Distributed Solution Approach for a Stackelberg Pricing Game of Aggregated Demand Response

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Abstract—**Demand-side management is a fundamental up-to-date strategy that transforms the traditional power grid to a modern smart grid where the flexible pricing mechanisms play a critical role in its successful implementation. In this paper, the pricing-demand response between a distribution system operator (DSO) and load aggregators (LAs) is modeled as a Stackelberg game, where the DSO is the price maker that adjusts its strategy based on observed responses from LAs. With the concerns of computational cost and privacy protection, two distributed solution approaches, particle swarm optimization and pattern search algorithm, are investigated and compared with the classical centralized backward induction approach. Numerical results on a small case study demonstrate the effectiveness of the proposed distributed solution approaches in leveraging flexible demand response potential.**

Keywords-Smart grid; transactive energy; demand response; particle swarm; Stackelberg game; pattern search; distribution system operator; load aggregator

I. INTRODUCTION

Being a measure of demand-side management, the demand response (DR) is defined by the U.S. Department of Energy as "changes in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized" [1]. Many advantages, such as load shifting and peak reduction, can be achieved with more participation of end-use customers. Different DR incentive approaches have been explored, including time-of-use (TOU) tariff, real-time tariff, critical peak tariff, demand-side bidding, interruptible load, and direct load control.

Extensive research has been conducted with regarding to different electricity pricing schemes, especially game-theoretic based approaches. For instance, in the two-step game with the objective of reducing electricity demand peak-to-average ratio (PAR) [1], the power companies pull consumers repeatedly in a round-robin fashion and provide them energy prices, then each customer optimizes its own schedule and updates it to the supplier. A one-leader, *N*-follower Stackelberg game has been formulated in [2], [3] to model the interaction between a utility company and multiple customers aiming at balancing supply and demand as well as smoothing the aggregated load. An *M*-leader and *N*-follower Stackelberg model is used in [4] to model the peer-to-peer energy trading process among prosumers in a community. A Nash-bargaining based cooperative model is formulated in [5] to let a distribution system operator (DSO) and demand load aggregators (LAs) collaboratively decide energy trade amount and allocate collective benefits fairly among participants.

With regards to solution approaches, the *centralized* backward induction (BI) approach [6] has been widely used since the establishment of game theory. It determines the next-to-last move by taking the last player's action as given. For example, in the pricing game between a utility company and different buildings [2], the price variable from the utility company is assumed as a given parameter to derive the best load response from buildings. In this paper, with the concerns of computational cost and privacy protection in centralized approaches, we explore *distributed* solution approaches for a pricing game where DSOs broadcast price and LAs adjust their actual loads accordingly. Unlike centralized solution approaches where all information from LAs is collected at a central unit to make global decisions, each LA in distributed solution approaches makes its own decision in a distributed way without revealing local information, while the DSO will make its next move based on the aggregated load feedback from all LAs.

The main contributions of this research are:

1. Investigating a heuristic *distributed* approach based on the particle swarm optimization (PSO) method to solve the pricing Stackelberg game.
2. Investigating a deterministic *distributed* approach based on the pattern search algorithm (PSA) to solve the pricing Stackelberg game.
3. Comparing the performance of the proposed two distributed approaches with the *centralized* BI solution approach.

The remainder of this paper is structured as follows. Section II presents the Stackelberg pricing model between a DSO and multiple LAs, while Section III presents three different solution approaches for the pricing model including one *centralized* and two *distributed* methods. Section IV presents detailed numerical results to validate the performance

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of the different solution approaches. Finally, Section V provides the summary and conclusion.

II. Stackelberg Pricing Model

In this section, a bi-level model is developed for the Stackelberg pricing game between a DSO and several LAs. There are several advantages to consider load aggregators instead of individual customers, for example, the dynamics of an aggregated load are slower and therefore more predictable. In addition, the communication burden in the electric distribution system will be reduced.

Generally, in DR programs, loads might be controlled (curtailed) during some periods of time by a system operator or customers might adjust their demand in response to real-time electricity price. In this study, the DSO is modeled as a leader while LAs are modeled as followers in a Stackelberg game model. The bi-level model of the Stackelberg pricing game is formulated in Eqs. (1)-(5), where at the upper level U_0 is the total utility value to be maximized for the DSO, and at the lower level U_n is the total utility value to be maximized for LAs. n, t are indices for number of LAs and time (in hours), respectively. $\underline{L}_{n,t}$ and $\overline{L}_{n,t}$ are the min and max demand limits for the nth LA at time step t. C_t is the incremental marginal cost and $\overline{P}$ is the max price reference. p_t and $l_{n,t}$ are the price and demand response variables, respectively, to be solved.

$$\max_{p_t, l_{n,t}} U_0 = \sum_{n,t} p_t \cdot l_{n,t} - \sum_{n,t} C_t \cdot l_{n,t} + \omega \cdot \sum_{n,t} S(l_{n,t}) - \theta \cdot \overline{P} \cdot m \quad (1)$$

$$\text{s.t. } C_t \leq p_t \leq \overline{P},\ \forall t \quad (2)$$

$$m \geq \sum_n l_{n,t}, \forall t \quad (3)$$

$$\max_{l_{n,t}} U_n = \sum_t S(l_{n,t}) - \sum_t p_t \cdot l_{n,t} \quad (4)$$

$$\text{s.t. } \underline{L}_{n,t} \leq l_{n,t} \leq \overline{L}_{n,t},\ \forall n \quad (5)$$

The objective function (Eq. (1)) has four terms. The first term is the total revenue from sold electricity. The second term is the cost of electricity to the DSO, different cost functions could be used, e.g. quadratic function [2], [7], [8]. The third term is the overall satisfaction value coming from LAs (in lower level). The fourth term is the penalty for peak demand. Note that ω and θ are weight factors for the customers' satisfaction and peak demand, respectively. The constraint in Eq. (3) makes sure the peak demand $m = \max_{t \in T} \sum_n l_{n,t}$ is greater than the total aggregated load at all times. The peak-to-average ratio is calculated by PAR $= \frac{m}{\frac{1}{T} \sum_{n,t} l_{n,t}}$.

The function $S(l_{n,t}) = \overline{L}_{n,t} \cdot \overline{P} \cdot (1 - e^{-\alpha_{n,t} \cdot (\frac{l_{n,t}}{D_{n,t}})})$ is used to represent monetary value of customers' satisfaction. $D_{n,t}$ is the nominal demand of LA n, which can be obtained from historical energy load profiles. This exponential utility function $f(x) = 1 - e^{-\alpha \cdot x}$ is a concave increasing function commonly adopted in utility theory to model users' preference [9]. Its value trend is illustrated in Fig. 1 with different preference coefficients $\alpha \in \{0.5, 2, 4\}$. The parameter α could be treated as the sensitivity towards energy consumption curtailment, for instance, the utility value of a relatively large value of $\alpha = 4$ reaches about 0.8 when $l_{n,t} = 0.5 D_{n,t}$, while it is only 0.2 for $\alpha = 0.5$.

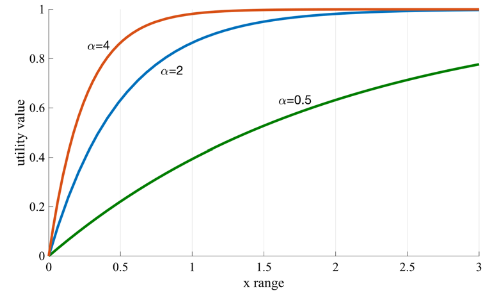

Figure 1. Utility value for different user preferences.

III. Solution Approaches

In this section, we initially present the BI *centralized* solution approach to the Stackelberg pricing game (Eqs. (1)-(5)). Then, we present two *distributed* solutions, based on PSO and PSA, for this pricing game problem.

A. Backward Induction (BI)

The classical BI method is a *centralized* approach to solve for the equilibrium solution of the Stackelberg pricing game (Eqs. (1)-(5)) by following the following two steps:

1. Derive optimal demand response to price.

The electricity price p_t from the DSO is assumed as a parameter, then the best load response $l^*_{n,t}$ is obtained by the first-order derivative of LAs' objective functions. The constant $\frac{\overline{L}_{n,t} \cdot \overline{P}}{D_{n,t}}$ can be redefined as a parameter.

$$l^*_{n,h} = \frac{D_{n,t}}{\alpha_{n,t}} \cdot \ln \frac{\alpha_{n,t} \cdot \overline{L}_{n,t} \cdot \overline{P}}{p_t \cdot D_{n,t}} \quad (6)$$

2. Derive optimal price based on user response.

After the optimal load $l^*_{n,h}$ is obtained, it can then be plugged into the upper level and convert the bi-level model into a single level model with p_t as the only variable. Eq. (6) needs to be substituted into Eq. (5) to get an additional range for p_t. If a fixed price structure is adopted, an additional constraint $p_t = p_{t-1}, t \geq 2$ should be added to the model.

Although the BI method provides elegant explicit solution for the optimization problem, it does suffer serious challenges including heavy computation cost for large-scale implementation, all information needs to be collected and privacy is not protected, less robust decisions due to natural uncertainties or information infidelity [10]. In the sequel, two *distributed* approaches will be introduced to overcome such challenges.

B. Particle Swarm Optimization (PSO)

The PSO is a widely used swarm intelligence-based approach, which mimics a flock of birds that communicate together as they fly across the solution space. It is adopted here because of its outstanding performance in nonlinear continuous solution space [10], [11], [12]. Its proposed distributed solving process is summarized as follows:

1. Randomly initialize vector p_t in the range of Eq. (2).
2. Given p_t, each LA computes U_n as in Eq. (4). Check $l_{n,t}$ in the range of Eq. (5), if not, adjust p_t upward or downward accordingly.
3. LAs pass their $l_{n,t}$ and $S(l_{n,t})$ to the DSO. U_0 is then computed as fitness for particle swarm.
4. Update velocity and position for each particle, check mutation and stop criteria.

C. Pattern Search Algorithm (PSA)

The PSA belongs to a set of direct search algorithms that do not use derivatives or approximations of derivatives to solve the problem [13]. Thus, it can be applied for non-differentiable non-continuous problems. For our case, in the p_t searching process of PSA, the corresponding load responses from LAs can be computed in a distributed way. PSA updates current iteration by sampling the objective function at a finite number of points along a suitable set of search directions with the aim of finding a decrease in the function value.

The following steps are used to solve for the Stackelberg pricing game in Eqs. (1)-(5) using the PSA [14]:

1. Perform the optional SEARCH step in order to find an improved point p_t; this step evaluates U_0 on a finite number of trial points on the mesh.

2. If the SEARCH step fails to generate an improved point, create the poll directions set $\{p_t + \Delta \cdot d, d \in \Gamma\}$($\Delta$ is the length and Γ is the finite direction set) and perform the POLL step again in order to find an improved point.

3. If an improved point ξ is found, update the current best point p_t and Δ.

The pattern search with the implicit filtering algorithm in [15] is adopted in this study.

IV. Numerical Experiments

A. Data Setting

Day-ahead hourly optimization is considered with three aggregators (n1, n2, n3). The nominal demand $D_{n,t}$, $\alpha_{n,t}$ in the satisfaction function (same for all aggregators), and marginal cost C_t are shown in Figs. 2 - 4. The parameter $\overline{P}$ is assumed to be constant equals to 30. The weight combination (ω, θ) in Eq. (1) is set to be (3,1). These settings are applied to the proposed three solution approaches. Here, we assume all aggregators have the same price determined by the DSO.

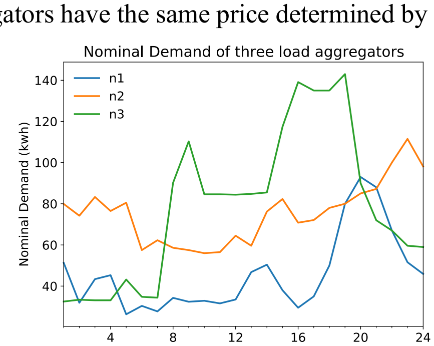

Figure 2. Nominal demand of aggregators.

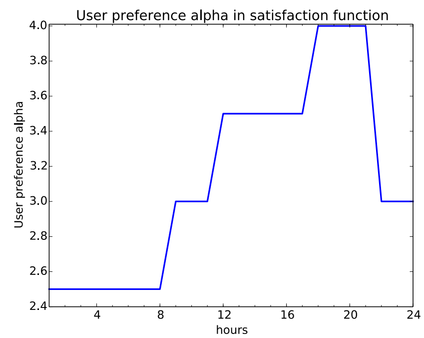

Figure 3. User preference in satisfaction.

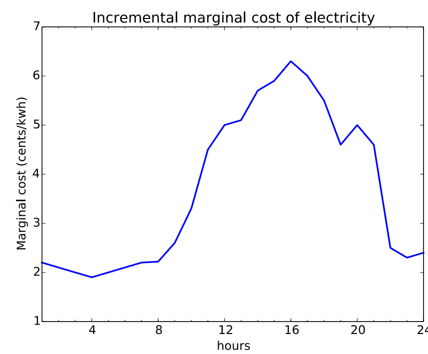

Figure 4. Incremental marginal cost of electricity.

B. Experimental Results

For the parameter setting of the PSO algorithm, w is set to be 0.6 and updated as (0.5 + unif(0,1))/2 in every iteration i, $c_1 = c_2$ is set to be 1.49 and updated by following the nonlinear acceleration strategy [10].

The maximum allowed iteration number is 100, swarm size is 30, total run time is set to 50. If the best fitness is not improved in consecutive 10 iterations, a random disturbance between [-0.5, 0.5] is added to the current position of each particle to mutate current particles. If the best fitness is not improved in successive 30 iterations, stop the algorithm.

The obtained price signals from PSO in 50 runs are shown in Fig. 5, and the corresponding DSO objective values are shown in Fig. 6, where the red line is the optimal objective value \$8316.39 obtained from the BI approach; see results summary in Table I.

Figure 5. Prices for 50 runs of PSO.

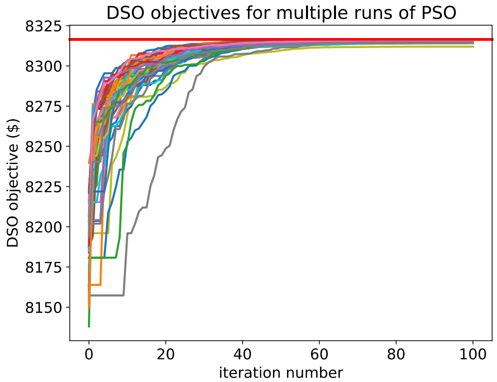

Figure 6. DSO objective values for 50 runs of PSO.

Since PSA is a direct search method and doesn't rely on derivative information, its performance greatly depends on the maximum allowable function evaluation number (here, one function evaluation means one complete negotiation process between a DSO and LAs). For instance, the objective iteration is shown in Fig. 7 with function evaluation number of 2000. The final objective value approaches the optimal value after about 40 iterations. The final objective value and solving time are shown in Fig. 8 along with a function evaluation number from 100 to 2000.

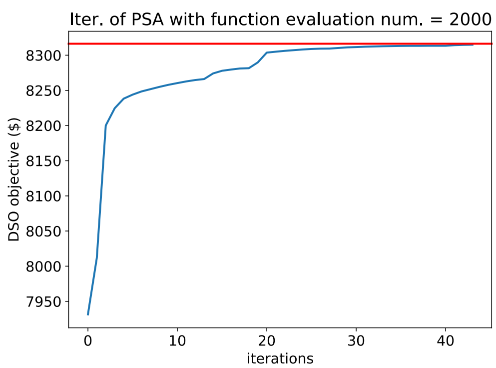

Figure 7. Iteration process of PSA with function evaluation number of 2000.

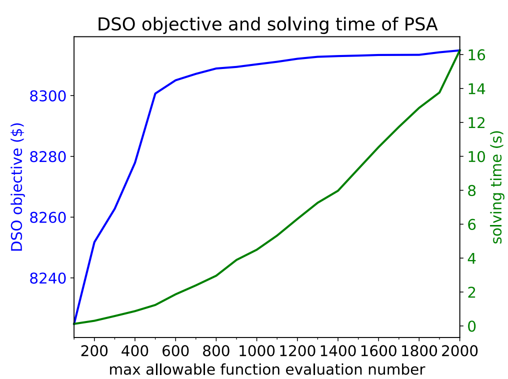

Figure 8. DSO objective and computational time of PSA with different function evaluation numbers.

For comparison purposes, the BI approach is implemented using a non-commercial nonlinear solver SCIP [16]. Both fixed price (FIX) and Time-of-Use (TOU) price structures are tested. The average price in multiple runs are calculated and the best price is picked based on the best objective value for PSO. Overall, as indicated by the price comparison in Fig. 9, the TOU prices are blow the fixed price before 11am and after 23pm, which will motivate LAs to shift demand from relatively peak hours 11am-23pm (see Fig. 2) or high generation cost hours (see Fig. 4) to off-peak hours before 11am. This is supported by the aggregated demand response outcome shown in Fig. 10. Note that this peak-load shifting effect is greatly determined by users' preference on satisfaction (see Fig. 3 and Fig. 1); small value of α before 11am means more load or regulation flexibility, and large value of α means that users are reluctant to the price response in that period.

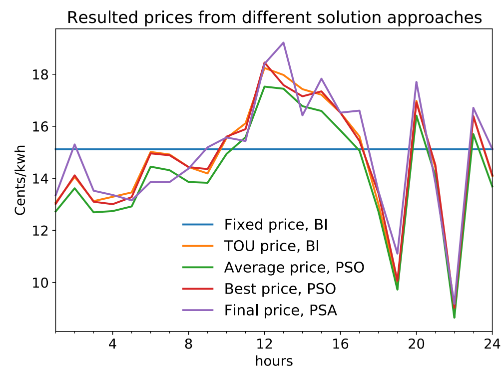

Figure 9. Resulted prices from the different solution approaches.

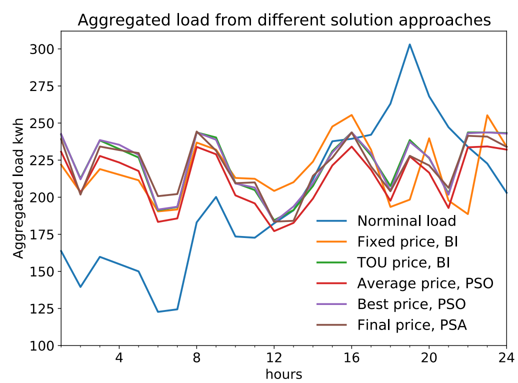

Figure 10. Resulted aggregated loads from the different solution approaches.

To better compare the different solution approaches, the detailed experimental results are summarized in Table I. The best price for PSO is used, and the function evaluation number is fixed as 2000 for PSA. Compared with fixed price, TOU prices from BI, PSO, and PSA have a lower PAR value and higher overall social welfare (objective sum of DSO and LAs) for one-day operation. The average price for each aggregator is calculated by $\frac{\sum_t p_t \cdot l_{n,t}^*}{\sum_t l_{n,t}^*}$. The BI (TOU) and PSO (Best) provide customers with a lower average price which means they could consume more energy with a similar bill payment and satisfaction level. The average price of PSA is higher than BI and PSO mainly due to the limitation of the function evaluation number.

As a centralized approach, BI will need to collect all sensitive information from customers, such as their load profile, satisfaction preference, etc. And its computational cost (solver SCIP) is higher than the distributed algorithms (PSO and PSA), for instance, it takes 300 seconds to reach a relative gap of 2.88% and 600 seconds to reach a relative gap of 2.69% for this small case study. Each iteration in PSO takes about 0.3 seconds for parallel execution and it will need around 30 seconds for max 100 iterations limit. Seen from Fig. 6, it starts to converge around 50 iterations. The solving time for PSA is shown in Fig. 8. Although PSO performs slightly better than PSA in Table I, its performance is not stable due to its stochastic nature and it requires a parameter tuning process and a large number of repetitions. On the other hand, PSA is a deterministic method and its convergence property is guaranteed given enough function evaluations.

TABLE I. PROFIT AND COST IN SUMMARY (PRICE: CENT, COST/REVENUE/SATISFACTION: $), (CUSTERMOR SATISFACTION: PEAK PENALTY = 3:1).

DSO Level	**BI (FIX)**	**BI (TOU)**	**PSO (Best)**	**PSA (Final)**
Peak Load	255.43	243.69	243.72	244.30
PAR Value	1.17	1.09	1.09	1.10
Total Satisfaction	2593.66	2600.74	2600.79	2597.71
DSO Revenue	790.33	780.87	780.70	786.89
DSO Gen. Cost	193.32	193.60	193.67	191.85
DSO Profit	597.01	587.27	587.03	595.03
DSO Objective	8301.39	8316.39	8316.31	8314.90
Aggregator Level	**BI (FIX)**	**BI (TOU)**	**PSO (Best)**	**PSA (Final)**
n1 Average Price	15.11	14.63	14.61	14.84
n2 Average Price	15.11	14.57	14.56	14.79
n3 Average Price	15.11	14.76	14.76	15.02
n1 Bill Payment	206.15	203.57	203.48	205.05
n2 Bill Payment	293.70	289.45	289.37	291.59
n3 Bill Payment	290.46	287.85	287.84	290.24
n1 Satisfaction	682.23	684.70	684.71	684.14
n2 Satisfaction	955.92	959.30	959.41	958.35
n3 Satisfaction	955.50	956.73	956.67	955.21

V. CONCLUSION

In this paper, a Stackelberg pricing model is formulated for a DSO and LAs, where in the upper level, the DSO maximizes its profit as well as its social obligation to customers, while in the lower level, LAs minimize their electricity bill payment and discomfort. To accelerate the solving process and protect sensitive information, two distributed algorithms, PSO and PSA, are implemented and compared. Numerical results show that the performance of the proposed distributed approaches is in close proximity to the optimal solution. The problem of how the committed aggregated demand response can be allocated to each individual customer will be explored in a future paper.

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