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A new time-dependent analytic compact model for radiation-induced photocurrent in epitaxial structures

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Abstract

A compact model for photocurrent in epitaxial structures (diodes and BJTs) is described, which represents a significant improvement over previous models. The relevant boundary value problem, based on the ambipolar diffusion equation is solved analytically.

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I. Introduction

Circuit simulators, such as SPICE or Xyce [1], are often used to analyze circuit-level photocurrent effects generated by ionizing radiation. Such simulation tools typically rely on compact models to represent the photocurrent response of individual components such as diodes and transistors. However, photocurrent compact models (e.g., [2], [3]) have often applied empirical assumptions or physical approximations with limited validity. As a result, the calibrations for many models can fail if applied to multiple time scales.

An improved photocurrent model [4], which was solved analytically and relied on fewer assumptions, addressed these calibration problems. However, the model was not formulated to include a highly doped sub-collector, rendering it inapplicable to many realistic devices. In this paper, the mathematical formulation presented in [4] is extended for the first time to include highly doped sub-collectors. The model is favorably compared to TCAD and an older compact model [2] for a 2N2222 BJT (which has an $npnn^+$ sub-collector structure).

II. Background

The transport behavior of excess carriers in semiconductors is often described using the well-known drift-diffusion (DD) equations [5], which are commonly used in device, a.k.a., TCAD, simulations. However, these equations are not amenable to exact analytic mathematical techniques, so many photocurrent compact models (e.g., [2], [3]) use the ambipolar diffusion equation (ADE) to model the behavior of excess carriers in the undepleted regions of a device.

The ADE is derived from the DD equations using two approximations [5]. The first is the electrical neutrality, or charge balance approximation, which states that the excess electron and hole densities are equal across the entire domain. The second approximation is the congruence assumption, where the flux of electrons and holes out of any region must be equal. The resulting ADE is given by

$$\frac{\partial u}{\partial t} = D_a \nabla^2 u - \mu_a \mathbf{E} \cdot \nabla u - \frac{u}{\tau} + g \tag{1}$$

where u is the excess carrier density (electrons or holes), D_a is the ambipolar diffusion constant, μ_a is the ambipolar mobility, τ is the carrier lifetime, g is the creation rate for electron-hole pairs, and $\mathbf{E}$ is the electric field.

The only analytic ADE solution for photocurrent in a sub-collector that we have found is by Long, Florian and Casey (LFC) [3]. To simplify their analysis, they assumed the sub-collector to be infinite, and addressed only steady-state conditions. The Fjeldly photocurrent compact model [2] also treats the sub-collector, but not through an ADE solution. It makes the assumption that photocurrent collection from the sub-collector is limited to one diffusion length from the boundary.

In many cases, photocurrent-producing structures—such as reverse-biased *p-n* junctions (the base-collector of a BJT, e.g.), and the drain-body regions of MOSFETs—are constructed with epitaxial layers. Since epitaxial structures regions lend themselves to analysis by one-dimensional models, we restrict the ADE to one dimension, though our approach could be extended to two, or even three dimensions. We also assume a uniform depletion region and negligible electric fields in the undepleted regions of the device.

Our derivation focuses on the undepleted nn^+ or pp^+ portion of a device. Similar to previous work [4], we use the finite Fourier transform technique [6] to solve the (1D) ADE and describe the carrier dynamics in the unbiased nn^+ sub-component of an epitaxial device experiencing a radiation transient.

III. Mathematical Development

A detailed description of the mathematics can be found in [7], so only a brief overview will be given here. For a reverse-biased 1D pnn^+ diode, the total photocurrent may be written as the sum of the photocurrents generated in each region, $J_{total} = J_n + J_{depl} + J_{pp}$ (the subscripts correspond to the minority carriers in each region). Formulas for the photocurrents J_n and J_{depl} are already known, and may be found in [4]. In the undepleted nn^+ region, the excess minority carrier density may be found by solving the ADE (Equation (1)) in one dimension (with $\mathbf{E} = \mathbf{0}$), using the finite Fourier transform method [4], [6]. The interface boundary conditions require that the excess carrier current be continuous through the nn^+ junction, and that the ratio of the excess carrier density on each side of the junction remains fixed at a value dictated by the ratio of the majority carrier concentrations, within the two respective regions. When the system is solved, the two composite layers within the region, coupled with the interface boundary conditions [3], produce a sequence of piecewise continuous eigenfunctions, in which the eigenvalues are given by a transcendental equation.

For an arbitrary time-dependent $g(t)$, the formula for the excess carrier density in the nn^+ region may be written as

$$u(x,t) = \sum_{n=1}^{\infty} w_n \int_0^t g(v) e^{-\lambda_n (t-v)} dv \frac{X_n(x)}{\|X_n\|^2} \tag{2}$$

The associated photocurrent (defined to be positive for convenience) is given by

$$J_{pp}(t) = qD_1 \left.\frac{\partial u}{\partial x}\right|_{x=0} = qD_1 \sum_{n=1}^{\infty} w_n \int_0^t g(v) e^{-\lambda_n (t-v)} dv \frac{X_n'(0)}{\|X_n\|^2} \tag{3}$$

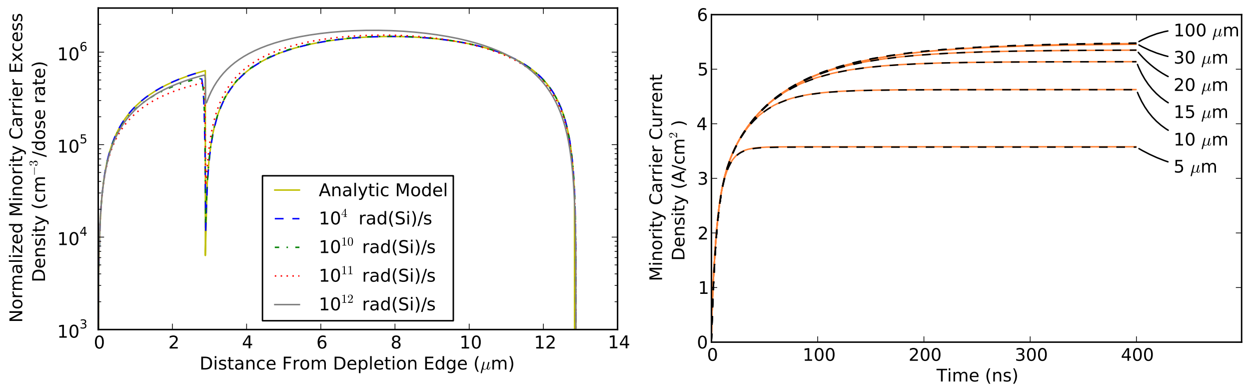
The X_n are the eigenfunctions, the λ_n are the associated eigenvalues, D_1 is the ambipolar diffusion coefficient in the *n*-doped region and $w_n = \langle 1 \cdot X_n(x) \rangle$. The integral in Equation 2 was also evaluated (and simplified) for the case when $g(t)$ is given in the form a discrete data set, thus enabling us to analyze an arbitrarily-shaped generation function [7].

IV. Code comparison

To evaluate the robustness of the analytic sub-collector model, we made several comparisons to TCAD simulations and the Fjeldly model [2]. The initial comparisons involved the characteristics of the nn^+ portion of a device. We then examined the simulated response of a 2N2222 BJT to a radiation pulse with the transistor in a realistic test circuit.

Figure 1a illustrates steady-state analytic and TCAD normalized excess minority carrier densities from an unbiased irradiated nn^+ doped silicon region. The TCAD excess carrier densities in the plot are normalized by the radiation dose rate. Continuous radiation pulses with dose rates between 10^4 rad(Si)/s and 10^{12} rad(Si)/s were simulated, and the excess carrier densities are plotted 100 μs after pulse initiation, which is well after steady-state conditions were achieved. The TCAD simulations show that the dependence of the excess carrier density on the dose rate is approximately linear for dose rates less than 10^{10} rad(Si)/s, and are in close agreement with the analytic solution over this range. The effect of the second boundary condition is evident in the discontinuity in the excess carrier profile, and the consistency with the TCAD results supports its use. (Note that no explicit boundary conditions are assumed at the *n*-n^+ interface in the TCAD simulations.) Higher dose rates (high-level irradiations) in the TCAD solutions show an increase in the excess carrier density in the n^+ region, and a decrease in the excess carrier discontinuity at the nn^+ interface, thus indicating a breakdown of the assumptions inherent in the ADE.

Figure 1b compares the analytic and simulated TCAD results for the minority carrier photocurrent densities in an nn^+ region for varying widths of a n^+ sub-collector substrate. The radiation dose rate is 10^9 rad(Si)/s, and the substrate width is indicated on the right side of the figure. The analytic and



(a) Excess carrier density for varying radiation dose rates

(b) Effect of sub-collector thicknesses on photocurrent

Fig. 1. Analytic model vs. TCAD simulation of the nn^+ region

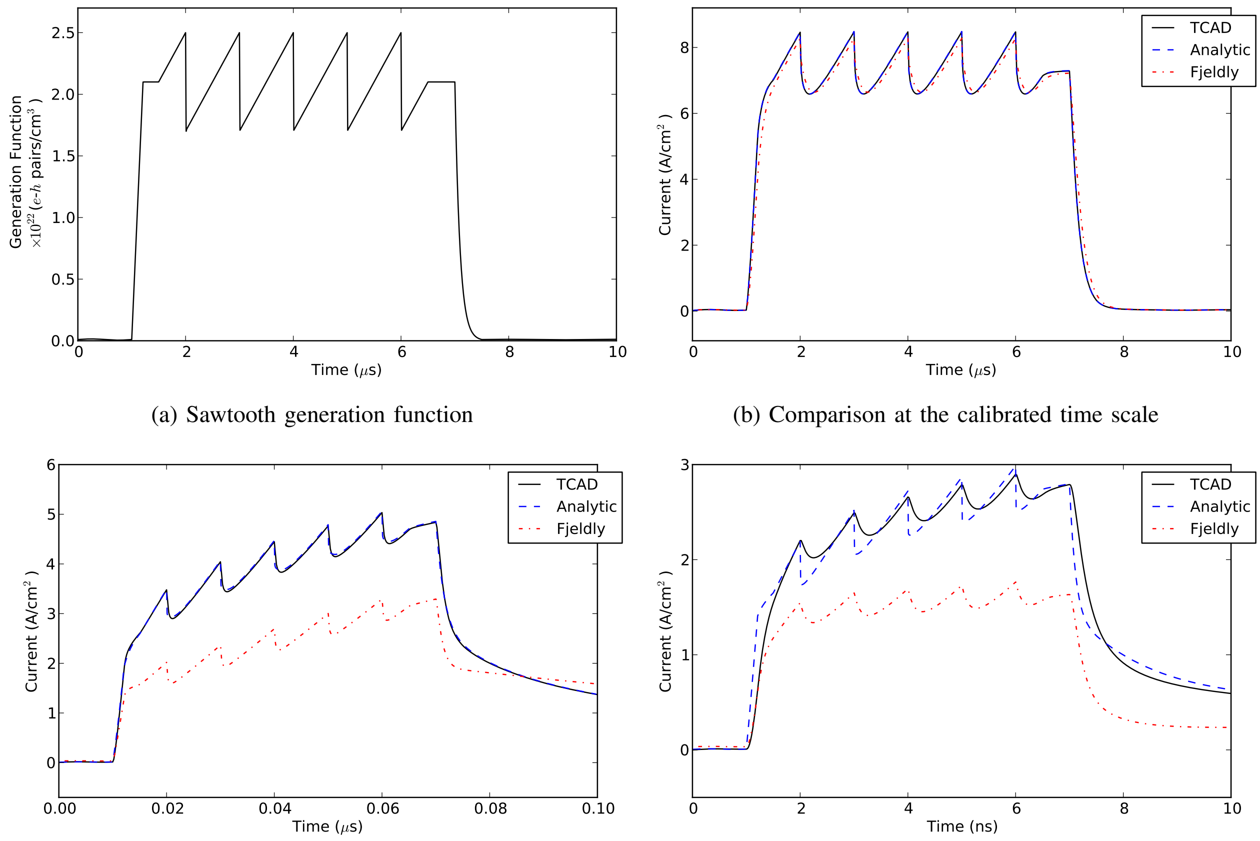
TCAD plots essentially overlap in each case. It is apparent that, for the parameters and doping levels used in these nn^+ simulations, a significant amount of charge is collected from deep within the substrate (in the range of 20–30 μm). Recall that the Fjeldly model [2] assumes photocurrent collection from the sub-collector only within one diffusion length. The diffusion length in this case is ~ 5 μm, which indicates that the Fjeldly assumption does not account for all of the delayed photocurrent contributions.

A comparison of the analytic model to the Fjeldly model and TCAD simulations is shown in Figure 2 for a 2N2222 ($npnn^+$) BJT. The transistor was simulated as part of a circuit that is representative of one that was used to test the 2N2222 at radiation facilities. Specifically, the base and emitter are shorted together, and attached to ground via a 50 Ω resistor. The collector is attached to a 5 V bias. A sawtooth waveform was chosen to represent the radiation pulse, and is shown in Figure 2a. The values of the minority carrier mobility and lifetime used in the analytic and TCAD models were obtained through calibration of the Fjeldly model to photocurrent data [8] taken at the timescale of the pulse shown in Figure 2a. For the analytic model, the photocurrent from the diode is computed as the sum of the photocurrents computed by solving the ADE in each of the undepleted regions, and adding in the current from the two depletion zones.

Figures 2b, 2c and 2d compare the analytic, Fjeldly and TCAD simulated current going through the 50 Ω resistor for input pulses on three different time scales. Figure 2b is the nominal time scale, and is representative of the time scale on which the Fjeldly model was calibrated. It is, thus, not surprising that the three models exhibit very good agreement. When the time scale of the pulse is compressed by a factor of 100, though, as in Figure 2c, the limitations of the Fjeldly model become apparent. The analytic model, however, still has very good agreement with the TCAD calculation. Finally, when the time scale is shortened to a pulse of a few nanoseconds of duration, agreement of the analytic model with the TCAD simulation begins to diverge, but is still good in a compact modelling sense.

V. Conclusion

In summary, we presented a new analytic solution that determines the current density coming from an irradiated finite 1D reverse-biased pnn^+ abrupt junction epitaxial diode. Our transient solution improves on the LFC solution [3], since it uses the correct nn^+ boundary conditions, is the solution for a finite diode, and takes into account an arbitrary time-dependent radiation generation density. We also developed the analytical solution for a piecewise linear generation function so that it may be used to analyze realistic pulses, including those based on experimental data. The analytic results compare favorably to TCAD



(a) Sawtooth generation function

(b) Comparison at the calibrated time scale

(c) Time-compressed by a factor of 100

(d) Time-compressed by a factor of 1000

Fig. 2. Comparison of the photocurrent density from the 2N2222 BJT, as computed with TCAD, the analytic model, and the Fjeldly model

simulation, and represent an improvement over the Fjeldly model [2]. Comparisons of the analytic model to experimental data are in progress.

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