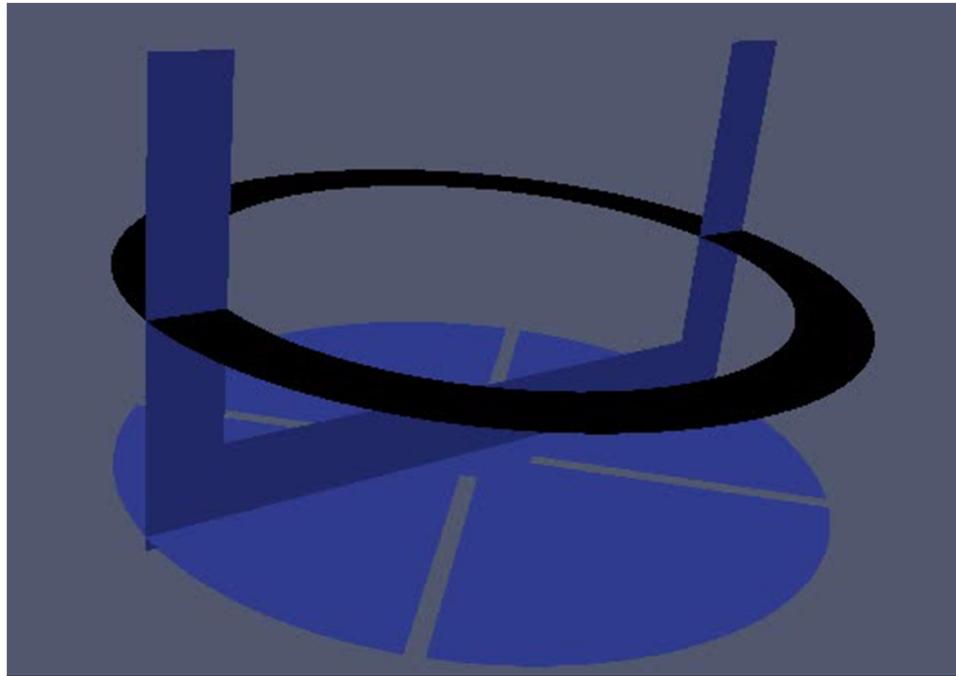


# Multiscale Models of Nuclear Waste Reprocessing: From the Mesoscale to the Plant-Scale

Rekha Rao, Chris Brotherton, Ben Cipiti, Stefan Domino, Lindsay Erickson, Anne Grillet, Paul Galambos, Carlos Jove-Colon, Jeremy Lechman, Harry Moffat, Martin Nemer, David Noble, Tim O'Hern, Roger Pawlowski, Christine Roberts, Scott Roberts, Veena Tikare, Greg Wagner, Nick Wyatt  
Sandia National Laboratories

Michael Loewenberg  
Yale University

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

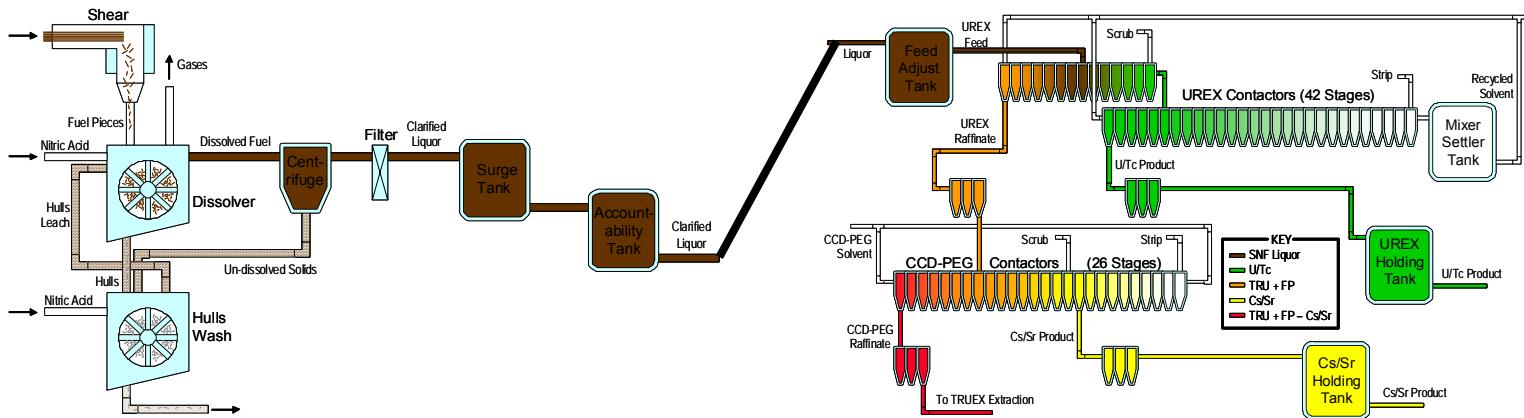


Turbulent (LES) single phase simulations of centrifugal contactor from Sierra Mechanics



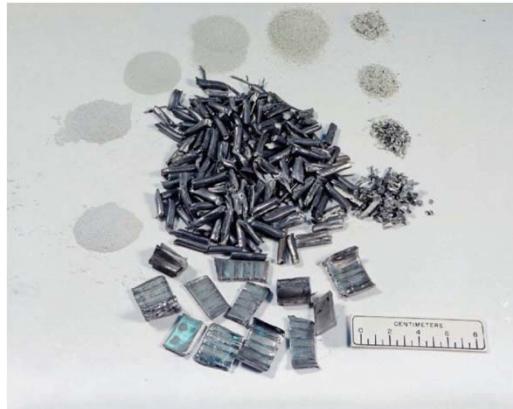
# Project Vision: A Scientific Approach to Nuclear Waste Reprocessing

- The amount of nuclear waste produced and stored can be greatly reduced through reprocessing, where fuel rods are separated into various streams, some of which can be reused in reactors
- Current process developed in the 1950s and is dirty and expensive
  - U/Pu separation is the most critical => PUREX
- Our approach => use science and simulation-based approach to develop a modern reprocessing plant
  - Leverage unique Sandia capability, SIERRA Mechanics, to model coupled physics via high performance computing
  - Models of reprocessing plants are needed to support nuclear materials accountancy, nonproliferation, plant design, and plant scale up

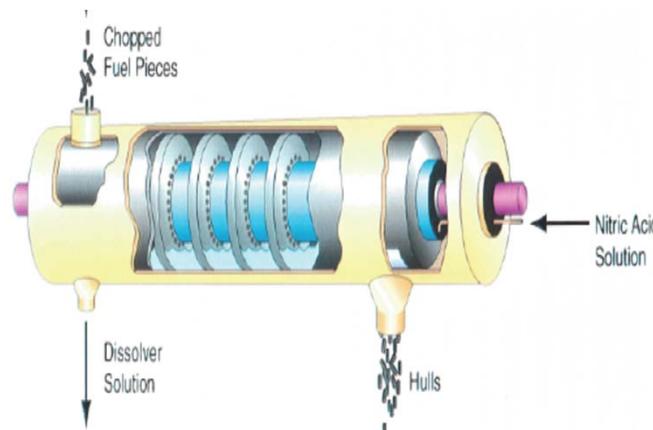


Secretary Energy Chu is very supportive of the “closed fuel cycle”

# Detailed Model of PUREX-type Processes

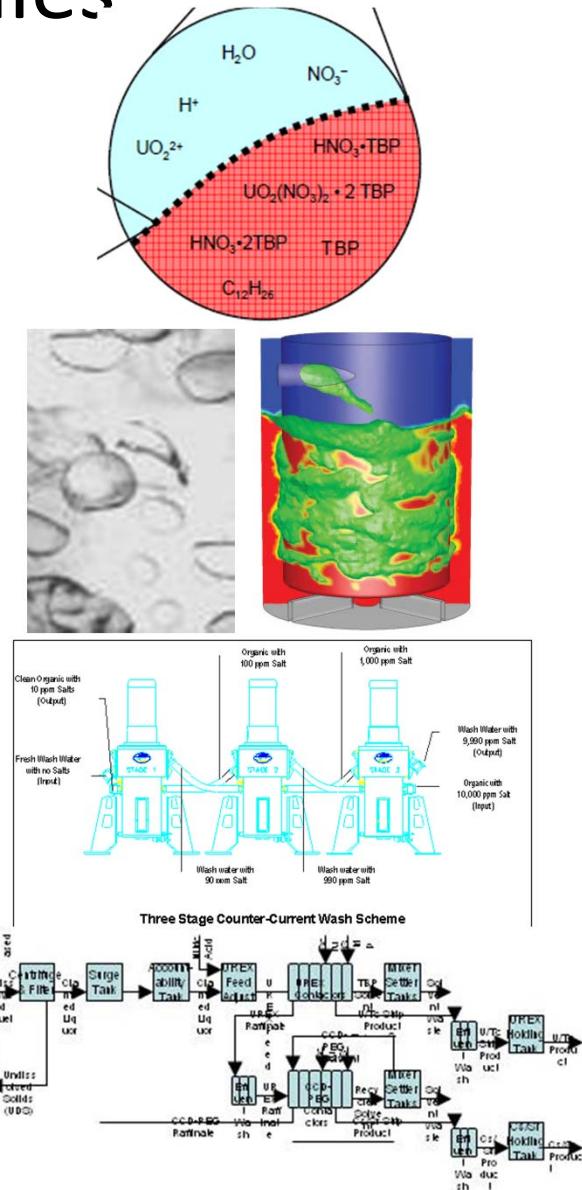


- Spent fuel rods are sheared and dissolved in acid
- Concentrated aqueous solution is then emulsified in a liquid-liquid extraction continuous contactor with a organic solvent continuous phase (TBP/dodecane)
- Uranium and plutonium oxide ions are then preferentially taken up by organic phase
- Centrifugal contactors used to enhance mixing and form aqueous droplets in organic continuous phase
- Maximize surface area for potential mass transport across the interface
- Up to five phases can occur simultaneously
- Criticality issues determine scale-up

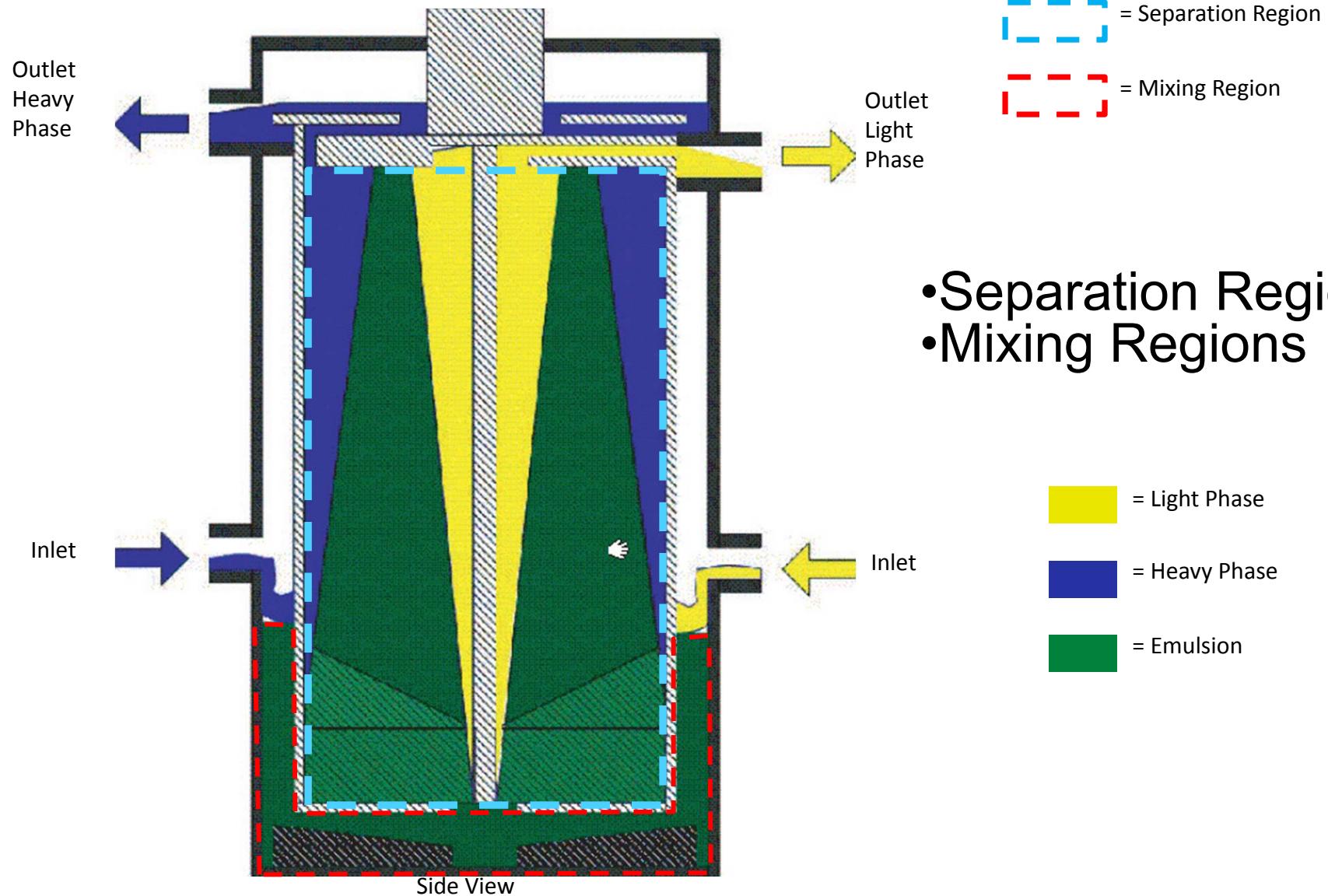


# Missing Physics Models To Be Developed at Multiple Length Scales

- Droplet-scale model
  - **Interfacial mass transfer and fluid mechanics**
  - Surface variables, surfactants and Marangoni effects
  - Advection and diffusion in concentrated ionic, multiphase solutions
    - Stefan-Maxwell equation for mass flux
- Single-stage models
  - **Direct numerical simulation of multiphase, turbulent, immiscible flows with interfacial mass transport**
  - Integration of neutronics for criticality
  - Solid-fluid interactions models for complex rotors
- Unit-operations, column scale simulations
  - Optimization
  - Solvent evaluation, virtual radioactive experiments
  - **Reduced-order column model based on population balances**
- Integration with plant flow sheets models for nonproliferation
  - **High-fidelity plant model** for non-proliferation, process control, and accident mitigation
- Validation at all scales
  - **Droplet-scale experiments**
  - **Contactor-scale experiments**

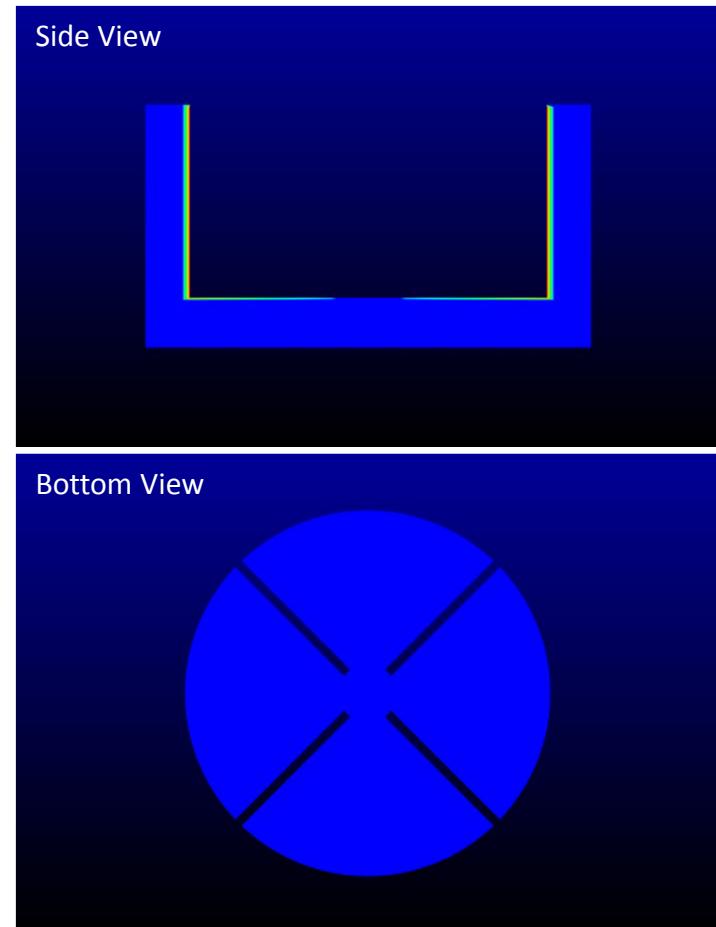
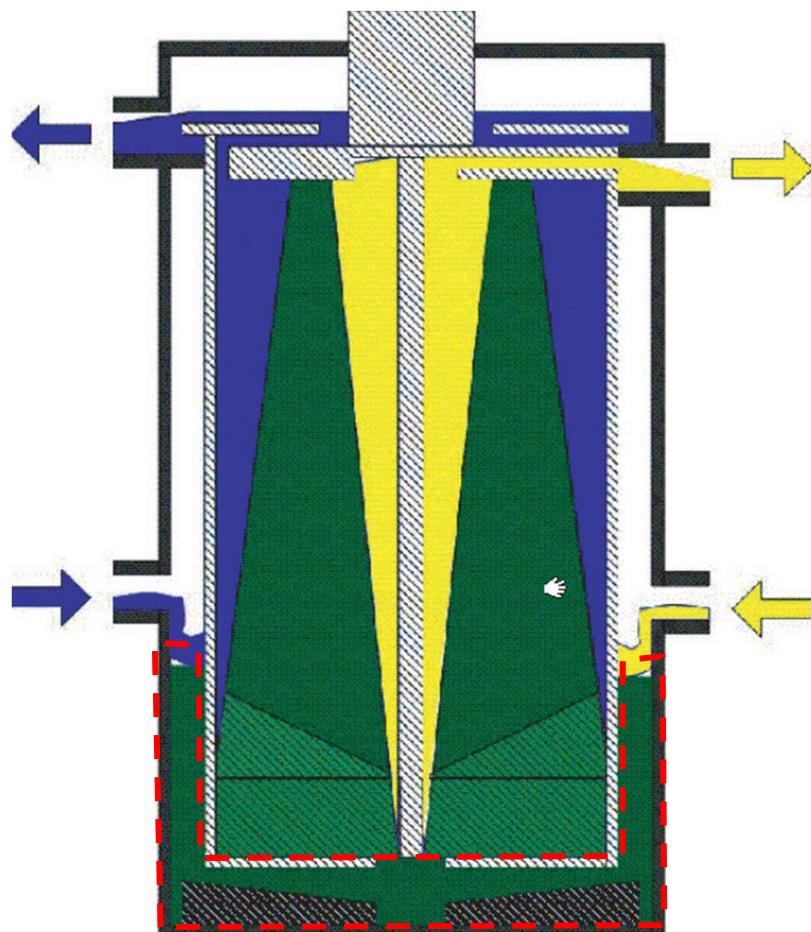


# Annular Centrifugal Contactor



# Contactor – Mixing Region

ksgs Turbulence Model with a Single Phase



- Sierra/Aria transient turbulence model of contactor – largest production Aria calculation to date
- Currently working on two-phase models of contactor mixing zones
- Working on diffuse interface and CDFEM implementation

# Droplet-Scale Methodology: Conformal Decomposition Finite Element Method

## Simple Concept

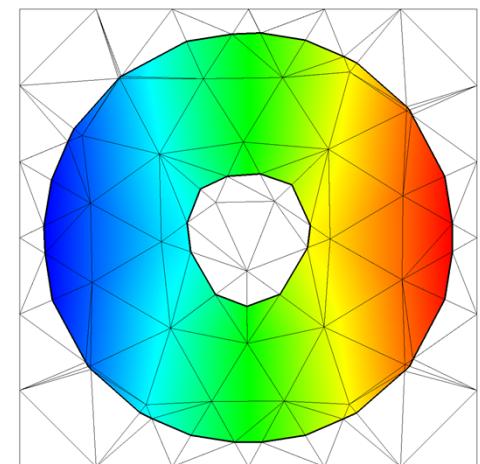
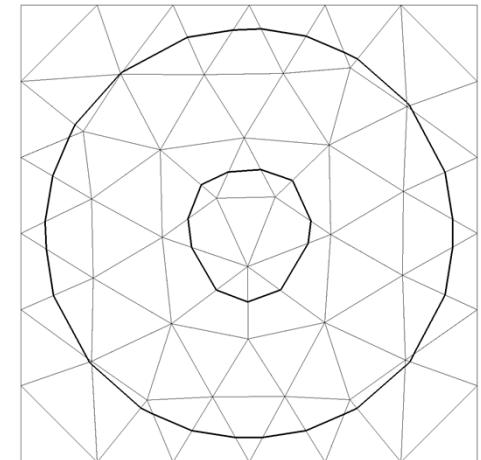
- Use one or more level set fields to define materials or phases
- Decompose non-conformal elements into conformal ones
- Obtain solutions on conformal elements

## Related Work

- Li et al. (2003) FEM on Cartesian Grid with Added Nodes
  - Focus on Cartesian Grid. Considered undesirable because it lost original mesh structure.
- Ilinca and Hetu (2010) Finite Element Immersed Boundary
  - Focus on solid-fluid with Dirichlet BCs

## Properties

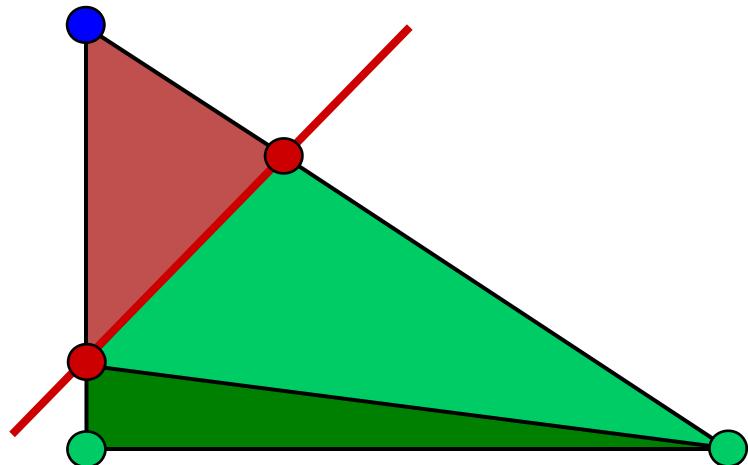
- Supports wide variety of interfacial conditions (identical to boundary fitted mesh)
- Avoids manual generation of boundary fitted mesh
- Supports general topological evolution (subject to mesh resolution)
- Similar to finite element adaptivity
- Uses standard finite element assembly including data structures, interpolation, quadrature
  - CDFEM shown convergent for steady flow, Noble et al, IJNMF, 2010
  - Extension to moving boundary problems



# CDFEM – Constrained Spaces for Stability and Robustness

- Discrete Space Considerations in CDFEM

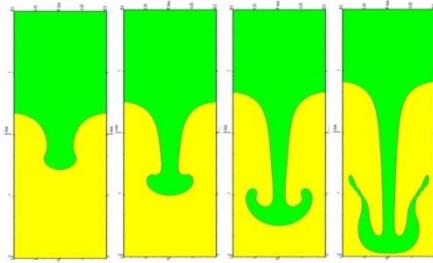
- Anecdotal evidence for space requirements
  - Static, diffusive problems have shown optimal convergence rates using subelements
  - Dynamic, advection problems have shown poorly controlled modes in pressure-velocity and level set fields
- Since interface is discretized as piecewise linear (PL) function on parent element, it follows that level set field should be constrained to be piecewise linear
- A sufficient condition (but possibly overly strict) for a piecewise linear level set function is constraining velocity to be piecewise linear
- Piecewise linear velocity suggests piecewise linear pressure
- Constraints are imposed using linear system infrastructure used for hanging node constraints
- Space used by Fries et al, 2010 for XFEM of two-phase and free surface flows since unconstrained space was unstable



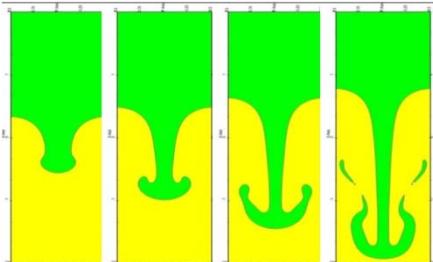
Discrete spaces used in this work

- Level set is PL on parent element
- Velocity is PL on parent element
- Pressure is PL on parent element for each phase (separate PL field for each phase)

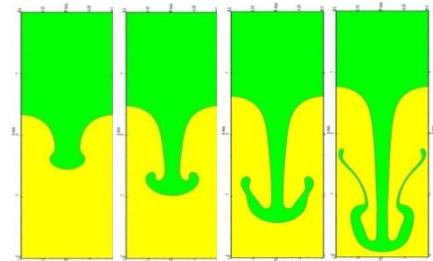
# 2D Rayleigh-Taylor Instability using CDFEM



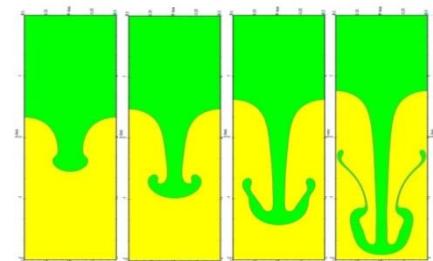
2.254% (1.95493)  
 $h=1/20; \Delta t=h/3$



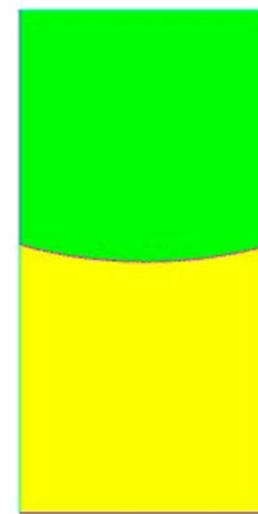
1.014% (1.97972)  
 $h=1/40; \Delta t=h/3.0$



0.89% (1.9822)  
 $h=1/80; \Delta t=h/3.0$

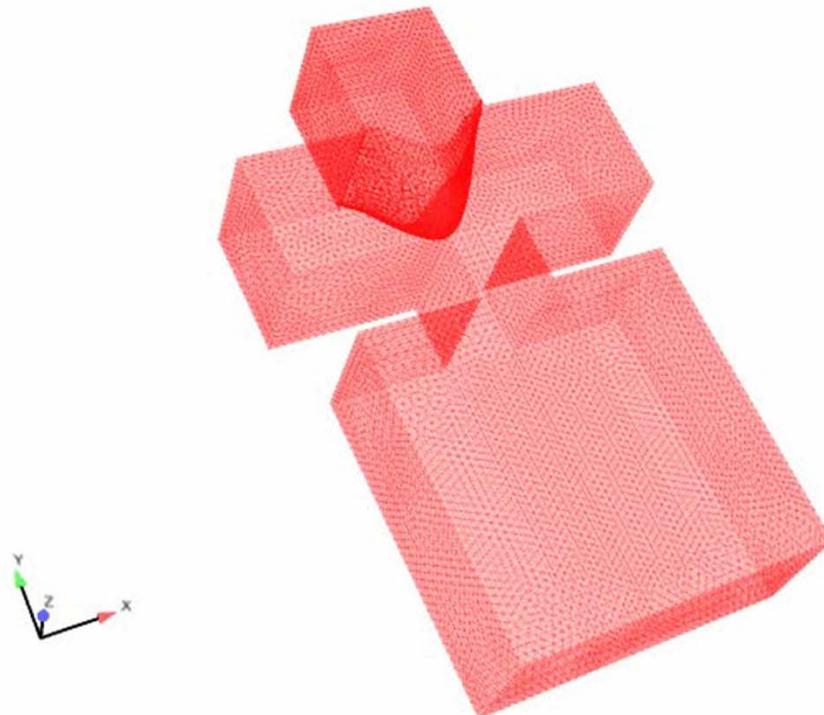
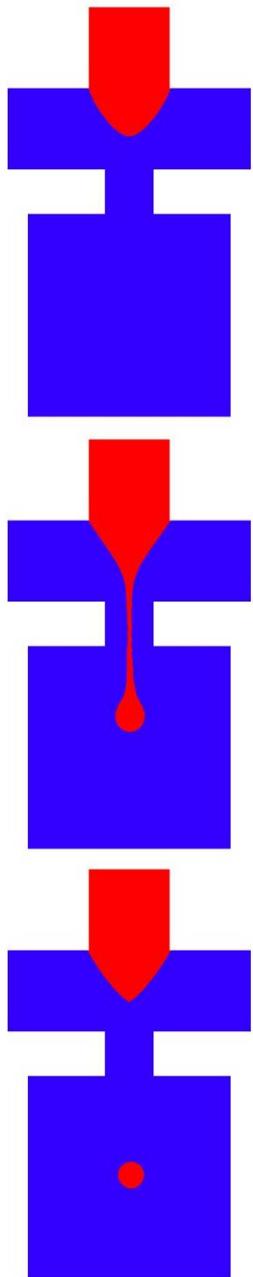


0.145% (1.9971)  
 $h=1/160; \Delta t=h/3.0$



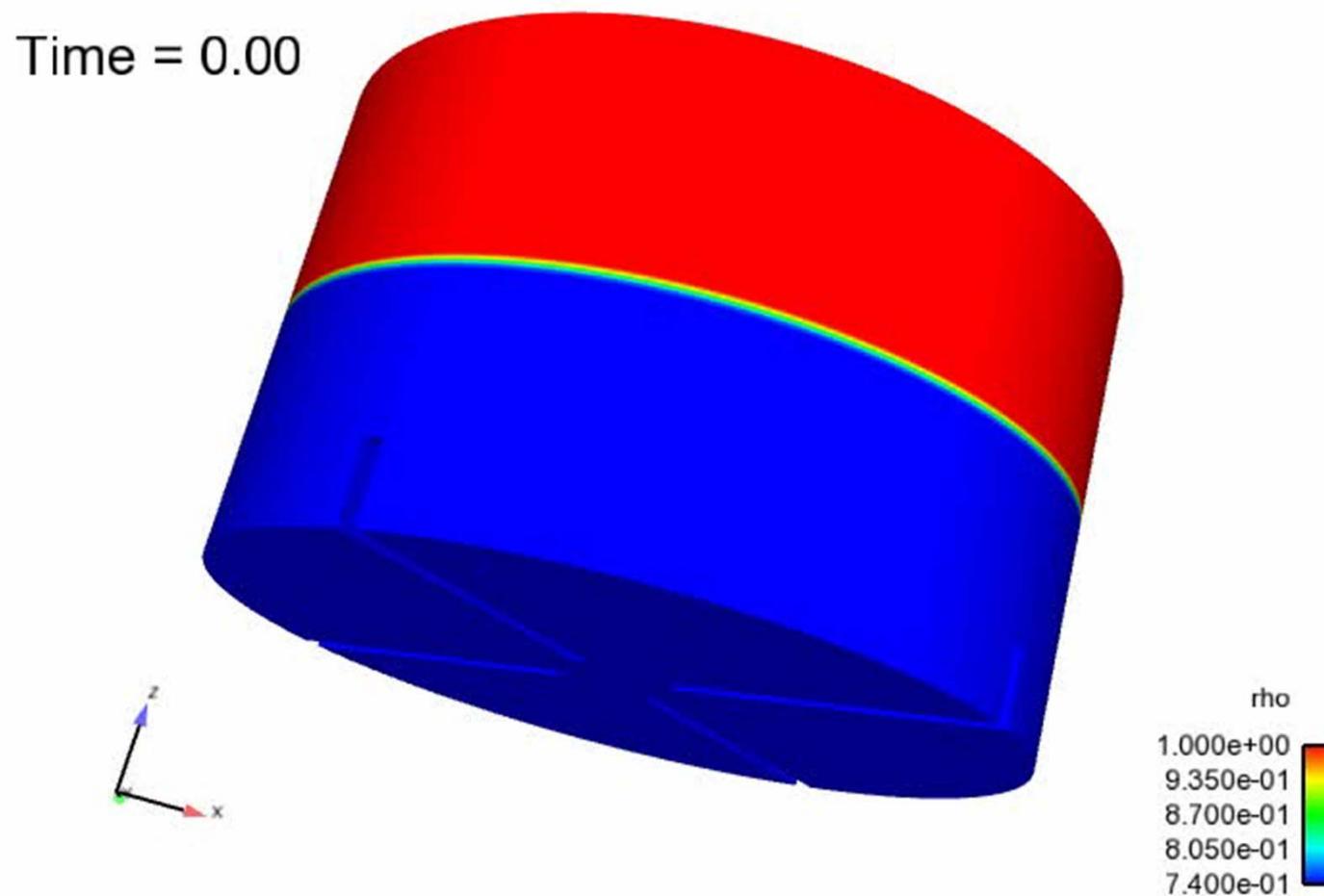
$x$

# $\mu$ Fluid Flow Focusing Droplet Generator

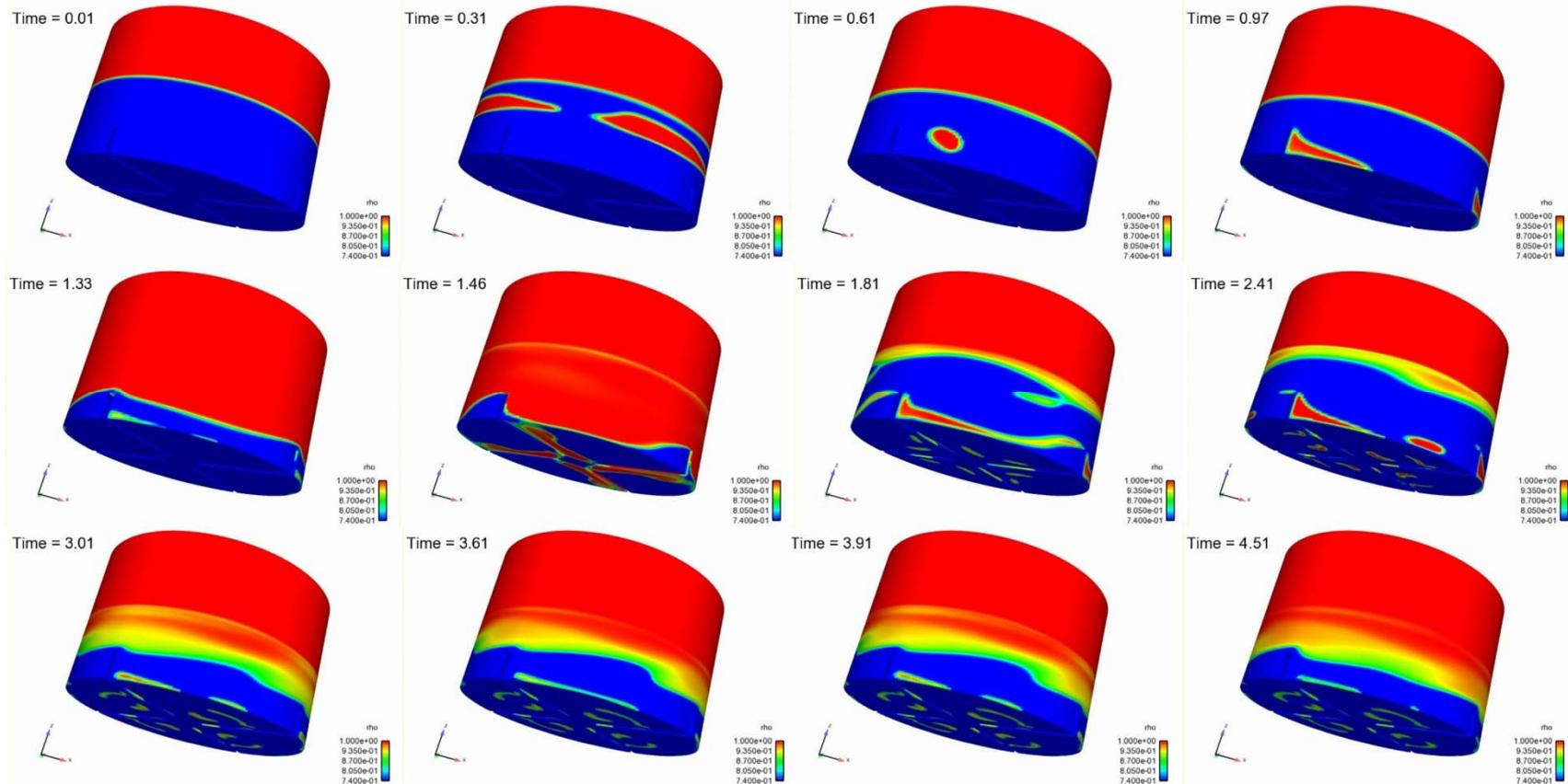


CDFEM method is able to transition from coflowing fluid to droplet generation regimes.

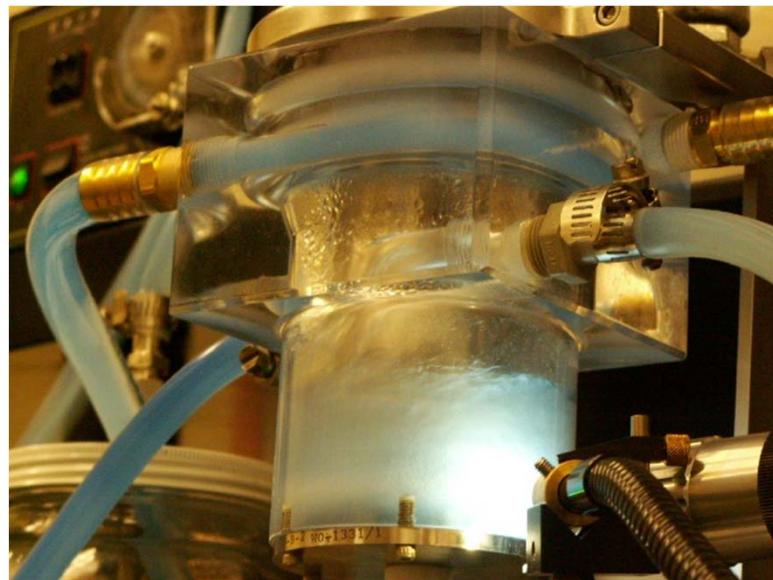
# Mixing Two Fluids in a Centrifugal Contactor (CVFEM/Level Set)



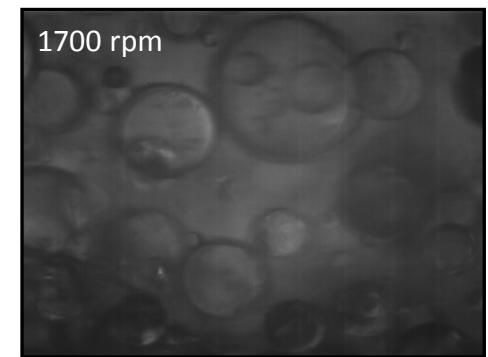
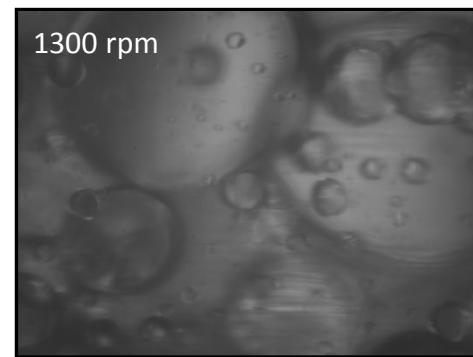
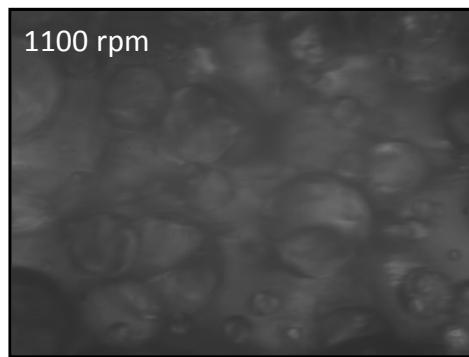
# 2phase Contactor: CVFEM/LS



# Centrifugal Contactor Experiments

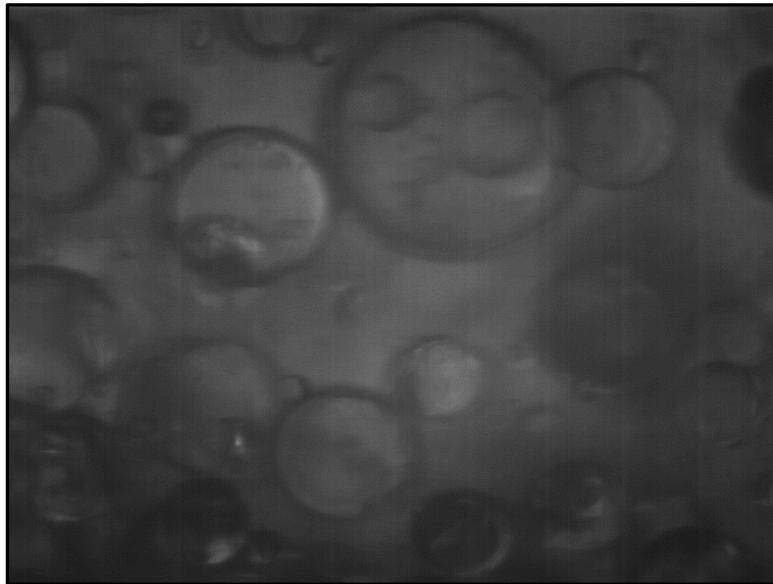


← High Speed Camera

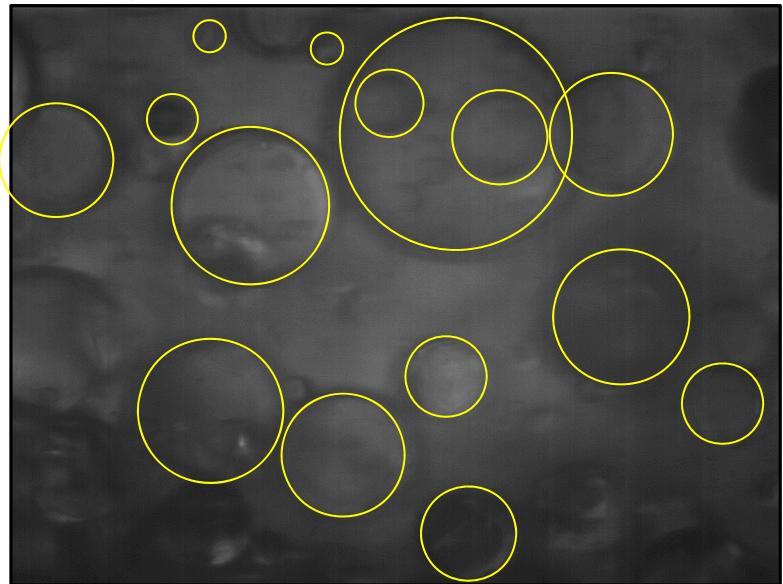


- Test Materials: PDMS and Water
- Rotor Speeds: 1100 – 2100 rpm
- Camera focused at inner surface of outer cylinder
- Camera is about 2 mm above bottom of rotor
- Total liquid height in gap is about  $\frac{3}{4}$  - 1in
- Drop size detection limit – 30  $\mu$ m

# Drop Size Analysis

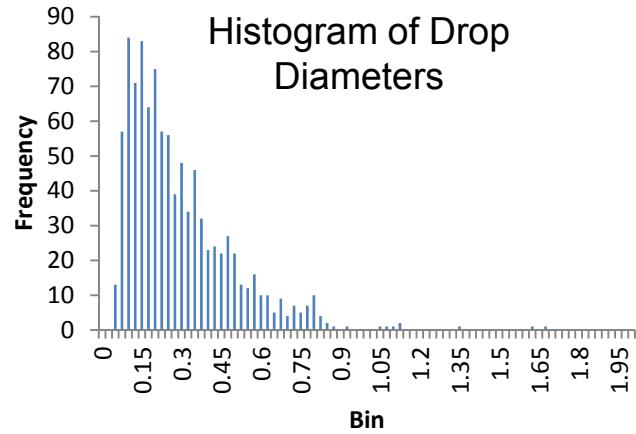


ImageJ  
→

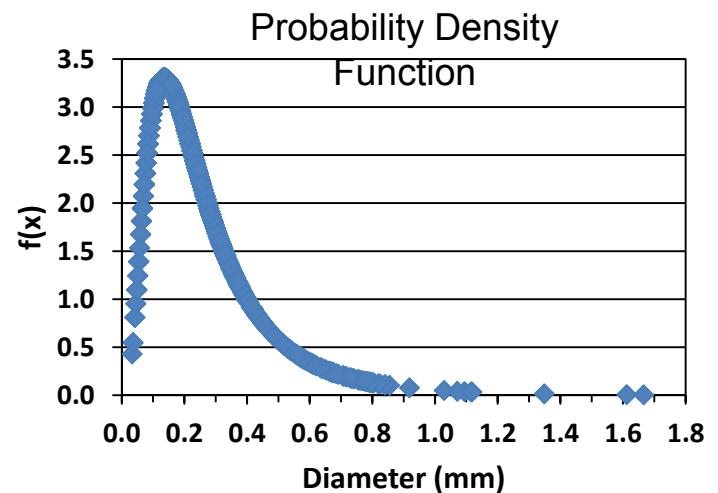


*\*For brevity here, not all droplets were circled*

Based on a calibration image (209 pixels = 0.625mm), ImageJ calculates the diameter of each of the indicated (yellow) circles. These data are then transferred into Excel where probability densities of the distributions are calculated.



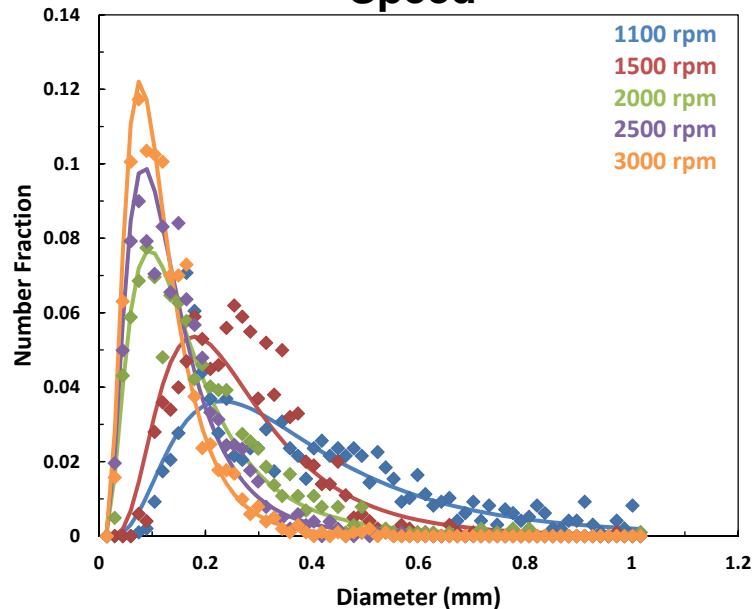
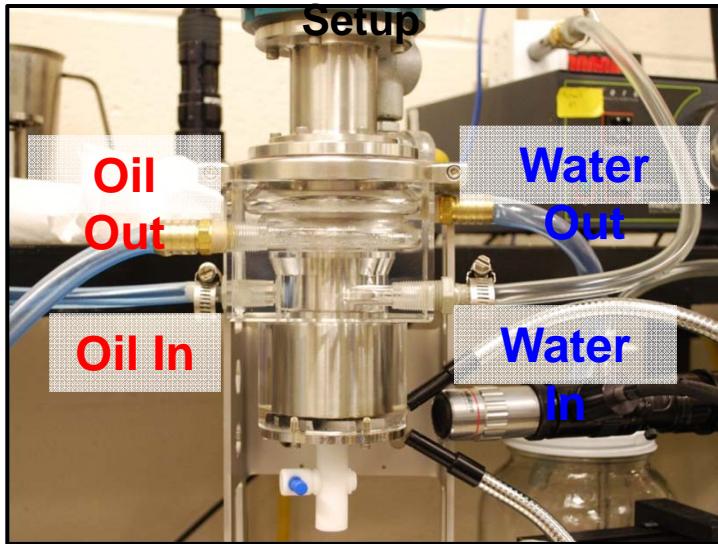
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# Drop Size Measurements in a Centrifugal Contactor

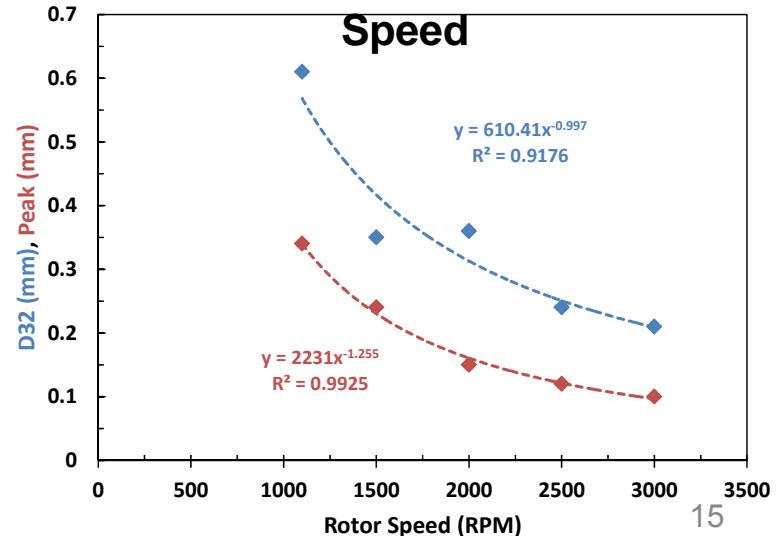
## Drop Size Distributions with Rotor Speed

### Centrifugal Contactor Experimental



- Laser fluorescence is used to measure oil droplet size distributions in a turbulent, three-phase mixing system
- Drop size distributions are the result of an equilibrium state between drop breakup and coalescence
- Drop size distributions are lognormal for all rotor speeds tested
- Distributions narrow and shift to smaller sizes with increasing rotor speeds as the larger droplets break
- Mass transport experiment underway
- Average drop size and Sauter mean diameter show a power law dependence on rotor speed
- Two presentations, collaboration with ANL, and AIChE paper underway

## Sauter Mean Diameter and Rotor Speed



# Population Balance Model of Drop Size Distribution and Mass Transport

## In mixing zone

- Turbulent shear
- Breakup, mass transport
- Population balance model developed

$$\frac{\partial n(v)}{\partial t} = v^{-1} \int_0^1 g(s) R'(v/s) M_1(v/s, t) ds - n(v, t) R(v)$$

$$M_1(v, t) = \int_v^\infty v_1 n(v_1) dv_1$$

breakup rate

$$R(v)$$

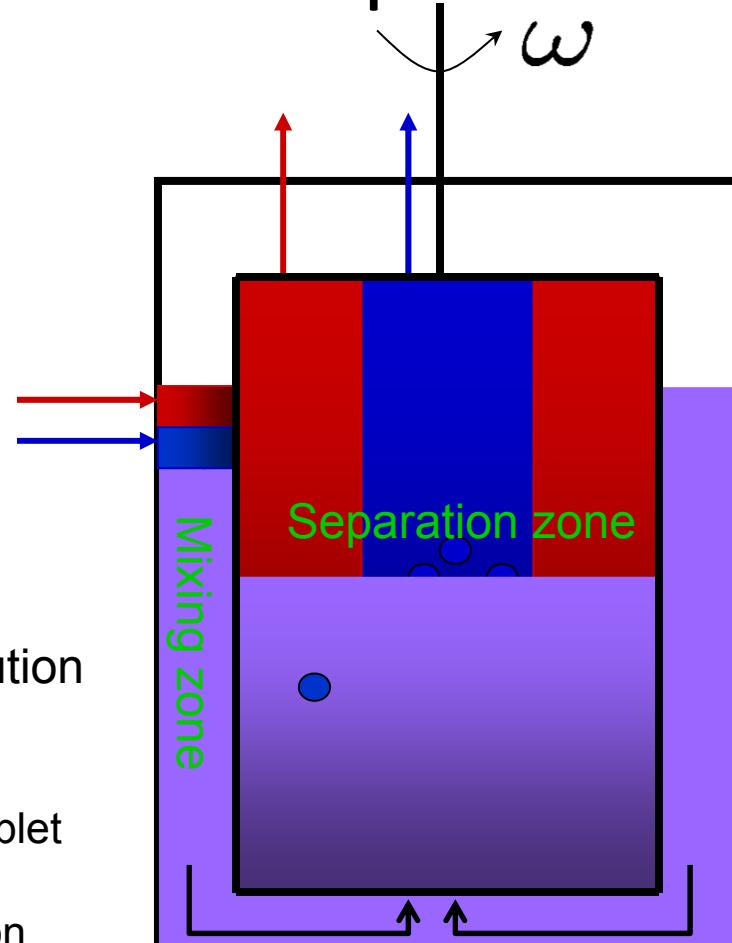
daughter drop distribution

$$G(v, v_1)$$

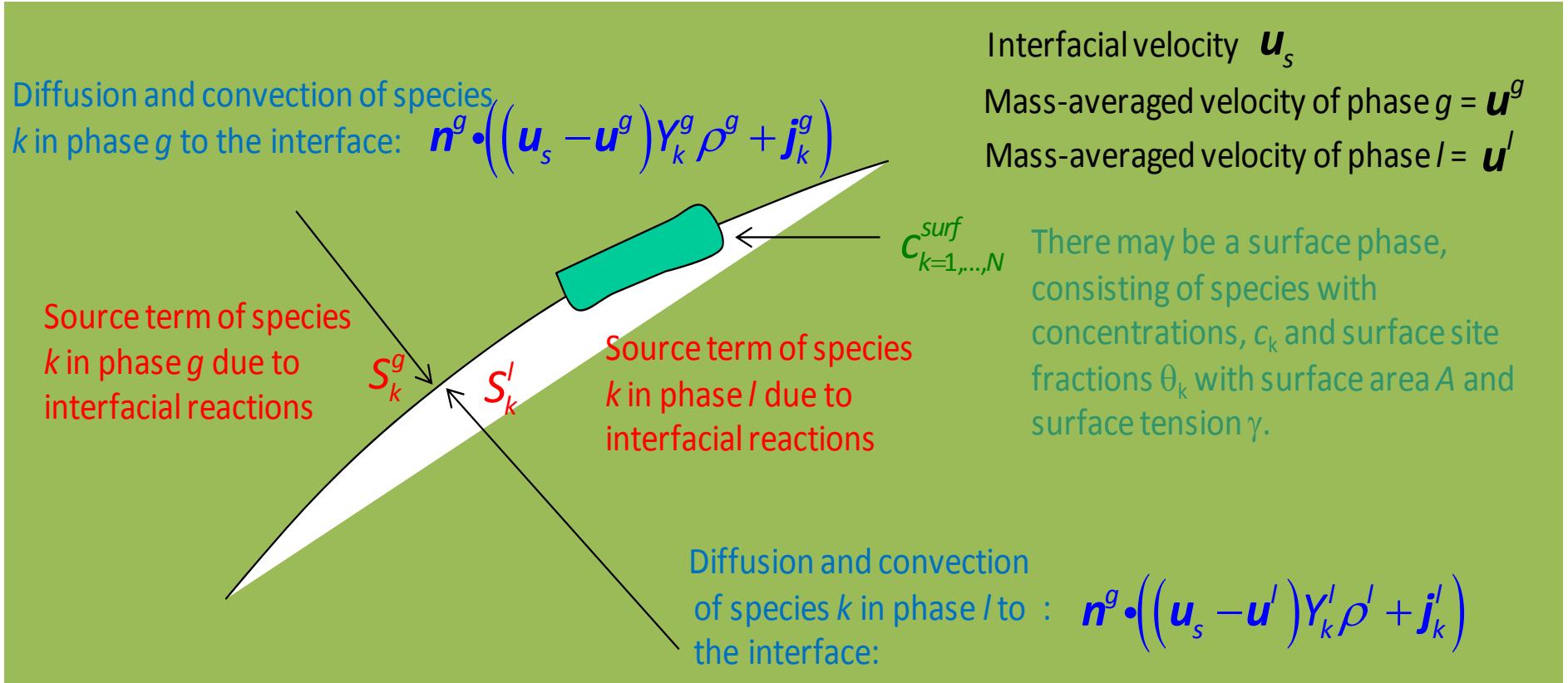
- Proceeding paper completed summarizing the droplet size distribution
- Chemical Engineering Science article in preparation
- Extension to mass transport is next step

continuous phase: **dense fluid**

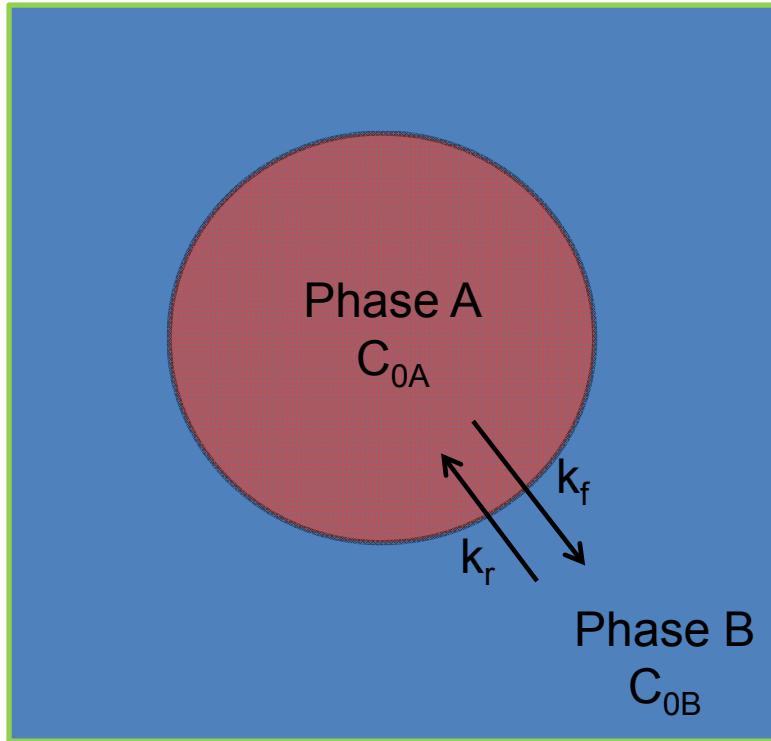
dispersed phase: **less dense fluid**



# A Formalism Developed for Interfacial Mass Transport



# Mass Flux Across the Interface



$$\text{Flux} = k_f C_{0A} - k_r C_{0B}$$

$k_f, k_r$  = Constants

$C_{0A}$  = Concentration of species 0  
in phase A at the interface

$C_{0B}$  = Concentration of species 0  
in phase B at the interface

- Species diffuse and advect on either side of the interface and experience a jump in concentration at the interface
- This is natural to implement with CDFEM

# Example Problem: Interfacial Mass Transfer with Moving Interface

Interfacial, initial conditions, and properties

$$D_{0A} \nabla C_0 = k_f C_0 - k_r C_1$$

$$D_{1B} \nabla C_1 = -(k_f C_0 - k_r C_1)$$

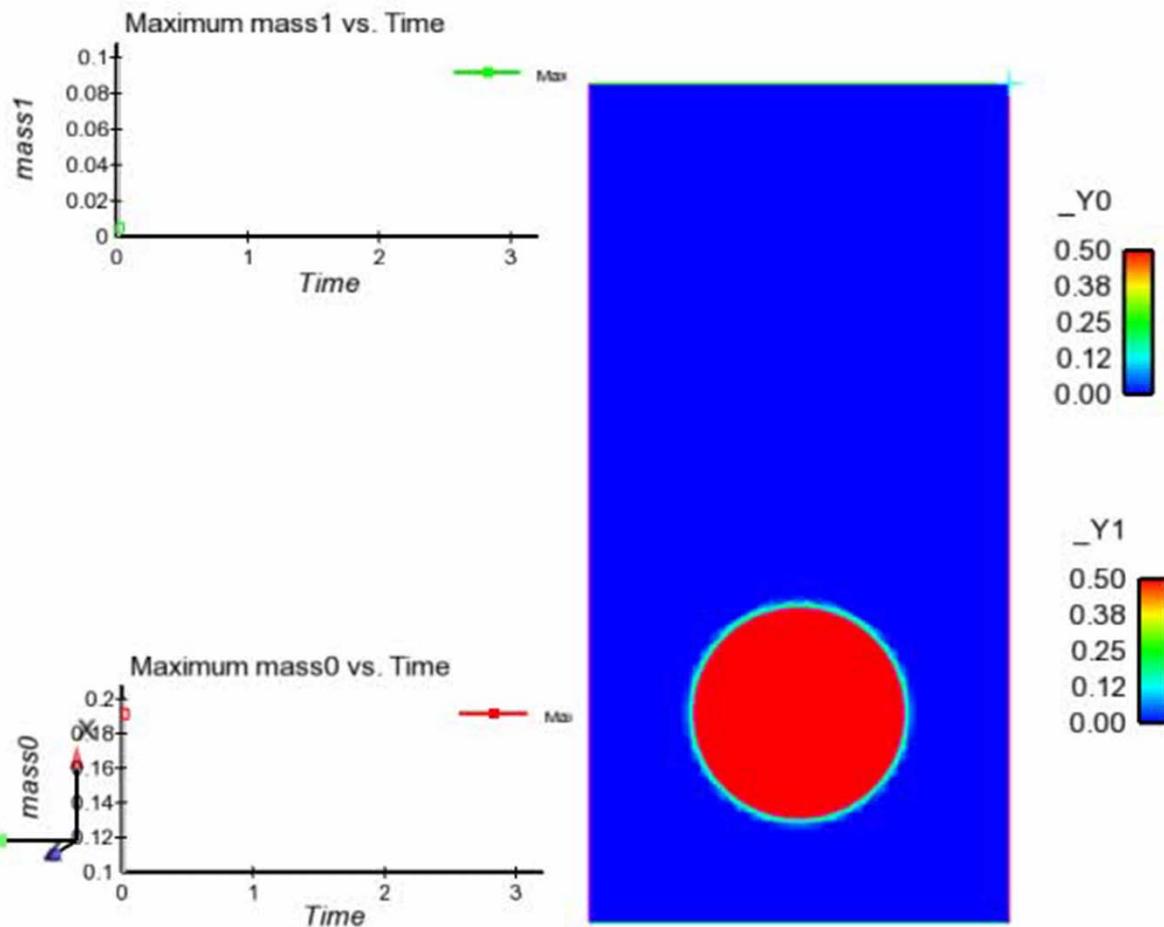
$$k_f = 1, k_r = 2$$

$$D_{0A} = D_{1B} = 0.05$$

$$C_0^{init} = 1, C_1^{init} = 0$$

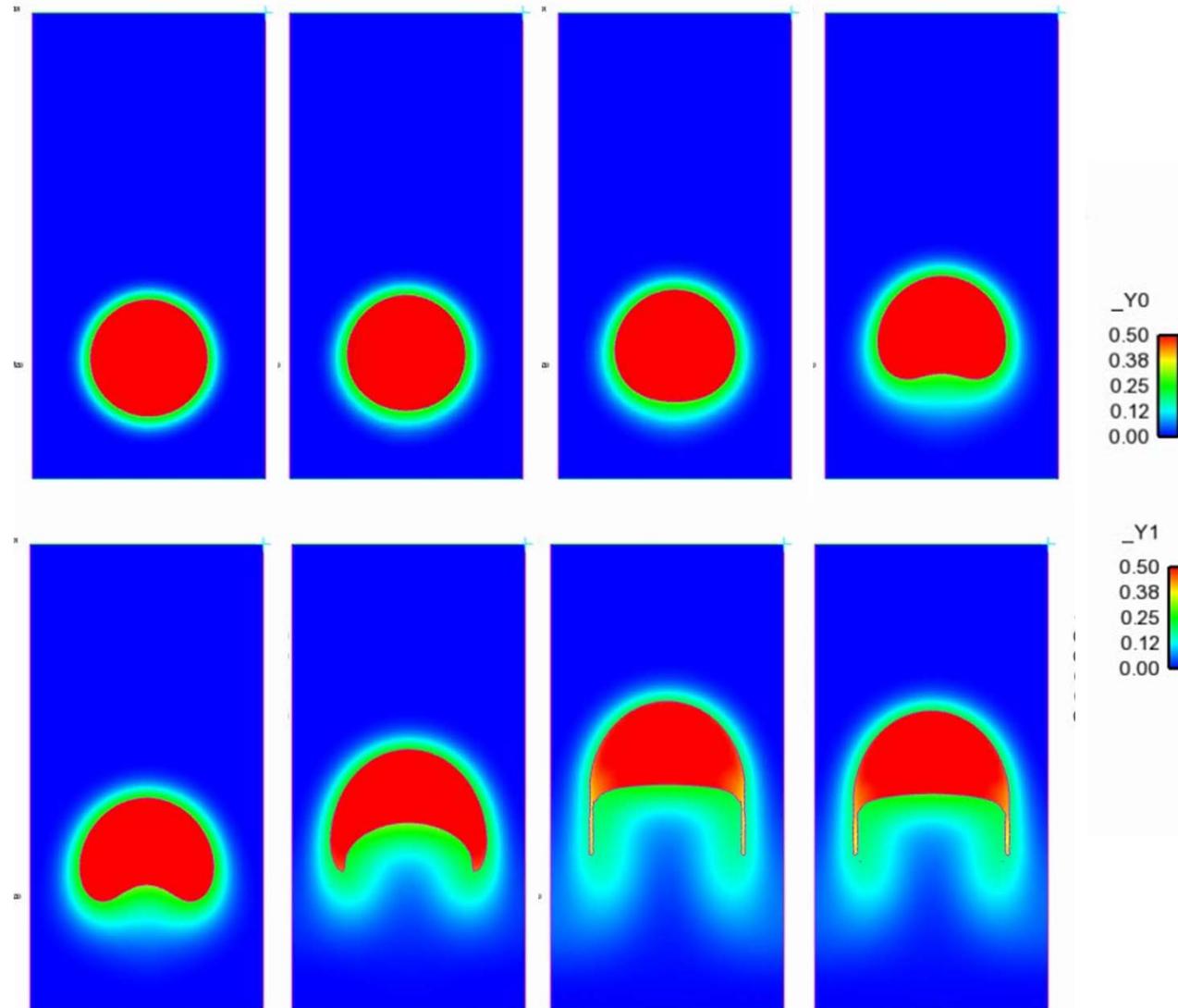
$$\text{mass0} = \int_{phaseA} C_0 dx dy$$

$$\text{mass1} = \int_{phaseB} C_1 dx dy$$



CDFEM implementation allows easy application of concentration jumps

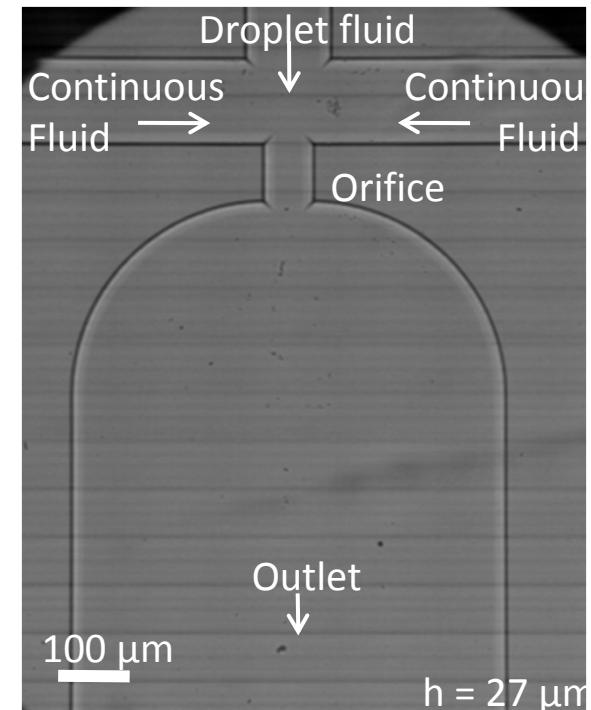
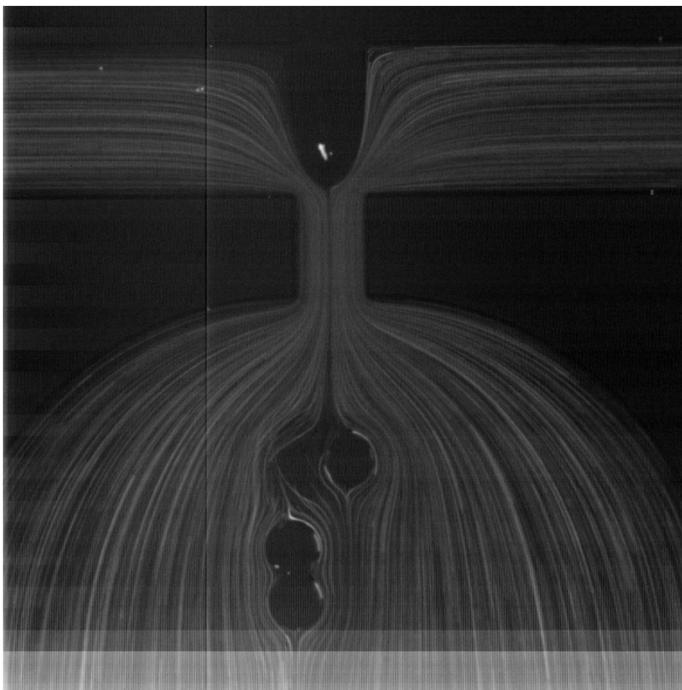
# Mass transport with advection



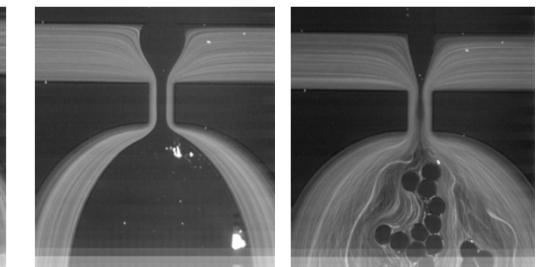
# Droplet-scale Experiment in Microfluidic

## Goals: Device

- Create uniformly sized droplets
  - *Flow Focusing Microchannel*
- Understand flow field inside/around droplets
  - *Phantom high speed camera*
- Understand liquid-liquid mass transfer
  - *Ocean Optics spectrophotometer*

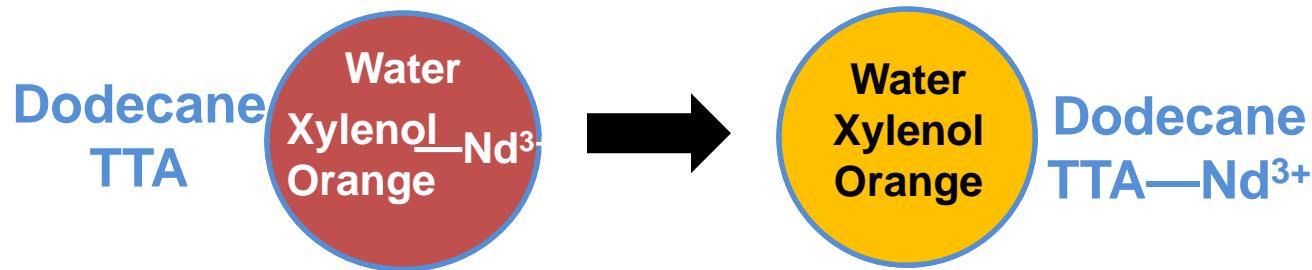


*Decreasing inner flow rate*

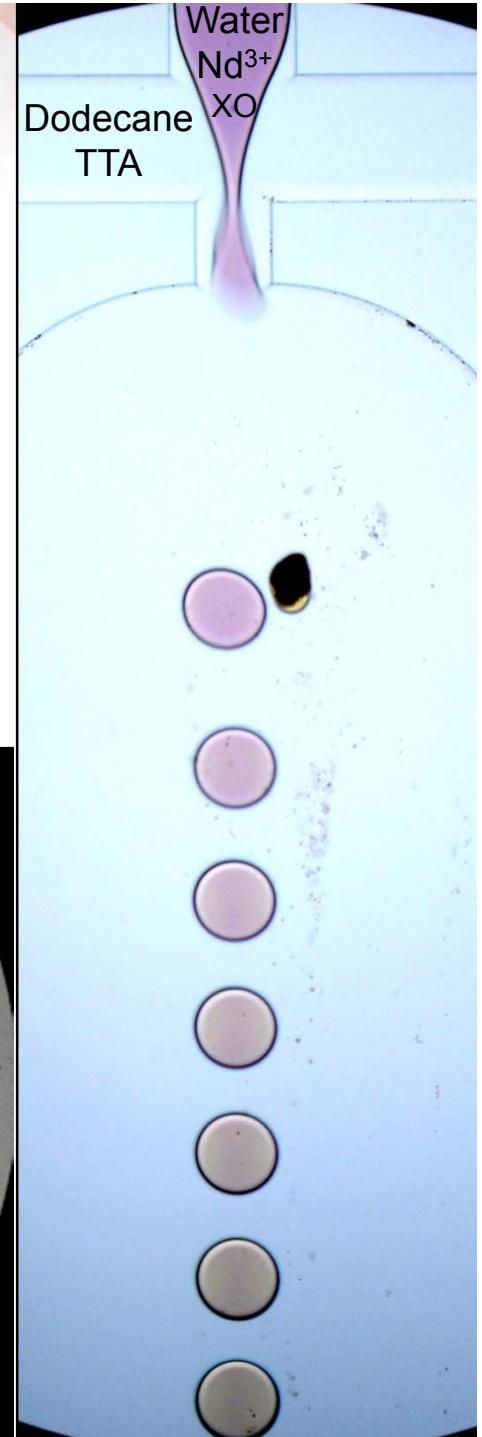


# Model System Developed for Mass Transfer Validation

- A nonradioactive model system was chosen based on Nd, which has behavior similar to Pu/U



- A spectrophotometer is used to watch Nd<sup>3+</sup> transfer from the water to the dodecane

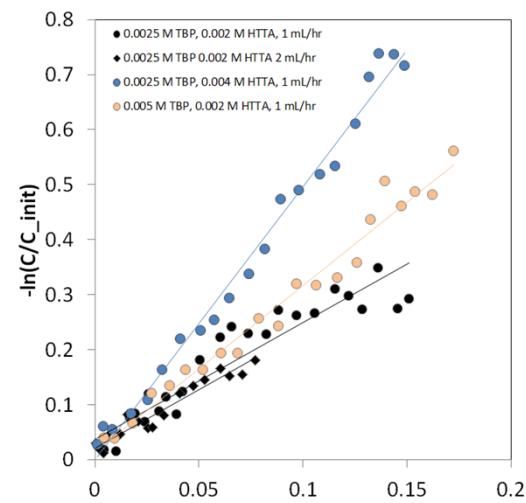
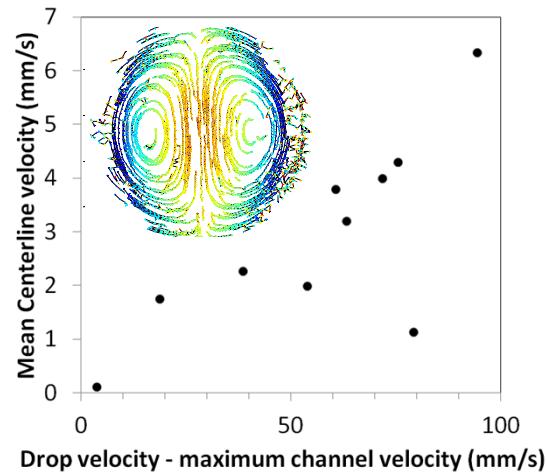


# Droplet Scale Experiments

**Monodisperse drop generation in a flow-focusing microfluidic chip studied: Droplet size correlated with flow, fluid conditions.**

C. C. Roberts, R. Rao, A. Grillet, C. Jove-Colon, C. Brooks, M. Nemer, *Lab Chip*, 2012, 12 (8): 1540

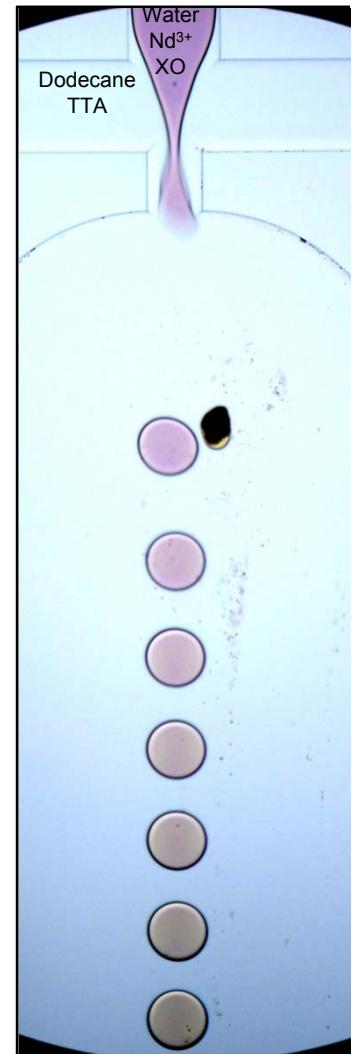
**Flow interior to the droplets visualized using particle tracking.**



**Model system mimics processing: Concentration quantified by color**



**Mass transfer coefficients measured for numerous flow rates, species concentrations to aid understanding contactor experiments.**

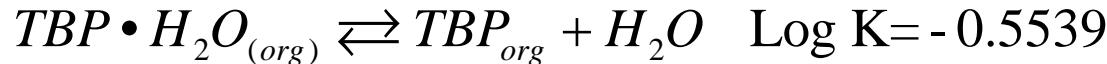
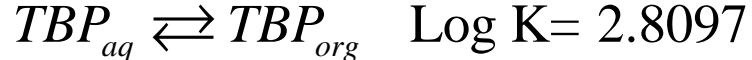


Presented: AIChE 2011, 2012; APS DFD 2011. Manuscript in preparation.

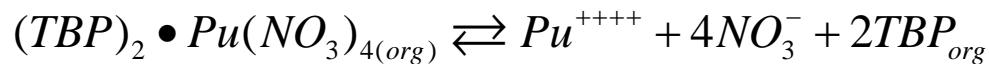
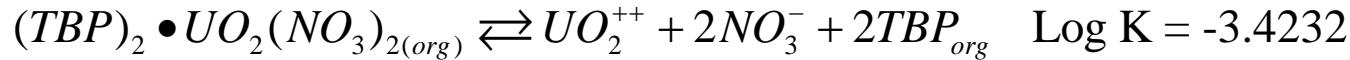
# Advanced Thermodynamics Model Developed For PUREX Extraction

## Reactions Considered for the $\text{HNO}_3\text{-UO}_2\text{-H}_2\text{O-TBP-AMSCO}$

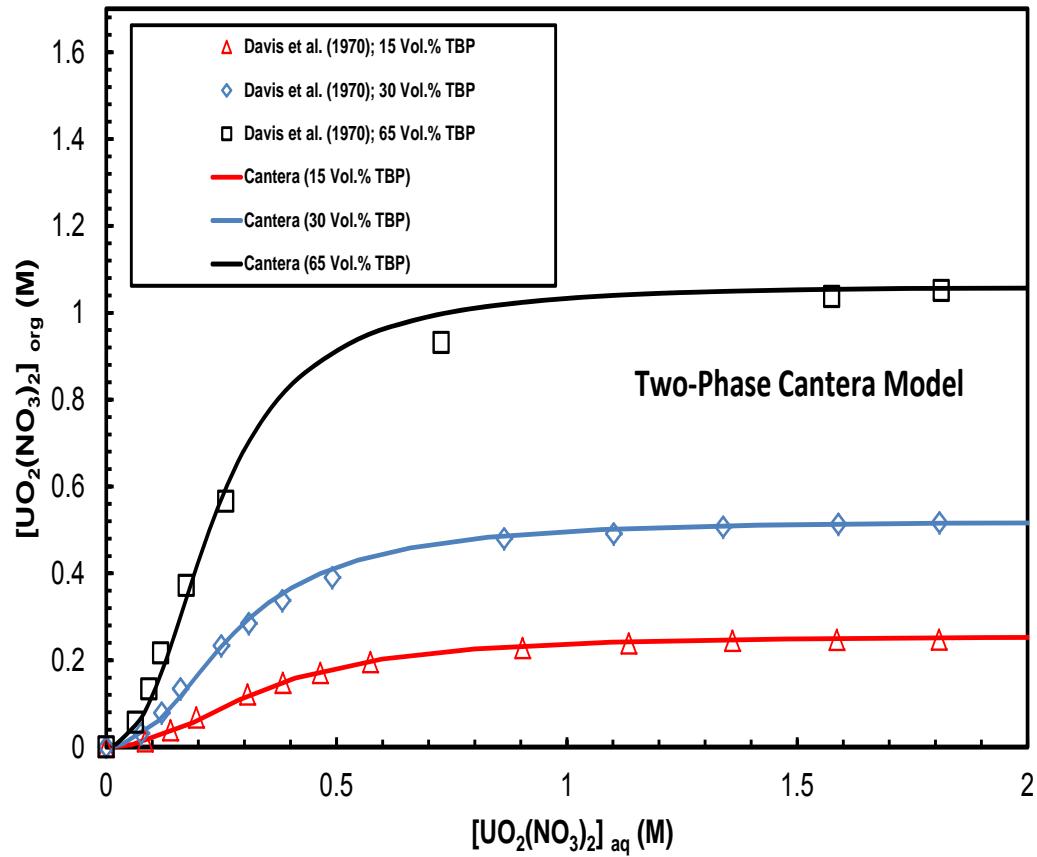
- Log K from extraction and solubility data



- Reactions for the extraction of U and Pu nitrates by TBP



# Extraction of Uranyl Nitrate by TBP/AMSCO

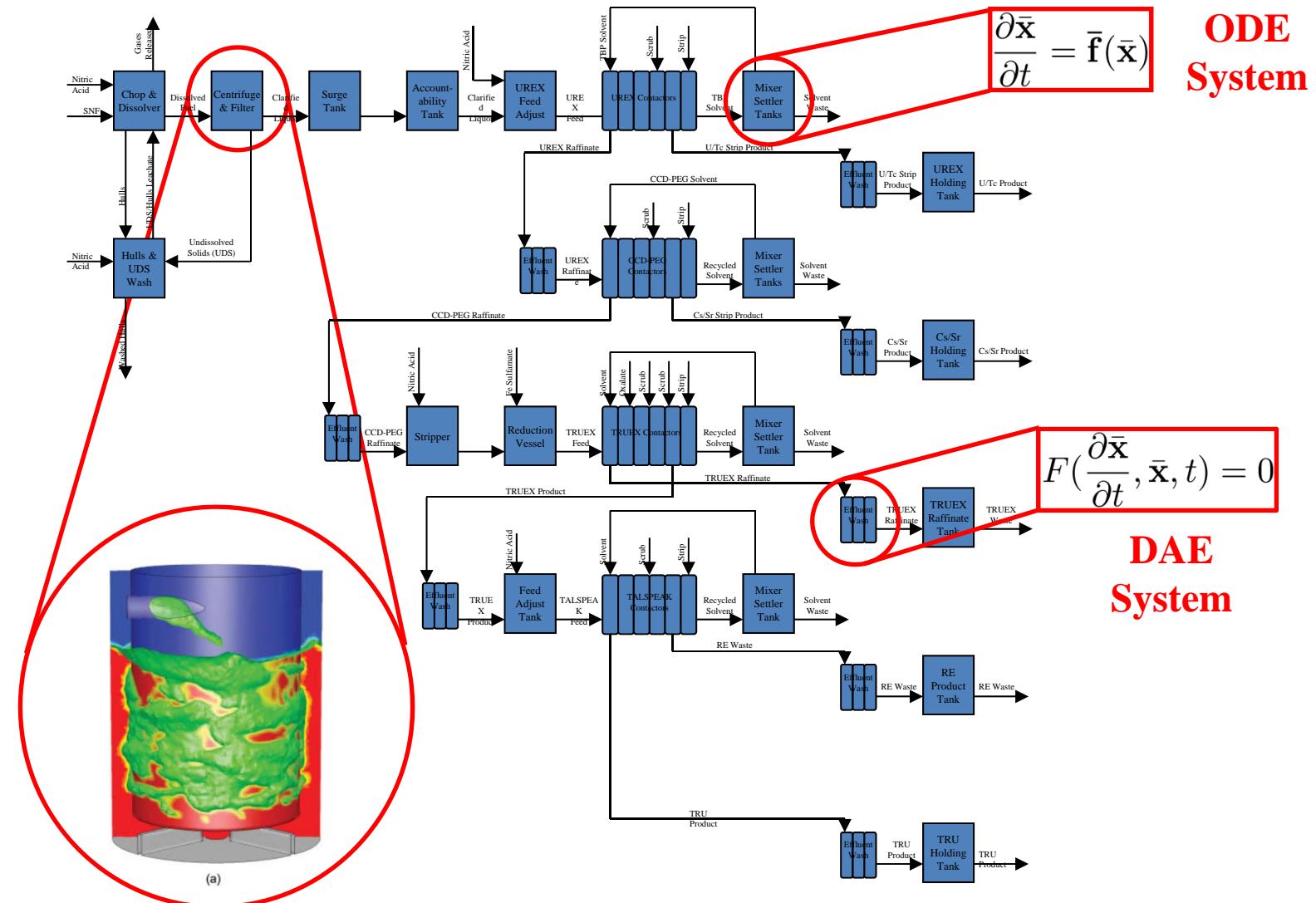


- Reaction set built in Cantera based on experimental data
- Cantera coupled to Dakota to determine Gibb's free energies for each species
- Proceedings paper and SAND report completed.
- Journal article submitted to *Separations Science and Technology*
- Cantera included as a library in Sierra Mechanics



# Mixed-Fidelity Plant Scale Model

(Embedded technology → UQ, Optimization, Bifurcation Analysis)

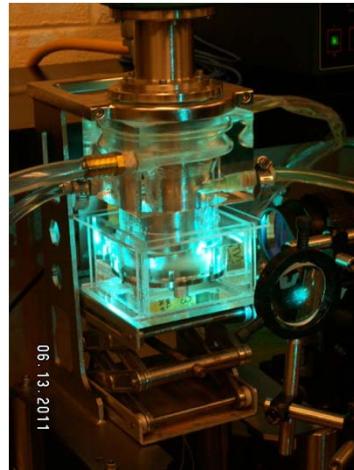


**Large-scale PDE  
System: SIERRA**

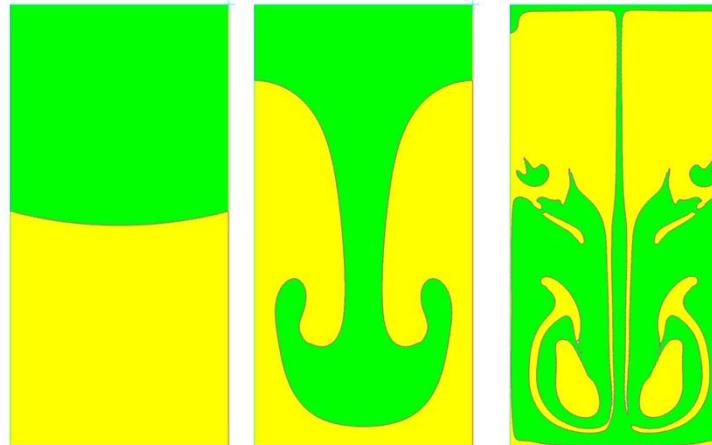
# Accomplishments and Future Work

- Droplet-scale model using CDFEM have been developed and verified on a published 2D benchmark problem from Hysing et al, 2009
- Mass transport has been incorporated into the CDFEM model
- A nonradioactive model system was developed using Nd and xlenol orange, with a new spectrophotometer to give quantitative mass transport data for the validation study
- A quantitative droplet-scale mass transport validation experiment is underway using the spectrophotometer
- Advanced thermodynamic models have been developed for the Uranyl Nitrate system and are under development for the Nd system
- Single-phase turbulent contactor simulations have been completed
- Working towards modeling two-phase flow in a centrifugal contactor using first a diffuse level set method and then CDFEM
- Collaboration with Marianne Francois (LANL) for DOE/NEAMS
- Hired post-docs Christine Roberts, Nick Wyatt and staff Martin Nemer
- Network modeling and criticality modeling will be a focus of next FY

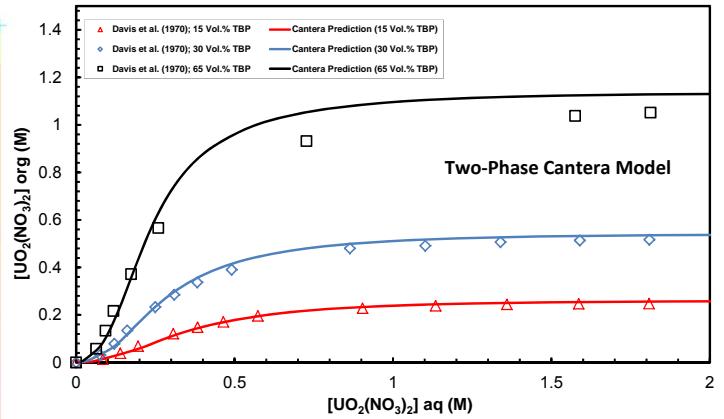
# Selected Highlights



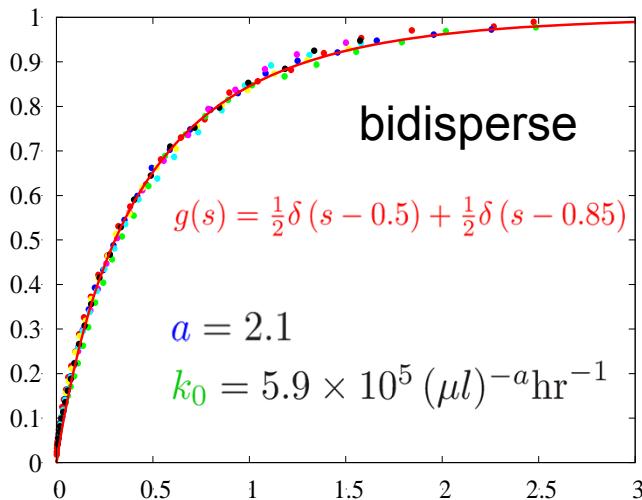
Contactor with laser light sheet to distinguish bubbles and drops



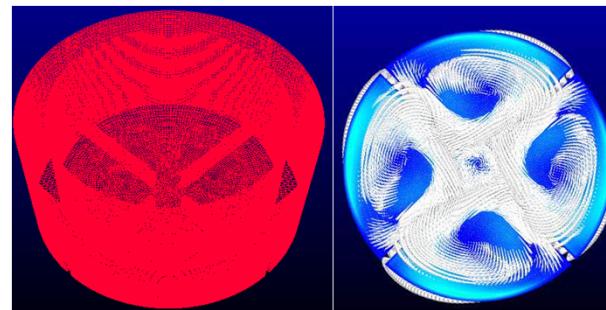
Novel CDFEM algorithm is robust through entire Rayleigh-Taylor instability



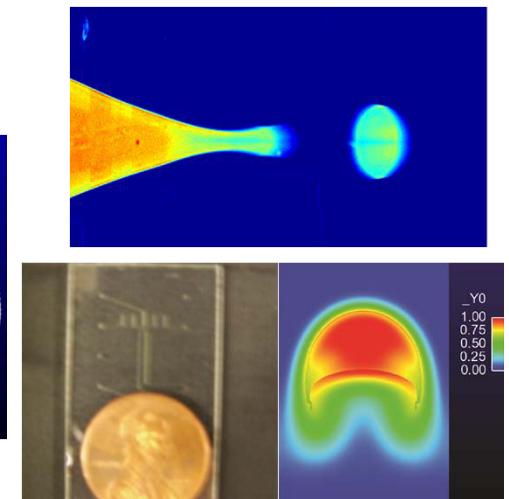
Cantera thermo model developed for U separation in the presence of TBP and H<sup>+</sup>



Population balance models predicts break-up data for classic experiment



Sierra/Aria 3D transient turbulent model of contactor



Microfluidic device for quantitative mass transport experiments shown with model