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Nonlinear Control Design for Nonlinear Wave Energy Converters

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ABSTRACT

This paper presents a nonlinear control design for nonlinear Wave Energy Converters (WECs). A nonlinear dynamic model is developed for a geometrically right-circular cylinder WEC design for the heave only motion or single Degree-of-Freedom (DOF). The linear stiffness term is replaced by a nonlinear cubic hardening spring term to demonstrate the performance of a nonlinear WEC as compared to an optimized linear WEC. A nonlinear resonance control based on energy (Hamiltonian for the system), power flow (Hamiltonian rate), and limit cycle operation is developed to exploit this nonlinear dynamical effect. A Complex Conjugate Control (C3) is implemented with a practical Proportional-Derivative (PD) controller for WEC devices to improve and optimize power absorption for off-resonance conditions for a linear WEC. Single frequency and Multi-frequency wave excitation input conditions are analyzed and evaluated. Numerical simulations demonstrate equivalent power capture for the nonlinear control design which incorporates and capitalizes on the nonlinear dynamics for the single DOF WEC. By exploiting the nonlinear physics in the nonlinear controller design, equivalent power and energy capture, as well as simplified operational performance is observed for the nonlinear cubic hardening spring term when compared to the optimized linear WEC.

WEC Model Development

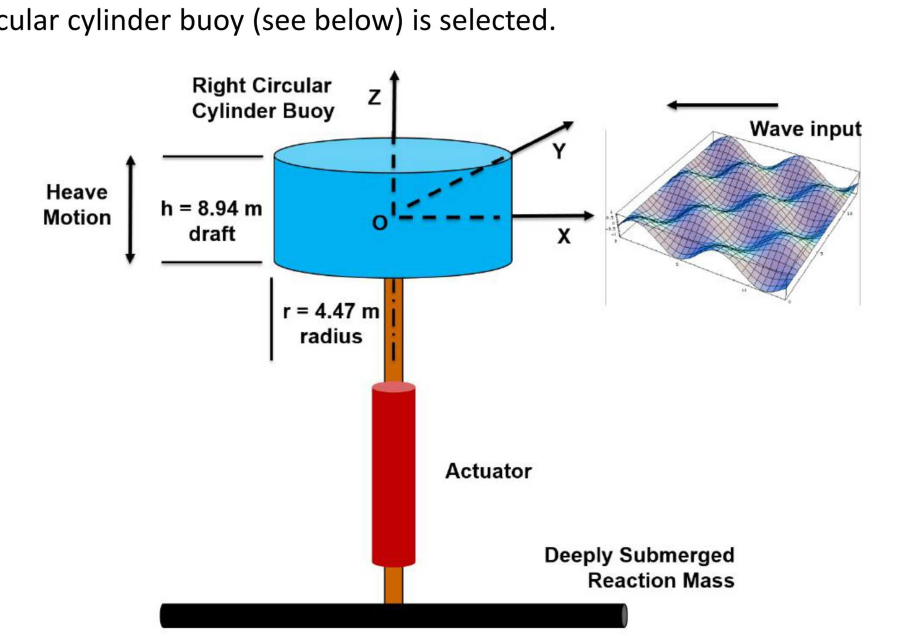
For this investigation an approximate hydrodynamic model for the WEC is assumed and for a heaving buoy the Cummins' equation of motion is given as

$$(m+\tilde{a}(\infty))\ddot{z}+\int_0^{\infty} h_r(\tau)\dot{z}(t-\tau)d\tau+kz=F_{ex}+F_u \qquad (1)$$

where m is the buoy mass, $\tilde{a}(\infty)$ is the added mass at infinite frequency, z is the heave position of the buoy's center of mass with respect to the mean water level, k is the hydrostatic stiffness due to the difference of the gravitational and buoyancy effects, F_{ex} is the excitation force, F_u is the control force, and h_r is the radiation impulse response function. With a state-space approximation for the convolution term in Eq. (1), the whole model can be rewritten as

$$\begin{Bmatrix} \dot{z} \\ \ddot{z} \\ \dot{x}_{r_1} \\ \dot{x}_{r_2} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/M & 0 & -C_{r_1}/M & -C_{r_2}/M \\ 0 & B_{r_1} & A_{r_{11}} & A_{r_{12}} \\ 0 & B_{r_2} & A_{r_{21}} & A_{r_{22}} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M \\ 0 \\ 0 \end{bmatrix} (F_{ex}+F_u) \qquad (2)$$

with a model of order two selected. The total model consists of four first-order ordinary differential equations. The equivalent mass is given as M = m + $\tilde{a}(\infty)$ with the state-space realization parameters listed in Table 1 (see paper). In the numerical simulations performed, a right circular cylinder buoy (see below) is selected.



CONTROL DESIGNS

Two separate control designs are reviewed and developed. The first is based on PDC3 and the other is based on a nonlinear spring effect NL.

PDC3 Development

A PD feedback control for each channel is introduced (see block diagram below). For a multiple frequency forcing function (as applied to irregular waves) the model becomes:

$$m\ddot{z}+c\dot{z}+kz=F_u+\sum_{j=1}^{N} F_{ex_j}\sin\Omega_j t$$

The PD controller is selected as:

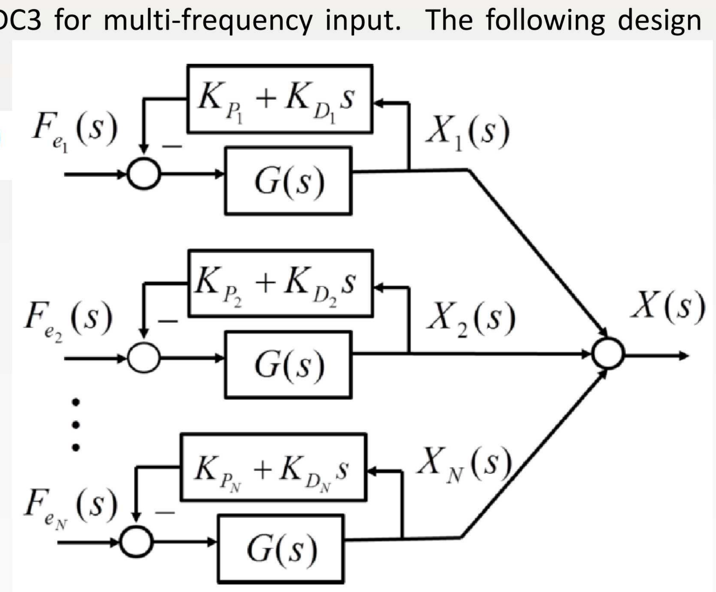
$$F_u=\sum_{j=1}^{N} F_{u_j}=\sum_{j=1}^{N}\left[-K_{P_j}z_j-K_{D_j}\dot{z}_j\right]$$

The resulting PDC3 dynamic model for each channel becomes:

$$m\ddot{z}_j+(c+K_{D_j})\dot{z}_j+(k+K_{P_j})z_j=F_{ex_j}\sin\Omega_j t$$

The final step is to resonate the PDC3 for multi-frequency input. The following design steps are:

1. Pick $K_{D_j}=c$ or $(c+K_{D_j}) = 2c = 2R$
2. Pick K_{P_j} such that $\omega_{n_j}^2=\Omega_j^2 = (k+K_{P_j})/m$ or $K_{P_j}=m\Omega_j^2-k$.



Nonlinear (NL) Cubic Spring Development

This feedback strategy focuses on NL oscillations to multiply and/or magnify the power/energy capture from the WEC. By introducing a cubic spring in the feedback loop an increase in power. This can be realized with a mechanical NL spring in combination with an ESS to help transmit reactive power between cycles. Alternatively, the cubic hardening spring can be realized by shaping the buoy to produce reactive power from the water and simplifying the design. The feedback law is given by

$$F_u=-R_{opt}\dot{z}-K_{NL}z^3+Kz$$

The maximum power/energy absorbed is

$$J=\int_0^t f\cdot\dot{z}d\tau=\int_0^t -R_{opt}\dot{z}^2 d\tau$$

NUMERICAL SIMULATION

The goal of PDC3 is to increase power/energy capture for off-resonance conditions. This requires additional filtering, individual frequency tuning, and associated power electronics and energy storage to meet the power requirements. For a nonlinear cubic spring these implementation components for PDC3 can be simplified and still produce equivalent power output. The numerical simulation results are presented for both a single frequency and multi-frequency wave force input. (Results shown to right).

Hamiltonian Surface Shaping Single-Frequency Case

A Hamiltonian surface defines the accessible phase space of the system. The dynamical system path/trajectory traverses this energy storage surface defined by the Hamiltonian as a result of the power flow. The Hamiltonian or stored energy for PDC3 is defined as

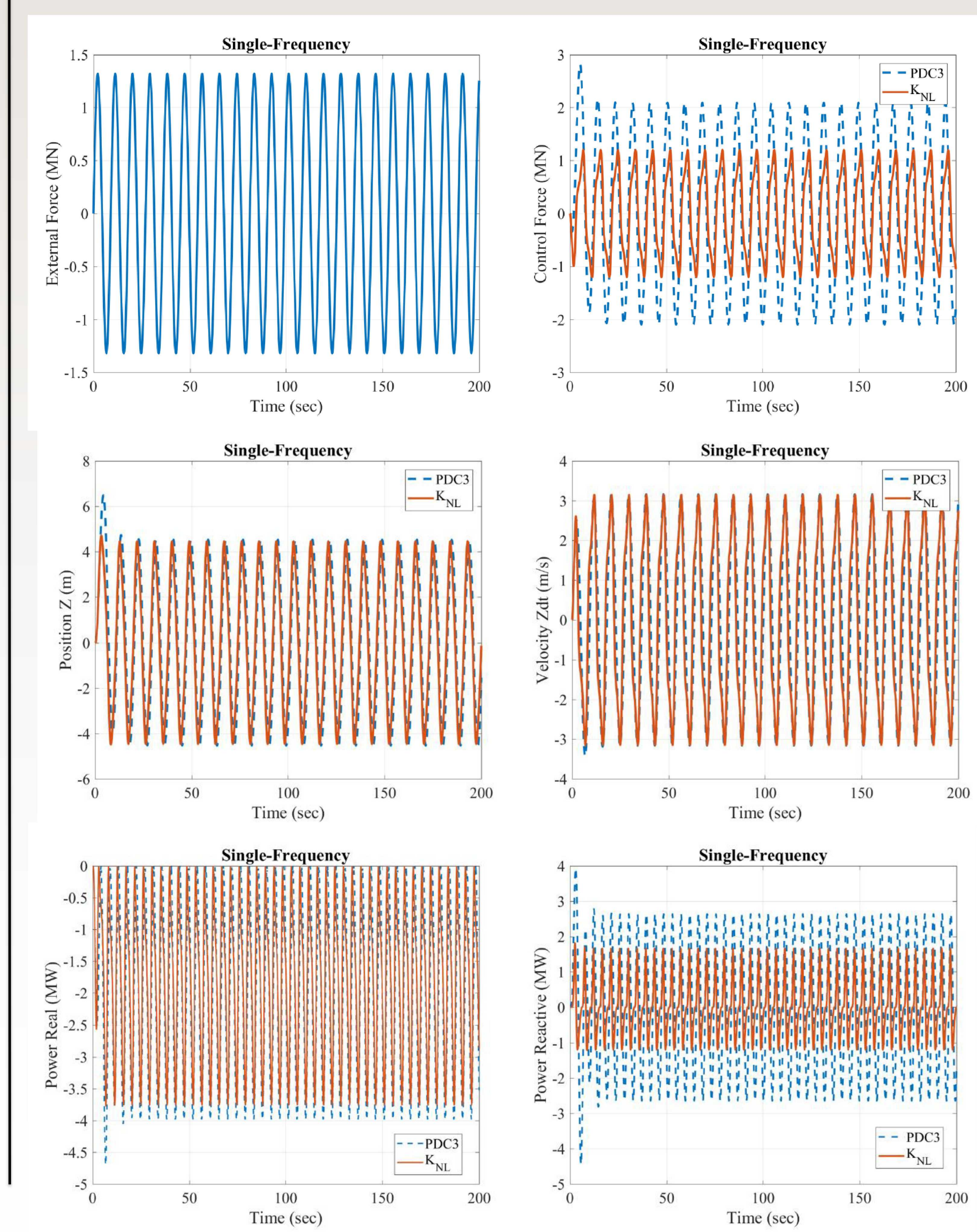
$$\mathcal{H}=\mathcal{T}+\mathcal{V}=\frac{1}{2}M\dot{z}^2+\frac{1}{2}(k+K_P)z^2.$$

The Hamiltonian for the NL cubic spring is defined as

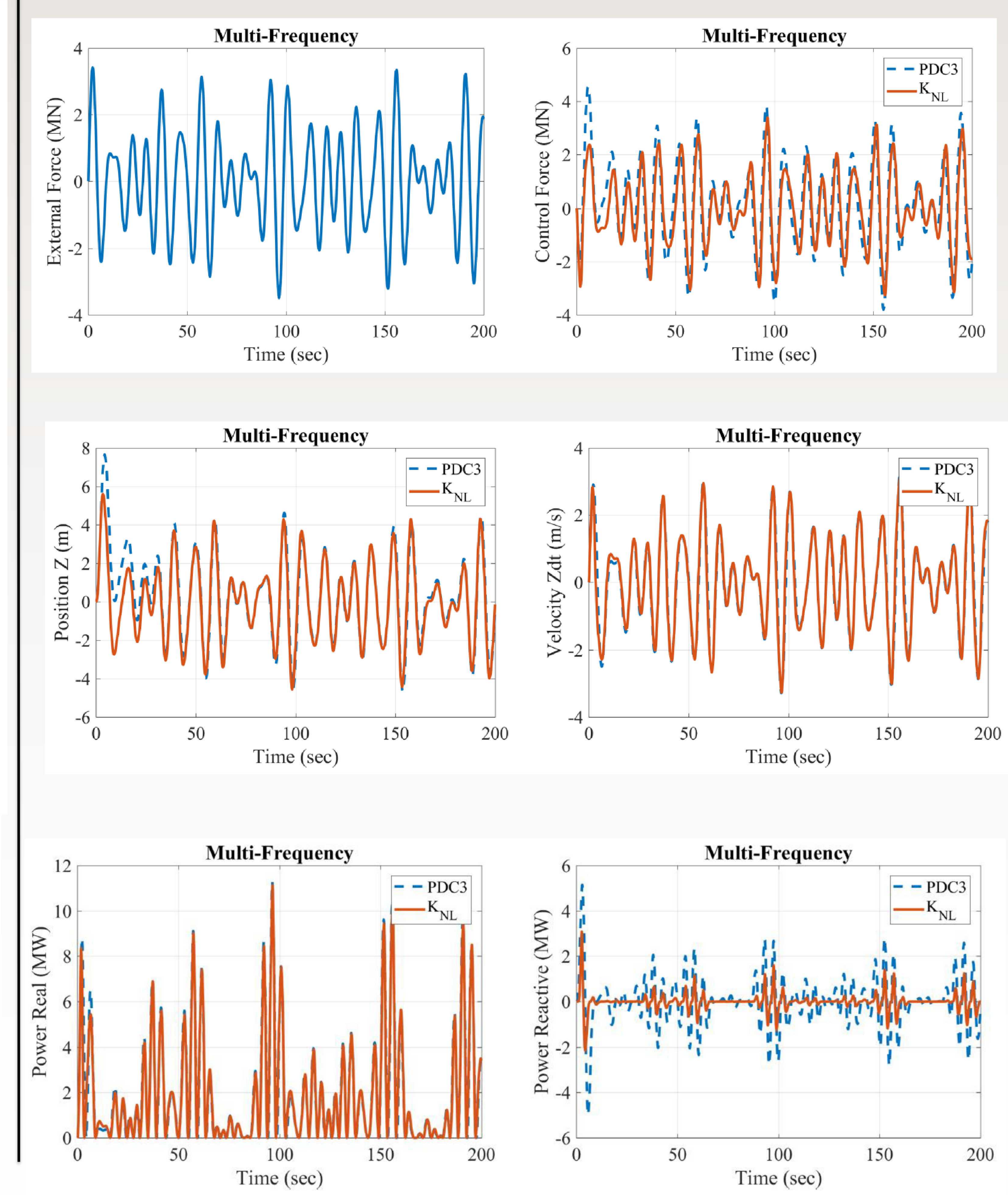
$$\mathcal{H}=\mathcal{T}+\mathcal{V}=\frac{1}{2}M\dot{z}^2+\frac{1}{4}K_{NL}z^4.$$

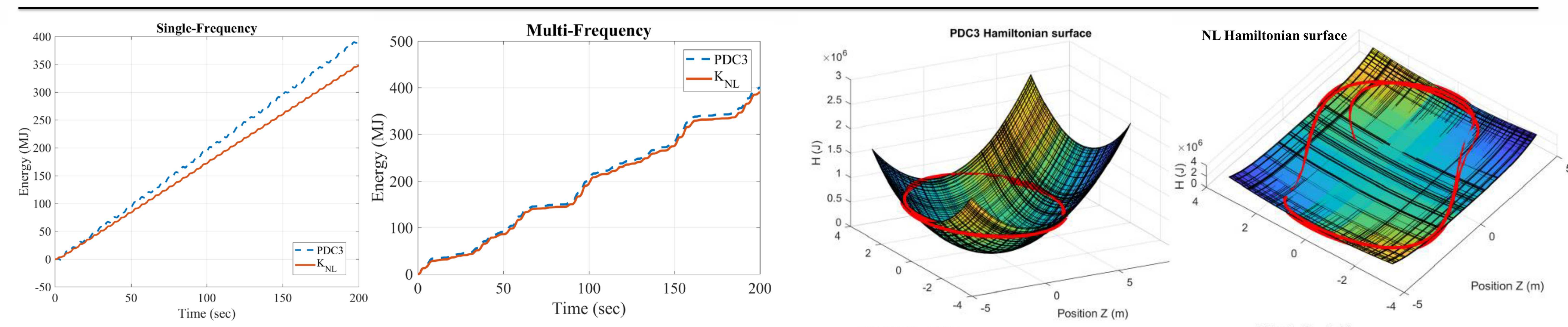
The profiles for each method are shown to the right. A limit cycle behavior is observed, where the goal for PDC3 is to resonate the WEC in off-resonance conditions. The trajectory demonstrates a tuned response or for electrical systems a power factor of one. For the NL hardening spring case, the limit cycle surface contour and shape are changed due to the potential energy provided by the NL cubic spring, generating a nonlinear resonance. The limit cycle is similar in response to a Duffing oscillator response

Single Frequency



Multi-Frequency





CONCLUSION

This paper presented a nonlinear control design for nonlinear WECs. A dynamic model was developed for a geometrically right-circular cylinder WEC design for the heave only motion. A practical C3 algorithm realization targeted both amplitude and phase through PD feedback and was developed from individual frequency components. For the NL design the linear stiffness term is replaced by a nonlinear cubic hardening spring term to demonstrate the performance of a NL WEC as compared to an optimized linear PDC3 WEC. Numerical simulations demonstrated the equivalent power for the nonlinear control design which incorporates and capitalizes on the nonlinear dynamics for the single DOF WEC. The comparison of PDC3 with a NL cubic hardening spring resulted in equivalent power/energy capture and improvements in reactive power requirements. In conclusion, the NL control design simplified the realization of the control law when compared to the PDC3 linear implementation. Future research will focus on optimizing the nonlinear controller for real wave conditions to demonstrate the viability of nonlinear WEC designs.

U.S. DEPARTMENT OF ENERGY

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