Resolved Sideband Emission of Quantum Dots Strained by Surface Acoustic Waves*

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and
National Institute of Standards and Technology (NIST)

* *Physical Review Letters* 105, 037401 (2010)

Resolved Sideband Emission of Quantum Dots Strained by Surface Acoustic Waves

- Introduction to semiconductor quantum dots.
- Schematic description of the experiment.
- Surface Acoustic Wave generation on-chip using microfabricated interdigital transducers.
- Images of the cryogenic experimental apparatus that was designed, constructed, and implemented.
- Experimental near-infrared spectroscopy of self-assembled quantum dots, with the quantum dot emission controllably modulated using surface acoustic waves generated on-chip.
- Resolved sideband regime experimentally demonstrated using resonant spectroscopy.
- Implications of the Resolved Sideband Regime: Quantum-Limited Spectroscopy, Quantum Cooling, Quantum Information, and Potential Optoelectronic / Photonic Device Applications.
- Summary.

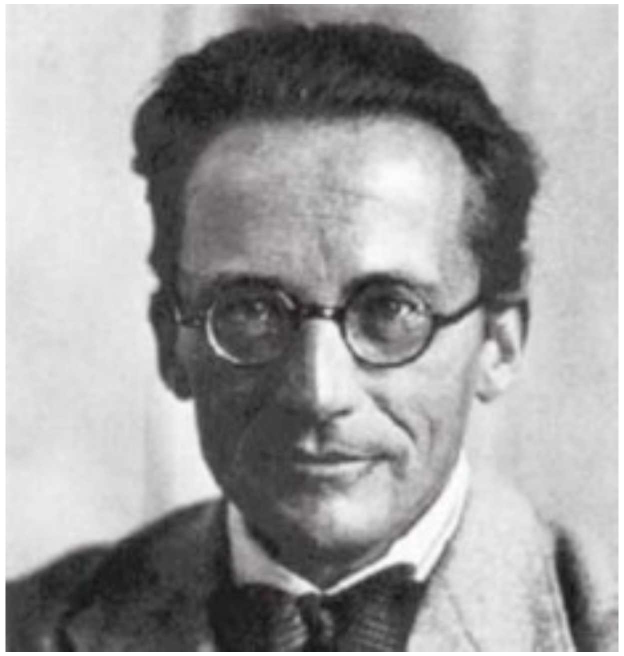
- Some history and background on the field from my perspective.
- A personal perspective on the evolution of the field of quantum mesoscopic mechanical systems.
- A description of the intriguing theoretical proposal that motivated my initial experimental work at NIST.
- Slides describing my initial experimental work at NIST that was published as:

Resolved Sideband Emission of InAs/GaAs Quantum Dots Strained by Surface Acoustic Waves

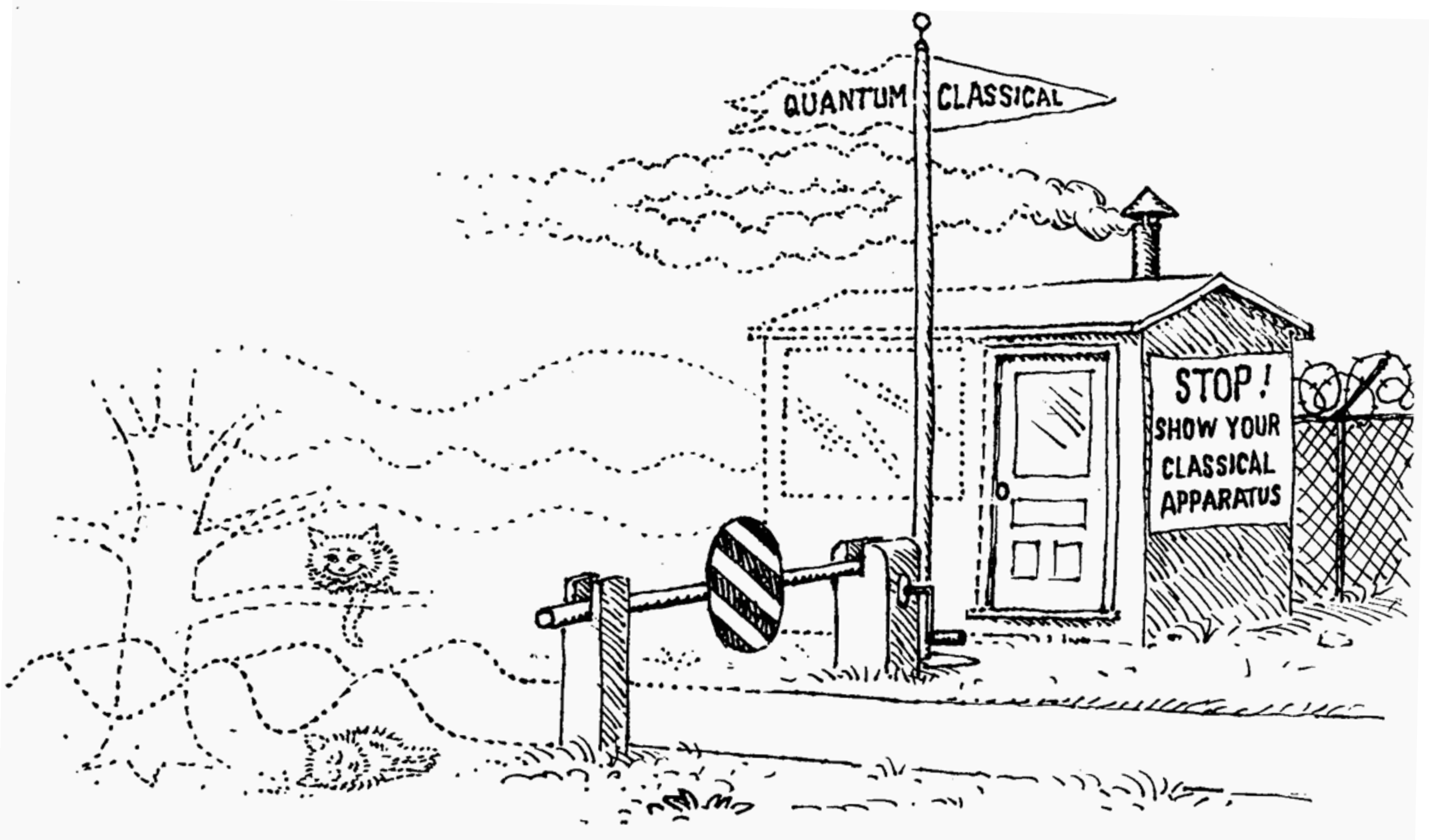
Physical Review Letters 105, 037401 (2010)

- Implications / Directions / Possibilities...
- A brief description of my current work at Sandia.

A Grand Challenge: Experimentally Probe the Quantum-Classical Interface



Erwin Schrödinger (1887–1961)

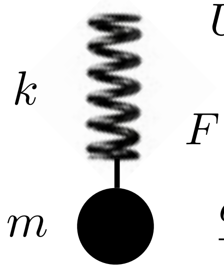


Delineating the border between the quantum realm ruled by the Schrödinger equation and the classical realm ruled by Newton's laws is one of the unresolved problems of physics. **Figure 1**

W.H. Zurek, *Physics Today*, 44,36 (1991)

Consider a specific system:

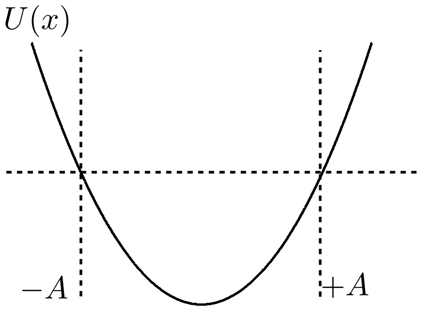
A Simple Harmonic Oscillator (SHO)



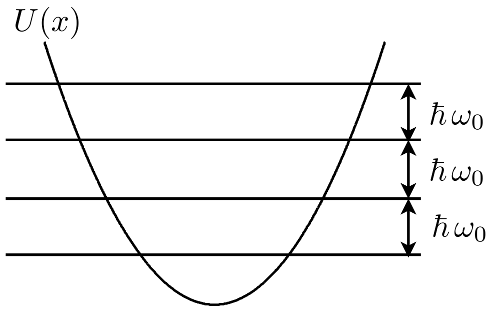
$$U(x) = \frac{1}{2}kx^2$$

$$F(x) \equiv -\frac{dU}{dx} = -kx$$

$$\frac{d^2U}{dx^2} = k = m\,\omega_0^2$$



Classical SHO



Quantum SHO

Compare $\hbar\omega_0$ to $k_B T$

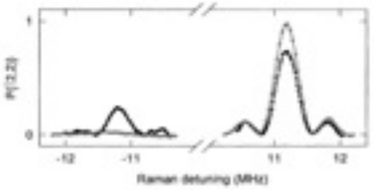
ν_0	$T_0 \equiv h\,\nu_0/k_B$
1 (MHz)	50 (μK)
10 (MHz)	500 (μK)
100 (MHz)	5 (mK)
1 (GHz)	50 (mK)
10 (GHz)	500 (mK)

A brief and incomplete recent history of the cooling of a mesoscopic mechanical mode to the quantum ground state

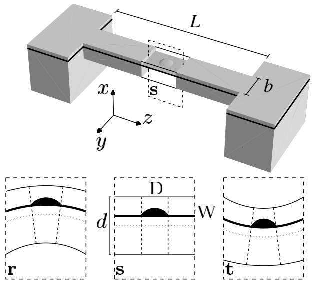
What particular system to use to try to reach the quantum ground state?

How should the system be engineered toward this goal?

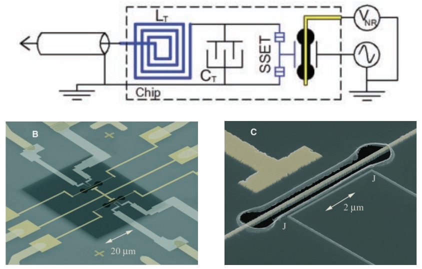
What are the key experimental and technical requirements?



Resolved-Sideband Raman Cooling of a Bound Atom to the 3D Zero-Point Energy, Physical Review Letters 75, 4011 (1995).

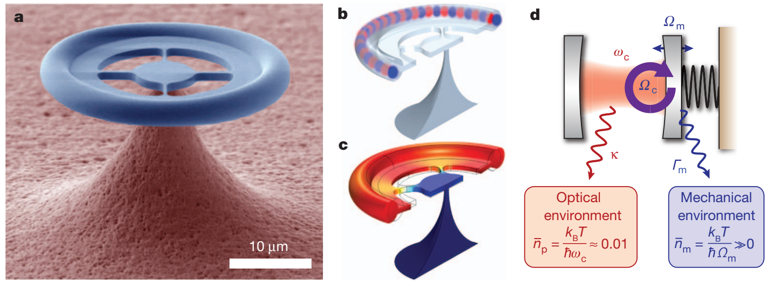


Laser Cooling of a Nanomechanical Resonator Mode to its Quantum Ground State, Physical Review Letters 92, 075507 (2004).

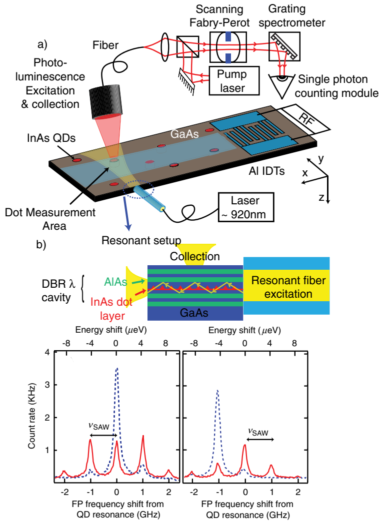


Approaching the Quantum Limit of a Nanomechanical Resonator, Science 304, 74 (2004).

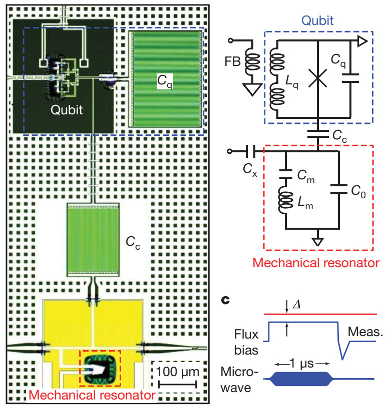
"Cavity Optomechanics"
"Quantum Electromechanics"



Resolved-sideband cooling of a micromechanical oscillator, Nature Physics 4, 415 (2008) . Regarded as the first demonstration of resolved-sideband cooling of a mesoscopic mechanical device. *Quantum-coherent coupling of a mechanical oscillator to an optical cavity mode,* Nature 482, 63 (2012).

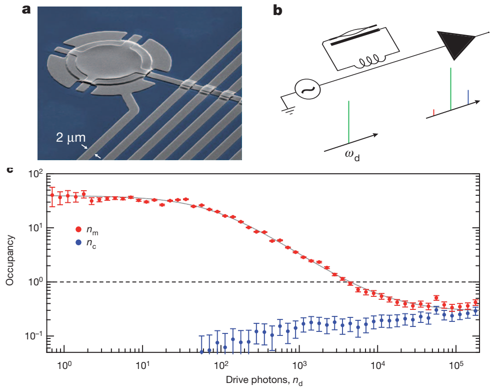


Resolved Sideband Emission of InAs=GaAs Quantum Dots Strained by Surface Acoustic Waves, Physical Review Letters 105, 037401 (2010).

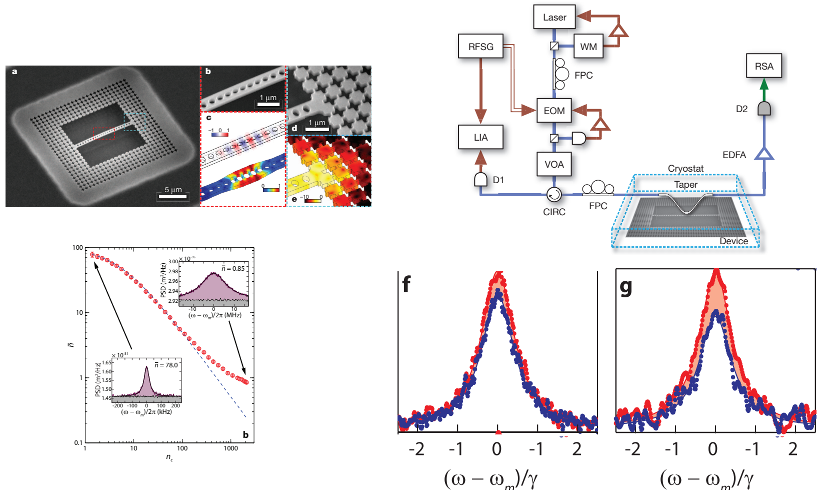


Quantum ground state and single-phonon control of a mechanical resonator, Nature 464, 697 (2010).

Selected by Science magazine as research breakthrough of the year for 2010.



Sideband cooling of micromechanical motion to the quantum ground state, Nature 475, 359 (2011).



Laser cooling of a nanomechanical oscillator into its quantum ground state, Nature 478, 89 (2011).

Observation of Quantum Motion of a Nanomechanical Resonator, Physical Review Letters 108, 033602 (2012).

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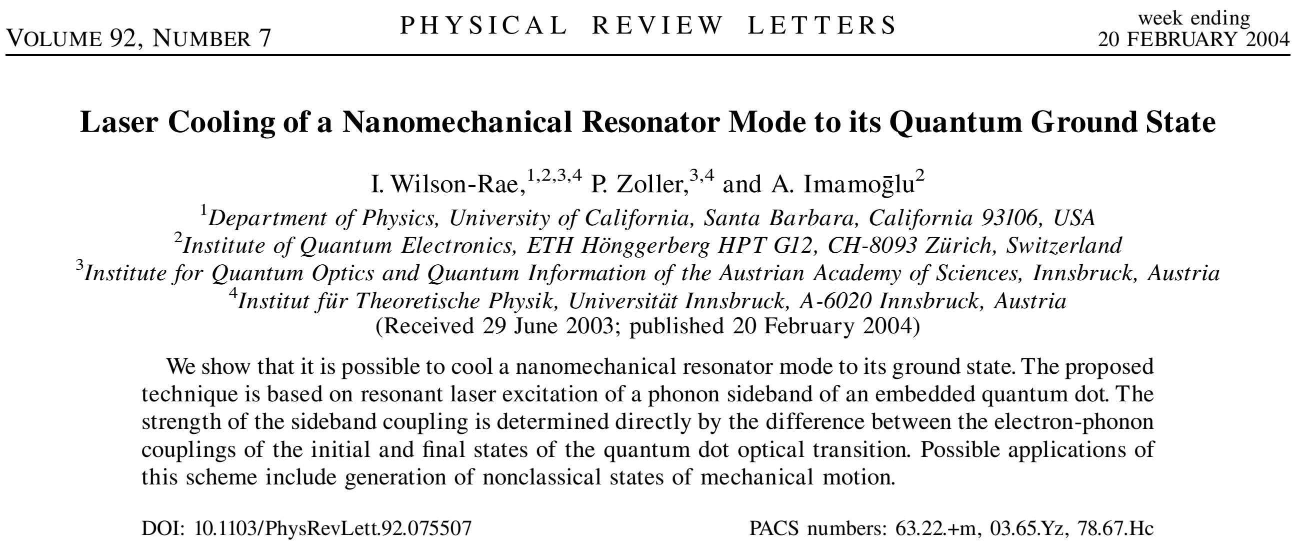
Center for Quantum Information and Control (CQuIC) Seminar

An intriguing idea: Coupling of a mesoscopic mechanical mode to an embedded quantum dot



VOLUME 92, NUMBER 7 PHYSICAL REVIEW LETTERS week ending 20 FEBRUARY 2004

Laser Cooling of a Nanomechanical Resonator Mode to its Quantum Ground State

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(Received 29 June 2003; published 20 February 2004)

We show that it is possible to cool a nanomechanical resonator mode to its ground state. The proposed technique is based on resonant laser excitation of a phonon sideband of an embedded quantum dot. The strength of the sideband coupling is determined directly by the difference between the electron-phonon couplings of the initial and final states of the quantum dot optical transition. Possible applications of this scheme include generation of nonclassical states of mechanical motion.

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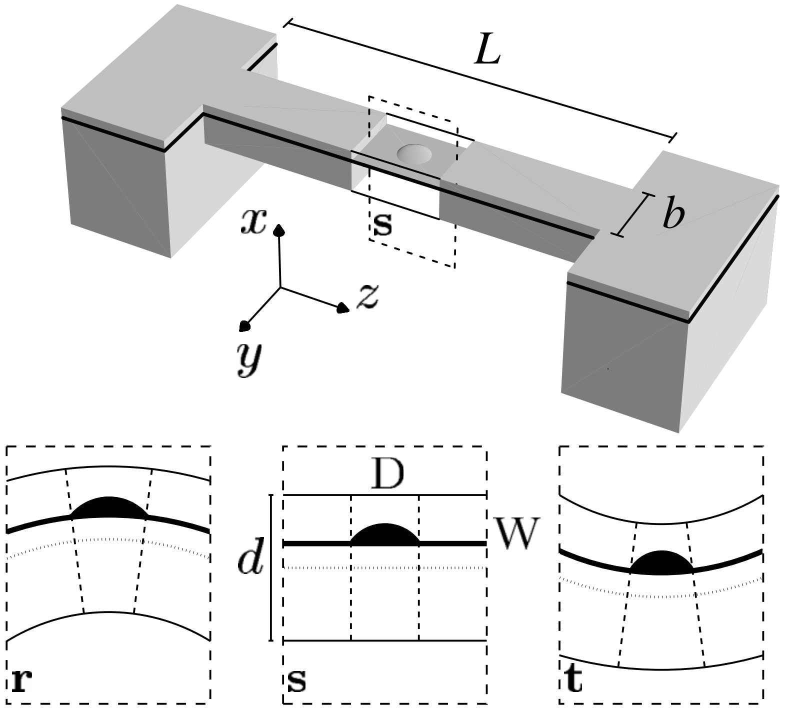
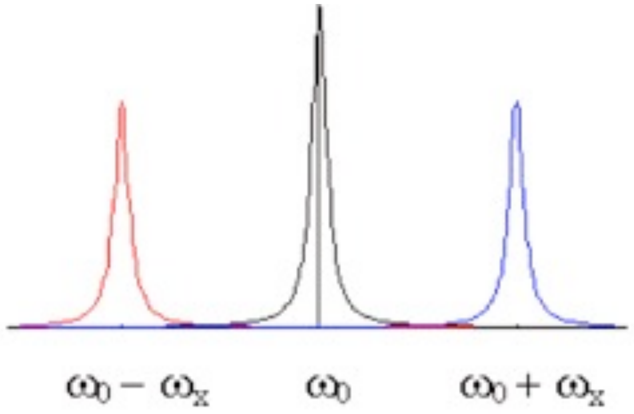
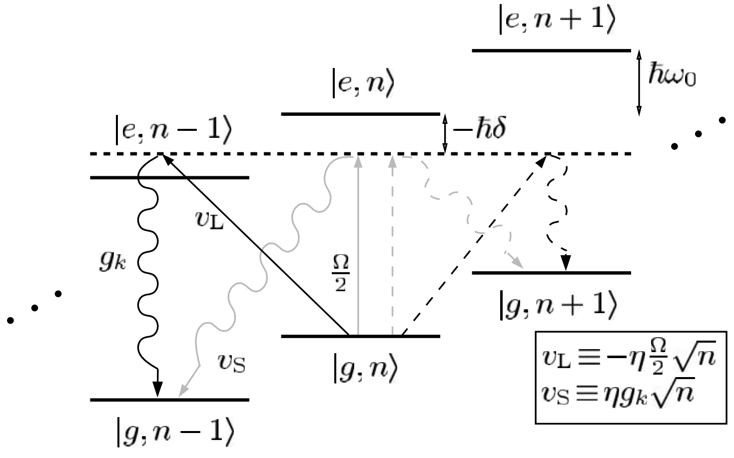
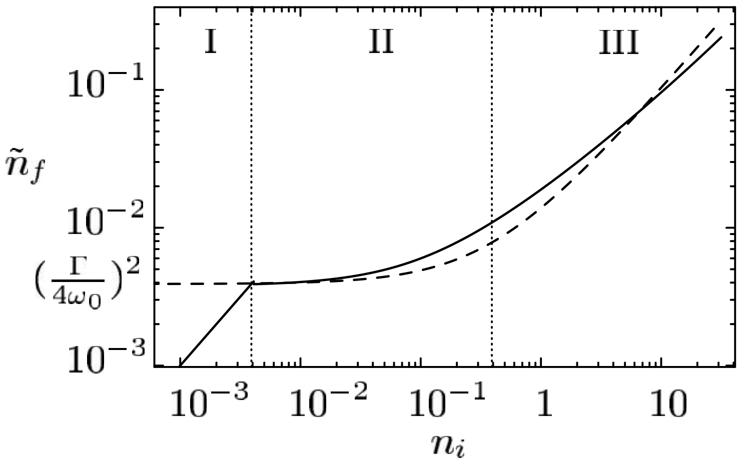


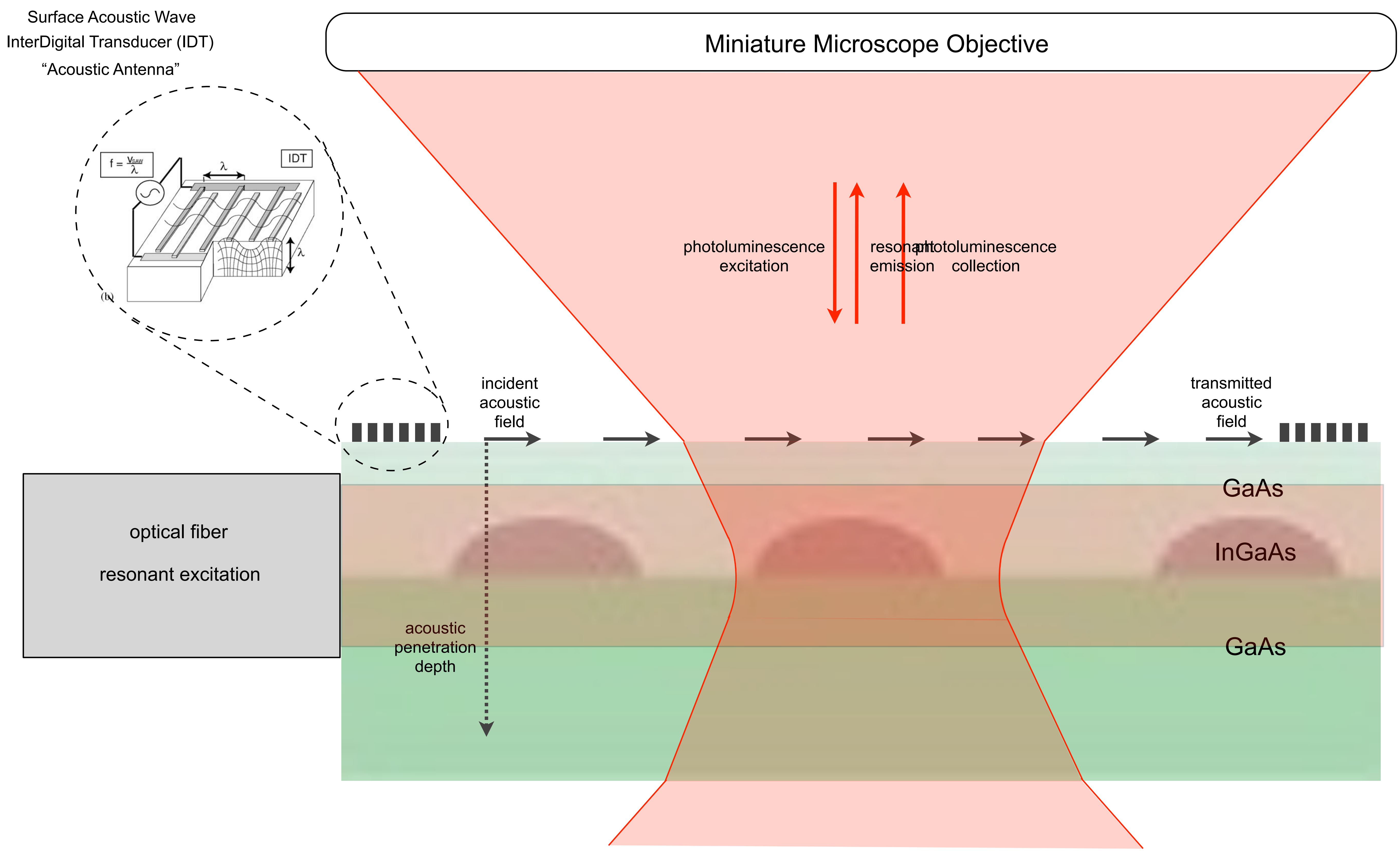






- “Photon recoil plays no role........Instead, here the coupling of the laser to the mechanical degrees of freedom is provided by the difference between the electron-phonon couplings of the ground state (g) and the excited state (e).”
- Message to the experimentalists: “We find that the resolved-sideband regime is a necessary condition for ground-state cooling.”
- Experimental implementation: Realization will be hard. How to achieve the first steps in an experimental demonstration?

Experimental Schematic (not to scale)



Surface Acoustic Wave
InterDigital Transducer (IDT)
“Acoustic Antenna”

Miniature Microscope Objective

photoluminescence
excitation

resonant
emission

photoluminescence
collection

incident
acoustic
field

transmitted
acoustic
field

GaAs

InGaAs

GaAs

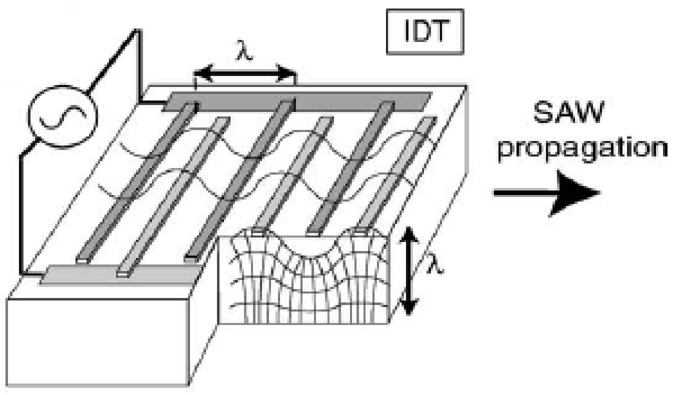
optical fiber

resonant excitation

acoustic
penetration
depth

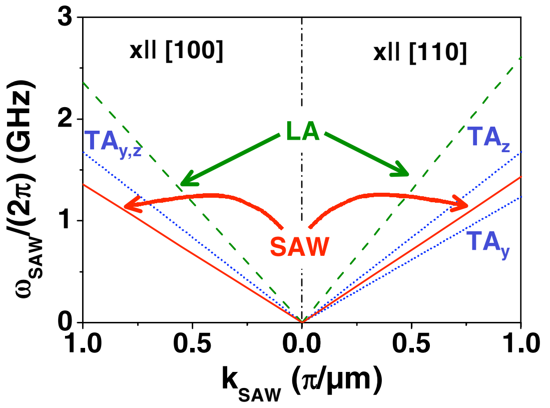
Surface Acoustic Waves: On-Chip Generation using Microfabricated Transducers

Device Schematic



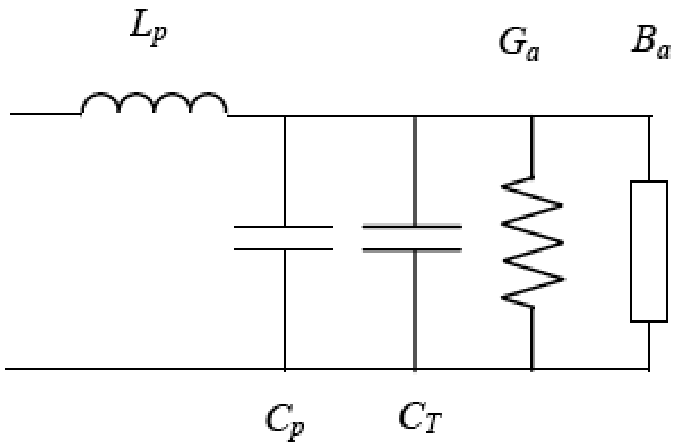
- Devices are lithographically-defined on the surface of the GaAs capping layer using electron-beam lithography and metal deposition.
- $\lambda_s \equiv$ wavelength of surface acoustic wave is determined by the period of the patterned device.
- Characteristic decay length (penetration depth) is given approximately by $(\lambda_s \; / \; \pi)$

SAW Dispersion Relation



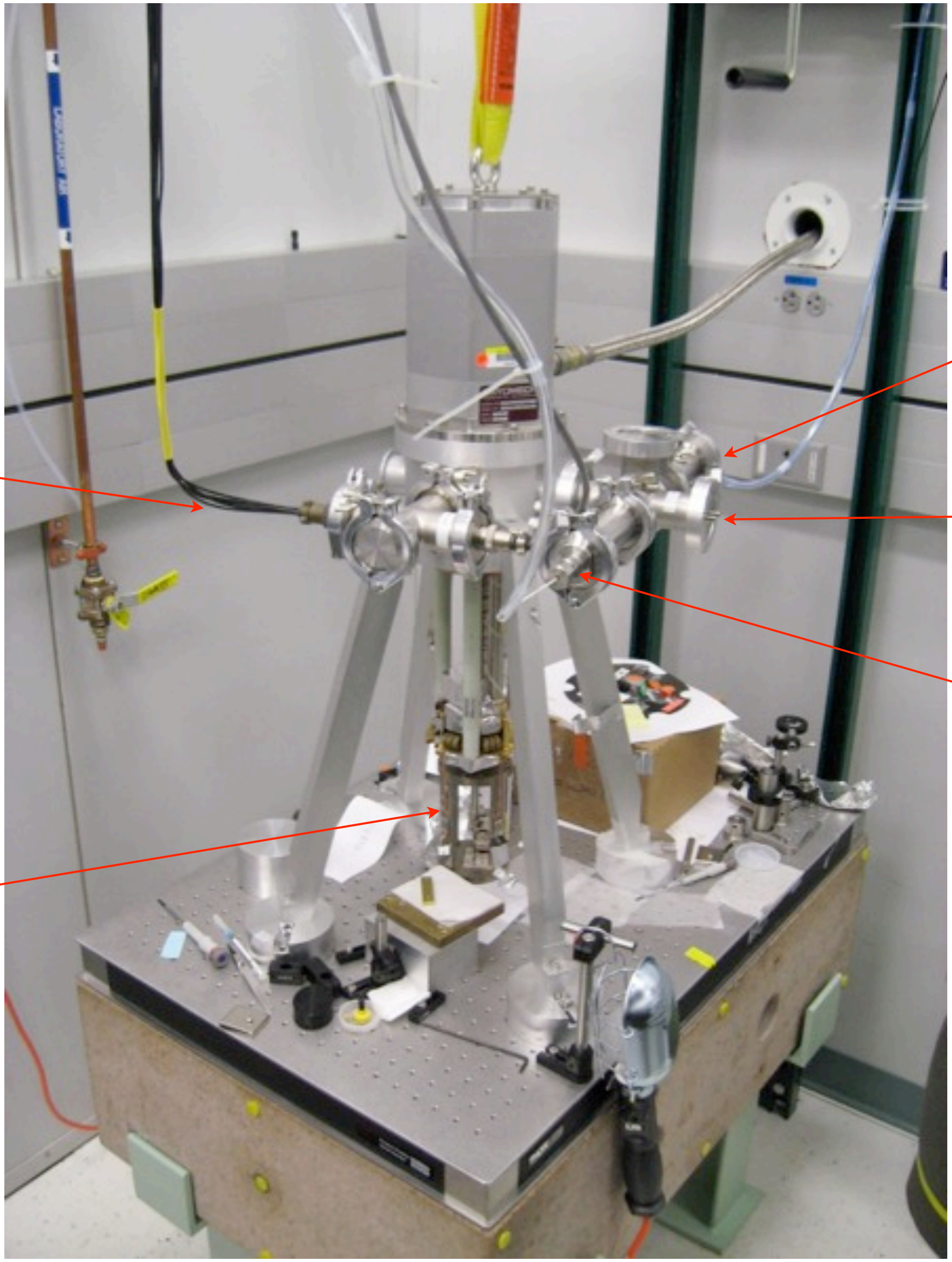
- Frequency and wavelength are simply related through the sound velocity of GaAs: $\nu_s\lambda_s = \omega_s / k_s = \mathrm{V}_{\mathrm{GaAs}}$
- For the device used in this work $\lambda_s = 2.9 \; (\mu\mathrm{m})$ corresponding to $\nu_s = 1.05 \; (\mathrm{GHz})$

Device Equivalent Circuit



- Device is capacitive at low frequencies. At radio frequencies, electromechanical resonance is possible such that electrical energy input can be converted to a propagating acoustic wave in the substrate.

Experimental Apparatus: 4K Pulse-Tube Cryostat



low-freq./DC electrical

heat sinking of RF lines
(two stages)

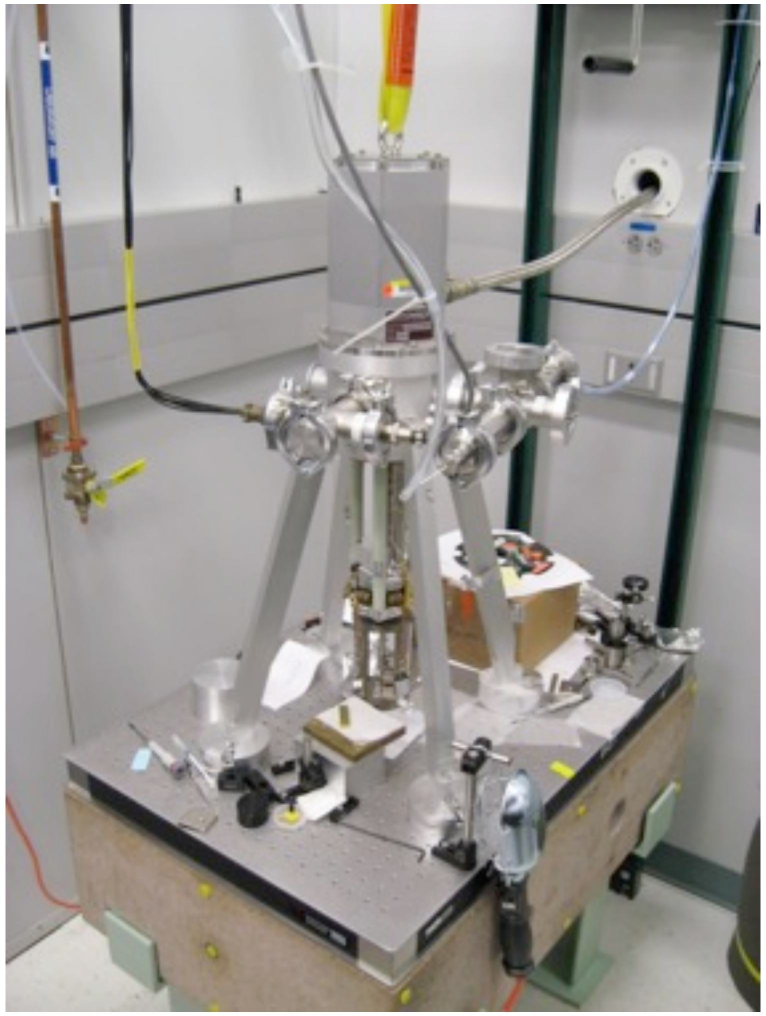
microscope housing and
sample space

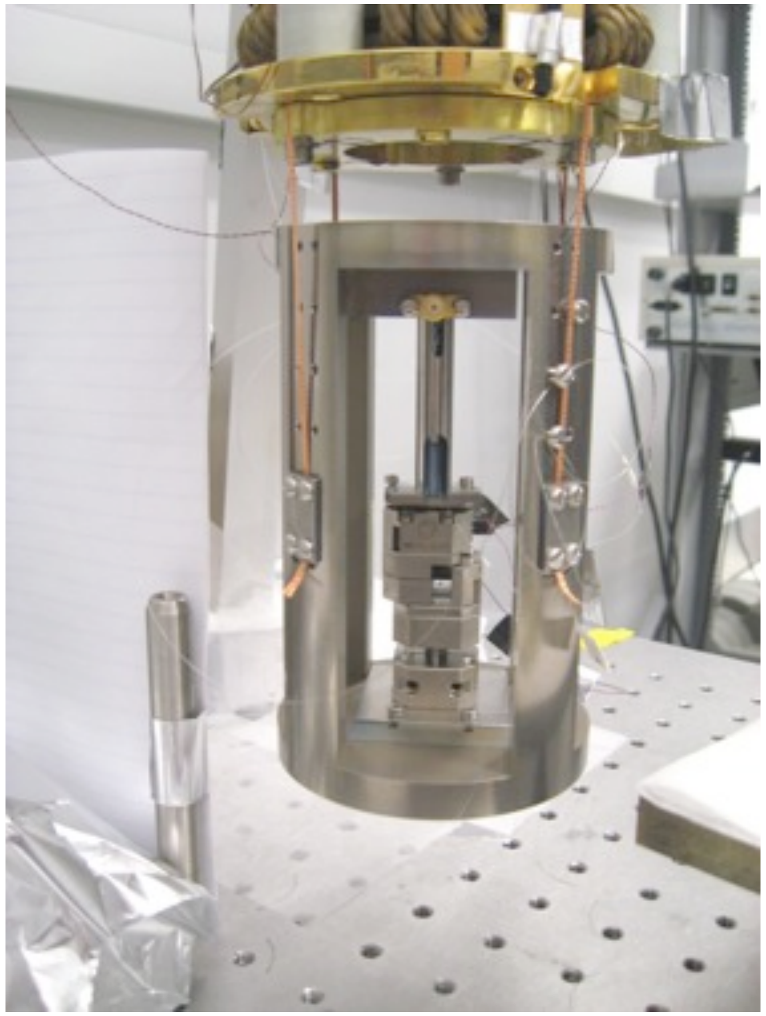
fiber optics
(experimental signals)

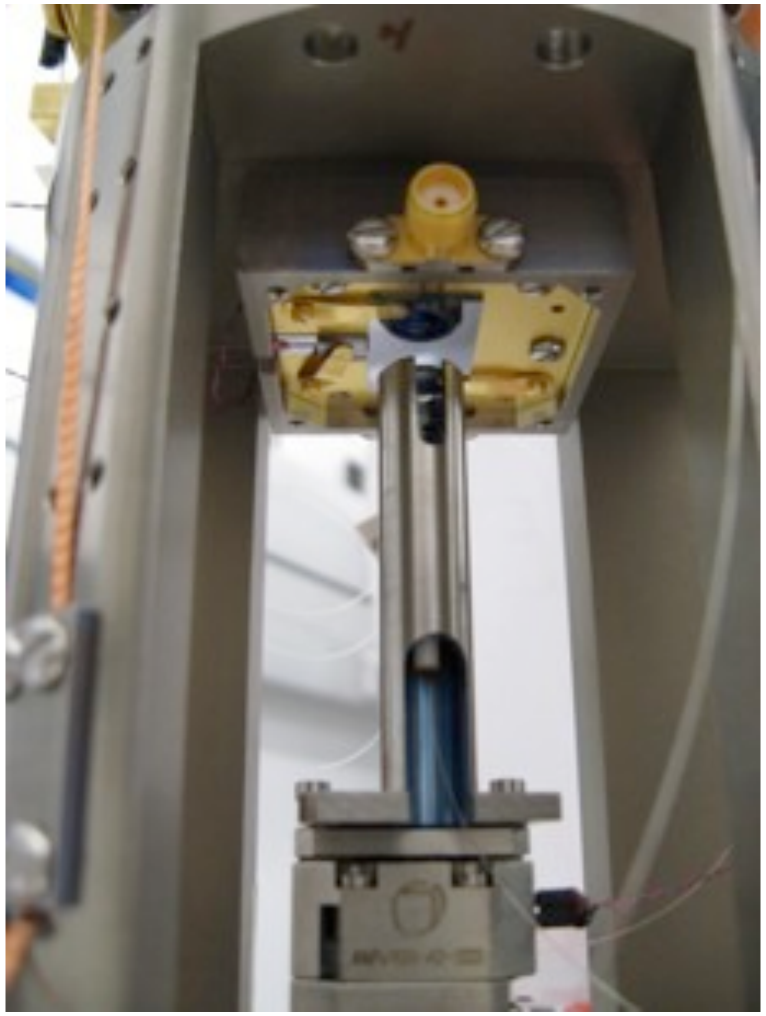
RF electrical

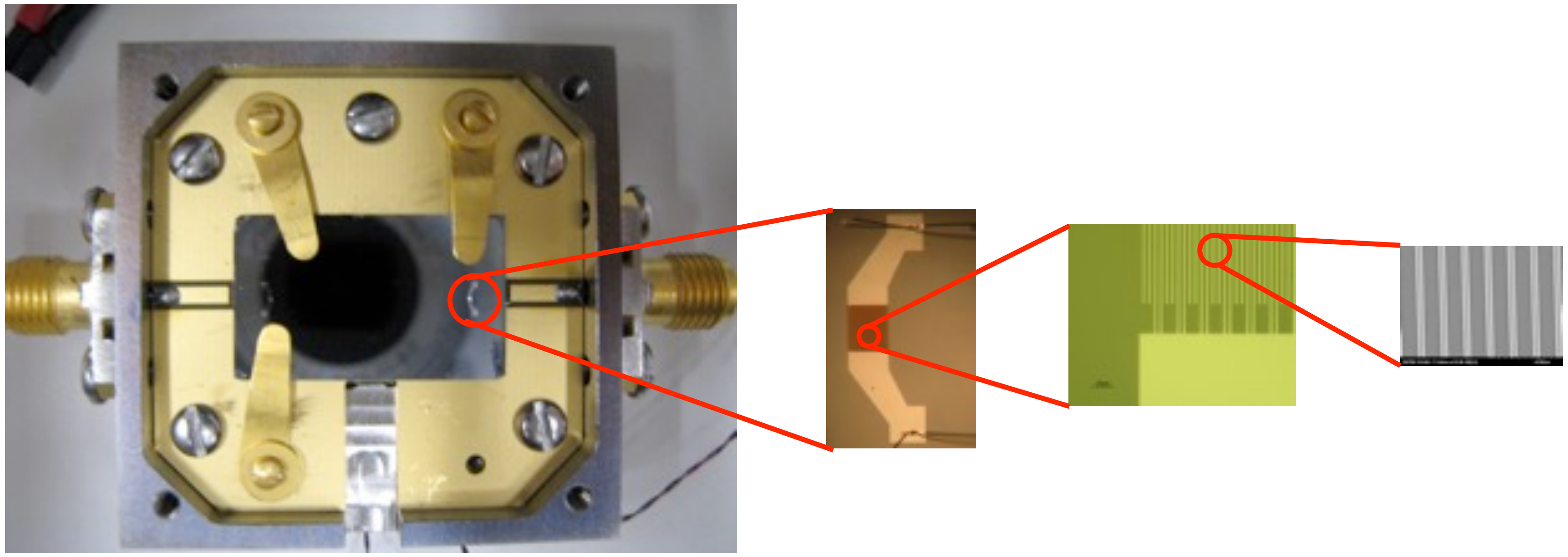
fiber optics
(nanopositioning feedback)

Experimental Apparatus: Views of Cryostat, Microscope Housing, and Sample Package

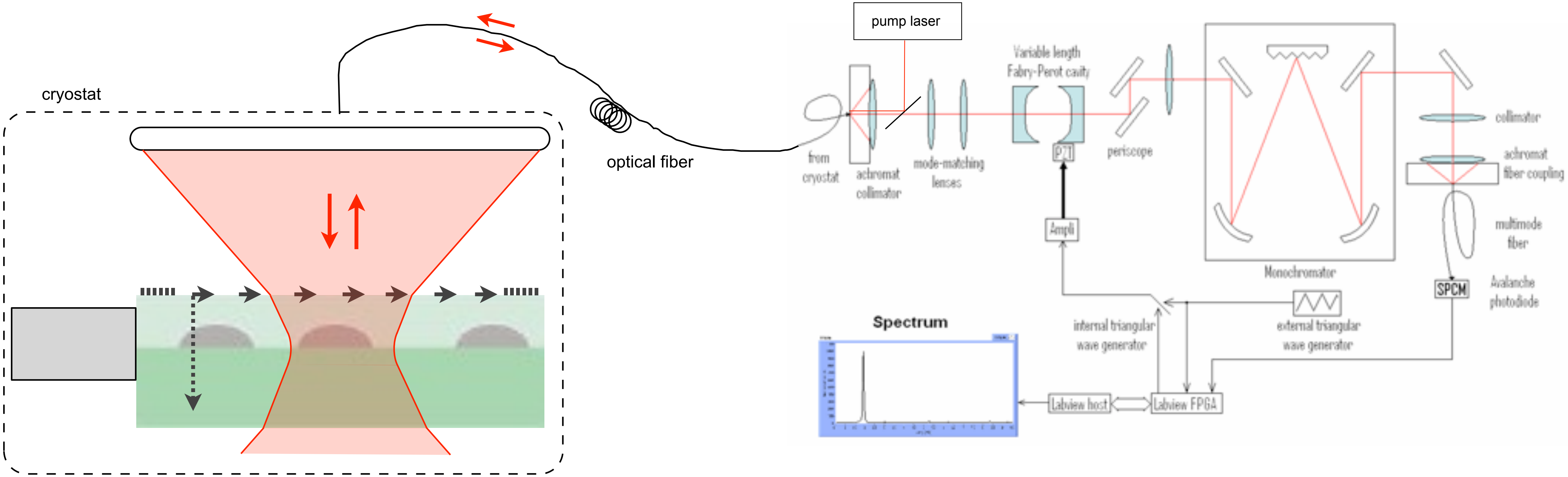








Non-Resonant Photoluminescence Spectroscopy



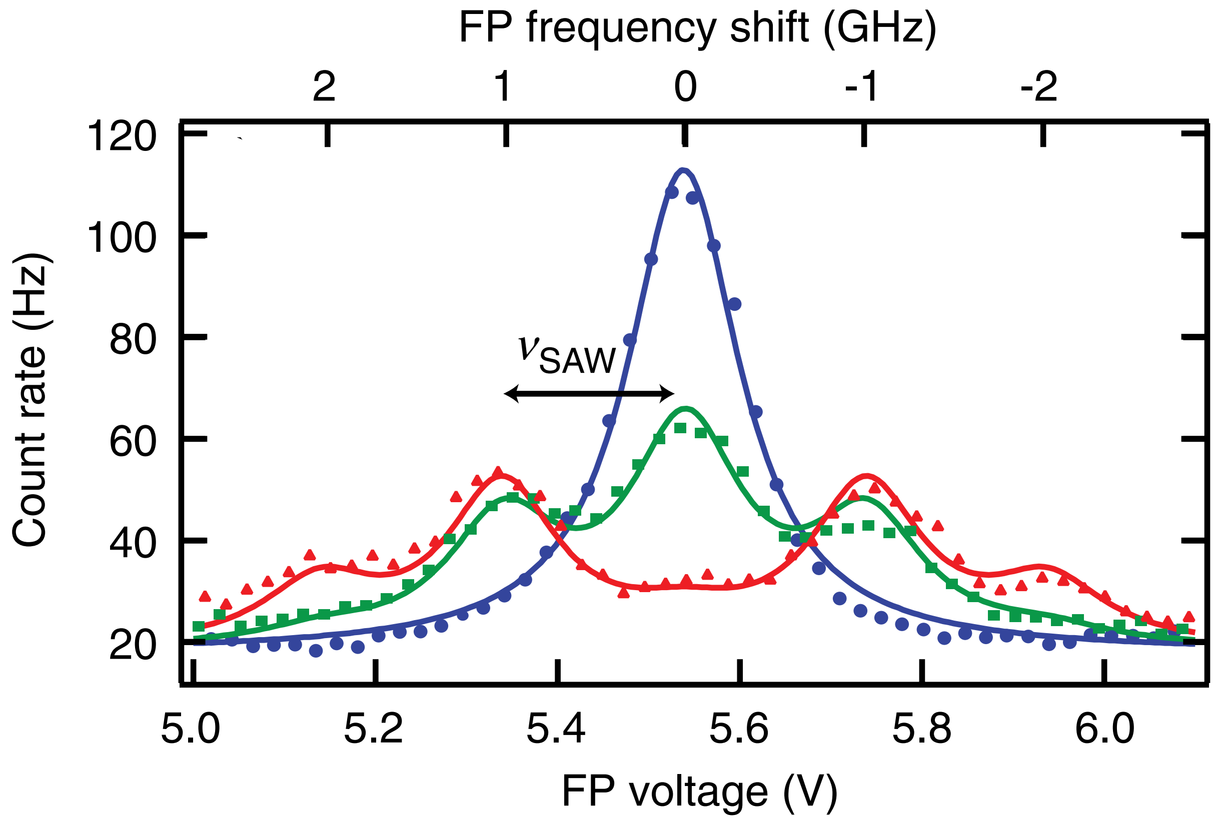
$$\lambda_0 = 921.5 \text{ (nm)}$$

$$\omega_0/2\pi = \nu_0 = 325.6 \text{ (THz)}$$

$$\text{FWHM} = 1 \text{ (GHz)}$$

$$\omega_s/2\pi = \nu_s = 1.05 \text{ (GHz)}$$

$$(\nu_s \equiv \nu_{\text{saw}})$$



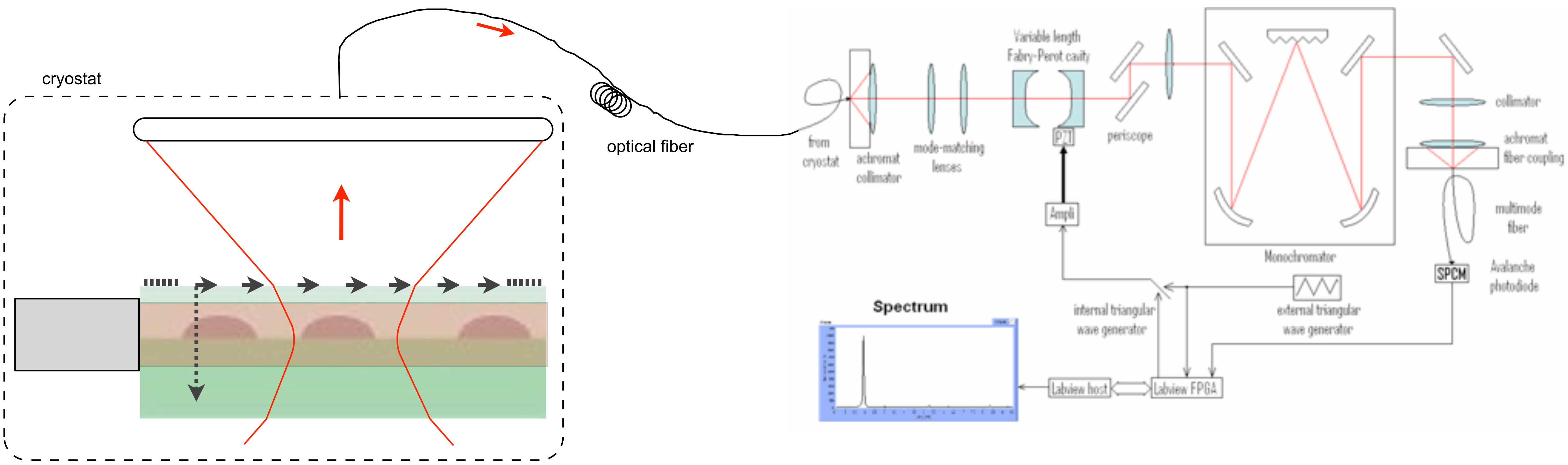
QD as Two-Level System with Hamiltonian

$$H = \frac{\hbar}{2}\left(\omega_0 + \chi\omega_s \sin(\omega_s t)\right)\sigma_z$$

leading to fluorescence Power Spectrum

$$P[\omega] = \sum_{n=-\infty}^{\infty} \frac{J_n^2(\chi)}{\gamma^2 + (\omega - (\omega_0 - n\,\omega_s))^2}$$

Resonant Spectroscopy: Resolved Sideband Regime



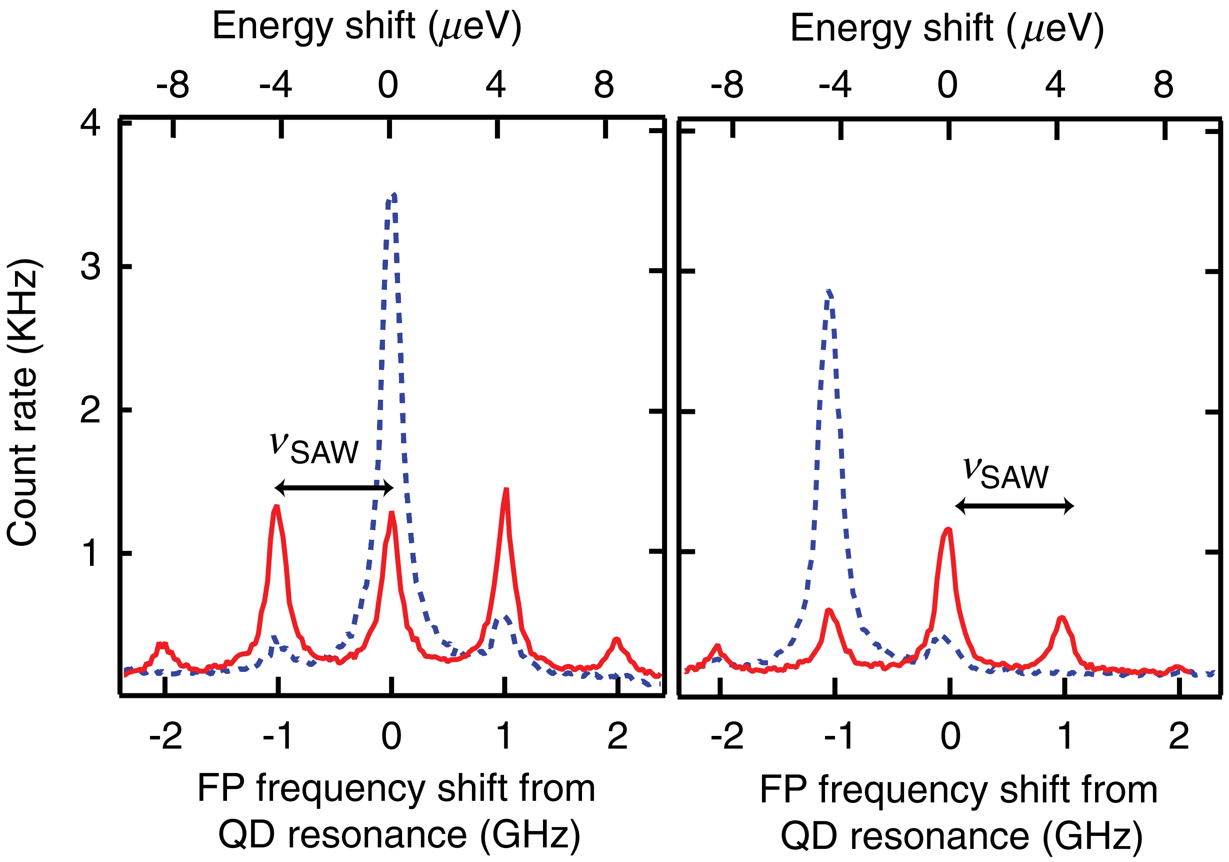
$$\lambda_0 = 924.0 \text{ (nm)}$$

$$\omega_0/2\pi = \nu_0 = 324.7 \text{ (THz)}$$

$$\text{FWHM} = 350 \text{ (MHz)}$$

$$\omega_s/2\pi = \nu_s = 1.05 \text{ (GHz)}$$

$$(\nu_s \equiv \nu_{\text{saw}})$$



QD as Two-Level System with total Hamiltonian $(H + H_{\text{int}})$ where

$$H = \frac{\hbar}{2}\left(\omega_0 + \chi\omega_s \sin(\omega_s t)\right)\sigma_z$$

$$H_{int} = -dE_0 \cos(\omega_L t)\sigma_x$$

leading to fluorescence power spectrum

$$P[\omega] = \sum_{n=-\infty}^{\infty} \left| \sum_{k=-\infty}^{\infty} \frac{J_{n+k}(\chi) J_k(\chi)}{\gamma - i(\omega_L - \omega_0 + k\omega_s)} \right|^2$$

$$\times \quad \delta(\omega - \omega_L + n\omega_s)$$

Implications: Quantum-Limited Spectroscopy, Quantum Cooling, Quantum Information, and Potential Device Applications

- *This work*: Experimental realization of the resolved-sideband regime for a single solid-state self-assembled quantum dot (published 2010):

PRL **105,** 037401 (2010) PHYSICAL REVIEW LETTERS week ending 16 JULY 2010

Resolved Sideband Emission of InAs/GaAs Quantum Dots Strained by Surface Acoustic Waves

- Approaching the regime of Quantum-Limited Spectroscopy of Solid-State Self-Assembled Quantum Dots:

Measured Linewidth from Resonant Spectroscopy: $\mathrm{FWHM} \equiv \Delta\nu = 350\ (\mathrm{MHz}).$ Planck's Constant: $h = 6.63 \text{ x } 10^{-34}\ (\mathrm{J\,s}) \equiv 4.14\ (\mu\mathrm{eV/GHz}).$

Thus the Measured Linewidth corresponds to an Energy Resolution of: $\boxed{\Delta\mathrm{E} = h\ \Delta\nu = 1.4\ (\mu\mathrm{eV}).}$

Alternatively, this can be expressed in terms of lifetime or quality factor: $\Delta\mathrm{E}\cdot\Delta t = \hbar \ \Rightarrow\ \Delta t \approx 0.5\ (\mathrm{ns}).$ Quality Factor $\equiv \mathrm{Q} = (\nu_0/\Delta\nu) = \left(\frac{325\ (\mathrm{THz})}{350\ (\mathrm{MHz})}\right) \approx 10^6.$

- Experimental realization of the resolved-sideband regime for a single trapped ion (published 1995):

VOLUME 75, NUMBER 22 PHYSICAL REVIEW LETTERS 27 NOVEMBER 1995

Resolved-Sideband Raman Cooling of a Bound Atom to the 3D Zero-Point Energy

- Optoelectronic / Photonic Device Application (I): Quantum-Dot Acoust-Optical Modulator (QDAOM) or Quantum-Dot Mixer

- Optoelectronic / Photonic Device Application (II): Experimentally Tune the Exciton-Biexciton Degeneracy-Splitting to realize source of Entangled Photons:

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **80**, 241303(R) (2009)

Entangled photons on demand: Erasing which-path information with sidebands

Summary

- A cryogenic experimental apparatus was designed, constructed, and implemented to enable precision optical measurements and radio-frequency electrical transmission to a semiconductor chip at low temperature.

- Self-assembled InAs/GaAs single quantum dots were investigated optically in the near-infrared. Surface acoustic waves in the GHz range were generated on-chip using microfabricated interdigital transducers. These transducers controllably induced phonon sidebands in the quantum dot emission spectrum, which were observed using non-resonant photoluminescence spectroscopy and resonant spectroscopy.

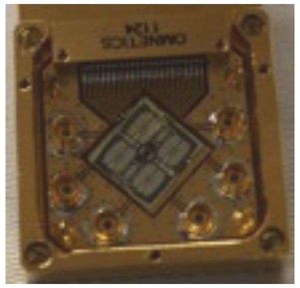
- The resolved sideband regime and optical frequency conversion were demonstrated using resonant spectroscopy. With the laser red-detuned by one phonon sideband, the observed asymmetric fluorescence spectrum reflects the fact that on average the energy of a scattered photon was greater than that of an incident photon, corresponding to the transfer of strain energy to the optical field. This is the physical basis of red-sideband cooling.

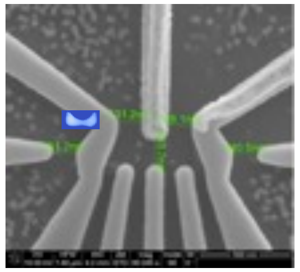
- Experimentally demonstrated spectral resolution is nearly quantum-limited: $\Delta \mathrm{E} = h\ \Delta\nu = 1.4\ (\mu \mathrm{eV})$.

- Potential optoelectronic / photonic device applications:
  (I) Quantum-Dot Acousto-Optical Modulator (QDAOM) or Quantum-Dot Acousto-Optical Mixer;
  (II) Dynamical tuning of the Biexciton-Exciton Degeneracy-Splitting to generate Entangled Photon Pairs.

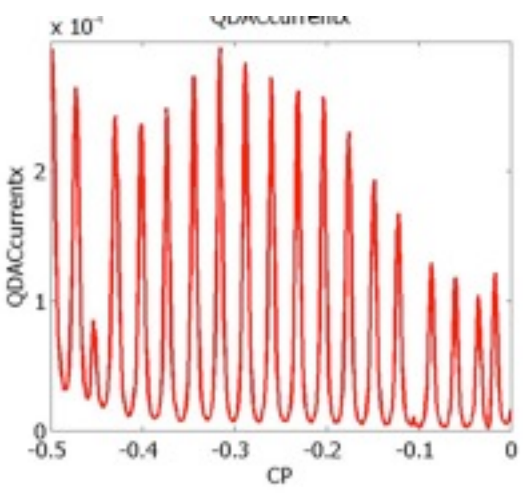
- The results of the work presented here are published as: *Physical Review Letters* 105, 037401 (2010).

A brief description of my current work at Sandia



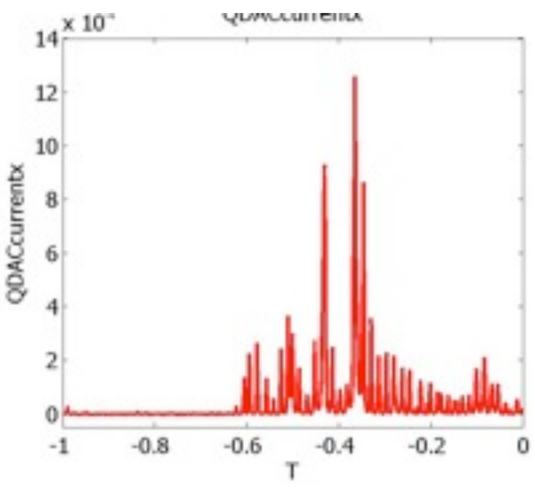


- The experimental apparatus for the cryogenic measurement of the devices is a dilution refrigerator (a) with a base temperature of 15 (mK). This ensures, for example, that the charging energy is significantly greater than the thermal energy.
- The cryogenic sample package (b) consisting of the device die [10 (mm) x 11 (mm)] on a printed-circuit-board housed in a metal enclosure. Low-frequency and RF/microwave electrical signals are integrated into the sample package.
- A Scanning Electron Micrograph of the MOS Device 485 investigated in this work, where the grey microfabricated structures are depletion gates used to electrostatically define a quantum dot. A simulated iQCAD image of a DQD is superimposed on the micrograph.

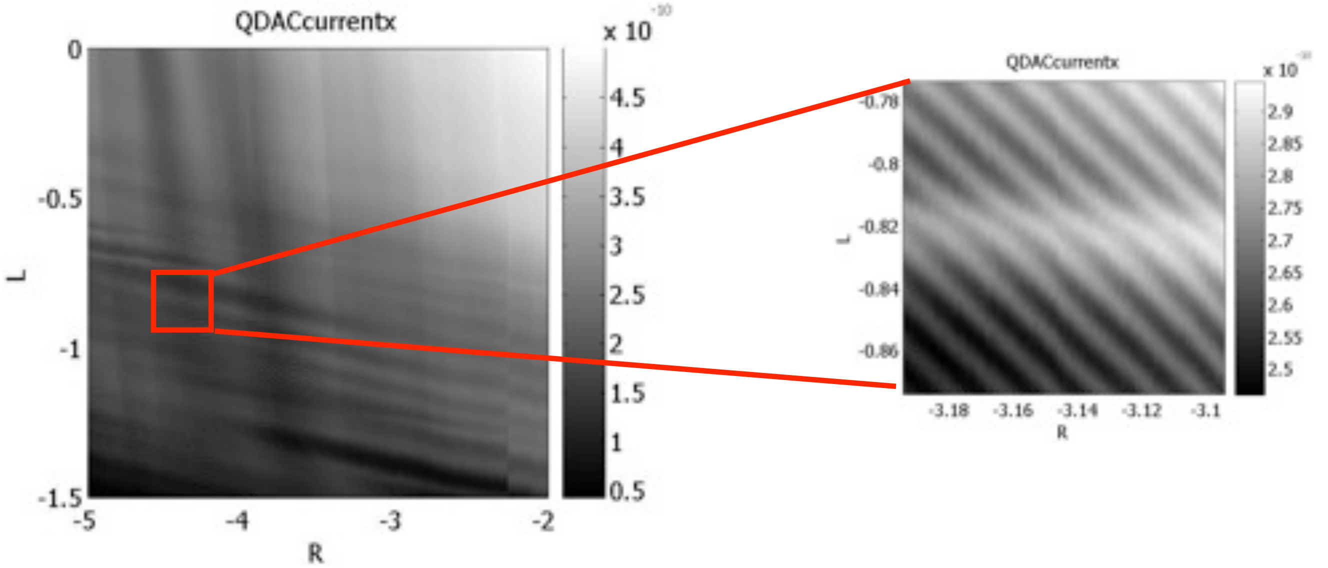


20 CB Oscillations

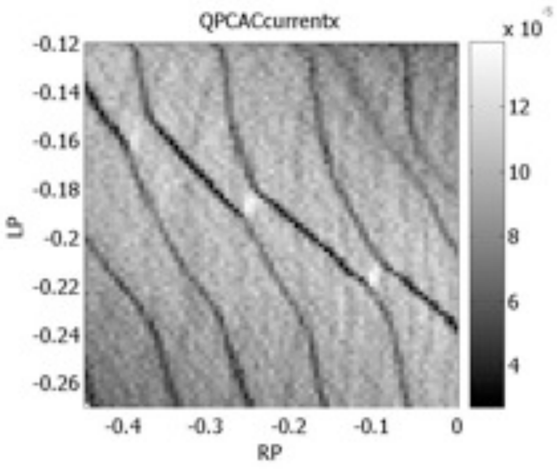
Average Spacing = 25 (mV)

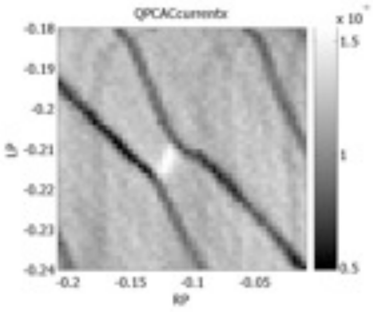


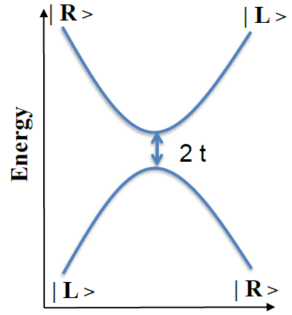
41 CB Oscillations
Average Spacing = 15 (mV)



The 2D background conductance oscillations are attributed to disorder resonances. The closer inspection above reveals the desired CB.







We are investigating the adiabatic evolution of a charge-qubit formed from the L and R states of a MOS DQD.

- AQUARIUS is a Grand Challenge Laboratory Directed Research and Development project at Sandia National Laboratory. The objectives of AQUARIUS include the demonstration of special-purpose two-qubit AQC optimization algorithms in:
  - Neutral atoms trapped by a nanofabricated optical array
  - Semiconductor electrons trapped by nanofabricated structures

- For a quantum mechanical evolution, the separation of external and internal time scales characterizing an adiabatic process may be written as:

$$T_e \gg T_i \quad \text{where} \quad T_i \equiv \frac{\hbar}{\Delta E} \quad \text{and} \quad \Delta E \equiv |E_m - E_n| \equiv \text{energy gap.}$$

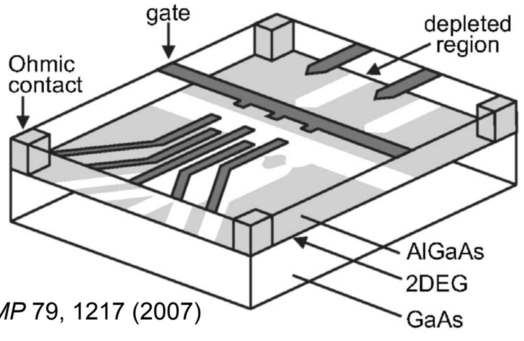
Introduction to Semiconductor Quantum Dots

- **Definition**: Quantum dots are systems where the elementary excitations are confined in three dimensions (3D) within a region comparable to or smaller than their de Broglie wavelength.

- **Artificial Atoms**: 3D confinement produces discrete energy spectrum, thus quantum dots are often described as solid-state "artificial atoms". However, unlike electrons in an isolated atom, carriers in semiconductor quantum dots can interact strongly with the solid-state environment, including phonons. Practical realization of the "particle(s)-in-a-box" problem from elementary quantum mechanics.

- **Experimental Realization**: Two distinct types of semiconductor quantum dots are *electrostatically-defined* and *self-assembled*:

Electrostatically-Defined Quantum Dots



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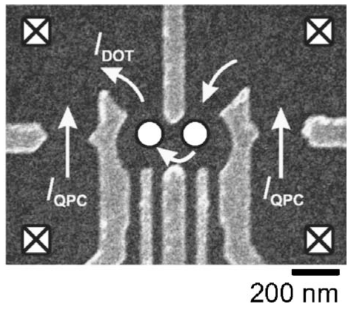
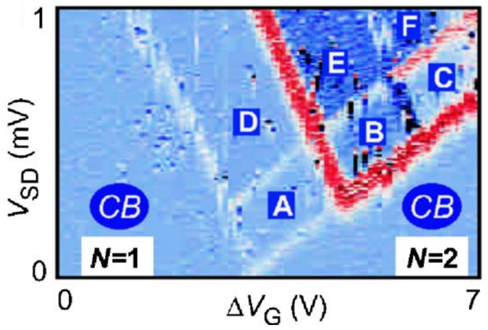
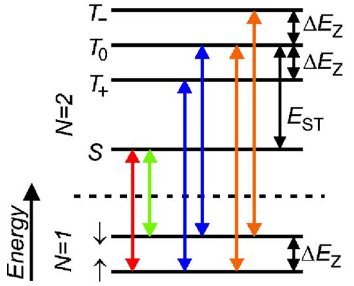
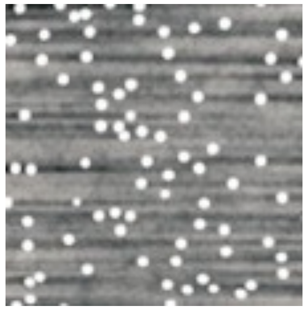




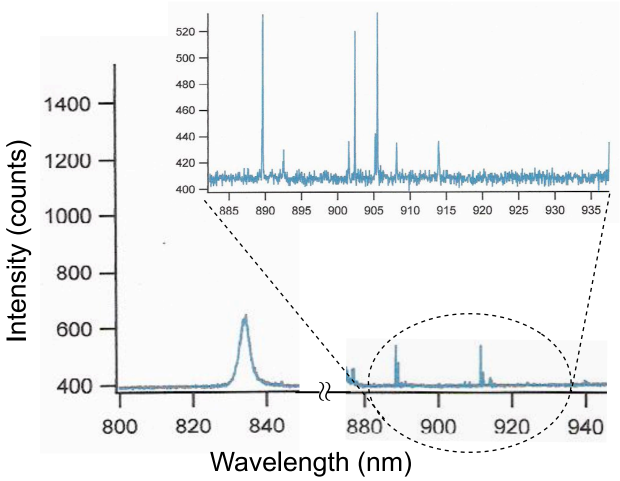


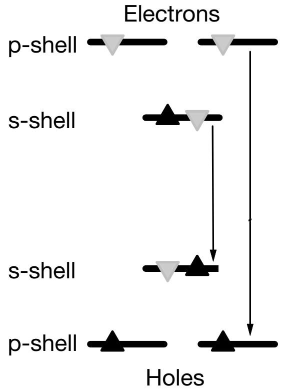
Self-Assembled Quantum Dots (SAQDs)

Cross-Sectional Schematic of SAQDs using GaAs substrate



AFM Image (1 μm x 1 μm) of Uncapped InAs QDs on GaAs





- **SAQD Growth**: SAQDs are 3D islands that form spontaneously during growth of lattice-mismatched materials using Molecular Beam Epitaxy (MBE). Typically 10-100 nm diameter, 2-5 nm epilayer thickness, containing a few thousand to tens of thousands of atoms arranged in a 3D crystal lattice.

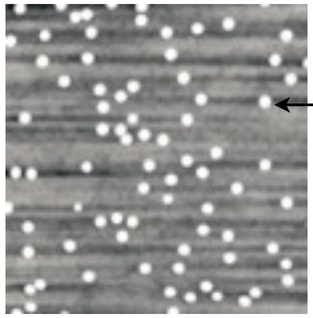
- **Motivations:** SAQDs can probed by optical spectroscopy (wavelength range: 850-1000 nm). For emerging applications in areas such as quantum information and potential integration of QDs with other mesoscopic systems, new experimental controls and techniques are desired.

- **Goal**: Experimental investigation of the previously unexplored resolved sideband regime for self-assembled quantum dots using controllable modulation.

Introduction to Self-Assembled Quantum Dots

- **Definition**: Quantum dots are systems where the charge carriers (electrons and holes) are confined in three dimensions (3D) within a region smaller than their de Broglie wavelength.

- **Growth**: Distinguish self-assembled and electrostatically-defined quantum dots. Self-assembled quantum dots are 3D islands that form spontaneously during growth of lattice-mismatched materials using Molecular Beam Epitaxy (MBE). 3D confinement is enabled through material selection: bandgap of epilayer < bandgap of substrate.



AFM Image (1 μm x 1 μm)
of Uncapped QDs on GaAs

InAs Quantum Dots

Cross-Sectional Schematic of QDs using GaAs substrate

- **Physical Structure of QDs:** typically 10-100 nm diameter, 2-5 nm epilayer thickness, containing typically a few thousand to tens of thousands of atoms arranged in a 3D crystal lattice.

- **Background**: 3D confinement produces discrete energy spectrum, thus quantum dots are often described as solid-state “artificial atoms”. However, unlike electrons in an isolated atom, carriers in semiconductor quantum dots interact strongly with the solid-state environment, including phonons. Practical realization of the “particle(s)-in-a-box” problem from elementary quantum mechanics.

- **Motivations**: The dipole moment of self-assembled QDs allows interaction with light and thus they can probed by optical spectroscopy (wavelength range: 850-1000 nm). QD energy levels have been manipulated by a variety of externally applied perturbations, including mechanical stress, electric fields, magnetic fields, and optical fields. For emerging applications in areas such as quantum information and potential integration of QDs with mesoscopic mechanical/electrical systems, new experimental controls and techniques are desired.

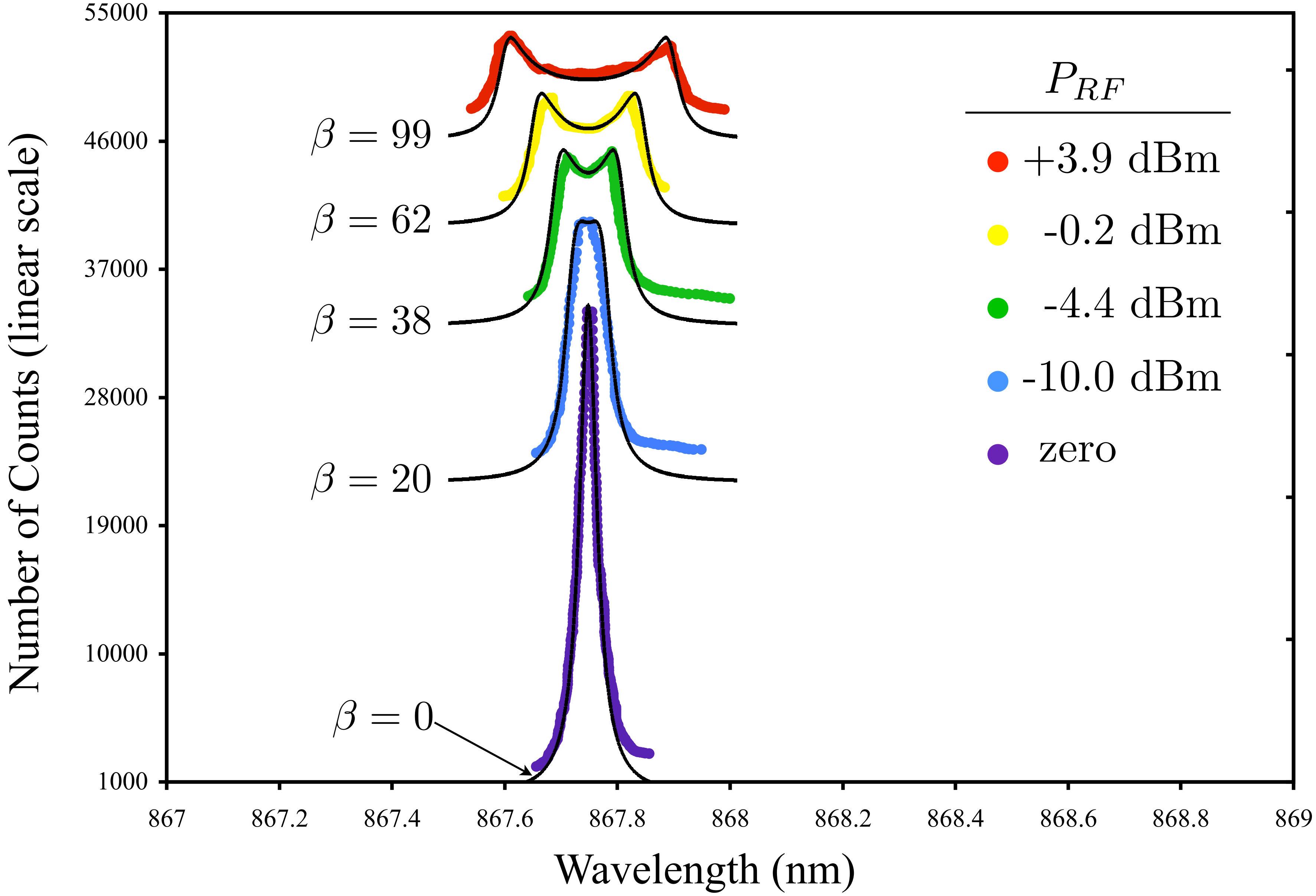
- **Goal**: Experimental realization of the resolved sideband regime for self-assembled quantum dots using surface acoustic wave modulation.

Demonstration of Optomechanical Coupling of a Self-Assembled Quantum Dot and a Radio-Frequency Surface Acoustic Wave

$$I(\omega) = J_0^2(\beta)L(\omega;\omega_0) + \sum_{n=1}^{n=\infty} J_n(\beta)^2\left[L(\omega;\omega_0+n\Omega) + L(\omega;\omega_0-n\Omega)\right]$$

There are two dimensionless parameters that appear in this problem: $\beta = \dfrac{eV_{ac}}{\hbar\Omega} \propto \dfrac{\sqrt{P_{RF}}}{f_{saw}}$ and $\varepsilon = \dfrac{f_{saw}}{\gamma}$

where $\Omega = 2\pi f_{saw}$, P_{RF} = RF source power, γ = FWHM of (unmodulated) Lorentzian QD spectrum

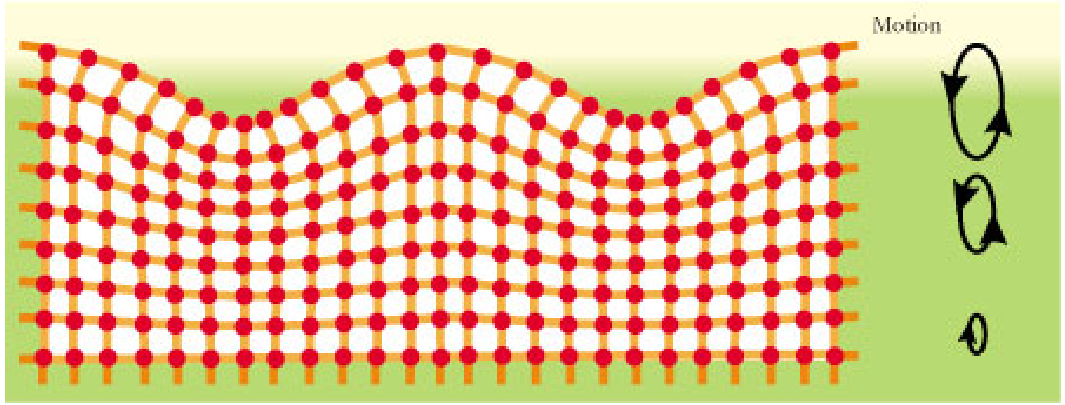


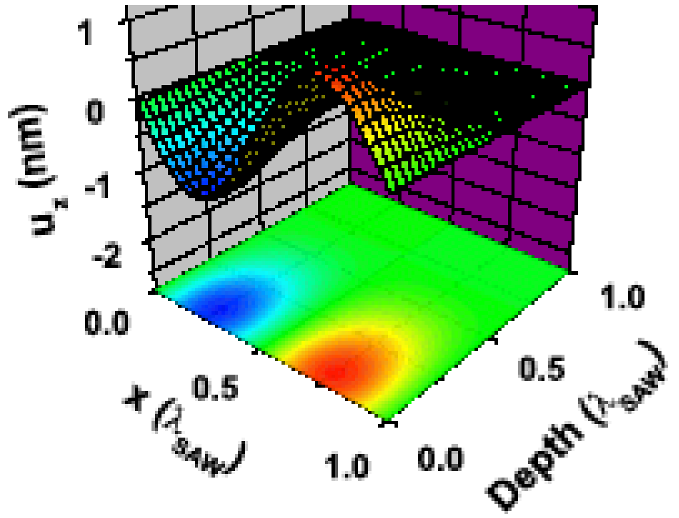
$f_{saw} = 1\,\mathrm{GHz}$

$\gamma = 15\,\mathrm{GHz}$

$\varepsilon = 0.067$

What is a Surface Acoustic Wave (SAW)?



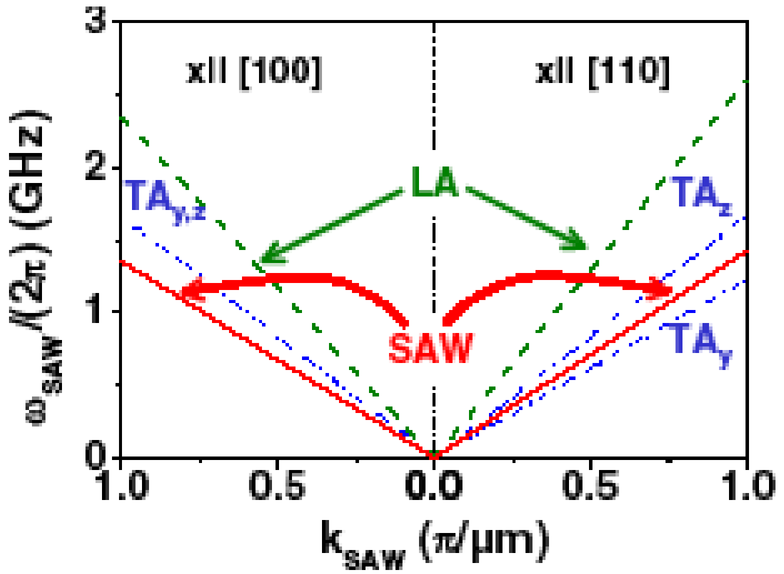


$$\vec{\nabla}\cdot\sigma = \rho\frac{\partial^2\vec{u}}{\partial t^2}.$$

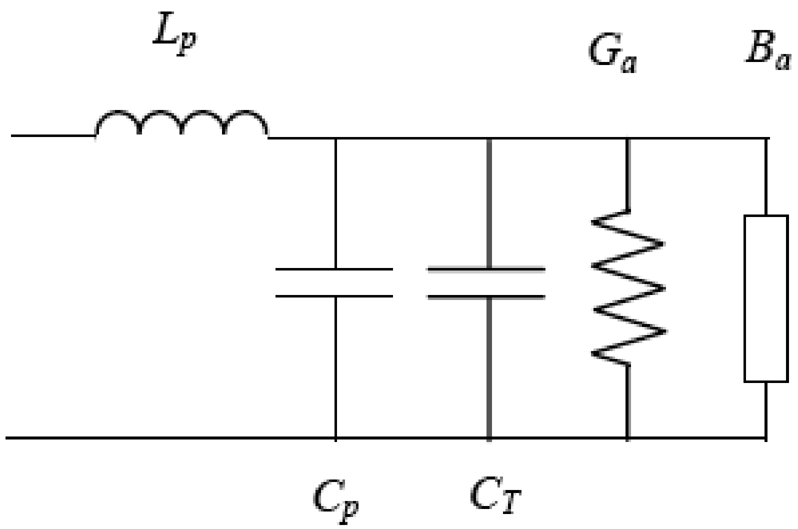
$$\sigma = \mathrm{c}\varepsilon - \mathrm{e}\vec{F}$$
$$\vec{D} = \epsilon\vec{F} + \mathrm{e}\varepsilon.$$

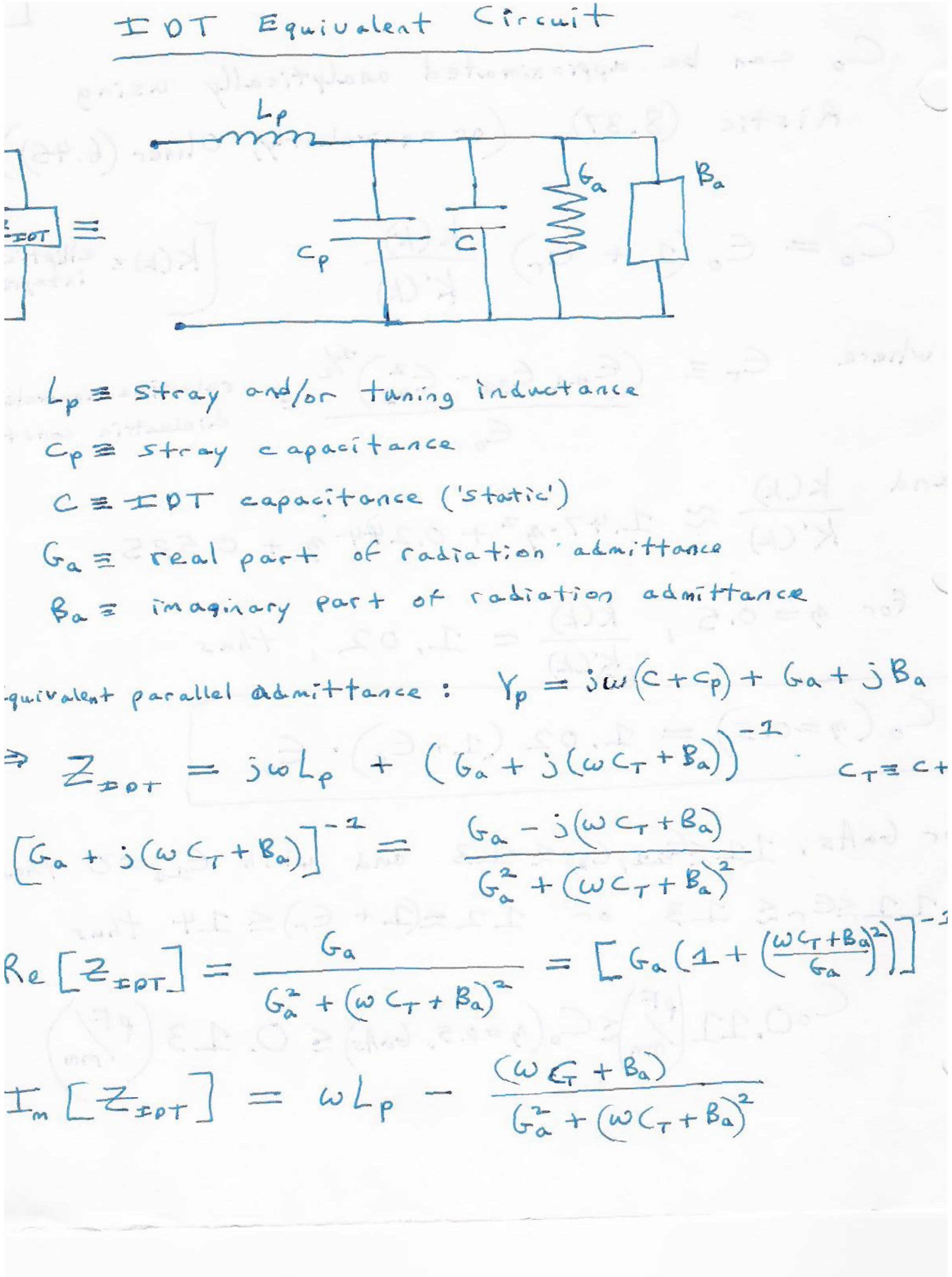
$$\sigma_{iz}(z^{+}_{\mathrm{int}}) = \sigma_{iz}(z^{-}_{\mathrm{int}}), \qquad i = x, y, z,$$
$$D_z(z^{+}_{\mathrm{int}}) = D_z(z^{-}_{\mathrm{int}}).$$

amplitude decaying to zero as $|z| \to \infty$



IDT Equivalent Circuit





IDT Equivalent Circuit

$L_p \equiv$ stray and/or tuning inductance

$C_p \equiv$ stray capacitance

$C \equiv$ IDT capacitance ('static')

$G_a \equiv$ real part of radiation admittance

$B_a \equiv$ imaginary part of radiation admittance

quivalent parallel admittance : $Y_p = j\omega(C + C_p) + G_a + jB_a$

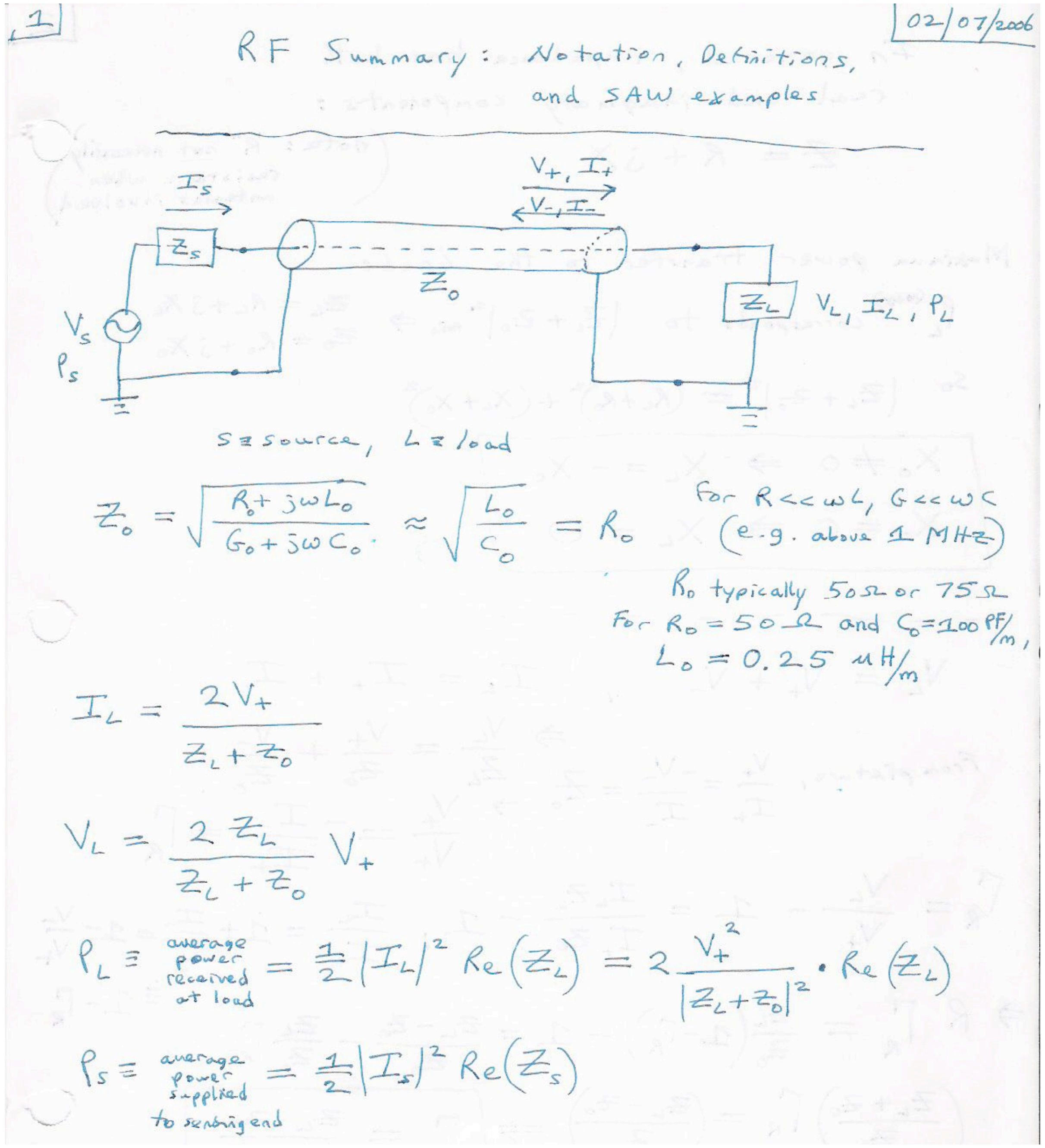
$\Rightarrow Z_{IDT} = j\omega L_p + \left(G_a + j(\omega C_T + B_a)\right)^{-1}$ $C_T \equiv C +$

$$\left[G_a + j(\omega C_T + B_a)\right]^{-1} = \frac{G_a - j(\omega C_T + B_a)}{G_a^2 + (\omega C_T + B_a)^2}$$

$$\mathrm{Re}\left[Z_{IDT}\right] = \frac{G_a}{G_a^2 + (\omega C_T + B_a)^2} = \left[G_a\left(1 + \left(\frac{(\omega C_T + B_a)^2}{G_a}\right)\right)\right]^{-1}$$

$$\mathrm{Im}\left[Z_{IDT}\right] = \omega L_p - \frac{(\omega C_T + B_a)}{G_a^2 + (\omega C_T + B_a)^2}$$

Problem #3: Impedance Matching to an Acoustic Antenna



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RF Summary: Notation, Definitions, and SAW examples

S ≡ source, L ≡ load

$$Z_0 = \sqrt{\frac{R_0 + j\omega L_0}{G_0 + j\omega C_0}} \approx \sqrt{\frac{L_0}{C_0}} = R_0$$

for $R \ll \omega L$, $G \ll \omega C$ (e.g. above 1 MHz)

R_0 typically $50\,\Omega$ or $75\,\Omega$

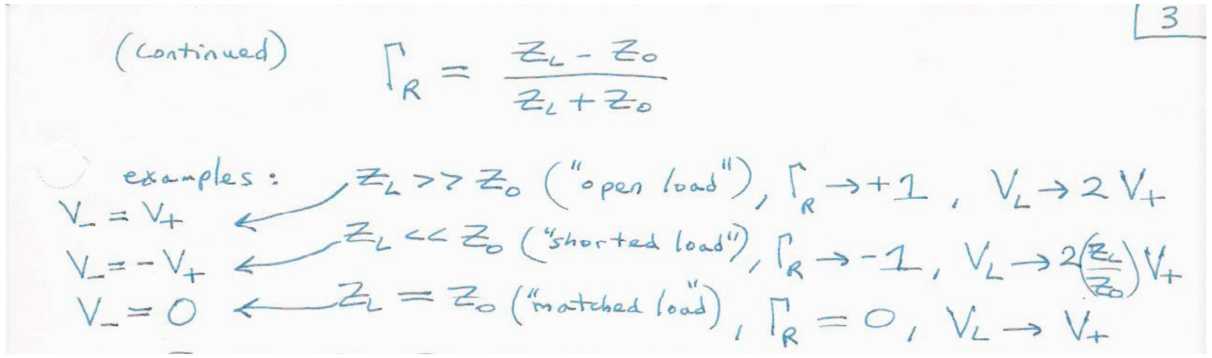
For $R_0 = 50\,\Omega$ and $C_0 = 100$ pF/m, $L_0 = 0.25\ \mu$H/m

$$I_L = \frac{2V_+}{Z_L + Z_0}$$

$$V_L = \frac{2Z_L}{Z_L + Z_0} V_+$$

$$P_L \equiv \text{average power received at load} = \frac{1}{2}|I_L|^2 \,\mathrm{Re}(Z_L) = 2\frac{V_+^2}{|Z_L + Z_0|^2} \cdot \mathrm{Re}(Z_L)$$

$$P_s \equiv \text{average power supplied to sending end} = \frac{1}{2}|I_s|^2 \,\mathrm{Re}(Z_s)$$



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(continued)

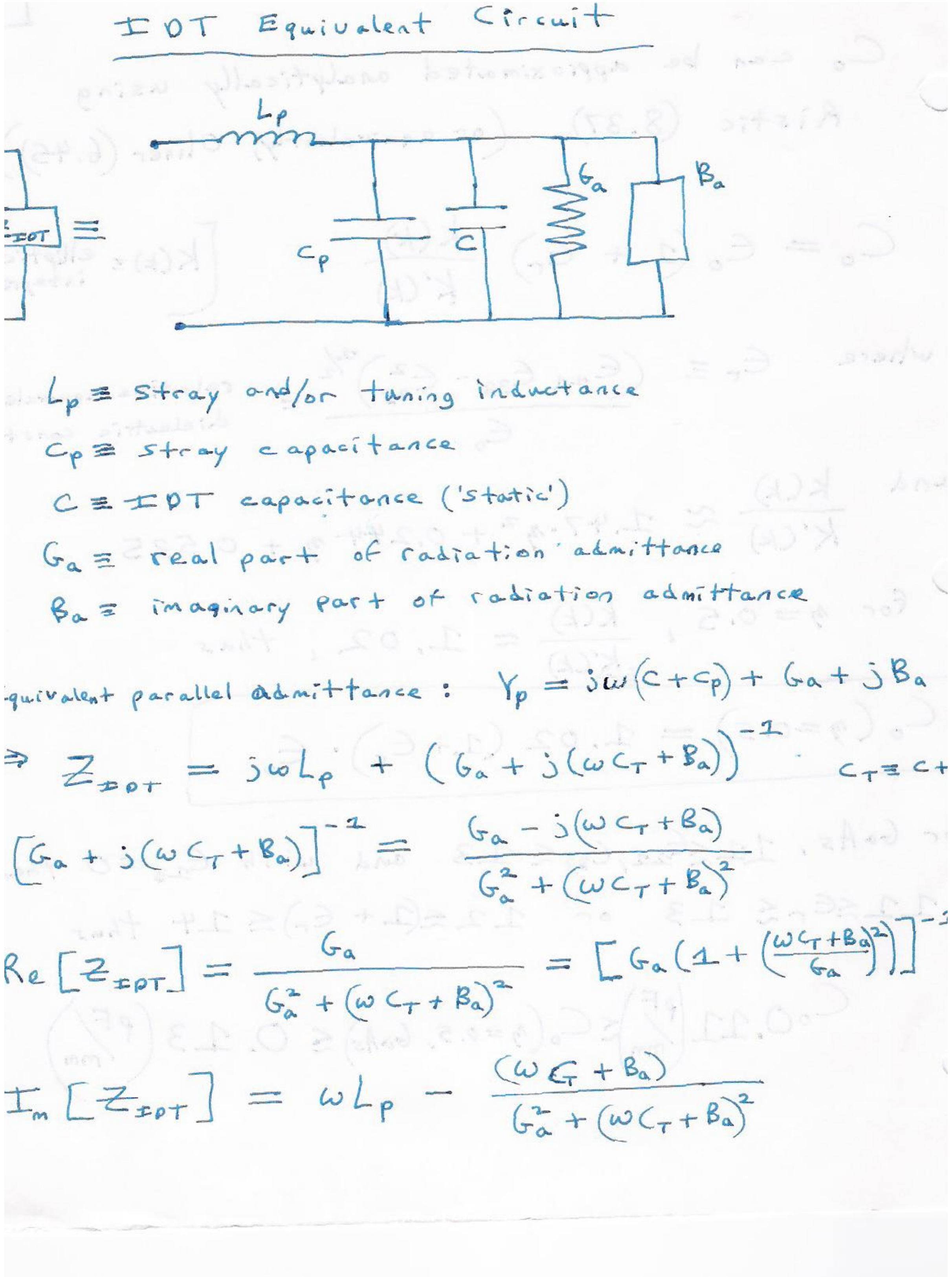
$$\Gamma_R = \frac{Z_L - Z_0}{Z_L + Z_0}$$

examples:

$V_- = V_+ \leftarrow$ $Z_L \gg Z_0$ ("open load"), $\Gamma_R \to +1$, $V_L \to 2V_+$

$V_- = -V_+ \leftarrow$ $Z_L \ll Z_0$ ("shorted load"), $\Gamma_R \to -1$, $V_L \to 2\left(\frac{Z_L}{Z_0}\right)V_+$

$V_- = 0 \leftarrow$ $Z_L = Z_0$ ("matched load"), $\Gamma_R = 0$, $V_L \to V_+$



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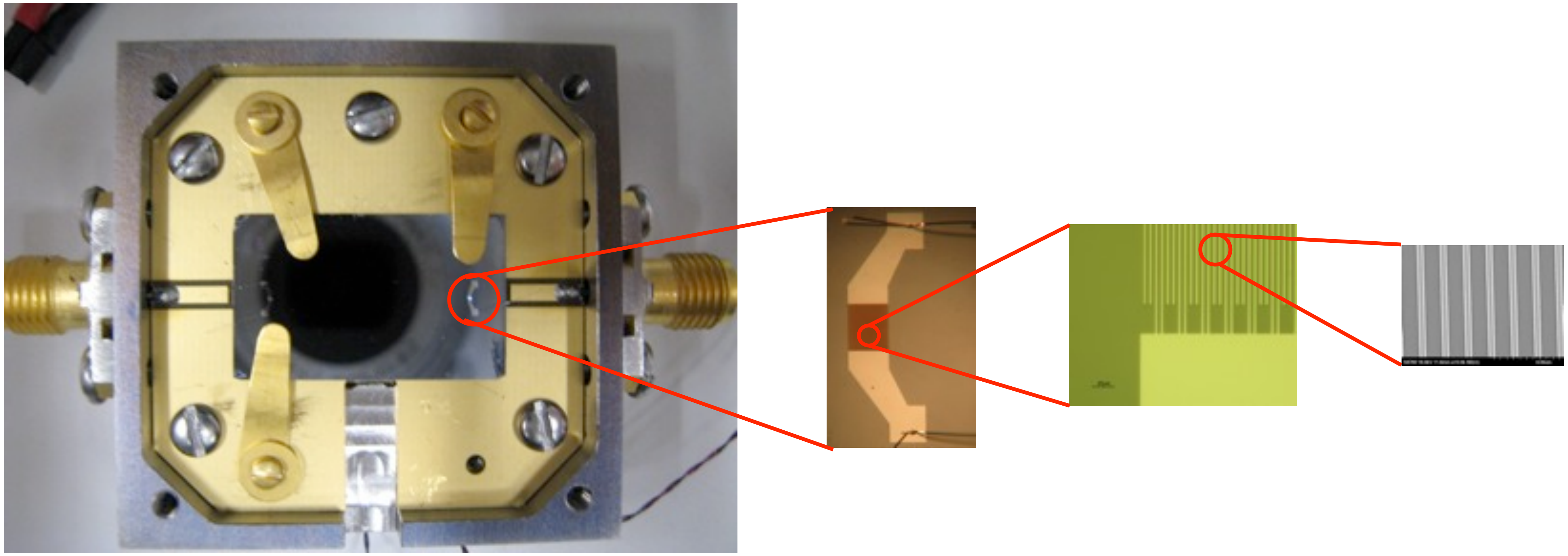
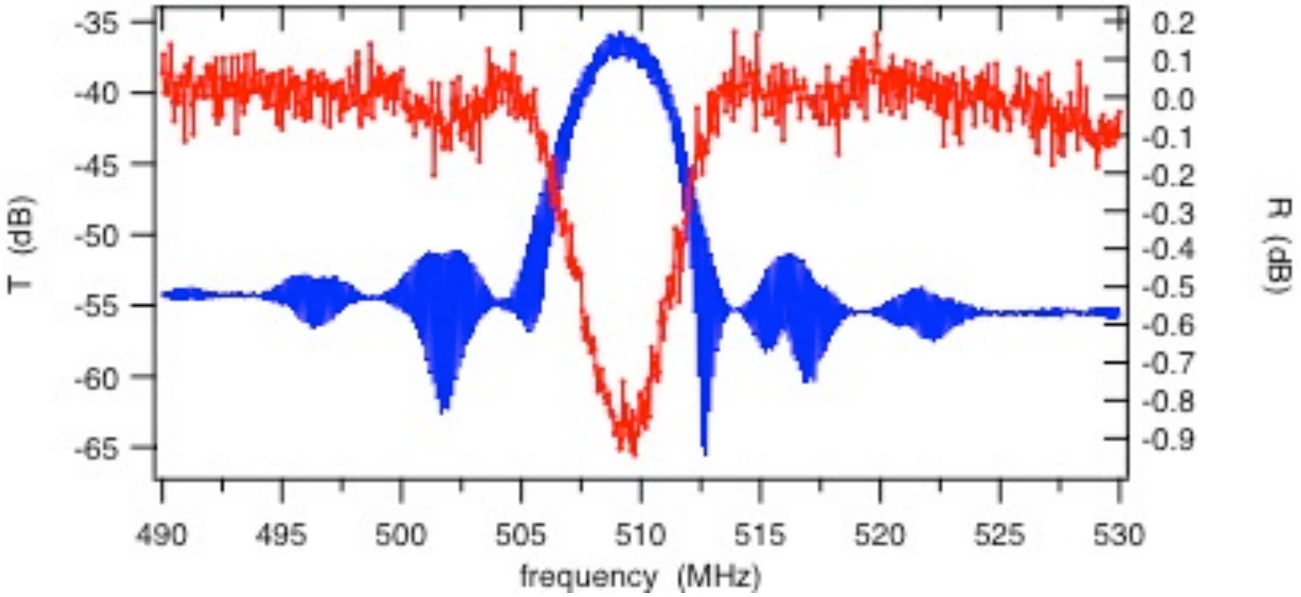
$$\left[G_a + j(\omega C_T + B_a)\right]^{-1} = \frac{G_a - j(\omega C_T + B_a)}{G_a^2 + (\omega C_T + B_a)^2}$$

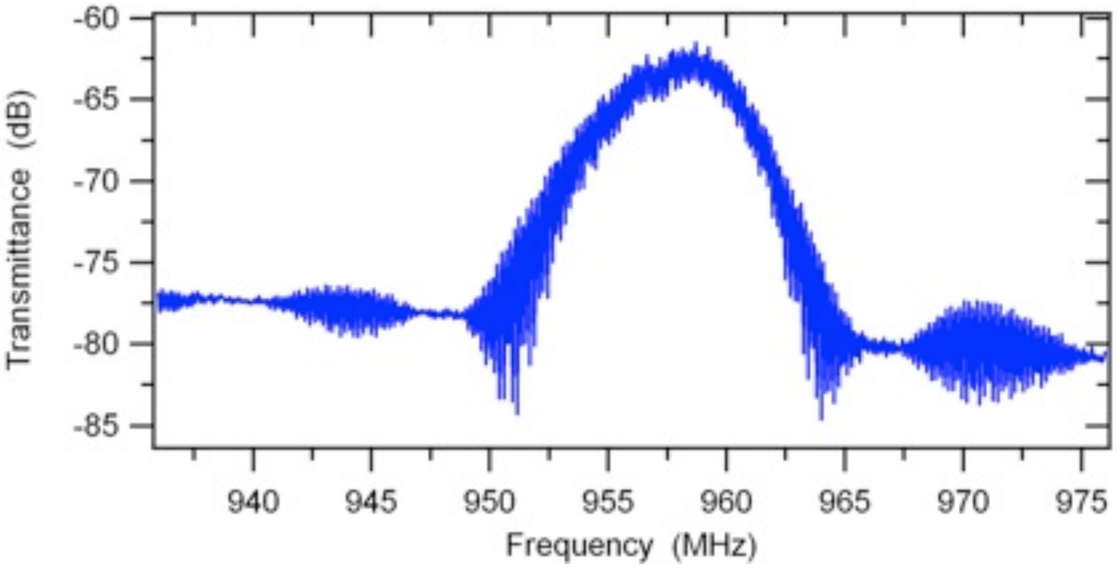
$$\mathrm{Re}\left[Z_{IDT}\right] = \frac{G_a}{G_a^2 + (\omega C_T + B_a)^2} = \left[G_a\left(1 + \left(\frac{\omega C_T + B_a}{G_a}\right)^2\right)\right]^{-1}$$

$$\mathrm{Im}\left[Z_{IDT}\right] = \omega L_p - \frac{(\omega C_T + B_a)}{G_a^2 + (\omega C_T + B_a)^2}$$

Surface Acoustic Wave Data







Theoretical Formulation to Understand Formation and Evolution of Sidebands

$$\vec{E} = \vec{E}_0 \exp(i\omega t + i\phi(t))$$

Traveling electric field accompanies propagating surface acoustic wave.

$$= \vec{E}_0 \exp(i\omega t + i\beta \sin(\Omega t))$$

Inserting sinusoidal phase modulation: $\phi(t) = \beta \sin(\Omega t)$

$\omega_{instant} = \omega + d\phi/dt$

$$= \vec{E}_0 \sum_{n=-\infty}^{n=+\infty} J_n(\beta) \exp[i(\omega + n\Omega)t]$$

Jacobi-Anger identity, where the $J_n(\beta)$ are Bessel functions.

$$= \vec{E}_0 \left\{ J_0(\beta) \exp(i\omega t) + \sum_{n=1}^{n=\infty} J_n(\beta) \left\{ \exp[i(\omega + n\Omega)t] + (-1)^n \exp[i(\omega - n\Omega)t] \right\} \right\}$$

Convolution of the (unmodulated) Lorentzian spectrum of the quantum dot with the above expression for the modulated excitation field gives for the normalized output power:

$$I(\omega) = J_0^2(\beta) L(\omega;\omega_0) + \sum_{n=1}^{n=\infty} J_n(\beta)^2 [L(\omega;\omega_0 + n\Omega) + L(\omega;\omega_0 - n\Omega)]$$