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# Quantum Tomography: State (& Process) of Play

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**Fault Tolerance**

Quantum Communication

**Error Correction**

**[Algorithmic] Cooling**

**Mastering Entropy**

- This is **THE** fundamental chore in QIP -- and almost all of our tasks equate to it.

Physics

Data Compression

Showing Off

**Quantum Information Processing**

Fidelity

**Relative Entropy** (Kullback-Leibler divergence)

Bregman divergences

Well-regularized metrics

2-norm (Euclidean, Hilbert-Schmidt)

Operationally meaningful quantities

1-norm

**See Optical Tomography!**

**See Model Selection!**

**Homodyne vs. Heterodyne Tomography**

**SICPOVMs**

**Uniformly Informationally Complete** Measurements

**Over-Complete** Measurements

- redundant statistics enable validation, model selection, etc.
- potential downside: naive data analysis (linear inversion) generally won't work.

**Mutually Unbiased Bases**

**Informationally Complete** Measurements

- provide *some* information about every dimension of state space.
- must have at least $d^2$ outcomes

[Limitations of] **Tensor Product Measurements**

**Undercomplete (but targeted)** measurements

**Why are we doing tomography, anyway?**

- requirements for QIP are very different from (say) the requirements for optical tomography in laser physics.
- e.g., fidelity vs. 2-norm; zero eigenvalues...

**Adaptive Tomography**

- Using a preliminary estimate to guide subsequent measurements (measure the most relevant dimensions) can dramatically improve ***fidelity*** -- but not 2-norm.

**"What Measurements Should We Make?"**

In a lot of qubit experiments, we just measure **local Pauli operators**. Or, if the system isn't *N*-qubits, we measure [tensor products of] **mutually unbiased bases**. MUBs and **SICPOVMs** are both **uniformly informationally complete**, which means they not only sample every dimension of density matrix space, but they are *equally sensitive* to each dimension (a.k.a. 2-designs).

However, **tensor products of local measurements cannot be UIC**, which poses challenges for *N*-qubits. And in certain systems (continuous variables) **UIC measurements don't always exist**.

**Overcomplete** measurements can provide valuable redundancy for *model selection* and detection of systematic errors. On the other hand, *undercomplete targeted measurements* can access specific information much more efficiently. And **adaptive tomography** can yield startling improvements in efficiency.

The absolute best bounds on what we can do

**Joint Measurements** on many samples

[Atomic] Ensembles

Coherent Measurements

**Error Metrics** ("How should we measure inaccuracy?")

This turns out to be a lot more important than a lot of people realize. It's seriously tempting to just pick any old metric... but even the well-motivated ones (i.e., ones with operational interpretations) can have very different properties.

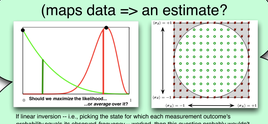
Different metrics for inaccuracy motivate different estimators. They suggest different shapes for uncertainty regions ("error bars"). They even motivate radically different measurements!

**Trace norm** is overrated -- its operational meaning is **strictly single-shot**, and when you regularize it (i.e., calculate it for $N \mapsto \infty$ copies), you get something more like...

...**Fidelity**, which is pretty good except that for many purposes it has **the wrong operational interpretation** -- it motivates reporting "the best lie" rather than "the honest truth". From which perspective...

...**Relative entropy** is great, except that it is **really finicky about zero probabilities**, which may be a feature or a bug.

**"Which Estimator?"** (maps data => an estimate?




If linear inversion -- i.e., picking the state for which each measurement outcome's *probability* equals its observed *frequency* -- worked, then this question probably wouldn't have come up. But it often gives negative eigenvalues (right figure). Which forced us to delve into statistics to answer the question "***Given certain data, what state is the best one to report?***" That turns out to be a very hard question...

**Bayesian Mean**

The mean (average) of the posterior distribution has some excellent properties, but computing it rigorously usually requires arduous Monte Carlo integration.

**Constraints:** are ***NOT*** priors ... (but carry some of the same dangers)

**PRIORS**:
- do we have them?
- do we need them?
- are they bad?
-are they useful?
- which ones are good?

**[Learn some] Measure Theory [please!]**

**Reference Priors & Least Favorable Priors**

**See "Absolute Bounds"!**

**Minimax Estimators**

**Objective Bayesian Methods**

**THE LIKELIHOOD FUNCTION**

$\mathcal{M} = \{E_1, E_2, \ldots, E_N\} \longrightarrow \mathcal{L}(\rho) \equiv p(\mathcal{M}|\rho) \longrightarrow \rho_{est}$

**Maximum Likelihood Estimation**

**Ad Hoc Estimators**

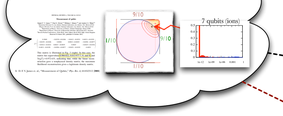
**L1 Regularization**

**See Compressed Sensing!**

**Hedged MLE**

**"Positivized" Linear Inversion**

**The Zero Eigenvalue Issue**




**See Model Selection!**

**Bayesian vs. Frequentist** controversy

On one hand, this messy business is worth knowing about.

But... on the other... it's maybe not a good use of time. :)

There are at least three levels:

(1) Bayesians say probabilities are *subjective* -- they represent beliefs that **you** (an agent) have about a system. Frequentists say probabilities are *objective* -- measurable physical properties of the system, like mass. **Both are insane (Bayesians somewhat less so), but there are no better alternatives!**

(2) Bayesian statisticians use Bayes' Rule and prior distributions to establish their post-measurement beliefs about the system, and use that to make an estimate. Frequentist statisticians get estimates by applying algorithms that usually work well. **Naive application of either is BAD -- but sophisticated methods of both schools end up nearly identical. You can be both!!!**

(3) There are also differences w/respect to interpretations of quantum theory. These are almost certainly not relevant to this discussion!

**Compressed Sensing**

**Matrix Product States**

**See Optical Tomography!**

**Submanifolds** of states

**Subsystems/ subspaces**

**Fidelity**
- with a desired state
-with a desired manifold

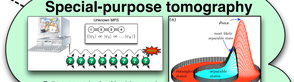
**Parameter Estimation**

**Failure probability** (twirling)

[Entanglement] **Witnesses**

**Randomized benchmarking**

**Special-purpose tomography**

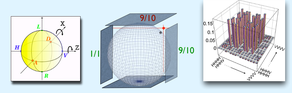



Full tomography for *N* qubits may be unfeasible. So we have specialized methods to extract relevant information efficiently, including:

- efficient tomography of restricted states/processes (subspaces, subsystems, or submanifolds).

- methods to extract specific parameters from arbitrary states/ processes (entanglement, fidelity, etc.)

**Quantum State Tomography**




...on qubits (or qudits)...
...with informationally complete measurements...
...using linear inversion...
...to estimate a $d \times d$ density matrix.

**Examples**: photon polarization (linear optics)
ion traps
NMR

**Ancilla-assisted** process tomography

**The Choi-Jamiolkowski Isomorphism**

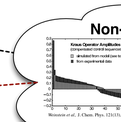
**Process Tomography**

Estimating the *dynamics* of a system -- its **quantum process** -- is far more important than state tomography. But, happily, the two are isomorphic -- **processes are isomorphic to states on two systems**. So we tend to use state tomography as a (simpler) playground.

There *are* some differences. Positivity is more fraught. Probing a process requires preparing a lot of different input states (perhaps in superposition, as in **ancilla-assisted** methods. And the matrices are just bigger, so scaling problems hit us faster.

**Selective Efficient** Quantum Process Tomography

**Non-CP Maps**




Quantum processes are completely positive (CP), in exact analogy to the positivity of the density matrix... ... but only if the system is initially uncorrelated with its bath! This is rarely true in experiments -- which means that we really can't impose (complete) positivity by hand!

Rötteler *et al.*, J. Chem. Phys. 123(13), 4117-4133 (2006)

**Direct Characterization** of Quantum Dynamics

**Likelihood Ratio Regions**

**Christandl and Renner's construction**

**Confidence Regions**

**See Error Metrics!**

**[Bayesian] Credible Regions**

**GOOD**

**Large Deviation Bounds**

**See Priors!**

**UGLY**

**"With your posterior"**

**Characterizing Uncertainty**

Every physicist knows that estimates require error bars. The problem is... **you can't just make them up!** Error bars -- or their more powerful generalization, ***region estimates*** -- must have a rigorous meaning. Otherwise, they are "security theater".

"GOOD" methods have [some] rigorous meaning that can be guaranteed without no/ few assumptions about the system. "BAD" methods are ad hoc... and while point estimators can be ad hoc (they don't promise much!), this is a mortal sin for error bars.

Unfortunately, the most-used method (bootstrapping) is "BAD".

**BAD**

**"Don't bother"**

**"Error bars" a.k.a. standard errors**

Undergraduate Statistics

Hypothesis Testing

**Likelihood ratios**

**Bayes Factors**

**Resampling Methods**

**Parametric Bootstrap**

**Nonparametric Bootstrap**

**See Zero Eigenvalues!**

**See Estimators!**

**Model Selection** -- "What is this system, anyway?"

Fundamentally, model selection addresses problems like inferring the Hilbert space dimension itself from the data. This is critical for **optical mode tomography**.

But model selection can also be used to rigorously test assumptions (e.g. compressed sensing), and to dynamically generate powerful ansatze for the system being studied.

It will be indispensable for teasing information out of **stubborn and difficult systems.**

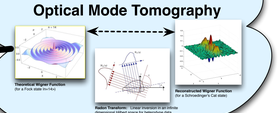
**See Relative Entropy!**

**Akaike Information Criterion**

**Good-Turing methods**

**See Data Compression!**

**Wigner Functions** and **Optical Mode Tomography**




**See Special-purpose Tomography**

**N-qubit Systems**

Solid State

Superconductors

Infinite-dimensional systems

Quantum simulators

Topological systems

Adiabatic Quantum Computers

**Stubborn and Difficult Systems**

So far, tomography has been applied -- mostly -- where it is relatively **easy** to apply. But there are a host of **difficult** applications out there -- and these are the real juicy challenges!

Some are just big (continuous variables, *N*-qubit systems).

Others have weak/noisy/slow/ambiguous measurements (superconductors, quantum dots, etc.)

Others do not support universal control (simulators, D-Wave, topological systems).

**Sophisticated statistical analysis**

**Measurement Tomography**

**Bootstrapping** (simultaneous estimation of all aspects of an experiment)

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