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Strategic Analysis with Simulation-Based Games

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Example 1: Complex Auctions

- A collection of items for sale (e.g., wireless spectrum licenses, slots in a keyword auction)
- The auctioneer fixes an allocation rule (who gets what) and a price rule (how much they pay)
- How does one bid?
  - for which collection of items to bid?
  - how much?

Example 2: Complex Security Settings

- Defender tries to protect a collection of interdependent assets
- Attacker chooses an optimal subset of assets to attack
- Complex consequences of joint defense and attack decisions
- How should attacker act?
- How should defender act?

What is common to these examples?

- Complex models of strategic interactions
  - Non-trivial to reason about agent behavior
  - Can simulate outcomes (payoffs)
- How can we systematically analyze such "complex" models?
- Answer: simulation-based game theory

Outline

- Preliminaries
- Nash equilibrium estimation
- Application to keyword auctions

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Some game theoretic formalisms

- **A game is defined by a set of agents, I (|I| = m), a strategy set S_i and a utility function u_i for each agent i**
- **A Nash Equilibrium is a joint strategy s of all agents such that for each agent i, s_i maximizes i's utility given that other agents' strategies remain fixed**
- **A useful technical construct is a *regret function* which measures for each strategy profile s the incentive that any player has to deviate from s:**

$$\epsilon(s) = \max_{i \in I} \max_{a_i \in A_i} u_i(a_i, s_{-i}) - u_i(s)$$

Simulation-based games

- A simulator takes as input a strategy profile s, and returns a sample of the payoff vector, $U = (U_1,\ldots,U_m)$
- Use $\hat{u}(s)$ to be an estimate of the true payoff vector, $u(s)$, based on simulation samples
- Assume that the game is symmetric: all players' strategy sets S_i and payoff function u_i are the same; so I will use u to denote a payoff function of an arbitrary player

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Finite (and small) games

- **Sample payoff for every strategy profile**
- **Estimate expected payoffs (sample means)**
- **Compute Nash equilibria of the estimated game**
- ***Theorem*: *Regret functions converge a.s.* uniformly on the space of mixed strategies**
- ***Theorem*: *Estimated equilibria converge to a subset of true equilibria***

Infinite games

- Focus on pure strategies only (hard enough)
- A strategy is a vector in R^n (finite dimensional)
- Can deal with Bayesian games
  - in general, strategies in such games are functions of private information
  - we restrict attention to finite-dimensional strategy spaces

Stochastic search methods for infinite simulation-based games

- Consider a symmetric strategy profile s
- Fix the strategies of all players except some player i
- Consider the problem of computing (or approximating) i's best response to a fixed strategy profile s_{-i} by all others
- Given a simulation-based game, the problem of computing a best response is a stochastic optimization problem (maximizing i's utility given simulations)
  - Suppose we can do this; we get both the best response strategy of i, as well as regret function value for this (symmetric) strategy profile

What good is knowing a best response?

- Approach 1: best response dynamics
- Approach 2: approximate regret minimization (ARM)

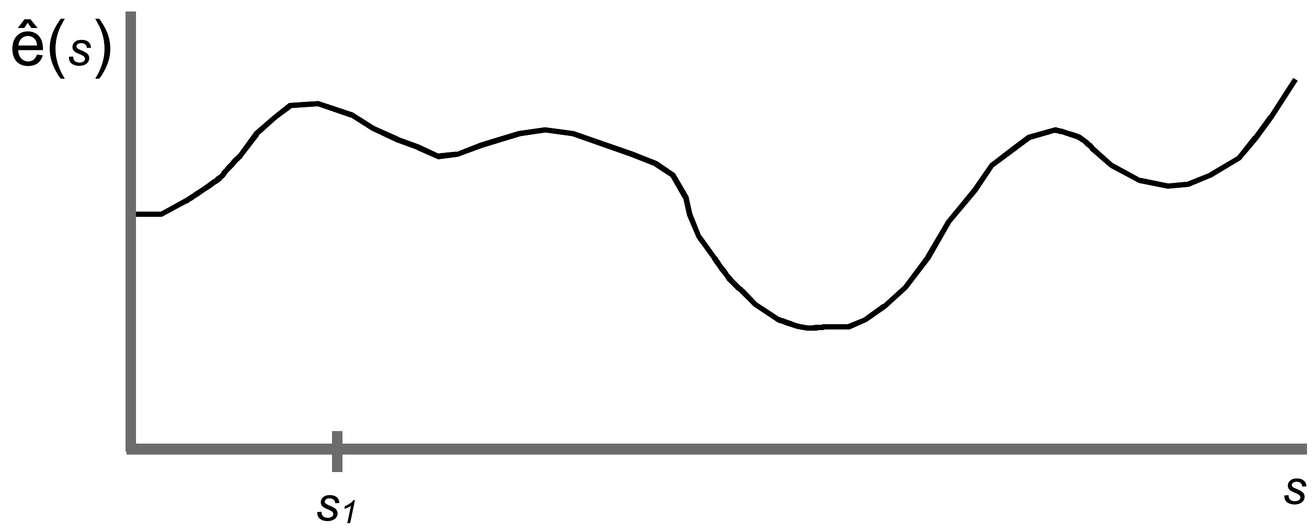
Best response dynamics

- Start with an arbitrary symmetric strategy profile s
- Find best response, br(s), to s of an arbitrary player
- Set s to br(s) in the next iteration
- Repeat
- Poor convergence properties, but has been observed to perform well in practice

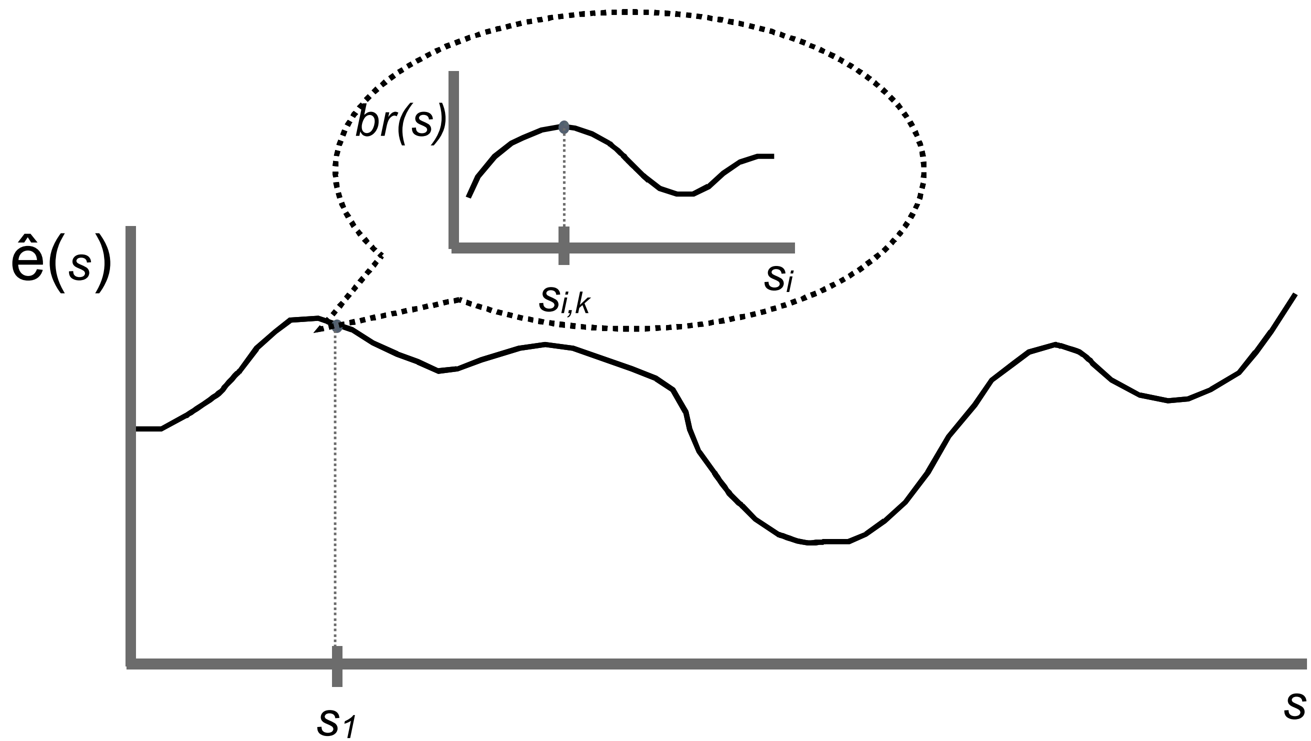
Approach 2: Approximate regret minimization (ARM)

- Computing a Nash equilibrium is equivalent to minimizing the regret function $\varepsilon(s)$
  – If a Nash equilibrium s* exists, then it is a global minimum of $\varepsilon(s)$
  – If a Nash equilibrium does not exist, a global minimum s* is a $\varepsilon(s^*)$-Nash equilibrium (and the best approximation of Nash)
- Now suppose that we only have an estimate of $\varepsilon(s)$, ê(s)
  – Finding the minimum of ê(s) gives us an approximate Nash equilibrium
- But we know how to estimate ê(s) for any s!

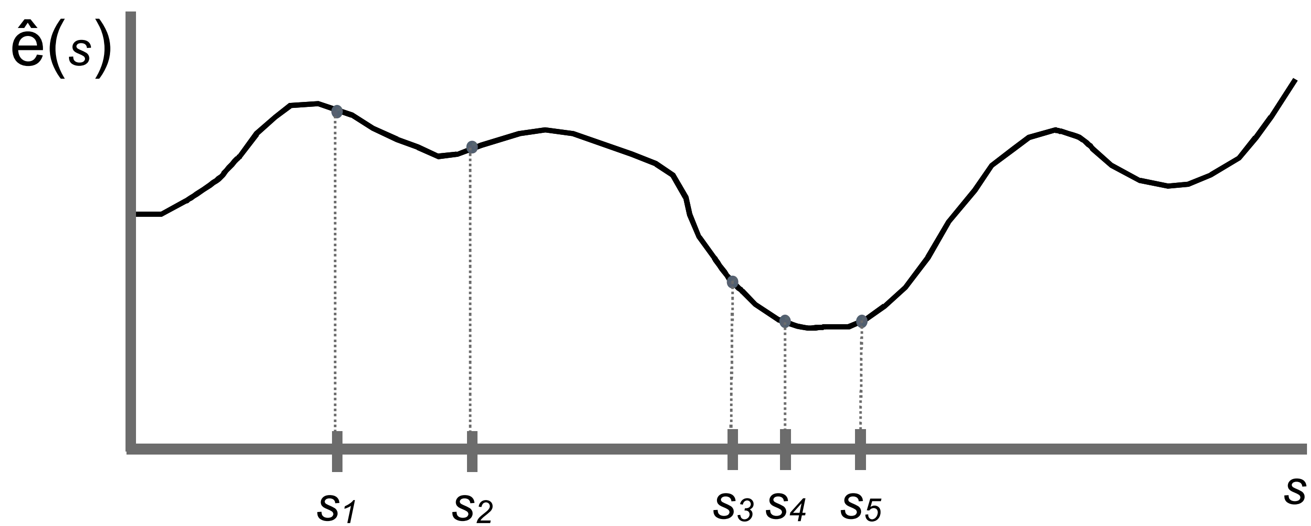
Approximate Regret Minimization



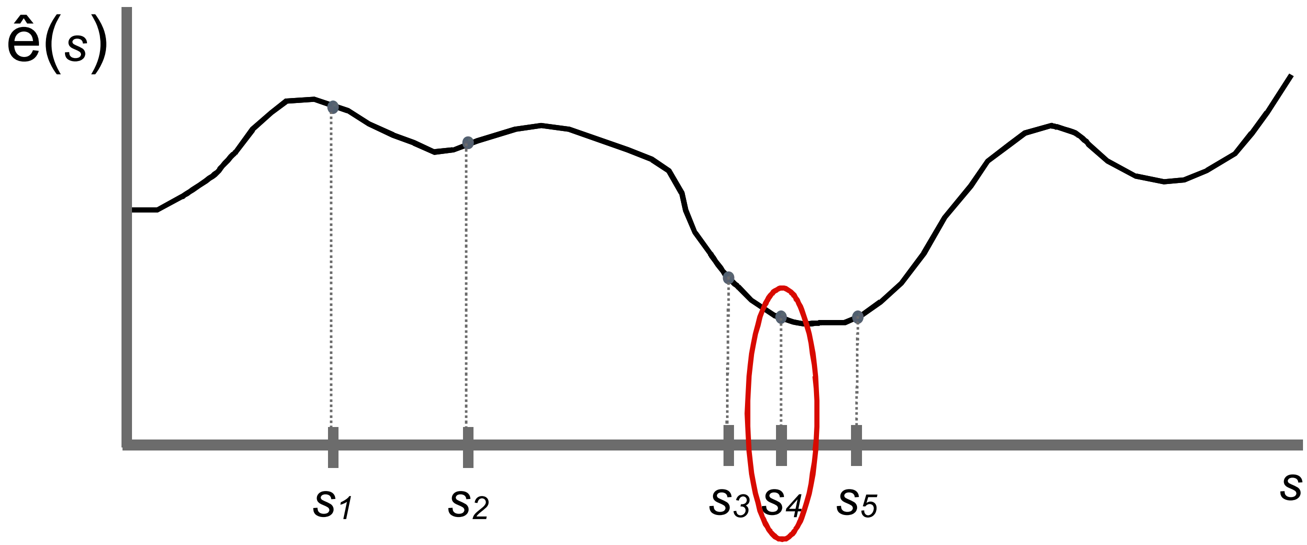
Approximate Regret Minimization



Approximate Regret Minimization



Approximate Regret Minimization



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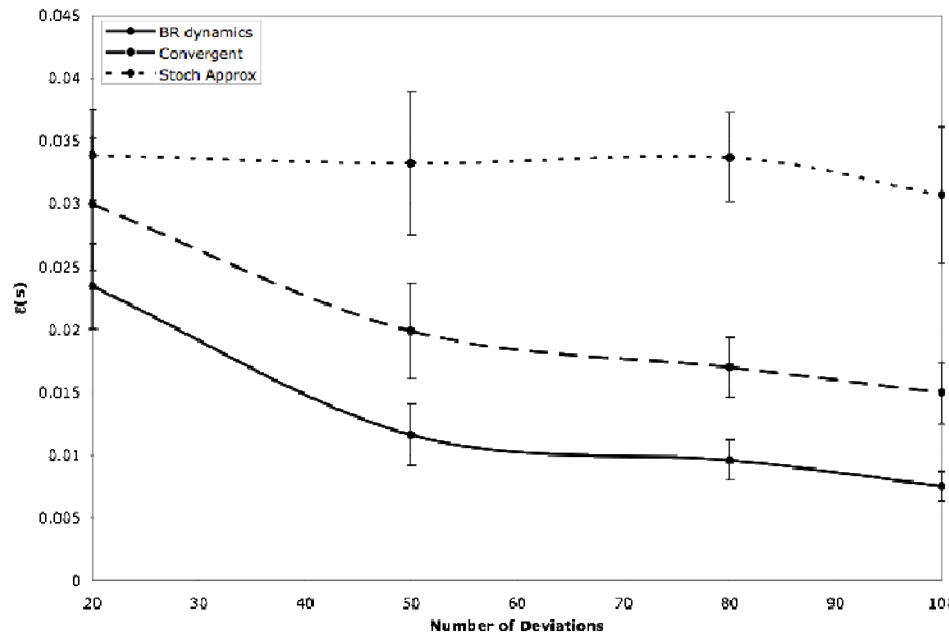
Approximate regret minimization

- *Theorem: ARM converges in probability (in terms of regret value) to the best approximation of Nash*
- *Corollary: if a Nash equilibrium exists, ARM converges to it (regret value goes to 0)*
- Modification of best response dynamics:
  - for each symmetric profile s visited, compute and keep $\hat{e}(s)$
  - at the end, choose a profile s with the smallest $\hat{e}(s)$

ARM vs. Best response dynamics

- **The ARM algorithm is provably convergent**
- **Best response dynamics need not converge**
- **Best response dynamics often very effective in practice**

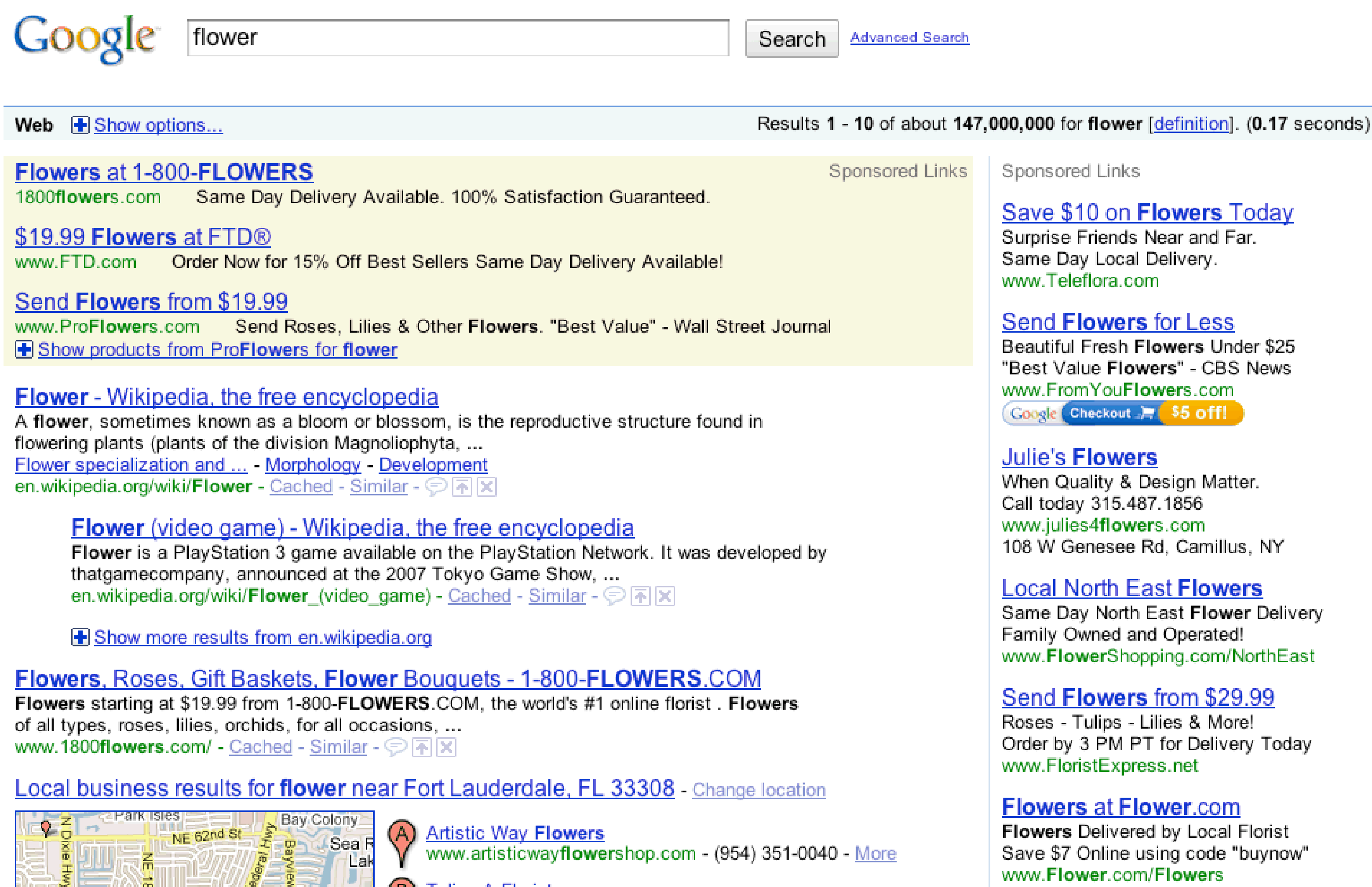


In practice, BR dynamics may be better

Application: Keyword Auctions

Keyword Auctions

Keyword Auctions



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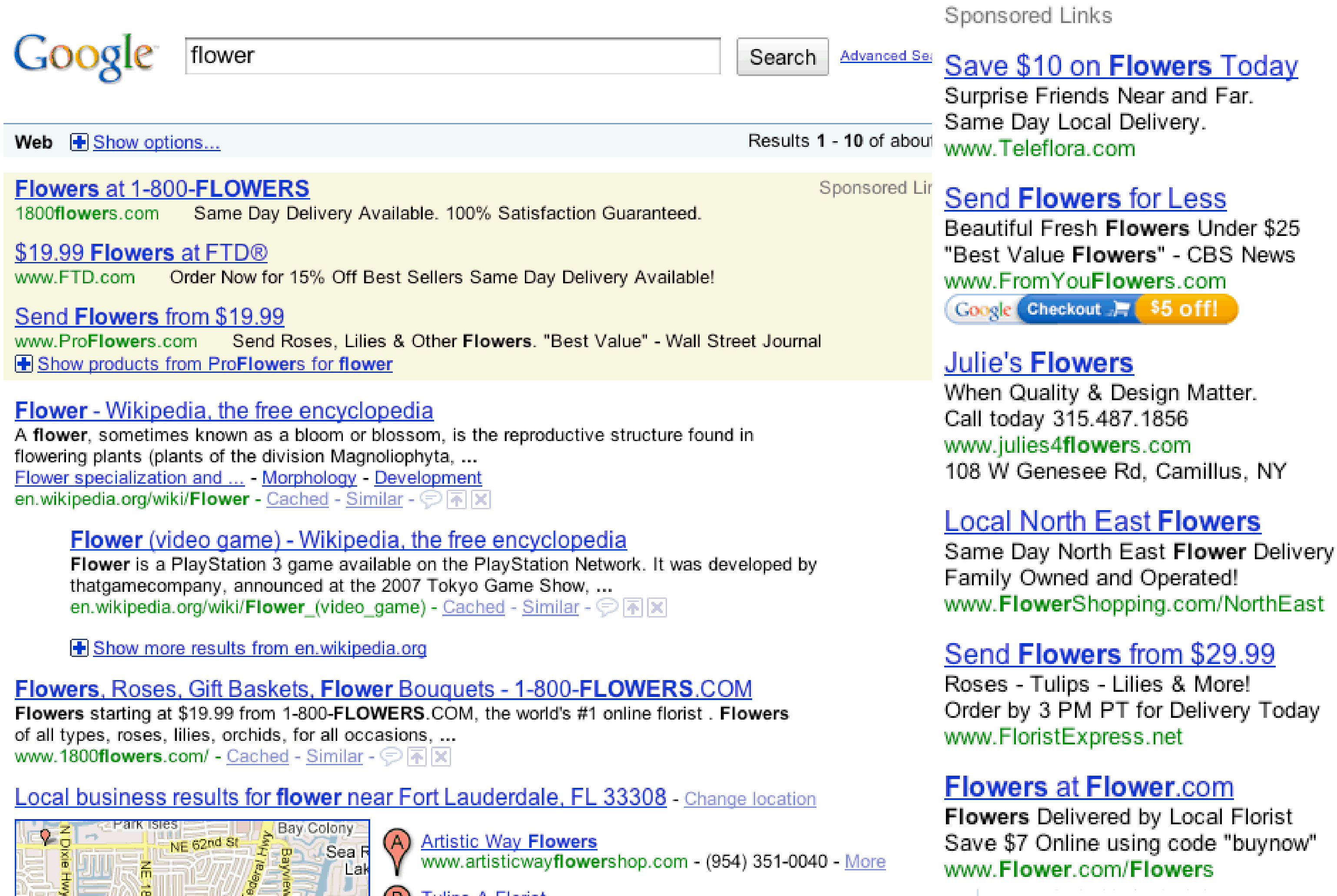
Keyword Auctions



Keyword Auctions

Sponsored Links

Save $10 on Flowers Today
Surprise Friends Near and Far.
Same Day Local Delivery.
www.Teleflora.com

Send Flowers for Less
Beautiful Fresh **Flowers** Under $25
"Best Value **Flowers**" - CBS News
www.FromYou**Flowers**.com
Google Checkout $5 off!

Julie's Flowers
When Quality & Design Matter.
Call today 315.487.1856
www.julies4**flowers**.com
108 W Genesee Rd, Camillus, NY

Local North East Flowers
Same Day North East **Flower** Delivery
Family Owned and Operated!
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Send Flowers from $29.99
Roses - Tulips - Lilies & More!
Order by 3 PM PT for Delivery Today
www.FloristExpress.net

Flowers at Flower.com
Flowers Delivered by Local Florist
Save $7 Online using code "buynow"
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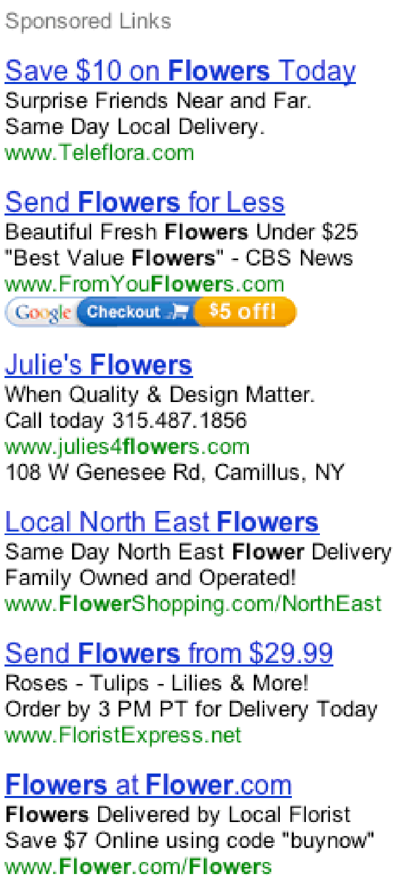
Keyword Auctions



- Advertisers submit bids b_i for keyword flower
- For each of them, the search engine computes a quality score e_i
- Advertisers are ranked according to a “score”:
  - bid * (quality score)^q, where q is a parameter in [0,1]
- Payment (per click) is the smallest amount that will keep the advertiser in his current slot
  - GSP (generalized second-price auction)

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Incentives to “Lie” in GSP

- While GSP was an attempt at generalizing Vickrey auctions, it is well-known to induce insincere bidding (i.e., $b_i \neq v_i$)
- How serious is the problem?
  - Are incentives to bid insincerely significant?
  - Are actual bid reductions large?
- Restrict strategies to be $b_i = \alpha v_i$
  - Large “bid reduction” <=> small α

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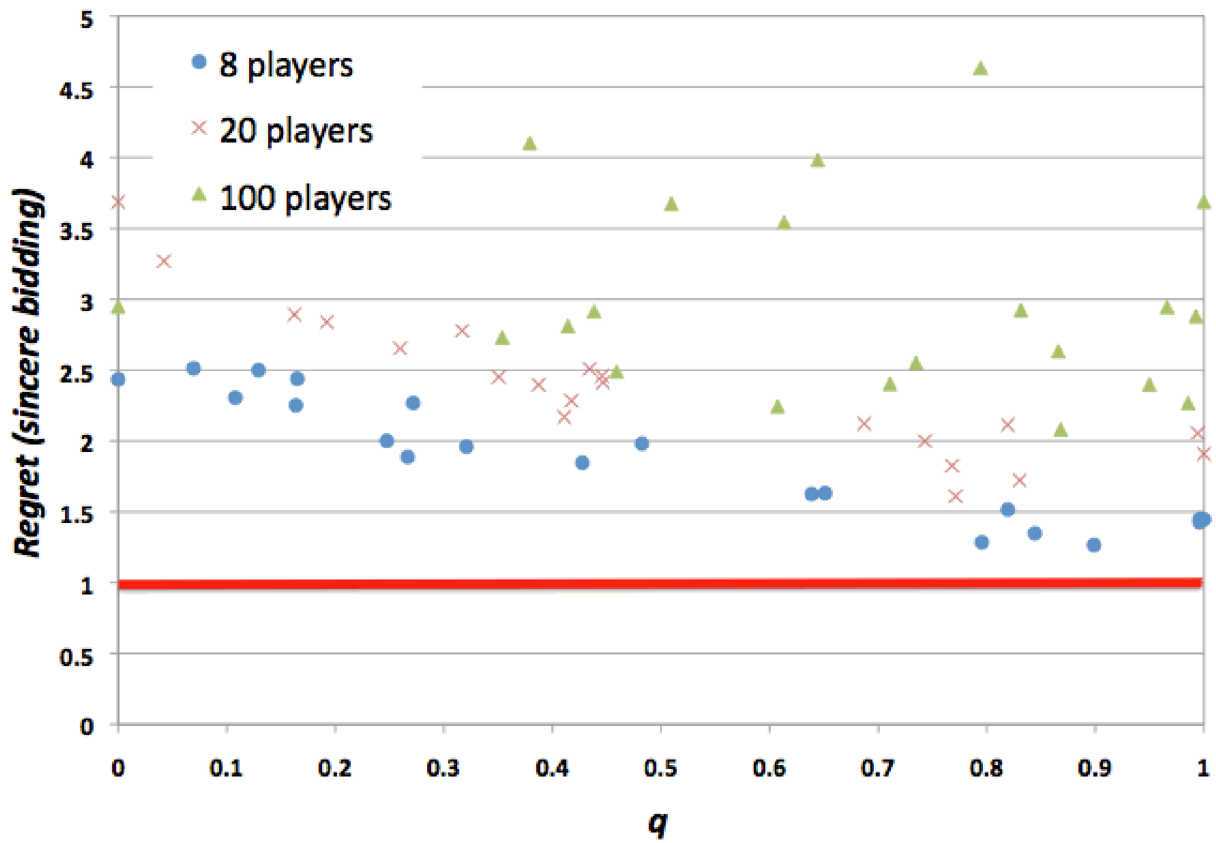
Incentives to Deviate from Sincere Bidding



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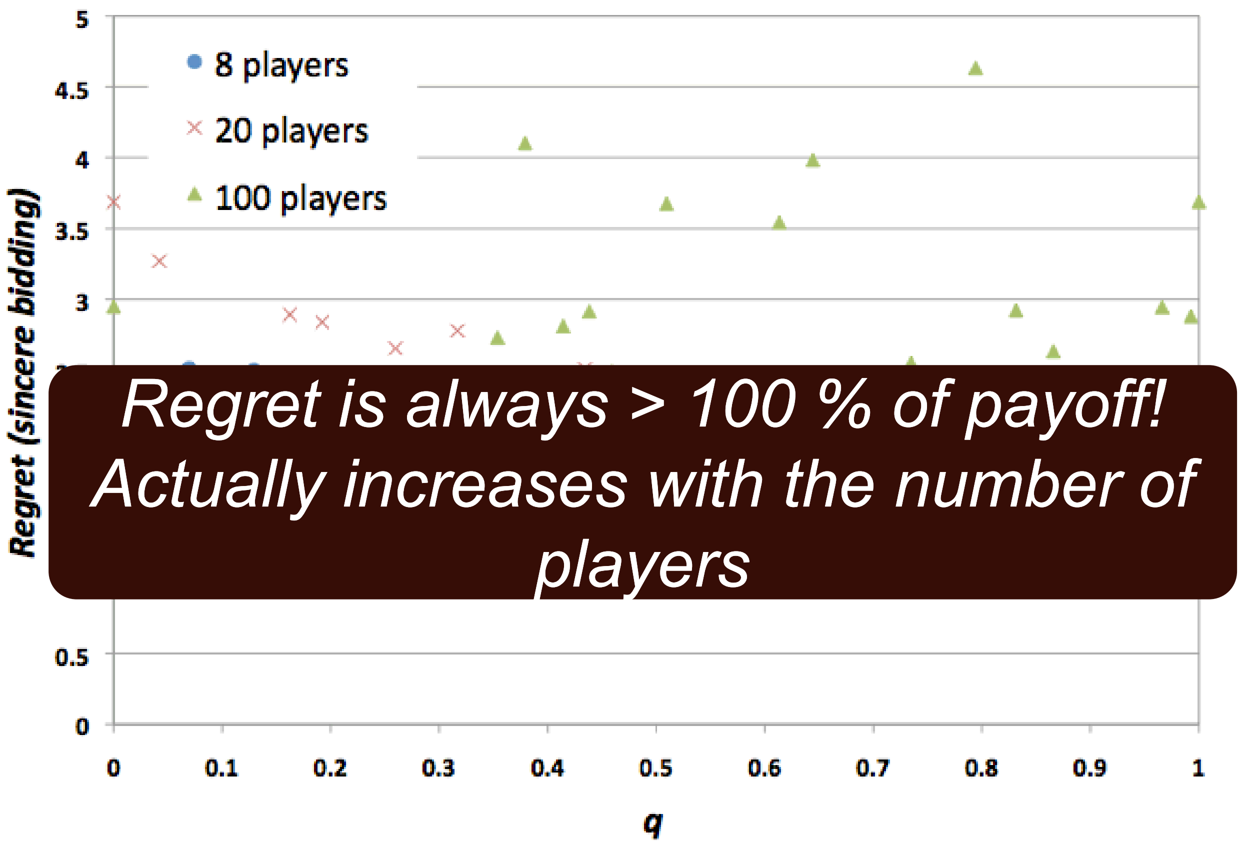
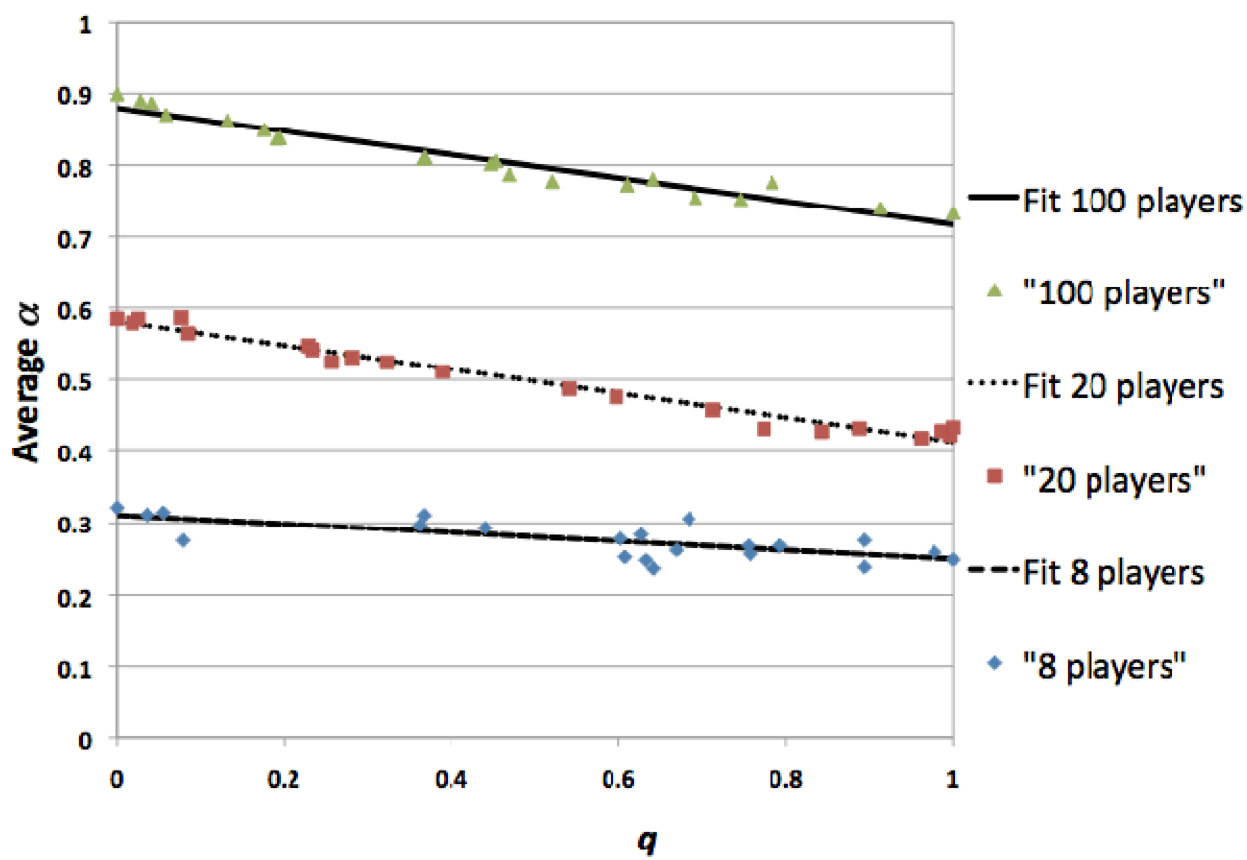
Incentives to Deviate from Sincere Bidding

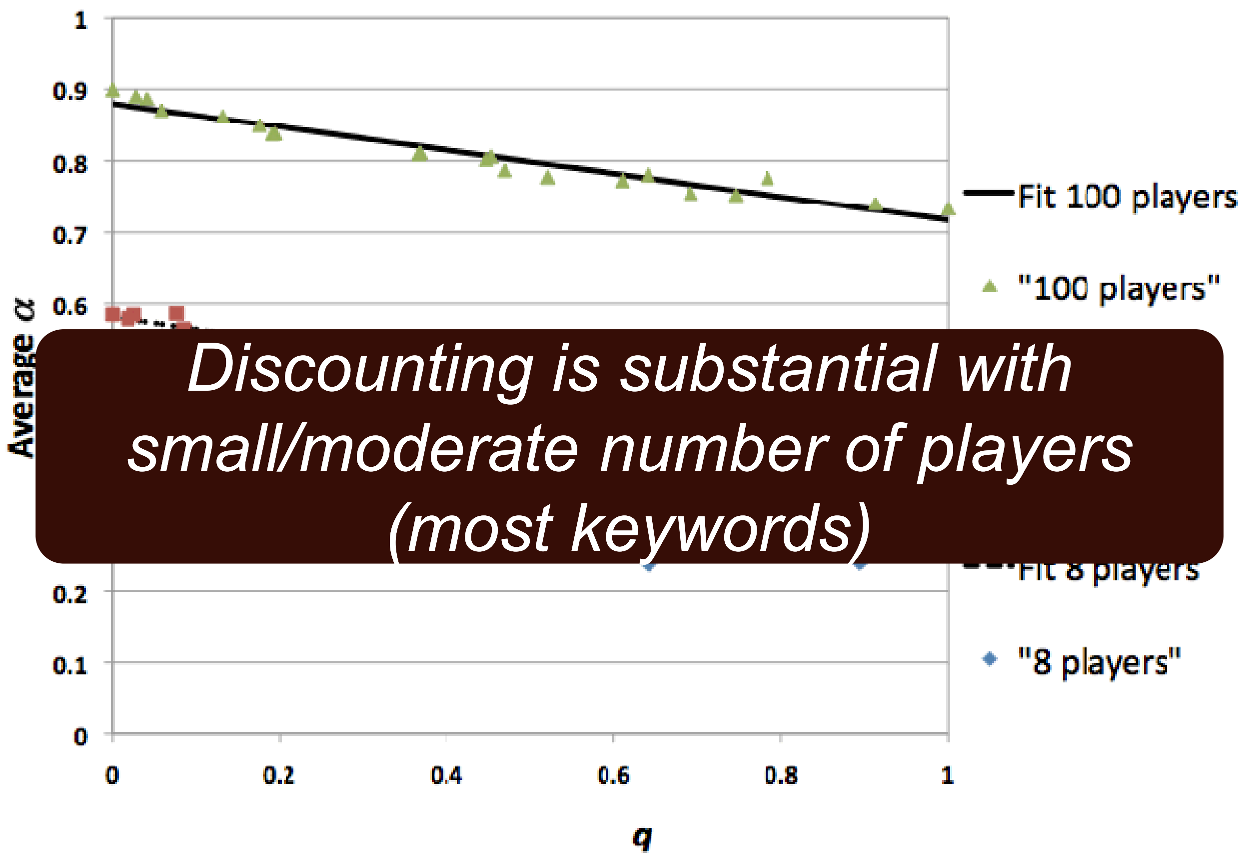


Regret is always > 100 % of payoff! Actually increases with the number of players

Equilibrium α



Equilibrium α

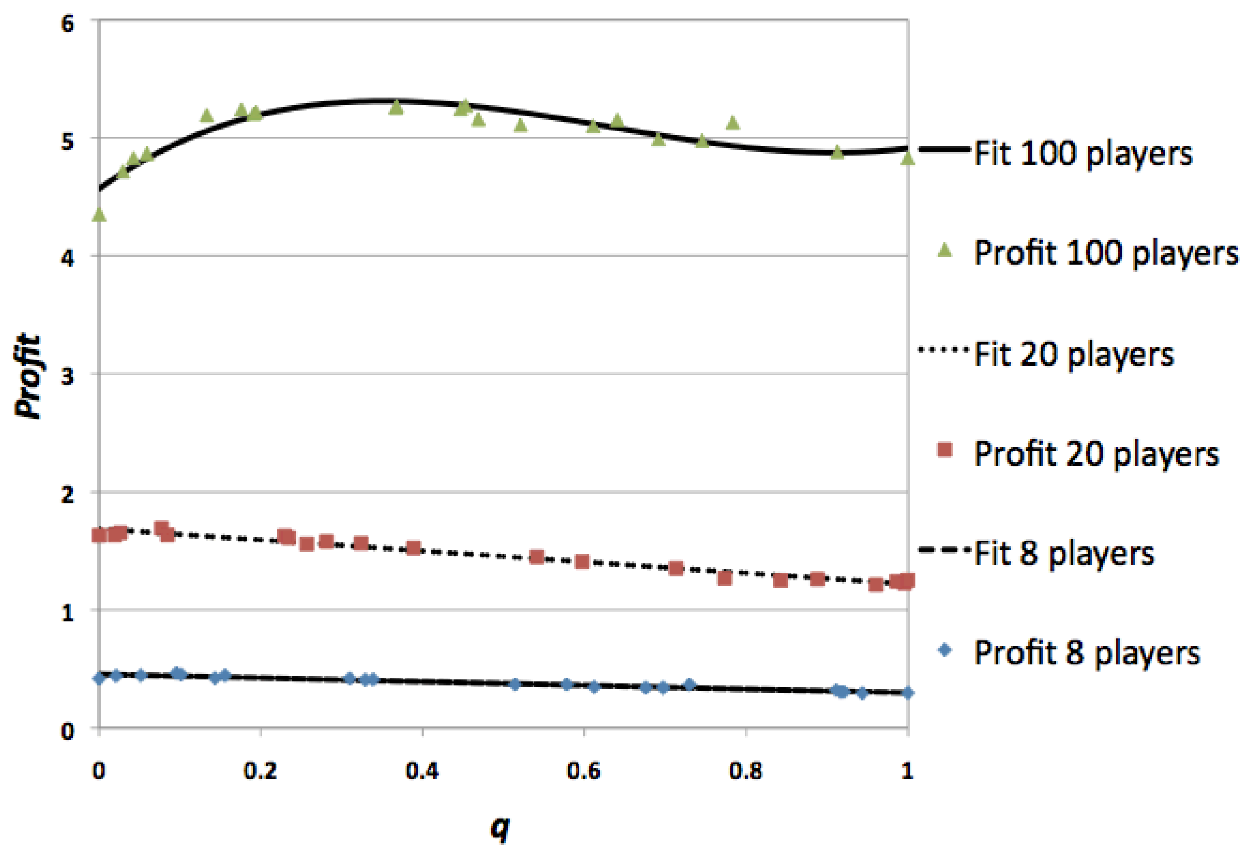


Discounting is substantial with small/moderate number of players (most keywords)

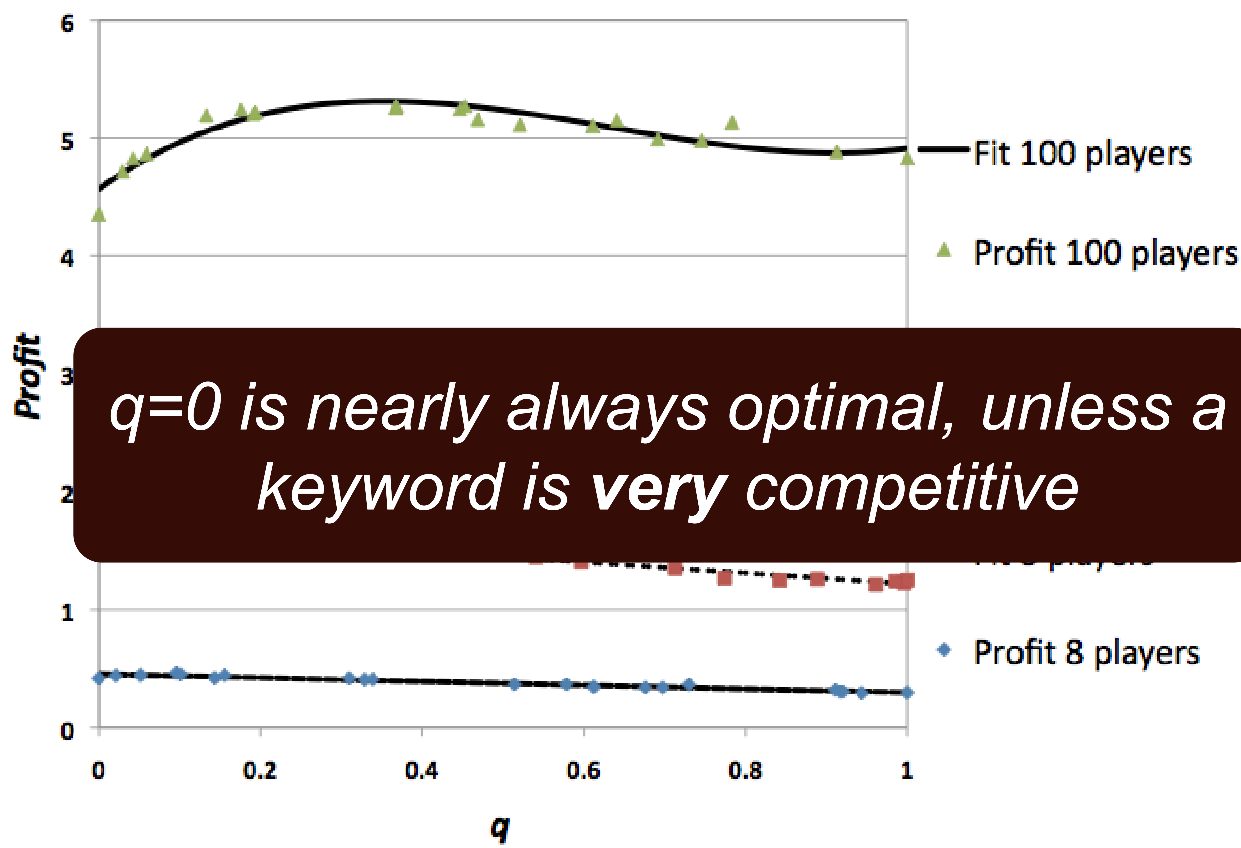
Maximizing Revenue in Keyword Auctions

- Would like to find a setting of q that maximizes revenue

Profit Function



Profit Function: Should Rank by Bid



*q=0 is nearly always optimal, unless a keyword is **very** competitive*

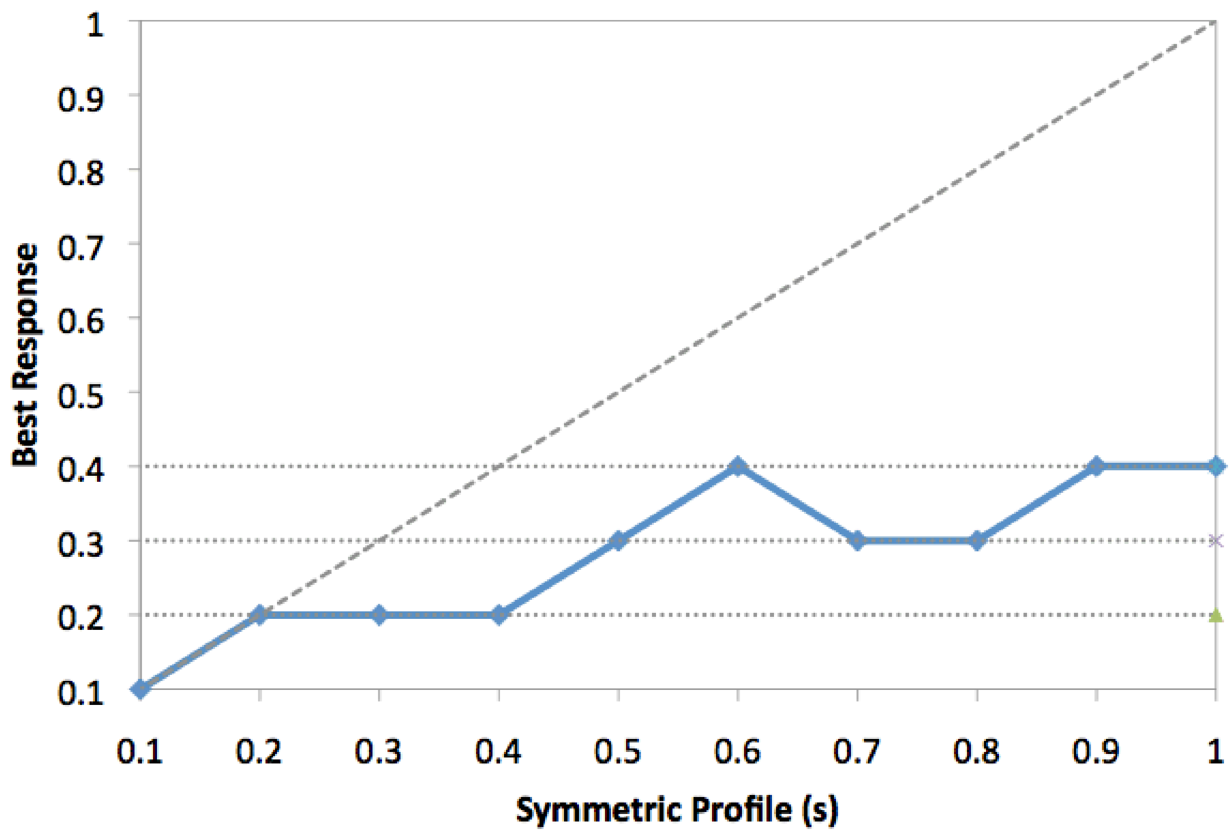
Bidding in TAC/AA

- Trading Agent Competition, Ad Auction Game
  - highly complex simulation environment for competition between autonomous advertising agents
- QuakTAC agent, based on simulation-based game theoretic analysis
  - Fourth place finish (6.78% from 1st place, 2.34% below second place, 1.25% below third place); simple design

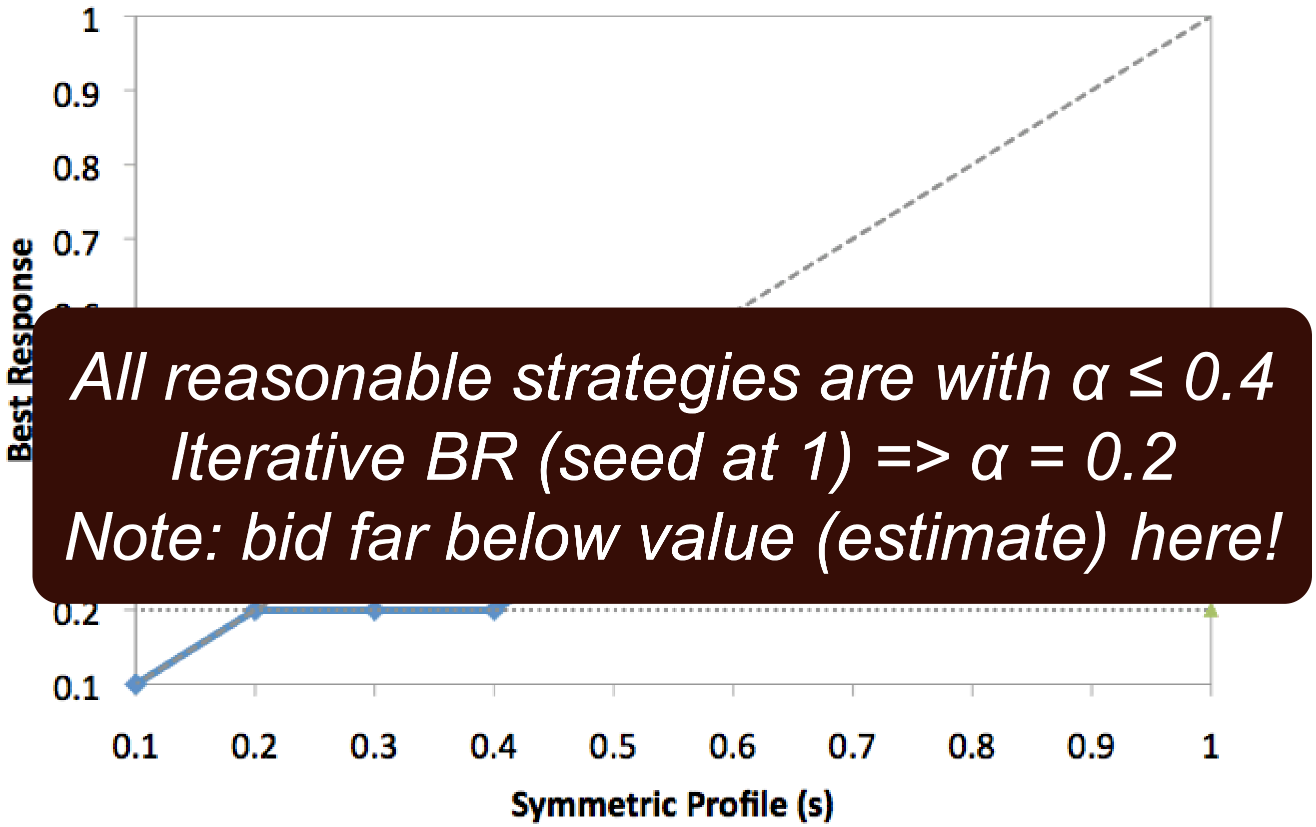
Bidding in TAC/AA

- **Focus on bidding strategy, linear form $b_i = \alpha_i v_i$**
- **Discretize $\alpha \in \{0.1, 0.2, ..., 1\}$**
- **Primary agent tasks: estimate values v_i and Nash equilibria**

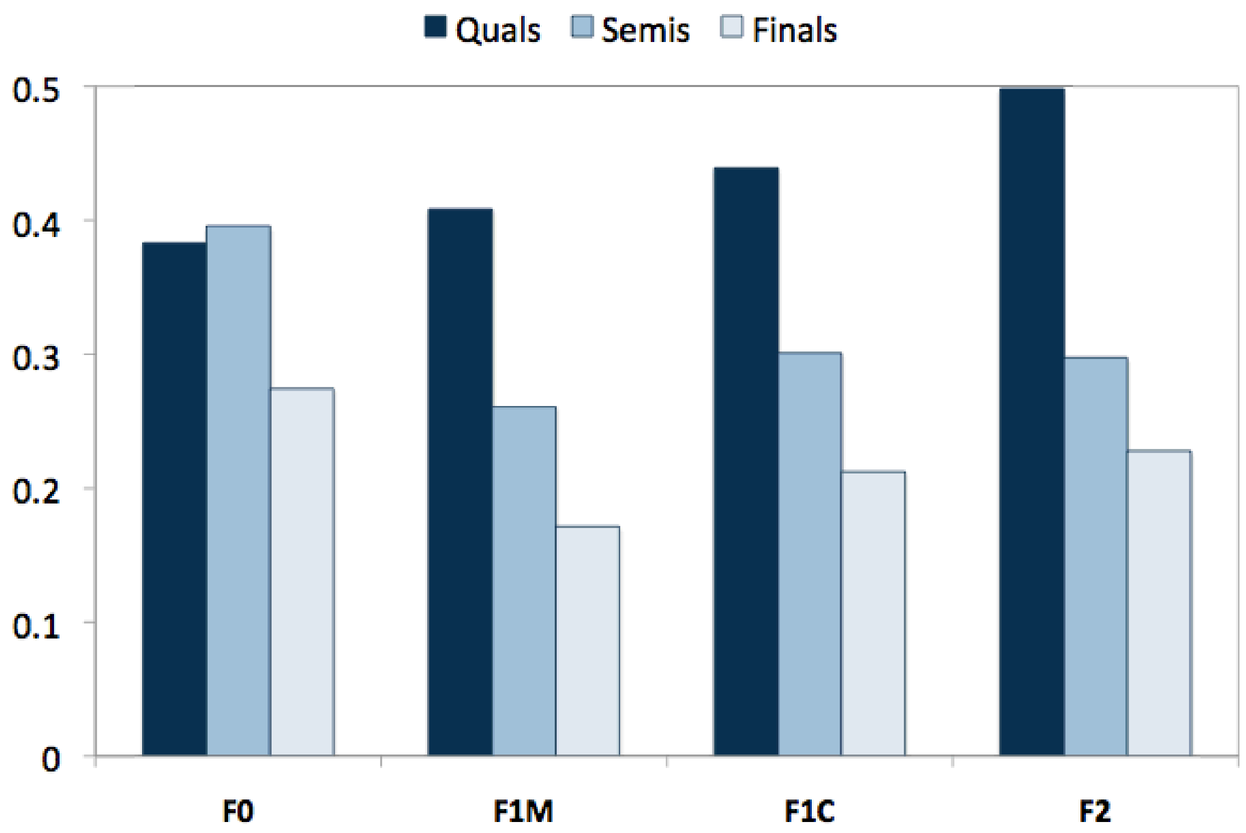
Best Response Function



Best Response Function



Equilibrium: Good Prediction?



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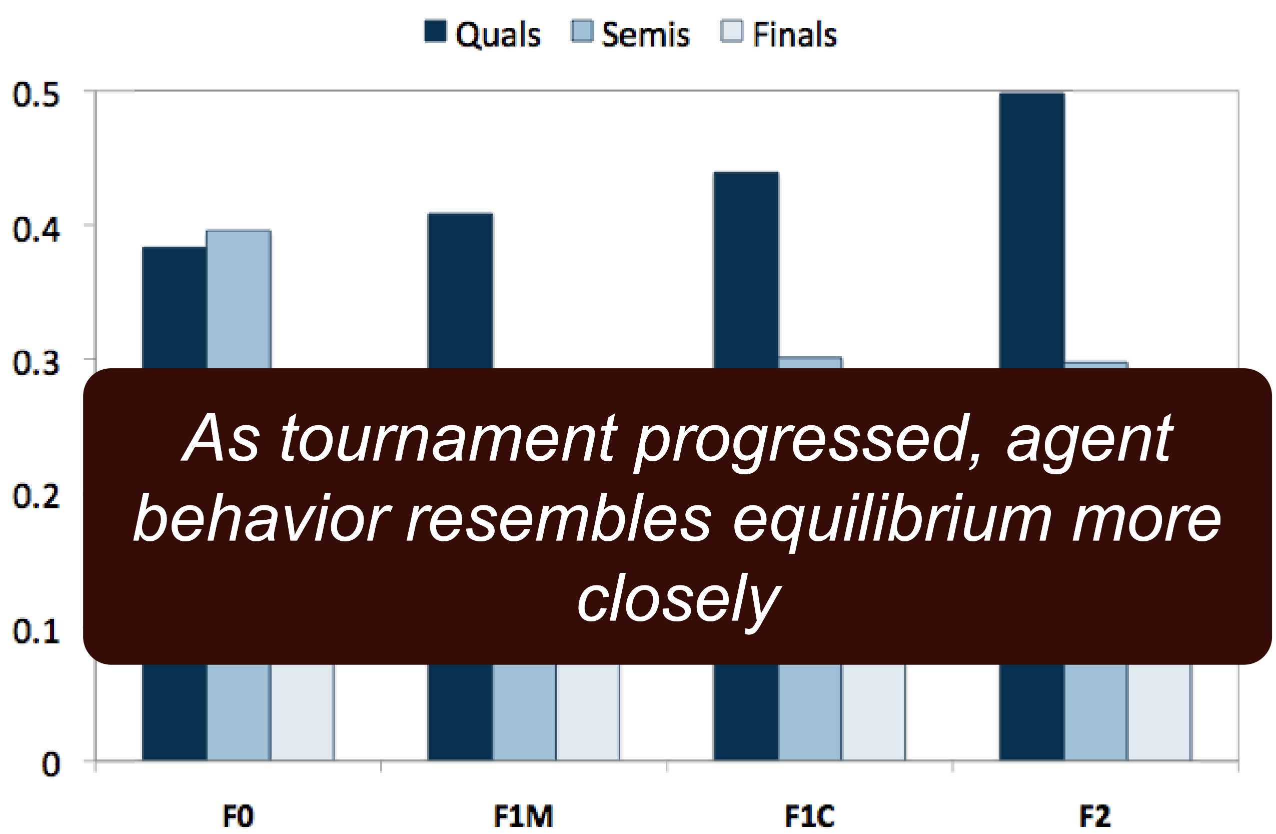
Equilibrium: Good Prediction?



As tournament progressed, agent behavior resembles equilibrium more closely

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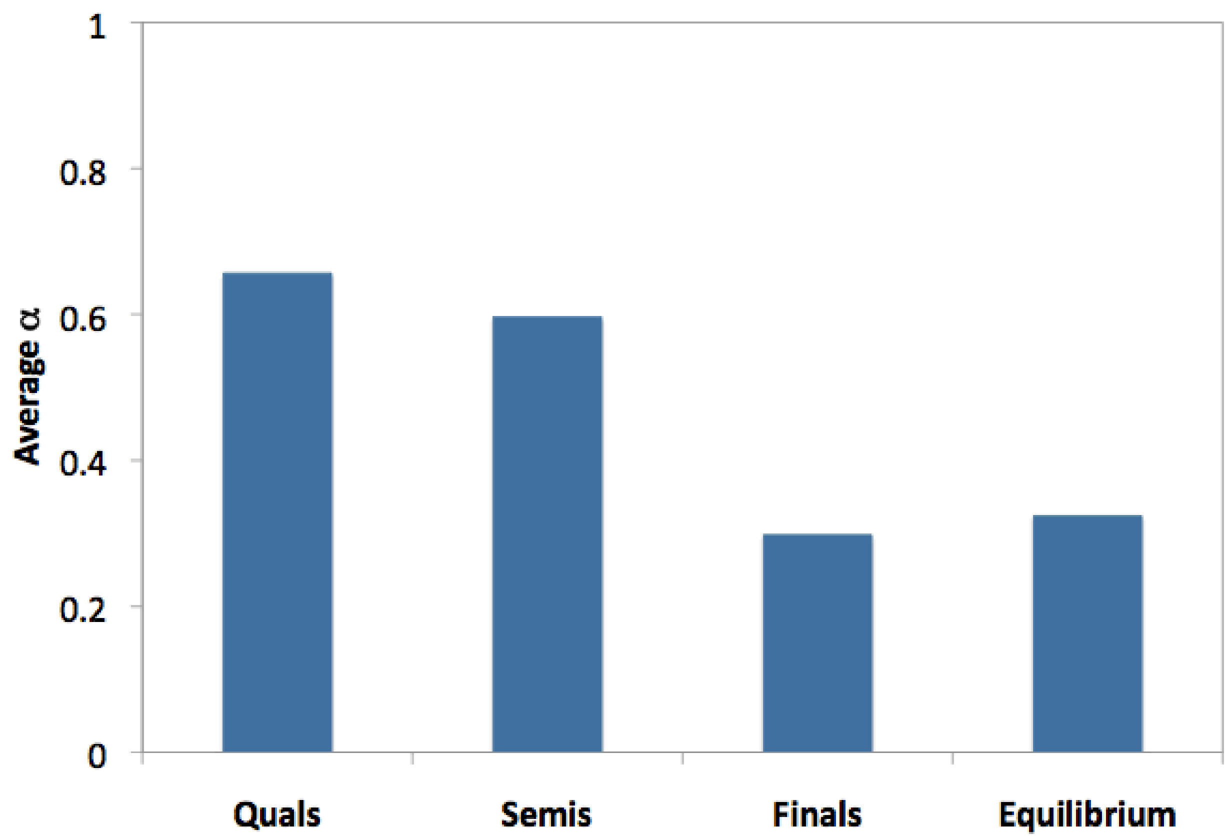
Equilibrium: Good Prediction?



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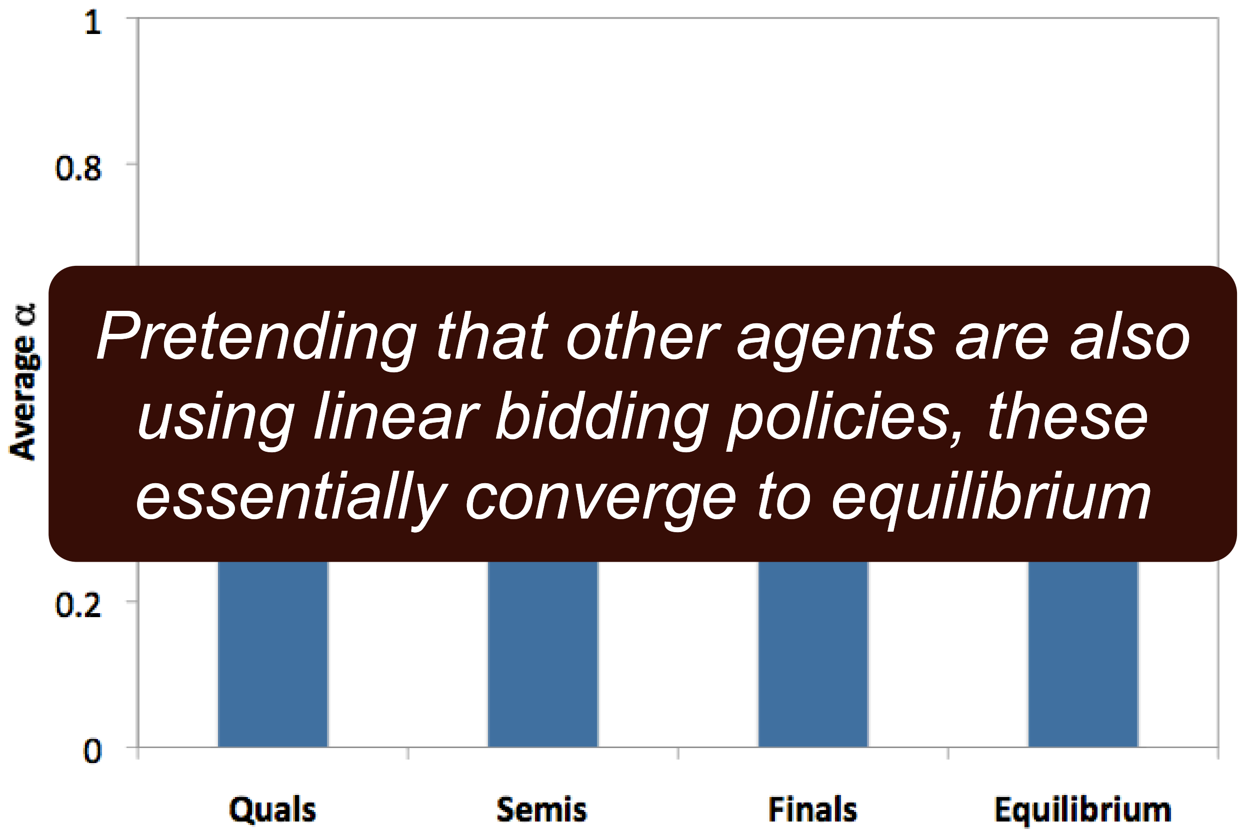
Equilibrium: Good Prediction?



Pretending that other agents are also using linear bidding policies, these essentially converge to equilibrium

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Summary

- **ARM: convergent algorithm for estimating a Nash equilibrium in infinite games**
- **Best response dynamics + regret minimization: empirically better**
- **Effective in analysis of a Bayesian model of keyword auctions**
- **Effective use of simulation-based game theoretic techniques in developing a successful TAC/AA agents**