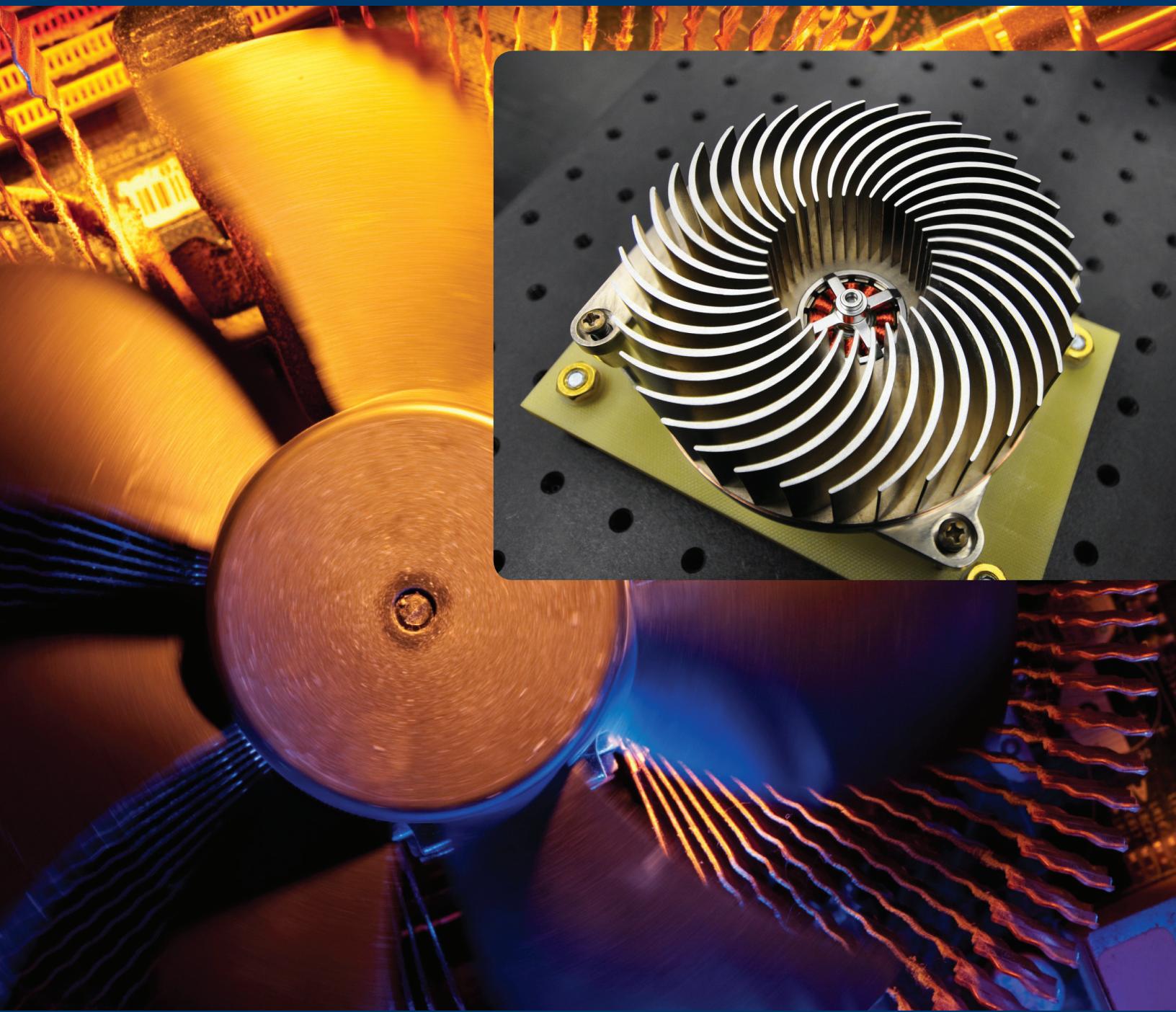


Sandia Cooler



Exceptional service in the national interest



Sandia
National
Laboratories

Sandia Cooler

1. Developer Information

A. Submitting Organization

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B. Joint Submitters

None

2. Product Information

A. Product Name

Sandia Cooler

B. Product Photo

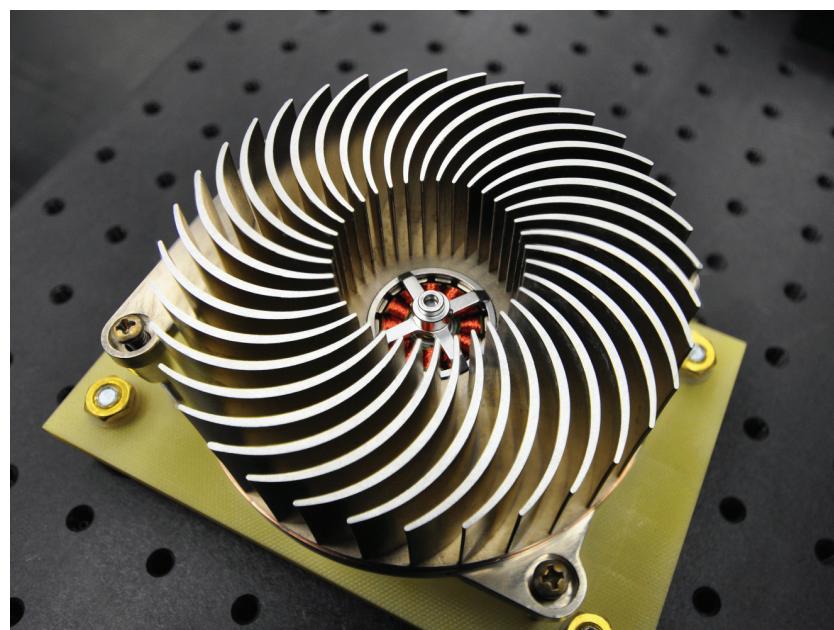


Figure 1. The Sandia Cooler is a groundbreaking alternative to conventional fan-plus-finned-heat-sink heat exchangers. This simple, rugged, cost-competitive device is 10 cm in diameter (3.9 inches) and significantly more efficient than conventional air-cooled heat exchangers.

3. Brief Description

The Sandia Cooler is 30-times more efficient than conventional air-cooled heat exchangers and is available for licensing to electronics and solid state lighting cooling manufacturers.

4. Product First Available for Licensing/for Order

The Sandia Cooler was available for licensing in 2011.

June 17, 2011 – Electronics Chip Cooling

Federal Business Opportunity (FedBizOpps.gov)

1. Solicitation Number 11_360, Sandia Corp. (DOE Contractor),

Posted June 17, 2011.

Announcement regarding licensing opportunities in the field of electronics chip cooling.

https://www.fbo.gov/index?s=opportunity&mode=form&id=f30333116c6736ba5c793c127db87e07&tab=core&_cview=1

2. Solicitation Number 12_366, Sandia Corp. (DOE Contractor),

Posted Nov 2, 2011.

Announcement for “Invite to Industry Day” at SNL/CA, November 17, 2011; Target audience included potential licensees and commercial partners/ industry reps looking for breakthrough air-cooling techniques in a variety of application areas.

https://www.fbo.gov/index?s=opportunity&mode=form&id=ea0c511279ed0e147cb5ab0f06acf8a7&tab=core&_cview=0

November 22, 2011 – Solid State Lighting

- A license option agreement in the area of solid state lighting was signed on November 22, 2011.
- Two additional license options are currently being negotiated in the area of solid state lighting.

5. Has this product, or an earlier version, been entered previously in the R&D 100 awards competition?

No.

*The Sandia
Cooler is a
simple, rugged,
cost-competitive
device that
completely
out-performs
conventional
fan-plus-finned-
heat-sink heat
exchangers.*

6. Principal Investigator

Jeff Koplow, Ph.D.

Principal Member of the Technical Staff

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7. Product Price

Initial offerings of this technology for the high-performance central processing unit (CPU) cooling market are expected to be priced between \$50 and \$80 – a level comparable to existing high performance coolers for that market. We expect that a mass-produced version for the original equipment manufacturer (OEM) cooling market could be sold for around \$15 to \$25

8. Patents

US Patent Application #: 12/185,570 Heat Exchanger Device and Method for Heat Removal or Transfer. Filed August 4, 2008. ORD. (SD# 10948.1)

Inventor: Dr. Jeffrey Koplow.

US Patent Application #: 61/448,655 Solid State Lighting Devices and Methods with Rotary Cooling Structures. Filed March 02, 2011. (SD# 11908).

Inventor: Dr. Jeffrey Koplow.

9. Primary Function

A recent Defense Advanced Research Projects Agency (DARPA) call for research proposals on new ideas for air-cooled heat exchanger technology stated: “Over the past 40 years, complementary metal–oxide–semiconductors (CMOS), telecommunications, active sensing and imaging and other technologies have undergone tremendous technological innovation. Over this same historical period the technologies, designs and performance of air-cooled heat exchangers has remained unchanged. The performance data for today’s state of the art heat exchangers and blowers is, in many cases, based on measurements performed in the 1960s.”

A more efficient cooling approach is clearly needed, and Sandia has developed an innovative, economical, robust solution to the air-cooled heat exchanger problem.

The Sandia Cooler is a fundamental breakthrough in air-cooling technology. The simple, rugged design transfers heat to rotating fins, greatly reducing the problem of “dead air” that is primarily responsible for inefficiency of conventional fan-plus-finned-heat-sink (FFHS) heat exchangers. This level of increased cooling performance will enable computer manufacturers/users to operate existing CPUs at up to twice their current clock rates without an increase in operating temperature.

While the Sandia Cooler can be used with any component requiring air cooling, we expect that the initial principal application of this breakthrough technology will be electronics and solid state lighting thermal management for consumer markets.

The problems we solve

Conventional FFHS heat exchangers are relatively inefficient at transferring heat from the surface of the heat exchanger fins to the surrounding air. The principal cause of this inefficiency is the boundary layer effect, wherein a heat transfer bottleneck is created by a boundary layer of “dead air” that clings to the cooling fins, acting like an insulating blanket. While increasing the air flow rate (fan speed) over a conventional heat sink results in only modest reduction in boundary layer thickness, tradeoffs involving fan power consumption and noise generation quickly become unfavorable.

The Sandia Cooler addresses inefficiency of FFHS heat exchangers by addressing the four most significant problems:

- 1) the boundary layer problem, as mentioned above,
- 2) heat-sink fouling by dust and other foreign matter [Figure 2],
- 3) the poor efficiency of small high-speed turbo-machinery (the amount of air moved in the direction desired is small compared to the amount of energy used), and
- 4) fan noise.



Figure 2. A conventional FFHS (left) and one clogged with dust.

**The Sandia
Cooler is
30-times more
efficient than
conventional
air-cooled heat
exchangers and
is available
for licensing
to electronics
and solid state
lighting cooling
manufacturers.**

How the Sandia Cooler revolutionizes heat exchangers

The Sandia Cooler efficiently transfers heat from a stationary base plate to a rotating structure that combines the functionality of cooling fins with a centrifugal impeller. The boundary layer of air that surrounds the cooling fins is subjected to a powerful centrifugal (to an observer in the rotating frame, where the boundary layer resides) pumping effect, providing an order-of-magnitude reduction in boundary layer thickness at a speed of a few thousand revolutions per minute (rpm). Additionally, high-speed rotation completely eliminates the problem of heat exchanger fouling. The “direct drive advantage,” in which relative motion between the cooling fins and ambient air is created by imparting motion directly to the heat exchanger, provides a drastic improvement in aerodynamic efficiency. In contrast to a conventional vane-axial fan, the heat-sink-impeller provides a great deal of flexibility with regard to fin geometry. Thus it is possible to design the fins to cleanly separate and cleanly rejoin the flow field at the entrance and exit of the fin array. Minimal flow disturbance translates to extremely quiet operation.

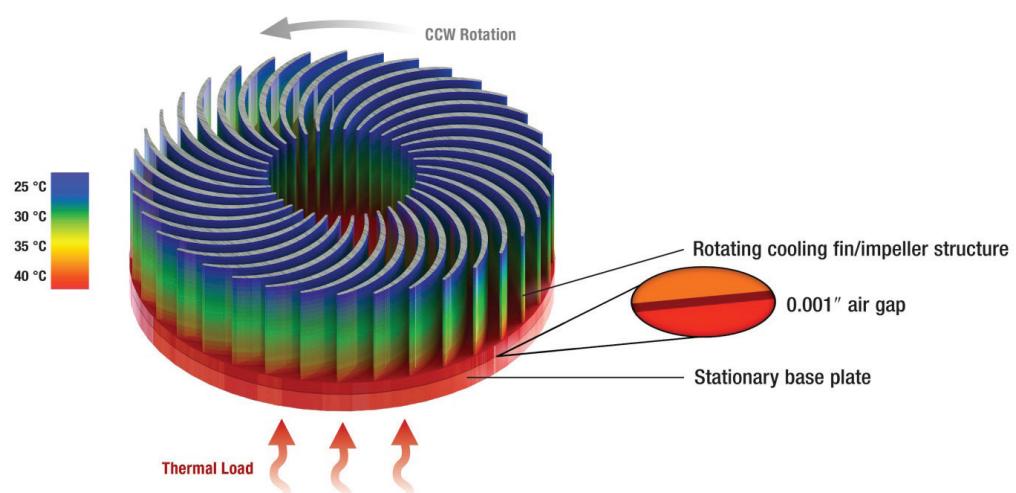


Figure 3. Schematic showing the heat path and approximate temperature distribution for the Sandia Cooler.

The schematic in Figure 3, above, demonstrates the heat flow path in the Sandia Cooler. As in a conventional heat sink, the thermal load is placed in contact with the stationary base plate that functions as a heat spreader. The base plate/heat spreader may be fabricated from a solid block of material (refer to Figure 4), or may be configured as a vapor chamber to further improve heat spreading.

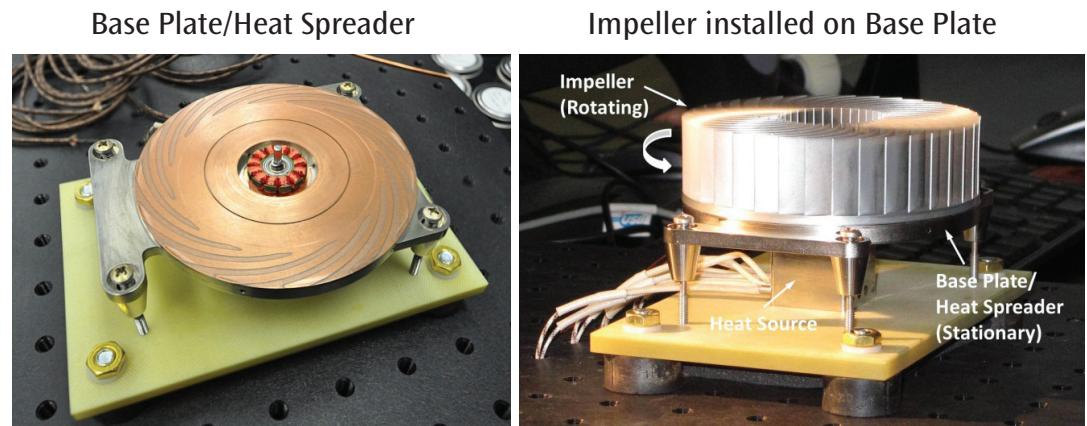


Figure 4. Sandia Cooler test bed. Base plate/heat spreader without impeller to show hydrodynamic air bearing grooves (left). Base plate with impeller installed (right). Heat source used to simulate a CPU shown below base plate.

The monolithic “heat-sink-impeller” (the finned, rotating component) consists of a thin plate populated with fins on its top surface. It functions like a hybrid of a conventional finned metal heat sink and an impeller. Air is drawn in the downward direction into the central region having no fins, and expelled in the radial direction through the dense array of fins. A high-efficiency, brushless motor mounted directly to the base plate is used to rotate (at several thousand rpm) the heat-sink-impeller structure.

The bottom surface of the impeller and top surface of the base plate are flat. During operation, these two surfaces are separated by a thin ($\sim 20 \mu\text{m}$) air gap, much like the bottom surface of an air hockey puck and the top surface of an air hockey table. This air gap is created using a hydrodynamic air bearing, analogous to those used to support the read/write head of computer disk drive (but with many orders of magnitude looser mechanical tolerances). One important point about the air bearing is that the $\sim 20 \mu\text{m}$ air gap is not maintained by using extremely tight mechanical tolerances. Much like an air hockey puck on an air hockey table, or a hard disk read/write head, the air gap distance is self-regulating. For example, if the air gap distance increases, the air pressure in the gap region drops, which causes the air gap distance to decrease. This built-in

corrective feedback provides excellent mechanical stability and an extremely stiff effective spring constant (important for ruggedness). Unlike an air hockey table, which relies on gravity to counter-balance the pressure force acting on the puck, in the air-bearing heat exchanger the hydrodynamic lifting force is counterbalanced by a stiff compression spring, such that the force of gravity acting on the heat-sink-impeller exerts little influence. The device can therefore be mounted upside-down, sideways, et cetera. The two air bearing surfaces are treated with a dry (ceramic) anti-friction coating to allow intermittent sliding contact during start-up and shutdown without damage to either surface.

Heat flows from the stationary base plate to the rotating heat-sink-impeller through this 20 μm planar air gap. The air gap has very low thermal resistance and is in no way a limiting factor to device performance because its cross-sectional area is large relative to its thickness, and because the air that occupies the gap region is violently sheared between the stationary lower surface and the rotating upper surface. The convective mixing provided by this shearing effect provides a several-fold increase in thermal conductivity of the air residing in the gap region.

10. How It Operates

This rotating impeller of the Sandia Cooler places the thermal boundary layer in an accelerating frame of reference. Placing the boundary layer in this non-inertial frame of reference adds a new force term to the Navier-Stokes equations, whose steady-state solution governs the functional form of the heat-sink-impeller flow field [Schlichting, 1979].

At a rotation speed of several thousand rpm, the magnitude of this centrifugal (in the frame of reference of the boundary layer) force term is such that as much as a factor of ten reduction in average boundary layer thickness is predicted [Cobb, 1956]. Unlike techniques such as air jet impingement cooling, the mechanism for boundary layer thinning in the air bearing heat exchanger does not rely on a process that entails dissipation of significant amounts of energy, nor is the boundary layer thinning effect localized to a small area. Rather, the centrifugal force generated by rotation acts on all surfaces simultaneously, and all portions of the finned heat sink are subject to the resulting boundary layer thinning effect. For the limiting case of a flat rotating disk, an exact solution of the Navier-Stokes equation is possible and indicates that the magnitude of the boundary-layer thinning effect is constant as a function of radial position.

Annual electricity costs in the U.S. for the information technology (IT) sector alone is ~\$7B. According to Dr. Koplow, the inventor, the comparably priced but much more efficient Sandia Cooler will reduce IT sector electricity needs and provide a significant contribution to energy conservation in an increasingly “wired” world.

11. Building Blocks of Our Technology

The Sandia Cooler comprises three components (refer to Figure 4):

- 1) Base plate: The base plate in Figure 4 is machined from copper but it could also be forged. In high-performance applications, a planar heat pipe or vapor chamber such as those used in commercial CPU/GPU coolers would be used to more efficiently spread the heat across the bottom surface of the impeller.
- 2) Heat-sink-impeller: The heat-sink-impeller is typically fabricated from copper or aluminum by single-stroke cold forging at a per-unit cost of a few dollars in large quantities.
- 3) Brushless motor: The 3 Watt brushless motor is similar to those used in conventional CPU cooler fans.

12. Product Comparison

There are hundreds of OEM CPU cooler manufacturers and models in the marketplace. Examples of OEM manufacturers include AVC, Taisol, AeroCool, Cooler Master, to name a few. The best of these OEM coolers operate at the thermal resistance of $\sim 0.3^{\circ}\text{C}/\text{W}$, approximately 6-times less efficient than the Sandia Cooler.

One of the current favorite fan/finned heat sink for the overclocker crowd (i.e., PC and electronics users who modify a computer/electronic device to operate faster than the manufacturer's specifications) is the Noctua NH-D14. It operates at a thermal resistance of $\sim 0.10^{\circ}\text{C}/\text{W}$, which is about twice that of the Sandia Cooler. The Sandia Cooler is a factor of ten smaller, however. In addition, the Sandia Cooler is quieter and much more resistant to dust fouling.

*The Sandia
Cooler is a
revolutionary
new approach
to cooling in
field that has
seen little
innovation over
the past four
decades.*

Table 1. Comparison Chart

	Sandia Cooler	Intel Stock Core 2 duo*	Noctua NH-D14*
Picture			
Volume (cm ³)	350	740	3800
Fin Area (cm ²)	810	1920	11800
Thermal Resistance (°C/W)		0.33 (150W)	0.10 (150W high) 0.11 (150W low)
Performance Overclocking rate or something			
Price	\$50 to 80 for high performance market \$15 to \$25 for OEM market	\$25	\$90

*Data for Intel and Noctua coolers taken from reviews at frostytech.com

Improvements over Competitors

The Sandia Cooler is a revolutionary new approach to cooling in field that has seen little innovation over the past four decades. Relocation of the fluid boundary layer to an accelerating (rapidly rotating) frame of reference reduces boundary layer thickness by a factor of ~ 10 , which is transformative from the standpoint of cooling performance. The Sandia cooler is also the first heat exchanger architecture that addresses the problem of heat exchanger fouling. Significantly lower noise is further compelling advantage.

Potential limitations

Initial impressions by potential manufacturers of the Sandia Cooler are that machining costs for the impeller and base plate will be quite high. However, the impeller shown in all of the photos was cold forged. A forging dye, manufactured for a few thousand dollars, can produce several hundred thousand parts for little more than the cost of material. Moreover, these parts meet the necessary tolerances as forged without additional machining. While the current base plate was machined, there is no reason that production versions could not be manufactured using inexpensive mass production techniques such as cold forging.

***The Sandia
Cooler has a
significantly
increased
cooling
capability that
will enable
computer
manufacturers
to operate
existing CPUs
at up to twice
existing clock
rates without
an increase
in operating
temperature.***

Reliability is the other concern for either the electronics/CPU cooling or solid state lighting markets. For CPU cooling, the manufacturers would like to see a demonstration that the cooler can operate continuously for 15 months. The solid state lighting market would like to see 50,000 hours. The overall lifetime of the Sandia Cooler is dependent on the reliability of the motor and motor bearings. The motor used on the Sandia Cooler is similar to those used in existing CPU coolers, such that the above lifetime specifications will be easily achieved.

Another area where licensees would like to see performance demonstrations concerns resistance to shock and vibration. Because air bearings are extremely stiff, we expect more detailed shock and vibration experiments will demonstrate performance in this area well beyond manufacturers' requirements. The inherent stiffness of air bearings should not only provide good shock and vibration resistance, but also enables the cooler to operate in any orientation (i.e. the effect of gravity is negligible).

13. Product Use

Principal Applications

The initial market we are targeting is CPU cooling. Figure 5 illustrates the concept of the “thermal brick wall” faced by CPU manufacturers. CPU clock speed reached a plateau around 2003. This plateau is a result of the inability of conventional CPU coolers to remove the additional heat generated by the increasing numbers of transistors employed in contemporary CPUs.

The significant increase in cooling capability of the Sandia Cooler compared to OEM coolers will enable computer manufacturers to operate existing CPUs at up to twice existing clock rates without an increase in operating temperature.

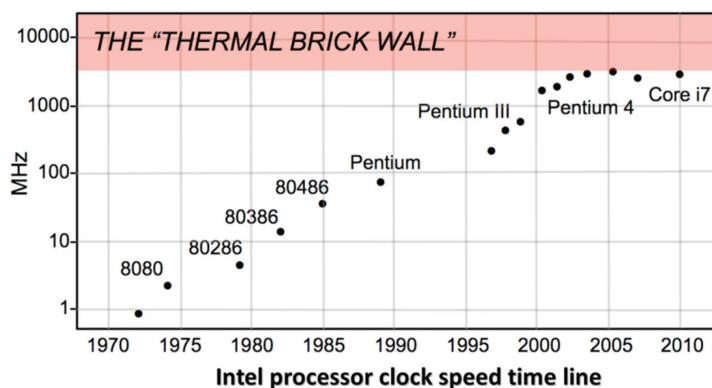


Figure 5. The thermal brick wall faced by the semiconductor industry.

***The Sandia
Cooler
represents a
paradigm shift
in air cooled
heat exchanger
technology,
with a potential
to significantly
improve the
efficiency of
lighting and
refrigeration.***

The second initial market application of this technology is Solid State Lighting (SSL). One of the biggest challenges for the SSL manufacturers is thermal management. A 10°C reduction in junction temperature can result in factor of two increase in lifetime. Current passive approaches to SSL cooling fall far short of what is required to allow proliferation of SSL technology. Other cooling approaches, such as synthetic jets, are complicated and costly. The Sandia Cooler provides the quantum leap in thermal management required to transform this industry.

We have signed a license option with Coolerix, LLC, to develop and market an LED light based on the Sandia Cooler. Instead of using a base plate/heat spreader, Coolerix will be placing LEDs directly on the bottom of the impeller, therefore transferring the heat directly to the impeller. They will also be using a light mixing approach where different color LEDs are attached to the bottom of the spinning impeller and the intensity of the individual LEDs is controlled to generate the desired color of light. Both of these variants on the original Sandia Cooler concept were invented by Dr. Koplow and are patent pending.

Other applications

The Sandia Cooler could potentially be applied to any system requiring a mechanism for transferring heat to air. Applications for this technology include: heating, ventilation, and air conditioning (HVAC), computer server farms (where the Cooler could be used to draw cool air in from outdoors and pump the hot air back outside), appliances, other electronics cooling, military and aerospace, automotive cooling, and battery cooling.

14. Summary

The Sandia Cooler represents a paradigm shift in air cooled heat exchanger technology. We have demonstrated better performance than one of the most popular higher-end CPU coolers. The potential impact of this technology is nearly limitless. Lighting consumes 20% of the electricity in this country. Air conditioning and refrigeration consume another 30%. A small improvement in efficiency in either of these areas would be dramatic in terms of energy and cost savings. The Sandia Cooler has the potential to make significant improvements in efficiency in both of these areas and beyond.

15. *Affirmation*

By submitting this entry to R&D Magazine I affirm that all information submitted as a part of, or supplemental to, this entry is a fair and accurate representation of this product.



Neal Fornaciari

APPENDIX

INDEX

Appendix A: Submitter Information

1. Contact person to handle all arrangements on exhibits, banquet, and publicity:

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Appendix C: Patents

HEAT EXCHANGER DEVICE AND METHOD FOR HEAT REMOVAL OR TRANSFER

CROSS REFERENCE TO RELATED APPLICATIONS

[0001] This application is a continuation of International Application No. PCT/US2009/044550, filed May 19, 2009, entitled “HEAT EXCHANGER DEVICE AND METHOD FOR HEAT REMOVAL OR TRANSFER”, which claims priority to U.S. Patent Application Serial Number 12/185,570, filed August 4, 2008, entitled “HEAT EXCHANGER DEVICE AND METHOD FOR HEAT REMOVAL OR TRANSFER”, and U.S. Provisional Patent Application Serial Number 61/164,188, filed March 27, 2009, entitled “HEAT EXCHANGER AND METHOD FOR HEAT REMOVAL OR TRANSFER”.

[0002] This application is also a continuation-in-part of U.S. Patent Application Serial Number 12/185,570, filed August 4, 2008, entitled “HEAT EXCHANGER DEVICE AND METHOD FOR HEAT REMOVAL OR TRANSFER,” which application claims the benefit of U.S. Provisional Patent Application Serial Number 61/008,271, filed December 18, 2007, entitled “COOLING DEVICE”.

[0003] This application also claims the benefit of U.S. Provisional Patent Application Serial Number 61/164,188 filed March 27, 2009, entitled “HEAT EXCHANGER AND METHOD FOR HEAT REMOVAL OR TRANSFER”.

[0004] These applications are each incorporated herein by reference.

Appendix C: Patents (cont.)

1

SOLID STATE LIGHTING DEVICES AND METHODS WITH ROTARY COOLING STRUCTURES

CROSS-REFERENCE TO RELATED APPLICATION(S)

[001] This application claims the benefit of the earlier filing date of provisional application No. 61/448,655, filed March 2, 2011 entitled “Rotary Cooled Solid State Lighting,” which application is hereby incorporated by reference in its entirety, for any purpose.

STATEMENT REGARDING RESEARCH & DEVELOPMENT

[002] The United States Government has a paid-up license in this technology and the right in limited circumstances to require the patent owner to license others on reasonable terms as provided for by the terms of contract No. DE-AC04-94AL85000 awarded by the U.S. Department of Energy to Sandia Corporation.

TECHNICAL FIELD

[003] Examples described herein relate to solid state lighting devices, methods and systems, and more specifically examples describe rotary-cooled solid state lighting devices.

BACKGROUND OF THE INVENTION

[004] Solid state lighting devices have been used to provide energy savings in lighting power consumption due to the increased efficiency of their source. However, solid state lighting devices known in the art may be expensive and may have insufficient operational life to offset their higher costs.

[005] In most commercially available LED bulbs, approximately 75-85% of the electrical power delivered to the bulb may be immediately converted to heat. This heat may limit the operational life of the device. For example, the insulating dielectric typically used in the

|

Appendix D: Letters of Support

An informational video including testimonials from our industry day can be found at: http://youtu.be/uGpV_VPUn8g



Coolerix, LLC

Feb 15, 2012

The Judging Committee
R&D Magazine

Dear Madam/Sir:

I am pleased to write this letter of support for the "Sandia Cooler" technology, developed by Dr. Jeff Koplow at Sandia National Laboratories, for the 2012 R&D 100 award.

I am a serial entrepreneur with a previous start-up (sold to Hangzhou Najing Technology, Ltd. in 2011) in the LED lighting space. As an industry participant, it is very clear to me that better thermal management – highly effective, highly reliable and low cost – is a critical and current industry need. When this innovation was brought to my attention, it immediately struck me as a potentially breakthrough solution for LED lighting applications.

Specifically, the rotational cooling architecture of this invention simultaneously addresses two major problems – (a) thermal resistance in confined spaces with limited air flow and (b) spatial mixing of color to achieve better uniformity. (a) There are over 6 billion Edison sockets globally. A significant fraction of these is found in ceiling fixtures designed to minimize air exchange with the surroundings as incandescent bulbs are more efficient at higher temperatures. However, as incandescent lighting is being phased out in favor of more efficient lighting technology like LED lights (that deliver superior efficiency, better light quality, superior controllability and longer lifetime), better heat dissipation is critical to improving efficiency and reducing cost. The rotational cooling architecture is able to deliver over an order of magnitude improvement in heat dissipation over other passive and active cooling solutions. (b) By mounting LEDs directly on the rotating impeller, spatial mixing of light to achieve better color uniformity is readily achieved. Color uniformity, or lack thereof, is a key dissatisfaction of LED lighting today.

Recognizing the commercial significance of this invention, I have put my money where my mouth is and have founded a start-up Coolerix, LLC that is focused on taking this innovation to market, as a licensee of this technology. Coolerix is now a 4 person endeavor looking to get to our first funding in summer 2012 and have our first product, based on this innovation for entertainment lighting applications, on the market in mid-2013.

Fundamental breakthroughs that can impact multiple product-markets are few and far between. I believe that this technology represents such a fundamental breakthrough innovation. I strongly recommend this for a 2012 R&D 100 award.

Sincerely,

Dr. Suresh Sunderrajan
Founder and CEO
Coolerix LLC

8 Featherstone Ct, Pittsford, NY 14534; +1-585-490-8833; www.coolerix.com

Appendix E: Press Coverage

C/Net News, “*Fanless heat sink design promises cooler, quieter CPUs*”

Rich Brown, July 12, 2011.

http://news.cnet.com/8301-17938_105-20078781-1/fanless-heat-sink-design-promises-cooler-quieter-cpus/?tag=mncol;2n

Wall Street Journal “*Fanfare for a new way to fan computer chips*,”

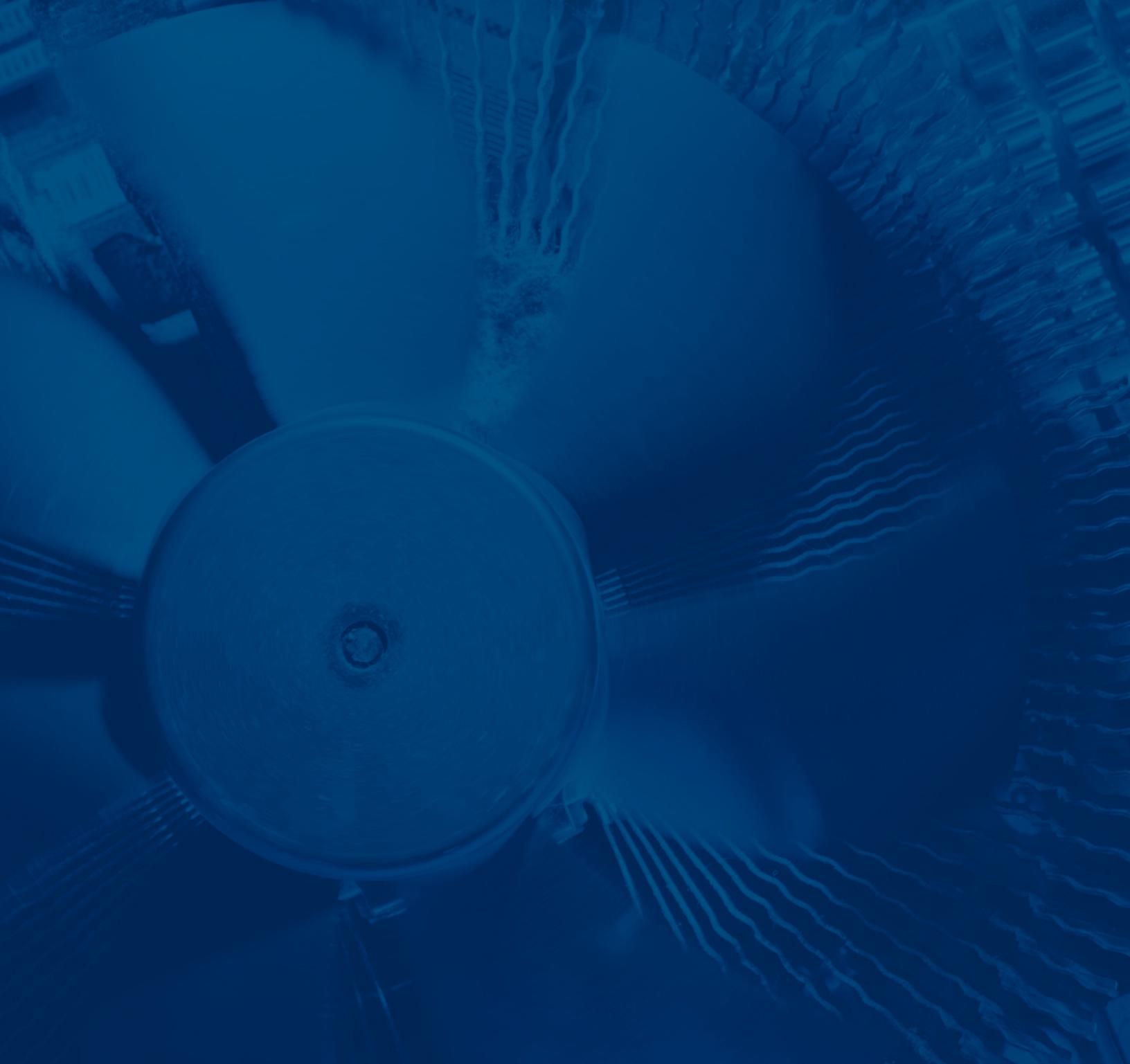
Don Clark, July 7 2011.

http://blogs.wsj.com/digits/2011/07/07/fanfare-for-a-new-way-to-fan-computer-chips/?mod=google_news_blog

Appendix F: References

Schlichting H., *Boundary Layer Theory*, McGraw-Hill, New York, 1979.

Cobb, E. C., Saunders, O. A., *Heat Transfer from a rotating disk*, Proceedings of the Royal Society of London. Series A, Mathematical and Physical Science, 1956.



Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000

SAND2012-xxxx