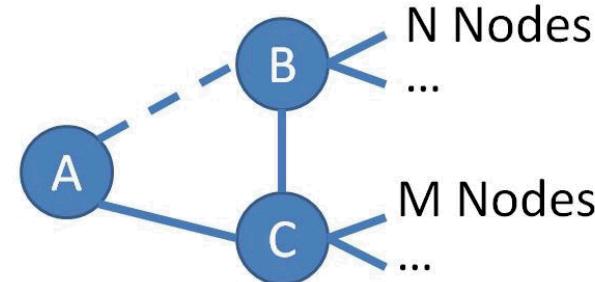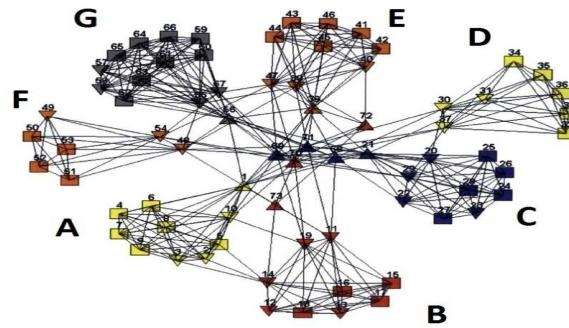*Exceptional service in the national interest*

Bayesian Security Games for Controlling Contagion
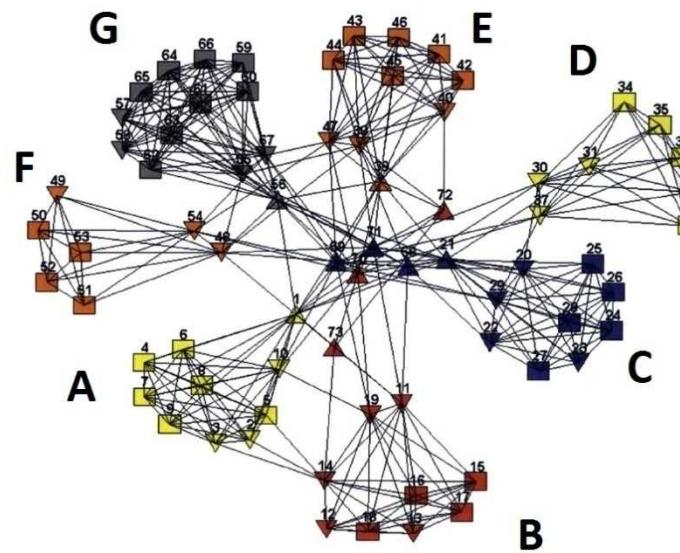
Jason Tsai[1], Yundi Qian[1], Yevgeniy Vorobeychik[2], Christopher Kiekintveld[3], and Milind Tambe[1]

[1]USC, [2]Sandia National Laboratories, [3]UTEP
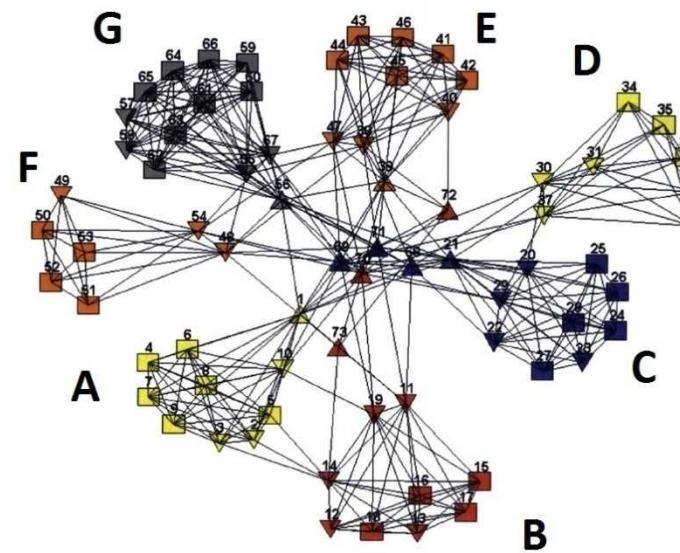
Setting

- Start with a network: edges represent possible influence transmission

- Two players, both aim to maximize influence, i.e., convert the majority of the nodes to their agenda

Motivation: Counterinsurgency

- U.S. vs Taliban in Afghanistan

- Taliban attempts to gain local support
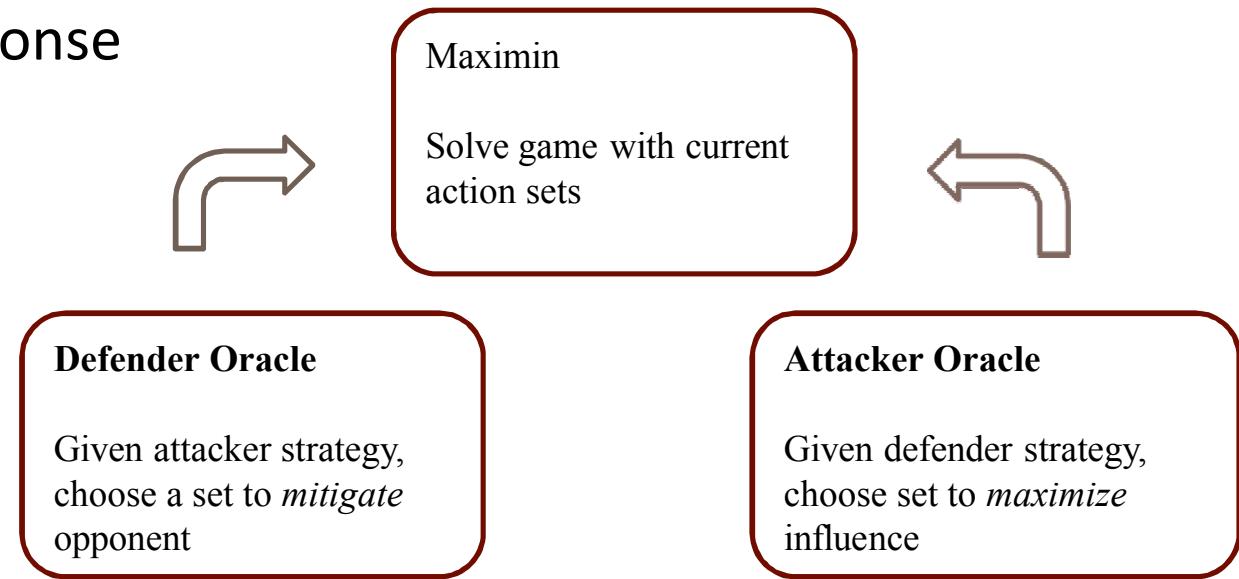
- U.S. attempts to mitigate Taliban support

Point of Departure: Baseline Model

- 2-players on a network: "influencer" (maximize contagion) vs "mitigator" (minimize impact of influencer); zero-sum

- Spread of influence: variation on independent cascades model (Kempe et al., 2003)

  - Both players simultaneously select up to R nodes

  - Activate each edge (i,j) with probability p_{ij}

  - Any uninfluenced node is "activated" if it has an edge to an influenced node

  - If node i has influenced neighbors of both kinds, flip unbiased coin

Computing Equilibria in Baseline Model

- Formulate as a (very large) Linear Program (actions = subsets of nodes to influence)

- Double-oracle algorithm
  - In each iteration, for each player, compute best response to the other, add this strategy to the LP, repeat

- Greedy best response
- LSMI heuristic
  to approximate
  influence

Maximin

Solve game with current action sets

Defender Oracle

Given attacker strategy, choose a set to *mitigate* opponent

Attacker Oracle

Given defender strategy, choose set to *maximize* influence

Uncertainty about the Network

- Previous art assumes complete certainty (or symmetric information) about the network structure

- In reality, the influencer typically knows the network much better than the mitigator

  - Influencer is "local", better attuned to culture, reputation, better understands how the influence actually flows

  - Mitigator is a foreigner, needs considerable effort to obtain accurate information, and may still be wrong due to misunderstanding local culture and linguistic nuances

- Model this information asymmetry as uncertainty of the mitigator about network structure

  - Existence of nodes/edges, influence probabilities, relative value of nodes (who are the real opinion leaders)

Model: Bayesian Zero-Sum Game

types are possible graphs
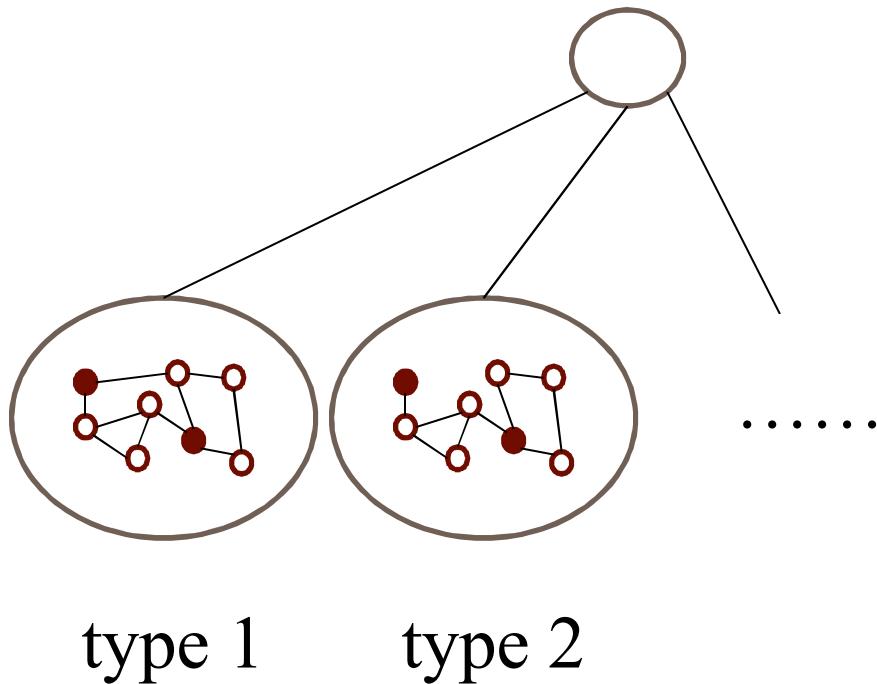
Assume that the influencer knows the true graph

type 1   type 2

Past Literature:
Graph Structure has Significant Impact

- Watts and Strogatz (1998) – a few short cuts make a dramatic difference

- Costenbader and Valente (2003) – numerous centrality measures shown to be highly sensitive to data errors

- Kossinets (2008) – Missing data can dramatically alter network-level statistics

- Lahiri et al. (2008) – network changes can have dramatic and unpredictable impacts on influence spread

Observation: Loss from Ignoring Uncertainty Can be Unbounded!
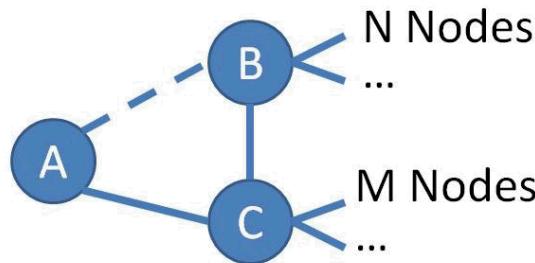
TODO: argument

#P-Hard even to estimate the impact of uncertainty about *a single edge*!

Scalability Challenges

- Can formulate our problem as an LP (since zero-sum), but there are 3 major scalability challenges

- Challenge 1: Estimating influence is #P-Hard
  - Solution: use LSMI heuristic (Tsai et al, 2012)

- Challenge 2: Strategy spaces of both players are exponential
  - Solution: Bayesian double-oracle (Halvorson et al., 2009)

- Challenge 3: Exponential number of influencer types (each type = graph)
 - Solution: ?

"Naïve" Idea

- Naïve idea:
  - take K sample types from the distribution over graphs
  - Use Bayesian double-oracle with only these types
- How well does it work? How large does K need to be (relative to the total number of possible types) to get good solutions?
- These are empirical questions

Experiments

- Consider variations of:
    - Graph topologies (generative models and real graphs)
    - Uncertainty/distribution over graphs
- Graph topologies:
    - "Scale-free": preferential attachment model
    - "Small-world": Watts-Strogatz model
    - BTER: generate K dense Erdos-Renyi subgraphs (communities), add random inter-community edges; can specify arbitrary degree distribution and clustering coefficient distribution
    - "Indian villages": social network for several Indian villages
- Models of uncertainty:
    - "Random Edge Uncertainty": uncertain about K randomly chosen edges
    - "Influential Node Uncertainty": uncertain about the identity of a highly influential node (add 4 edges to 3 randomly chosen nodes)
    - "Intercommunity Edge Uncertainty" (BTER only): uncertain about K edges that connect communities (dense subgraphs)
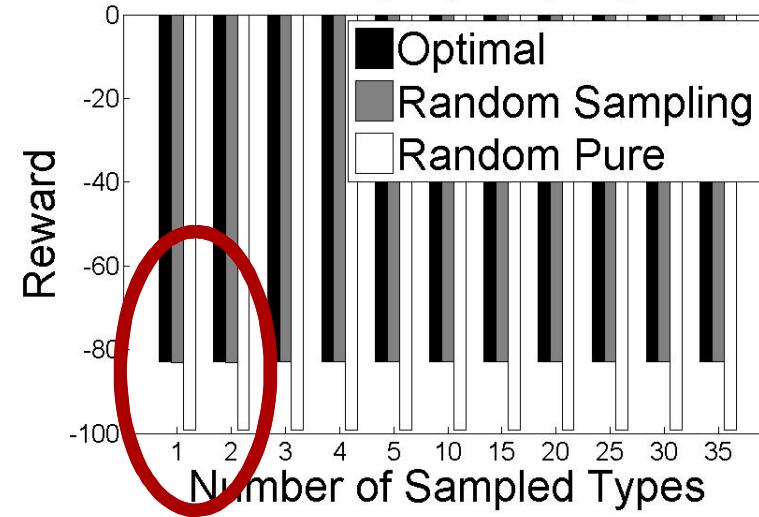
Scale-free, Small-world graphs
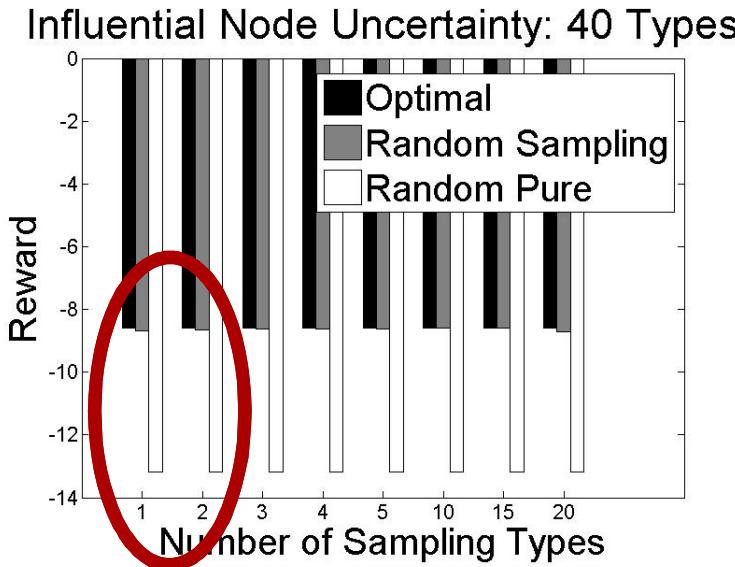## (random edge uncertainty)
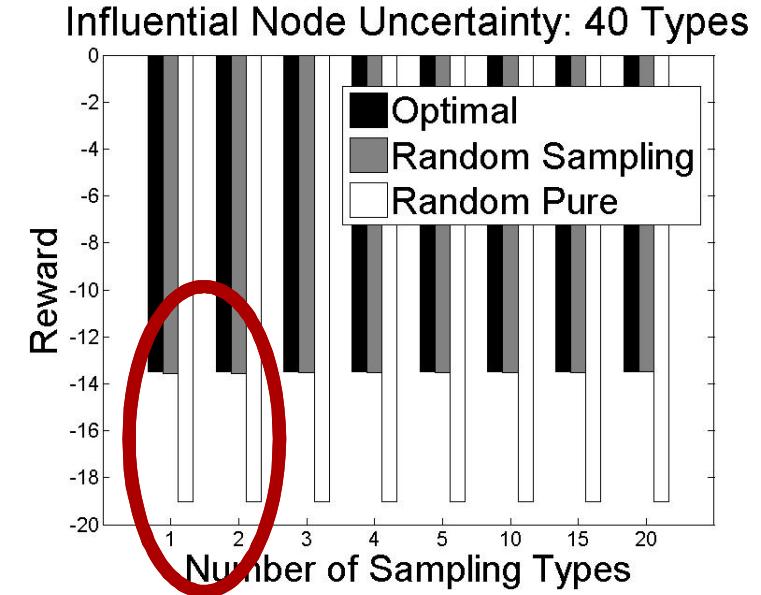

Scale-free Graphs
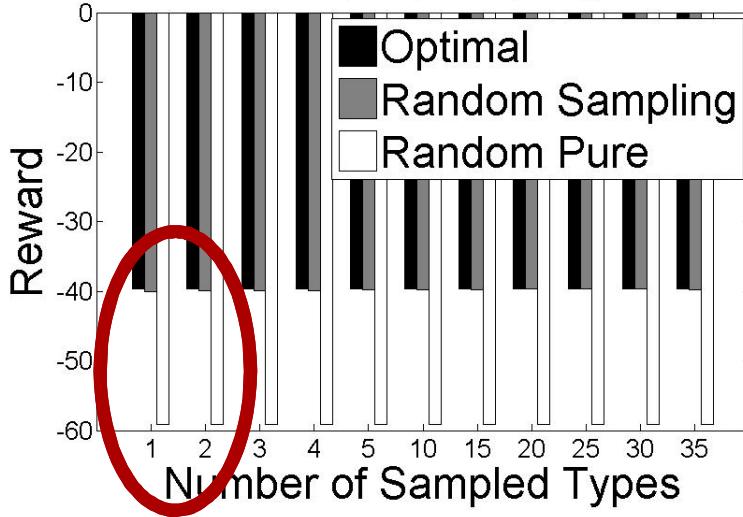
Small-world Graphs

Scale-free, Small-world graphs
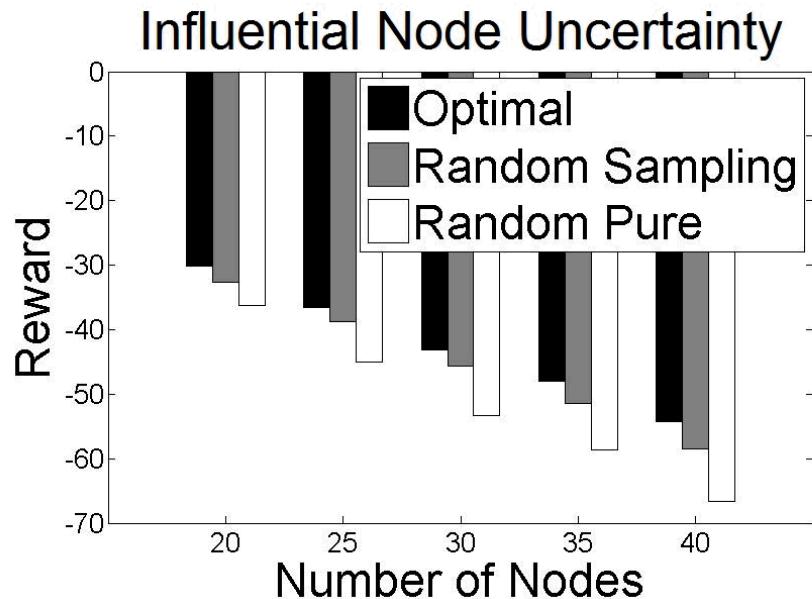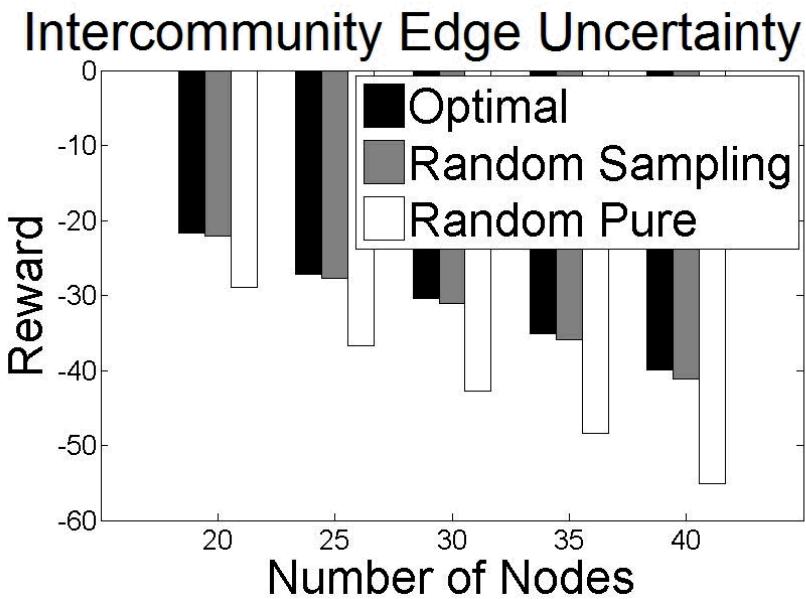## (influential node uncertainty)

Scale-free Graphs
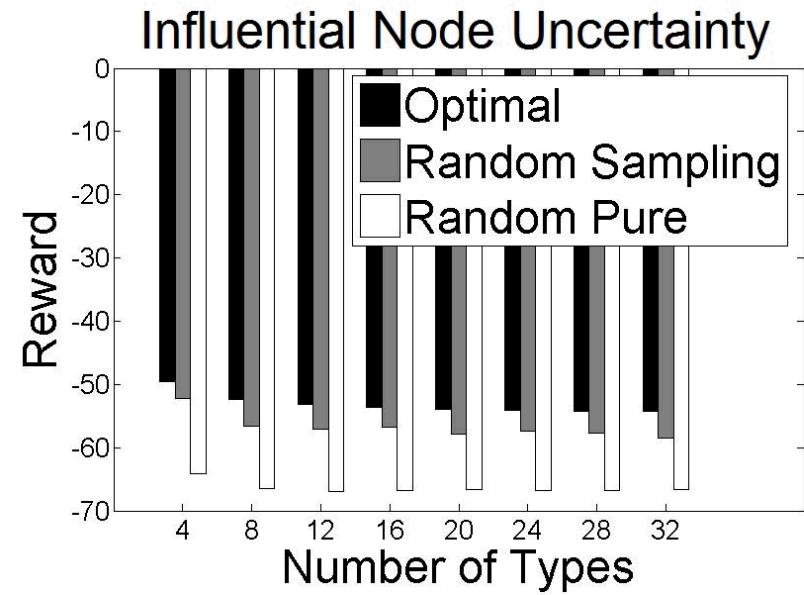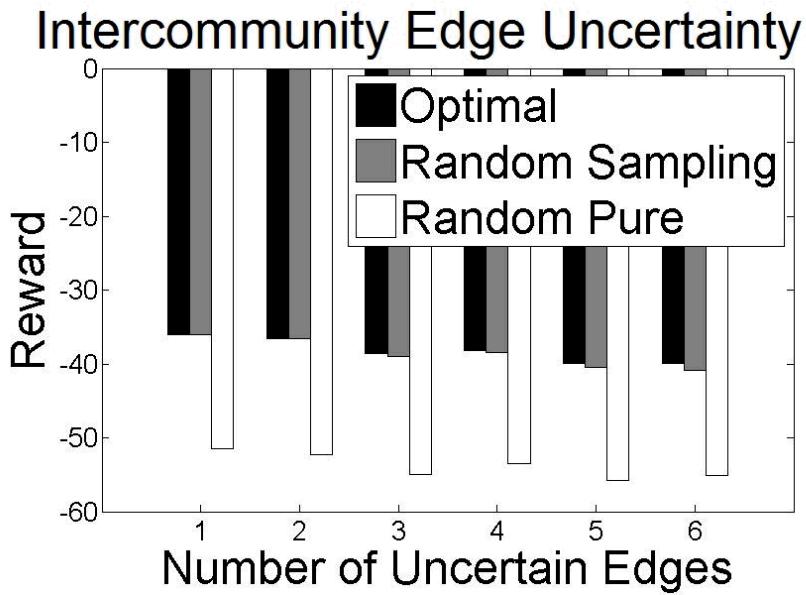

Small-world Graphs

BTER Graphs
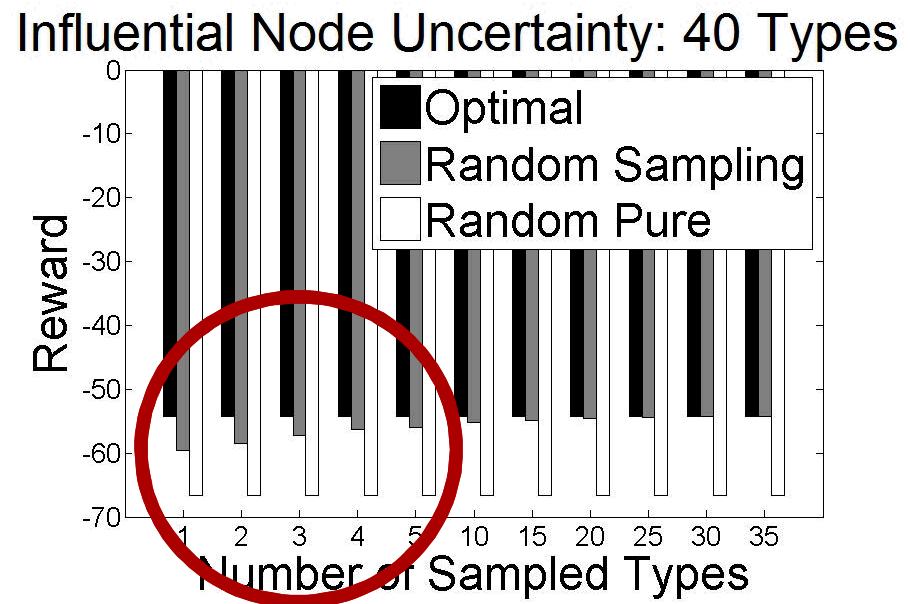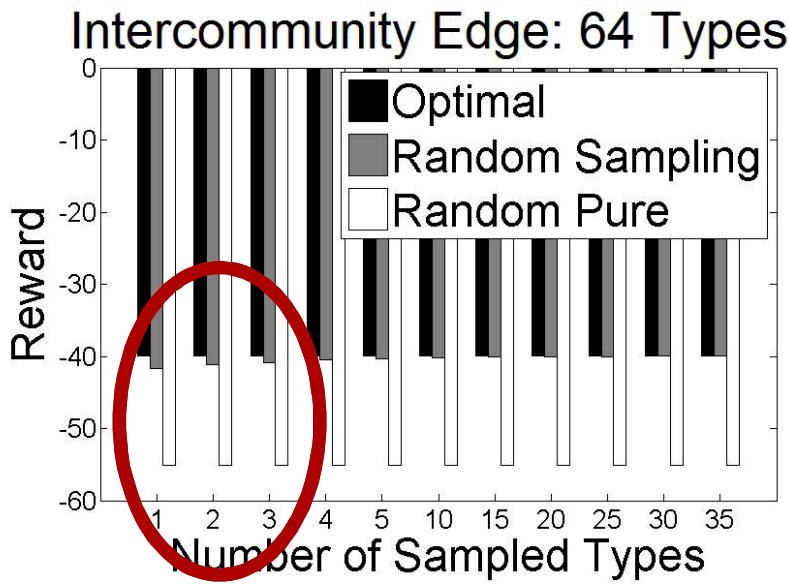
Suffices to sample a small number of types (2-5)
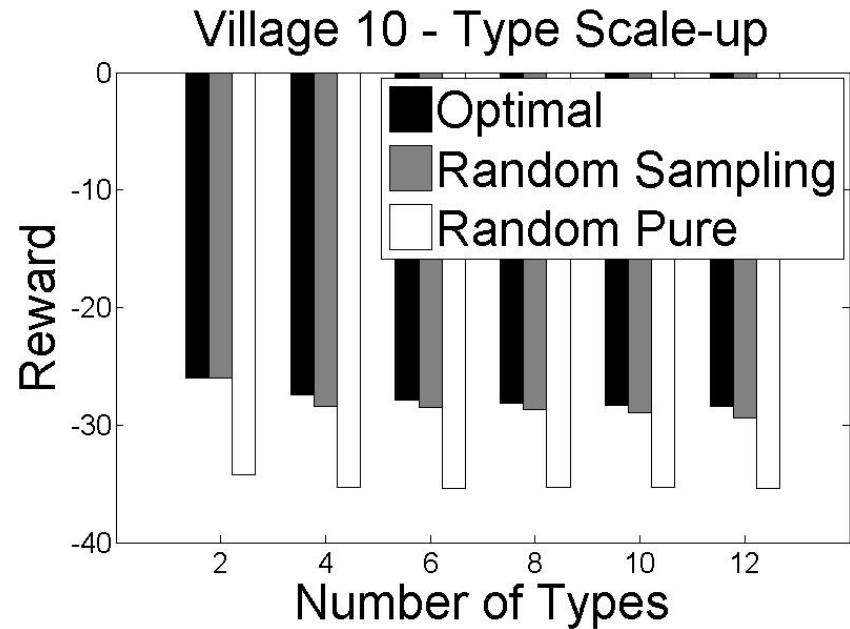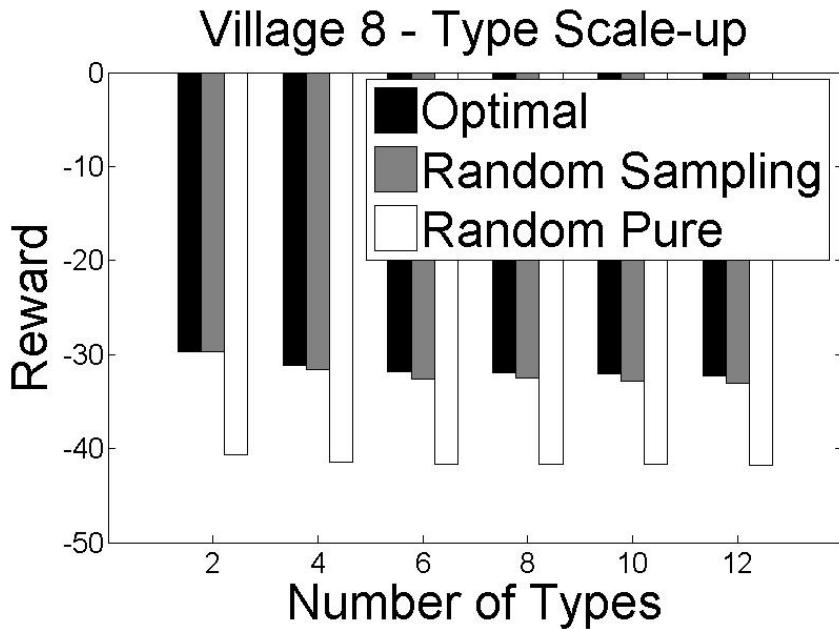
BTER Graphs

Suffices to sample a small number of types (2-5)

BTER Graphs

Not much affected as we increase the number of uncertain edges or total number of types

Indian Village Graphs

Village 8 - Type Scale-up and *Village 10 - Type Scale-up* bar charts comparing Optimal, Random Sampling, and Random Pure across Number of Types.

Same story if we look at Indian village graphs

Summary of Results

Parameter	Variations
Graph type	Scale-free, Small-world,
Graph size	
Uncertainty	Intercommunity edge, Intercommunity edge set, Inter/Intracommunity edge, Influential node
Sampled Types	1-40

Parameter	Variations
Number of types	4-40
	4, 10-50
	5-40
Avg contagion probability	0.1, 0.4, 0.7
Community density	0.6, 0.9
Indian village graph weight determination	Uniform, Weighted

Bottom-line: naïve strategy works!

Over 200 parameter variations tested

Why does it work?

consider overlap in nodes used by type-optimal strategies against:

a random type   *optimal strategy*
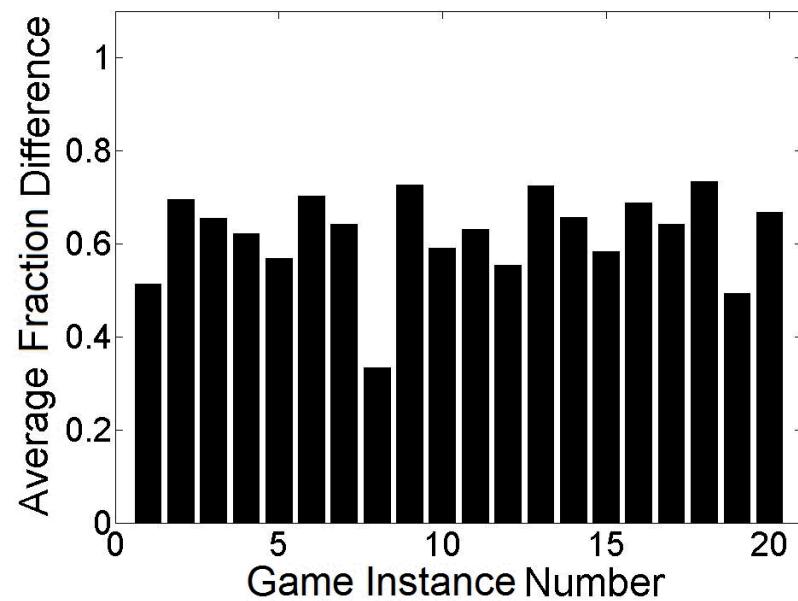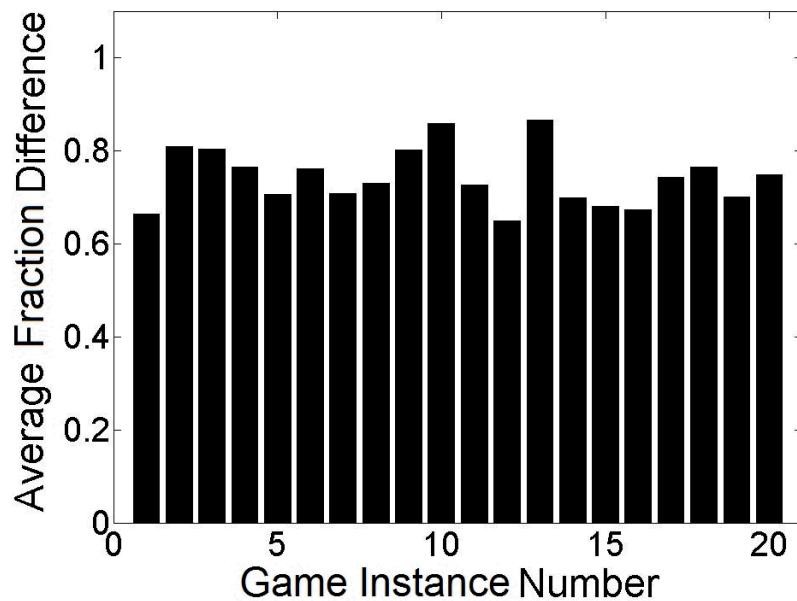
appears to be a core set of nodes used by optimal
strategies for individual types and optimal Bayesian strategy
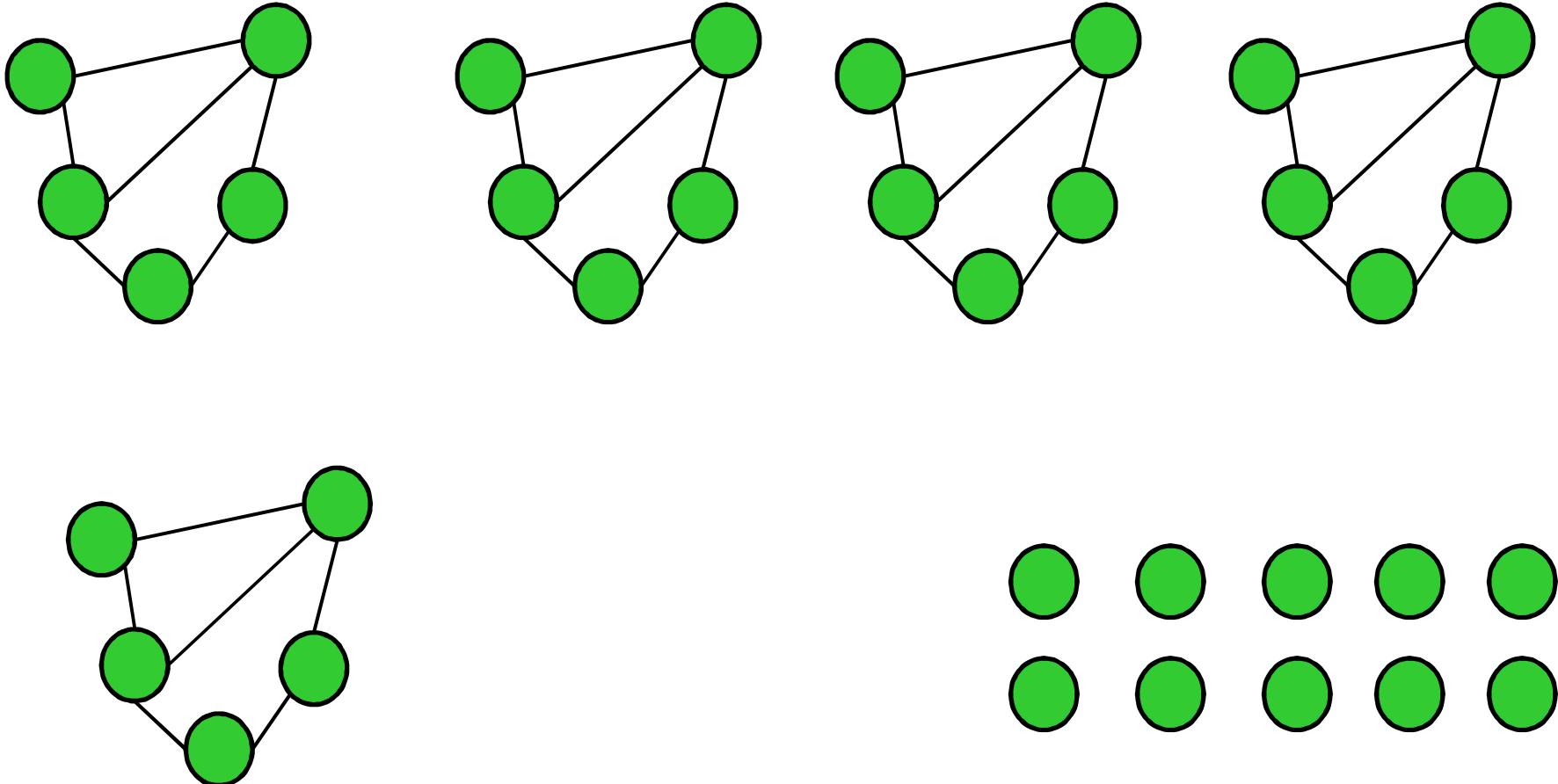
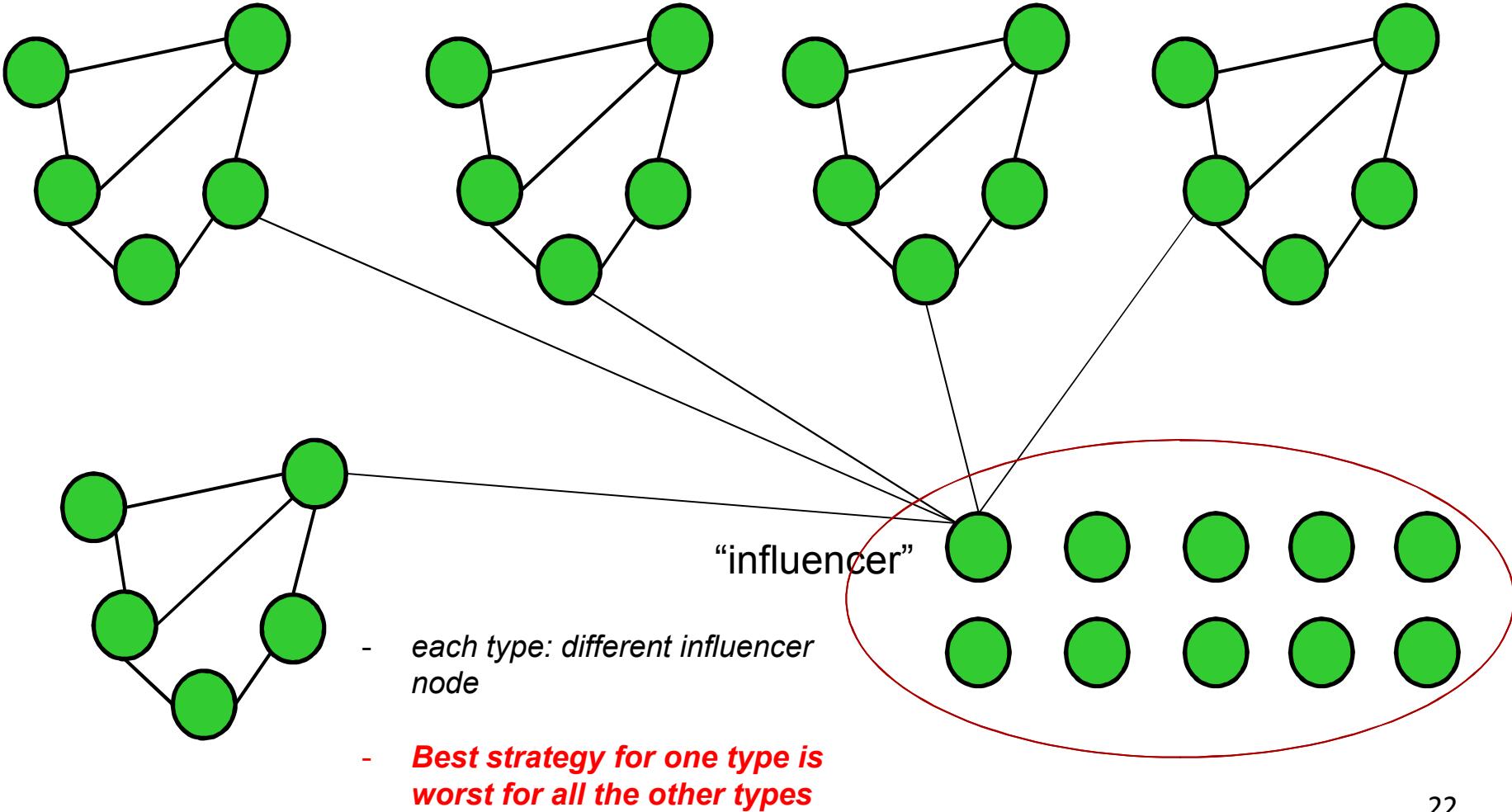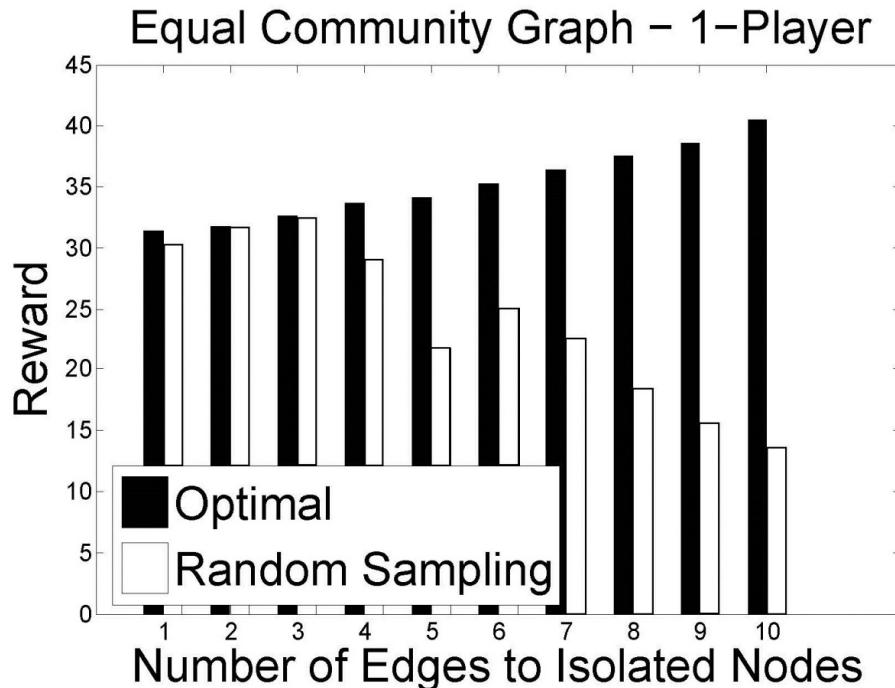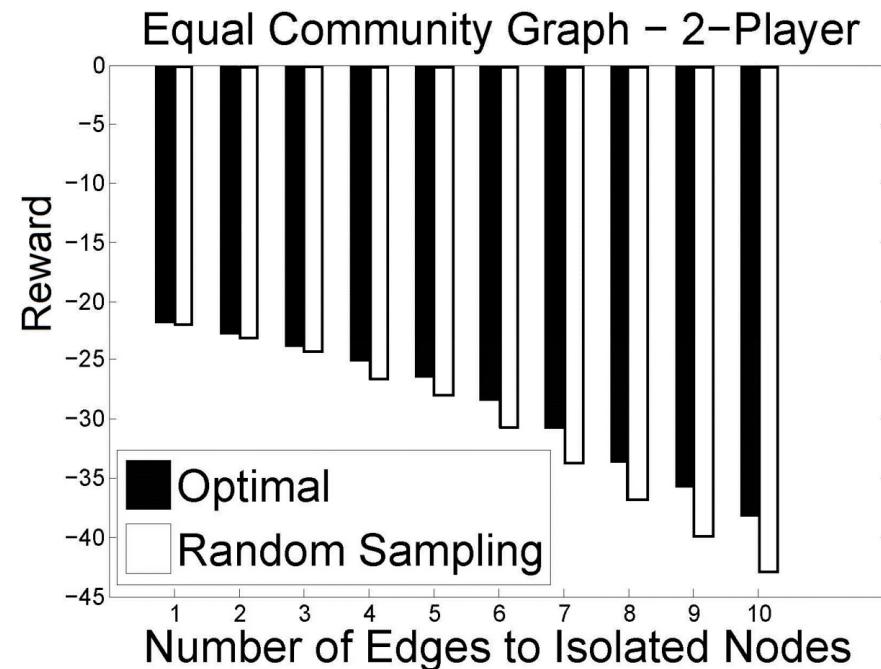# Illustration: "Equal-Community" Graphs

Illustration: "Equal-Community" Graphs

"influencer"

- each type: different influencer node

- ***Best strategy for one type is worst for all the other types***

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Real Answer: Games are Robust

Equal Community Graph – 1-Player

Equal Community Graph – 2-Player

Sampling a single type becomes arbitrarily worse than optimal

Sampling a single type remains close to optimal (remains robust)

Conclusions

- Uncertainty is pervasive in practice, and when uncertainty is about a graph structure, it presents severe scalability limitations in the worst case

- **While a single-player influence maximization can be very sensitive to uncertainty, influence games appear to be very robust to it**

- **As long as we have decent information about network structure, sampling a small number of types yields near-optimal mitigation strategies**