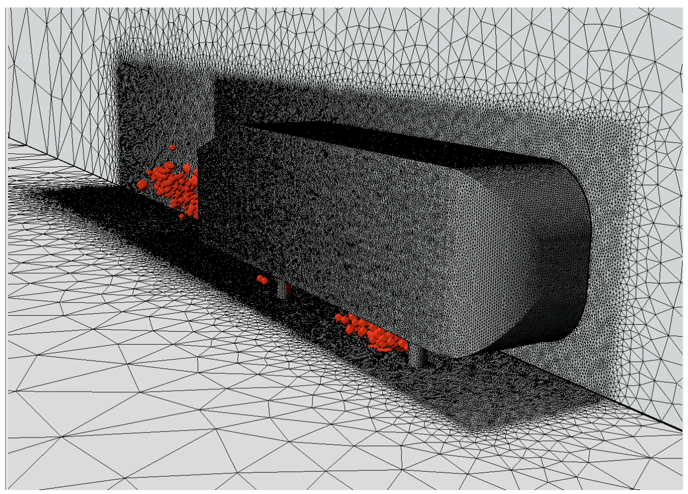
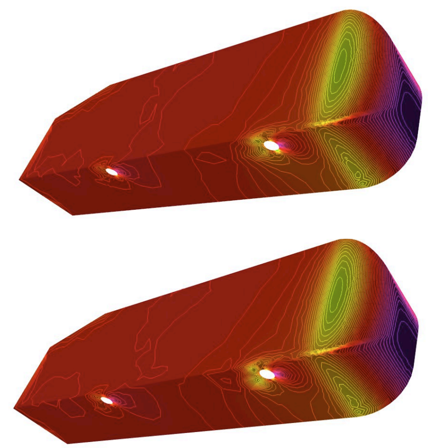
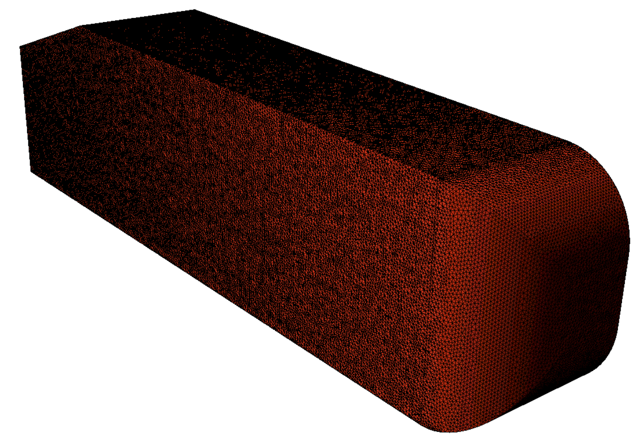
SAND2012-8958P

Model Order Reduction in Albany & Trilinos







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Albany Developer Meeting
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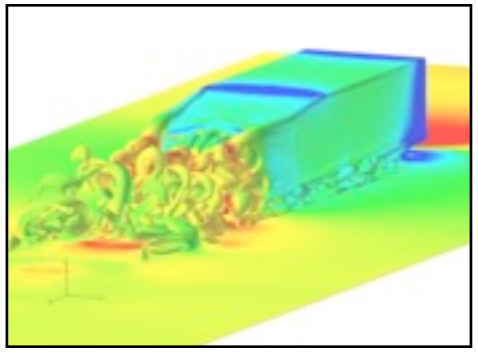
Motivation and objectives



High-fidelity simulation

- Detailed analysis of a few configurations

barrier

Time-critical applications

- Many query: UQ, optimization
- Near-real-time analysis: control

Goal: break barrier using model order reduction

Source: National Power Grid Simulation Capability: Needs and Issues

Project outline

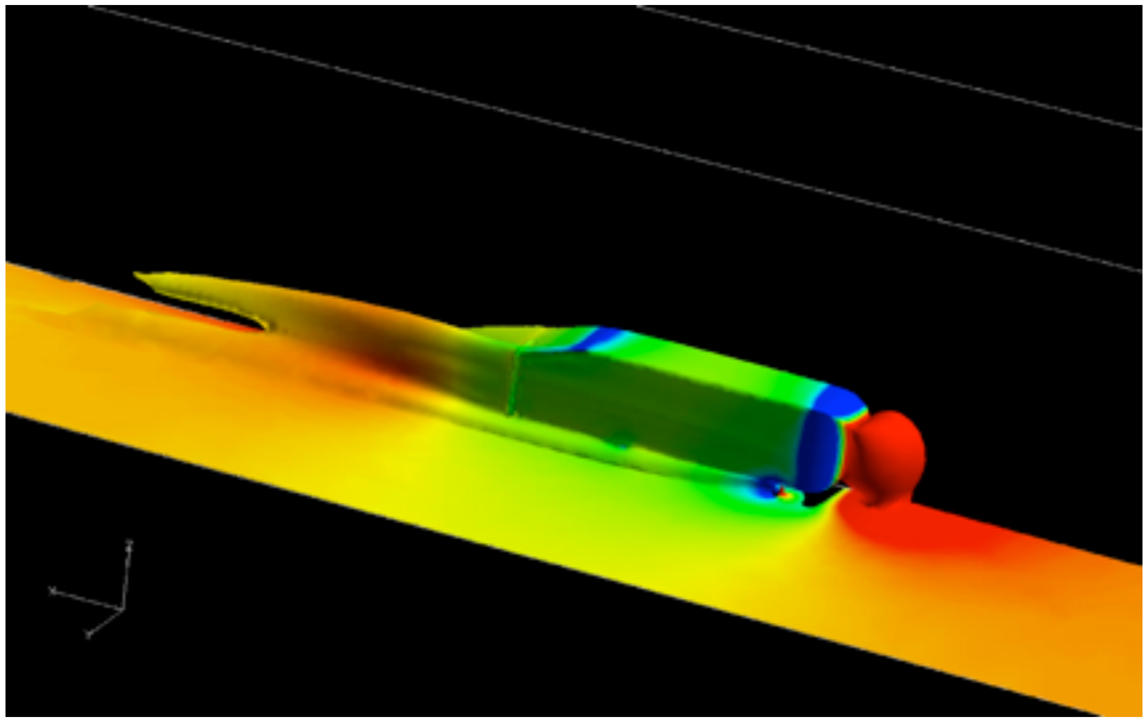
- Improve the state of the art
- Develop software compatible with Sandia's infrastructure
  - experimental support for ongoing research
  - extension to broad range of applications and analyses

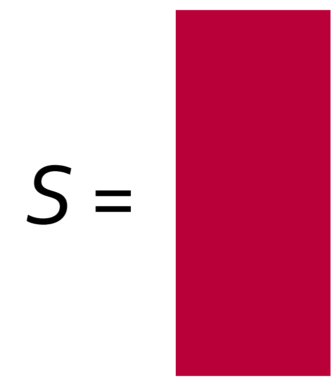
Model order reduction

- A class of surrogate-modeling method
  - preparatory offline phase
  - inexpensive online computations
- Extracts *dominant physical behavior* from high-fidelity model
- Based on algebraic formulation $F(x, \dot{x}, t, p) = 0$
  - basic methods are general, non-intrusive
- Better performance requires intrusiveness

Offline data collection

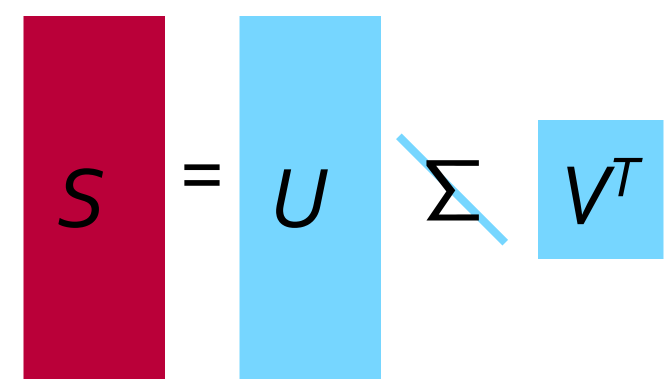
1. Collect snapshots of the state vector



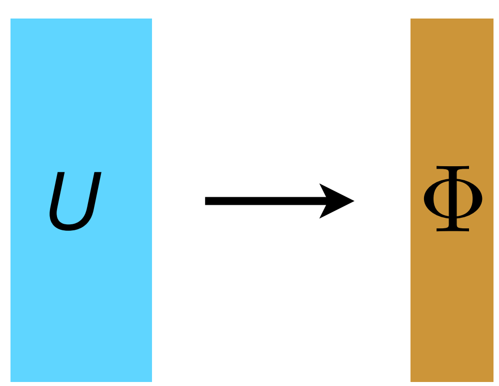


2. Compression

   a. compute singular value decomposition



   b. truncate



POD-Galerkin reduced-order model

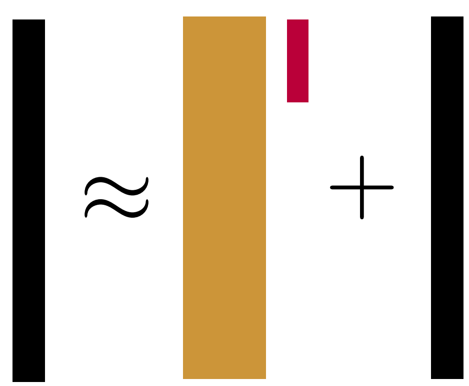
- High-fidelity model

$$\boxed{F(x, \dot{x}, t, p) = 0}$$

- Galerkin projection

<u>reduce # unknowns</u>

$$x \approx \Phi y + \bar{x}$$



<u>reduce # equations</u>

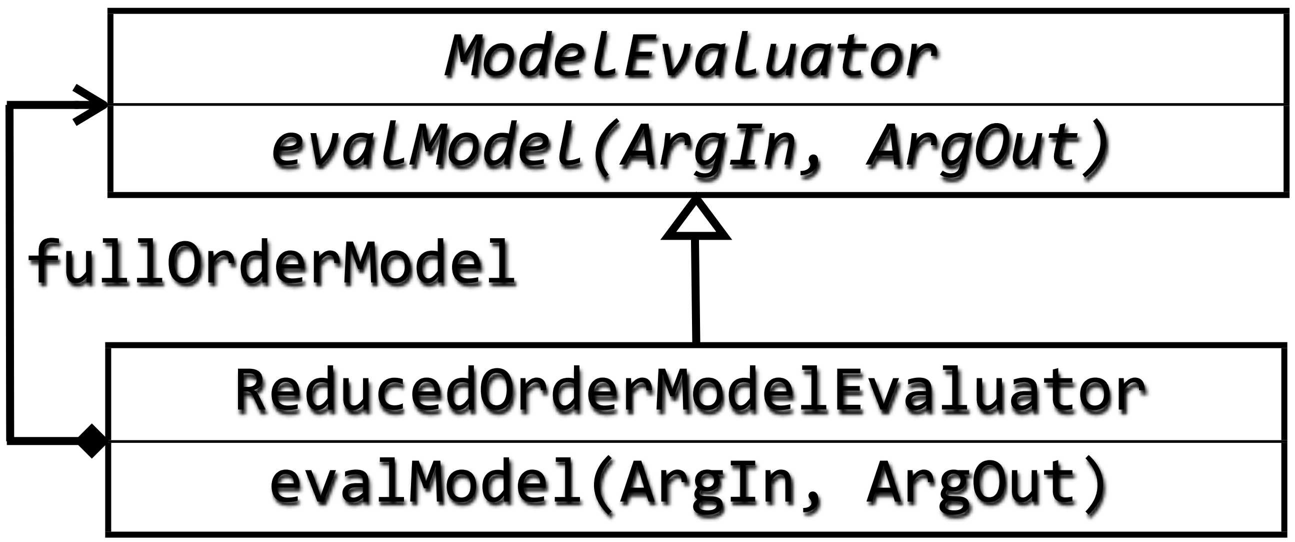
$$\Phi^T F(\Phi y + \bar{x}, \Phi \dot{y}, t, p) = 0$$

- Reduced-order model

$$\boxed{F_r(y, \dot{y}, t, p) = 0}$$

Non-intrusive implementation

- Exploit intrinsic modularity, standard interfaces in Trilinos / Albany
  - Loose coupling
  - Textbook use case for Composite / Decorator patterns
- ModelEvaluator hierarchy



$$F(x, \dot{x}, t, p) = 0$$

$$\Phi^T F(\Phi y + \bar{x}, \Phi \dot{y}, t, p) = 0$$
$$\Leftrightarrow$$
$$F_r(y, \dot{y}, t, p) = 0$$

- Observer hierarchies
  - snapshot collection
  - output for the reduced order model run

Complexity reduction

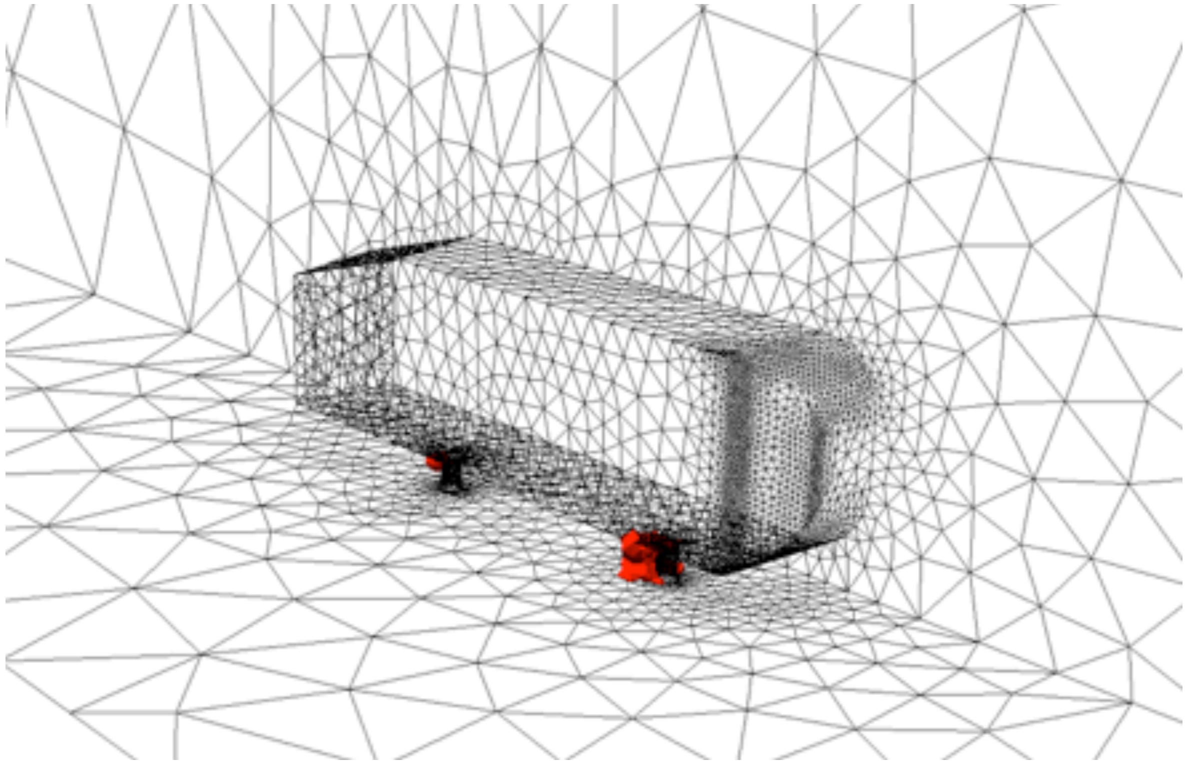
- Small dimension ≠ small cost
  - Newton iterations $$\left[\Phi^T J^{(k)} \Phi\right] p^{(k)} = -\Phi^T F^{(k)}$$
    - system of small dimension: inexpensive solve
    - system assembly scales with full-order dimension !

- Compute only a few entries of the full-order residual

$$\Phi^T F^{(k)} = \sum_{i=1}^{N} \Phi_i^T F_i^{(k)} \approx 0 \Leftrightarrow \sum_{i_s} \Phi_{i_s}^T F_{i_s}^{(k)} = 0$$

Sample mesh implementation

- *Goals*:
  - reuse existing simulation codes
  - minimize computing cores
  - scalability
- *Key*: evaluate only a few entries of the residual online
- *Idea*: sample a small subset of mesh



Data representation & storage

- Reduced-order basis representation

	RAM	File Format
Mesh-based field (Geometric)	STK	Exodus
MultiVector (Algebraic)	xPetra	Matrix Market

Application Trilinos

- Towards a mesh-based representation
  - \+ Physical interpretation
  - \+ Flexible computational topology
  - \- Complexity of implementation
  - \- Application dependent

Experience with Albany

- Enable MOR for a broad range of applications & types of analyses
  - versatility of the Albany platform

- Minimally intrusive Trilinos-based MOR software
  - general algebraic formulation ⟷ abstract Trilinos interfaces
  - rapid development in Albany with loose coupling

- Towards more efficient, specialized, intrusive implementations
  - existing software (e.g. STK): both constraint and leverage
  - Albany is a prime testing ground