta + \frac{1}{\rho}\nabla p_{bc}$$

  Note: Gradients are computed rather differently.

- As p_{bc} is a function of salinity & temperature, we ignore it for today's talk.

Continuity Equation

- Impose the constraint

$$\frac{\partial \eta}{\partial t} = \mathbf{v}_z \text{ at sea surface.}$$

- Question: How to compute $\mathbf{v}_z$ at surface?
- Answer: Use divergence theorem

$$\frac{\partial \eta}{\partial t} + \int_{-H}^{0} \left(\alpha_6 \frac{\partial u}{\partial \lambda} + \alpha_7 \frac{\partial v}{\partial \phi} \right) dz = 0$$

Problems with POP

- η discretizations are poorly conditioned
  - Double $\Delta t \rightarrow$ quadruple $\kappa(A)$.
  - We want to timestep across *centuries*. POP matrices are numerically singular well before then.
- Sparsity pattern of $\nabla\eta$ and $\nabla \cdot \mathbf{v}$ operator (as well as $\frac{d}{dz}\mathbf{v}$) can make things difficult for AMG.
- With these limitations, what's the best we can do?

Preconditioning

$$M \approx \begin{bmatrix} A & B^T \\ C & D \end{bmatrix}^{-1}$$

- Schur complement (Schur complement in η)
  - (1,1) solver: NSA-AMG (2-level) w/ Block ILU(0) smoother.
  - Schur complement solver: SIMPLE, Block-SIMPLE or Pressure Mass Matrix.
- Augmented Lagrangian (Schur complement in $\mathbf{v}$)
  - Solver: NSA-AMG (2-level) w/ ILU(0) or ILUTP smoother.
  - Solve η equations in postprocessing.

Preconditioning

- SIMPLE

$$S \approx D - CF^{-1}B^T \text{ where } F = diag(A).$$

- Block-SIMPLE

$$S \approx D - CF^{-1}B^T \text{ where } F = blockdiag(A, 2).$$

- Pressure Mass Matrix

$$S \approx \alpha I$$

- Augmented Lagrangian

$$A - B^T D^{-1} C$$

Since $D = I$ this is tractable.

Test Problems

- miniPOP — MATLAB “simplified” version of POP.

  No advection, no nonlinear terms, no z diffusion, flat sea bottom.

- POP ForcedChannelSmall — channel w/ surface wind.

  No z advection/diffusion, flat sea bottom, first Newton step.

- Domain size $\approx$ 1800km $\times$ 2000km $\times$ 2.6km.
- Time steps (days): 0.5–1024.

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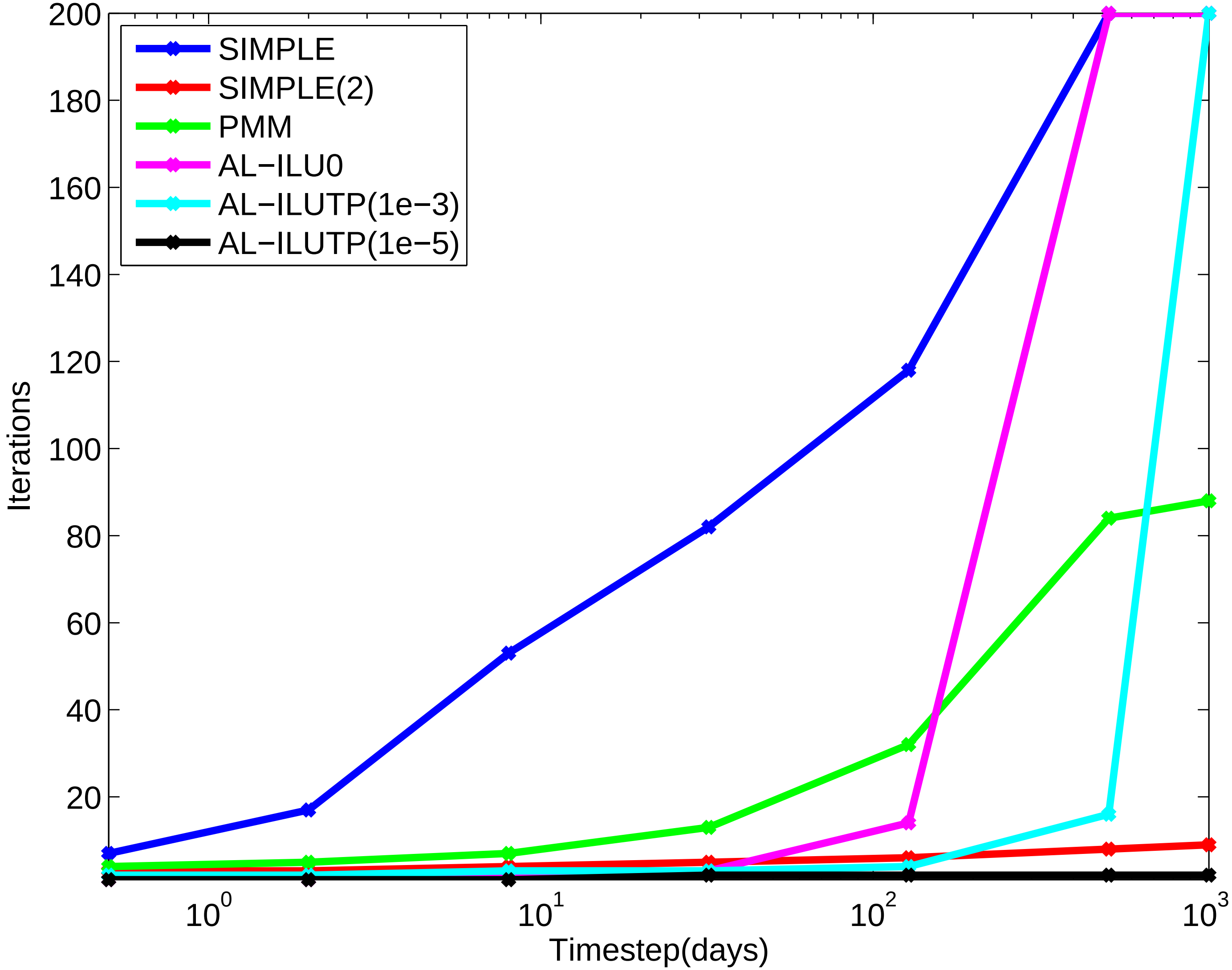
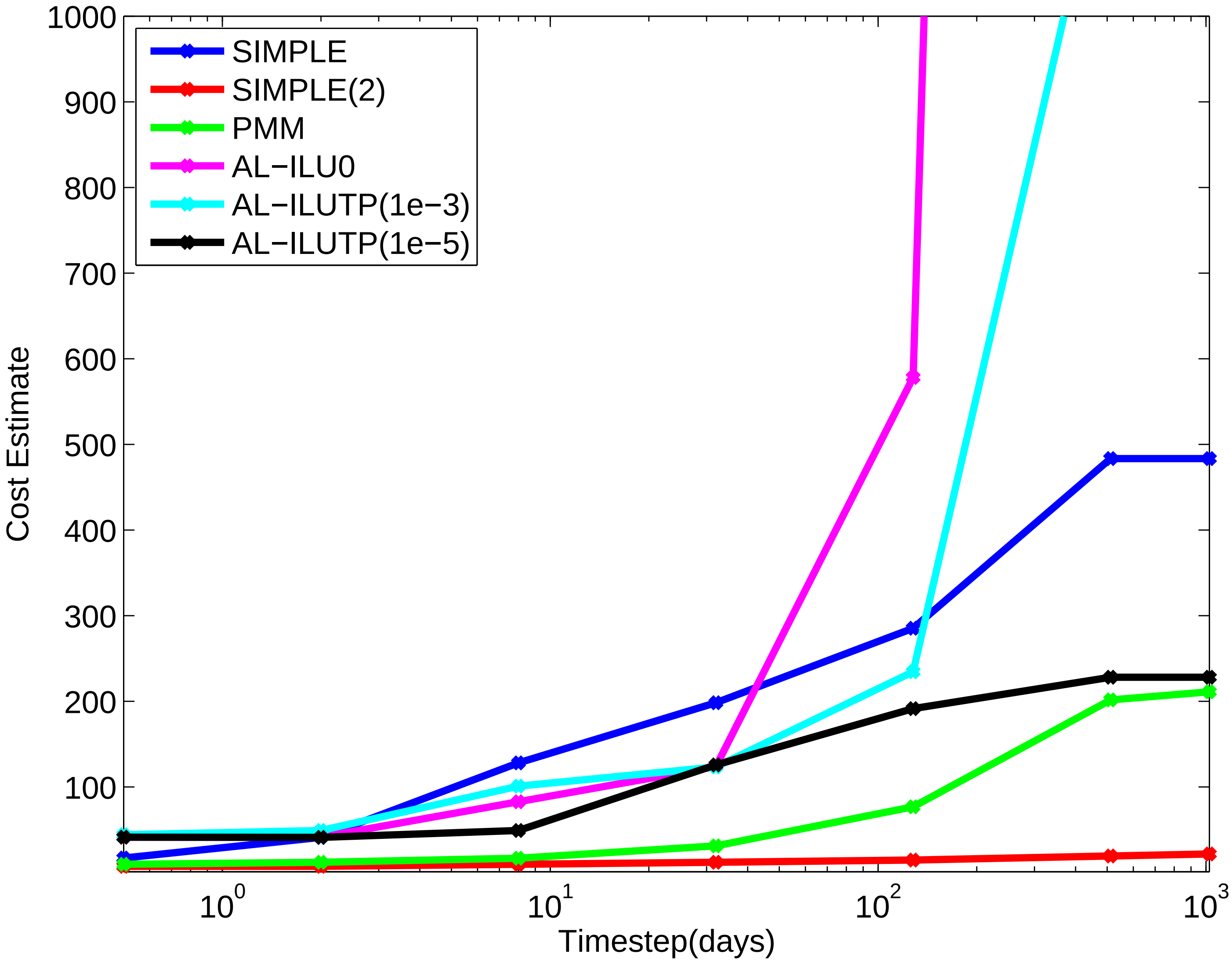
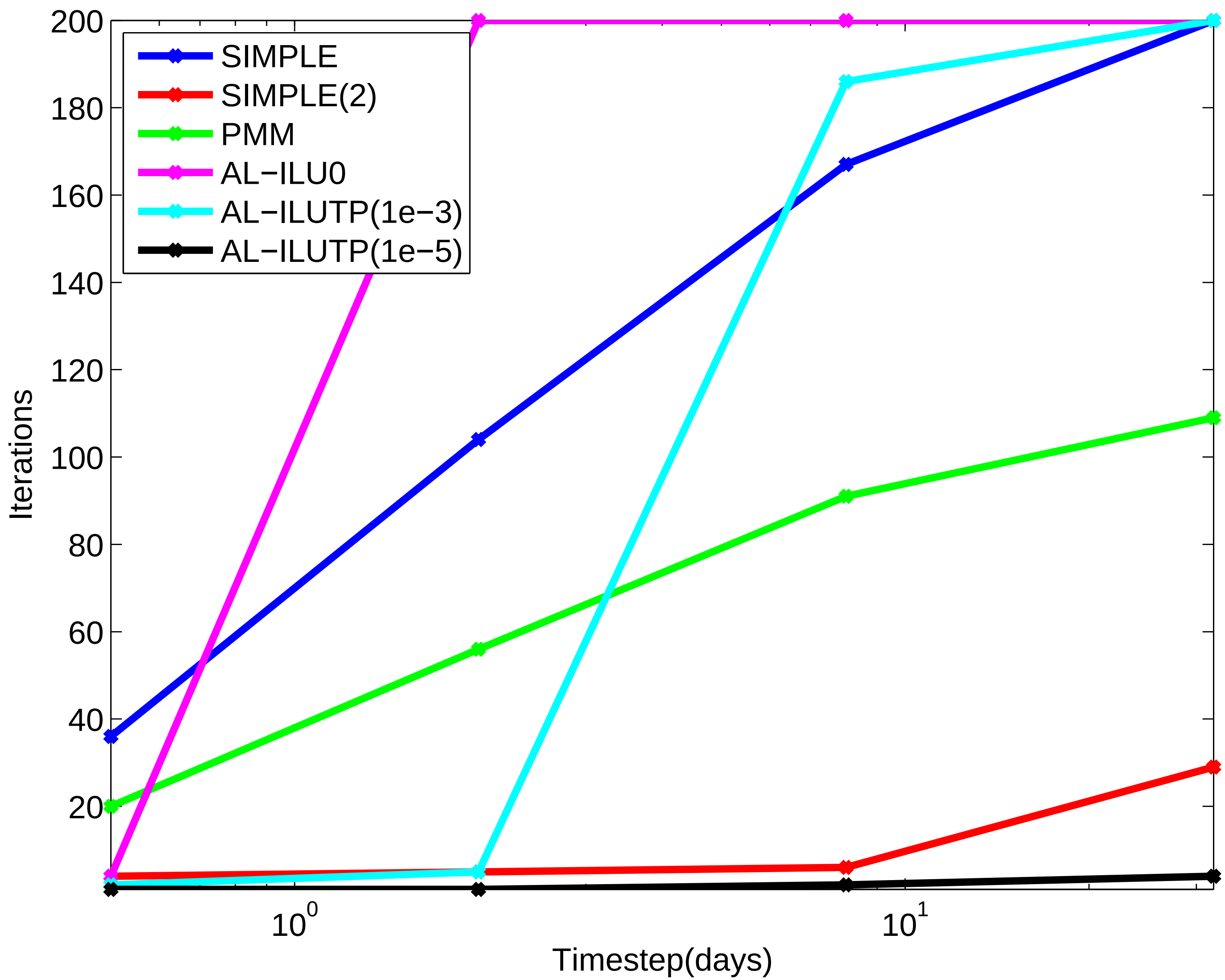
miniPOP 20x22x16: Iterations



miniPOP 20x22x16: “Cost”



POP 20x22x16: Iterations



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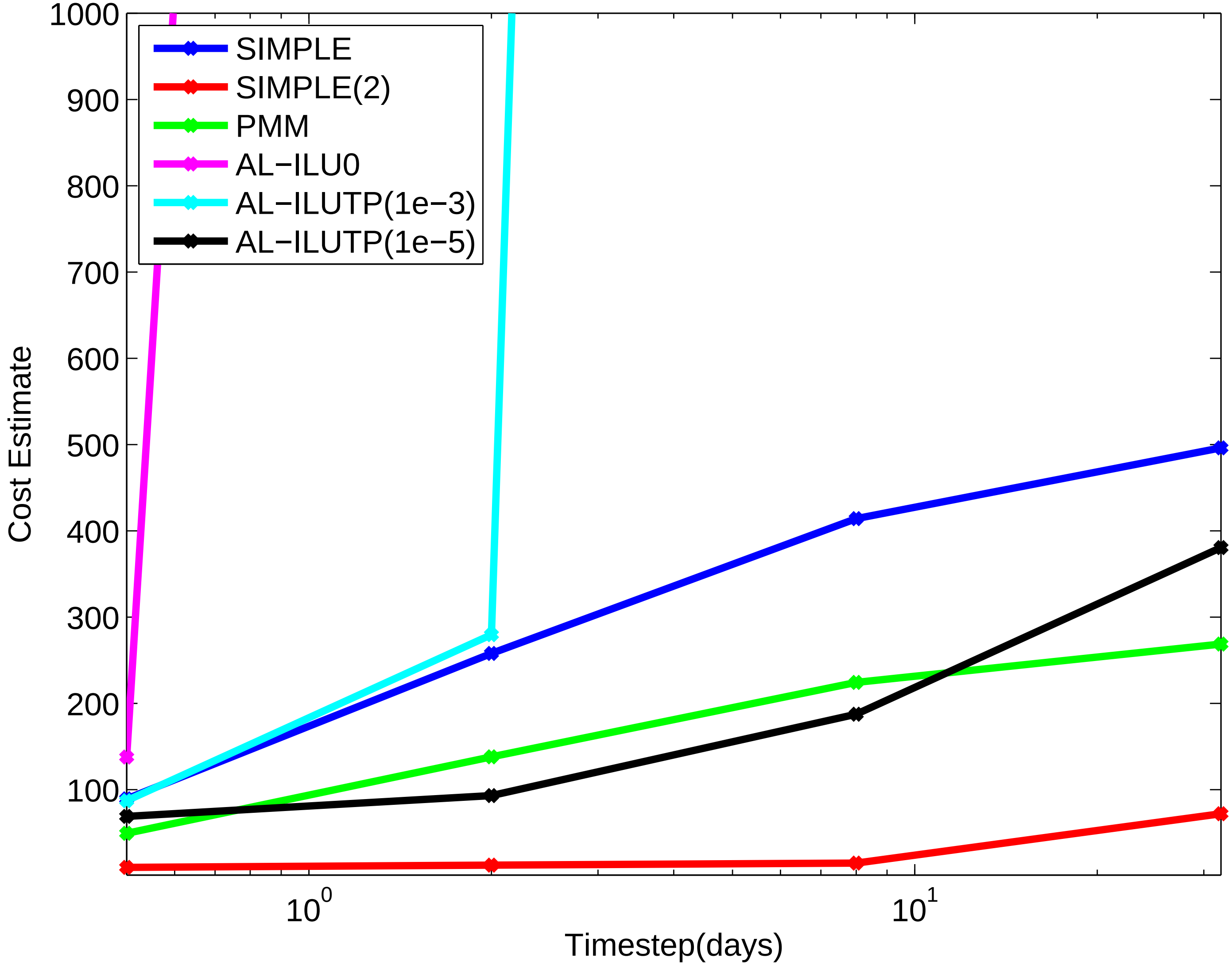

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Final Thoughts

- miniPOP is OK (if optimistic) predictor of POP preconditioner performance.
- Augmented Lagrangian methods
  - Good performance w/ very powerful smoother.
  - Can be expensive.
  - ILUTP w/ secondary fill limits (e.g. SuperLU) useful.
- Schur complement methods.
  - Pressure Mass Matrix OK for small Δt.
  - Block SIMPLE is good for “large” Δt.
    Q: Why is this so time insensitive?

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Future Work

- Approximate commutators may be better:

$$F_p \approx I + \Delta t \begin{bmatrix} -\Delta & \Omega \\ -\Omega & -\Delta \end{bmatrix}$$

  Not 100% clear how to do this.

- Work has begun at LANL to re-introduce z velocities (normal continuity equation).
  - This should improve conditioning substantially.
  - Removing the (2,2) block allows for use of incompressible flow preconditioners.

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