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Active Error Correction in Adiabatic Quantum Computation

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U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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Outline

- *Suppression schemes are equivalent*
- *Limitations of error suppression*
  - *Leakage into correctable space is bad, so need exponential suppression*
  - *Necessary to use many-body couplings (EGP)*
- *Stabilizer active error correction*
- *Extensions of active error correction to adiabatic computation*
  - *Timescale argument*
  - *Protected Hamiltonian*

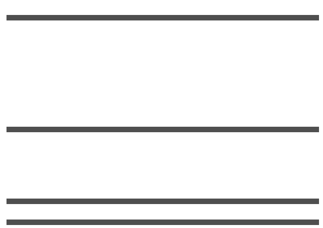
Effects of noise in AQC

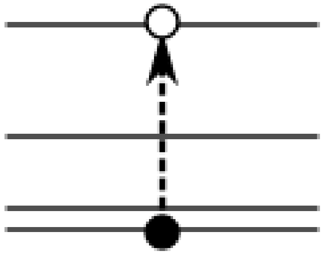
- We consider an instance of two qubit quadratic, unconstrained binary optimization (QUBO) as an example

$$H_{\mathrm{QUBO}}(t) = (\sigma_x^1 + \sigma_x^2)(1 - s(t)) + (\sigma_z^1 - \sigma_z^2 + \sigma_z^1\sigma_z^2)s(t) + H_\eta(t)$$

- Single qubit noise causes excitations from ground state

$$H_\eta(t) = \sum_i \eta(t)\sigma_z^i$$

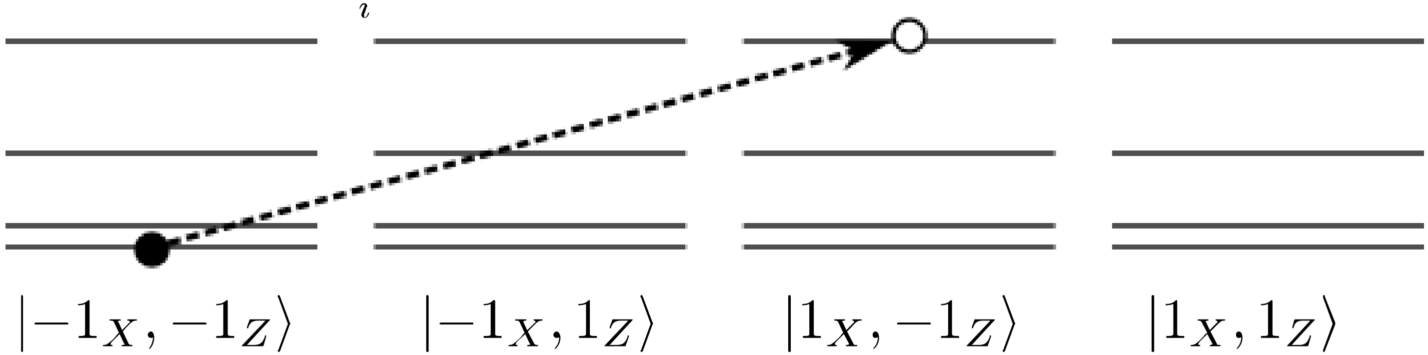




Error detecting codes in AQC

- We can encode the system with stabilizer code
- [[*N*,*N-2*,*2*]] error detecting stabilizer code
  - Two stabilizer generators: $\mathcal{S} = \left\{\sigma_x^{\otimes N}, \sigma_z^{\otimes N}\right\}$
  - Logical operators: $X_j = \sigma_x^1 \sigma_x^{j+1} \qquad Z_j = \sigma_z^{j+1} \sigma_z^N$
  - Encoded problem: $$H_{\mathrm{QUBO}}(t) = (X_1 + X_2)(1 - s(t)) + (Z_1 - Z_2 + Z_1 Z_2)s(t)$$

$$H_\eta(t) = \sum_i \eta(t)\sigma_z^i$$



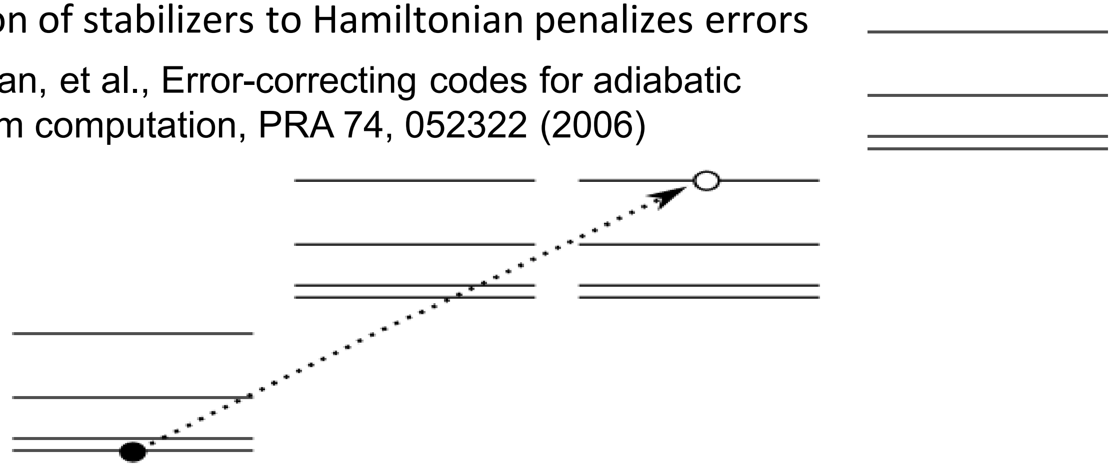
D. Gottesman, Ph.D. Thesis

D. Lidar. Towards Fault Tolerant Adiabatic Quantum Computation, PRL Vol. 100, 16 (2008)

Suppressing errors

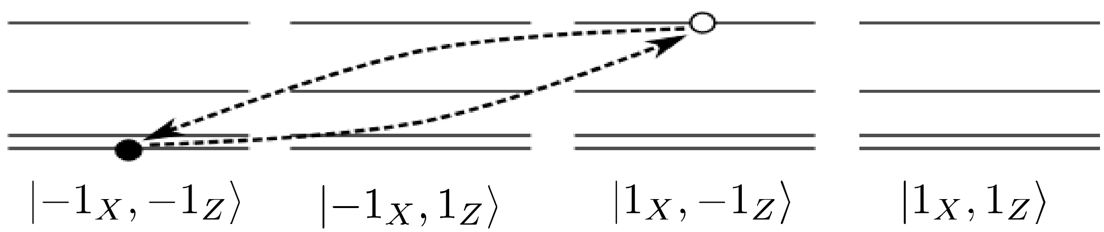
- Cannot correct errors due to diabatic transitions: $\left[\frac{dH}{dt}, \mathcal{S}\right] = 0$
- Can suppress single qubit errors due to noise:
  - Quantum error suppression (QES):
    Addition of stabilizers to Hamiltonian penalizes errors

    S. Jordan, et al., Error-correcting codes for adiabatic quantum computation, PRA 74, 052322 (2006)



  - Dynamical decoupling (DD):
    Control pulses refocus errors

    D. Lidar, Towards fault tolerant adiabatic quantum computation, PRL, 100, 160506 (2008)



DD and QES relationship

- The encoded system with control is,

$$H = H_{\mathrm{QUBO}}(t) + \sum_i \eta_i(t)\sigma_z^i + \mathrm{control}$$

$$\mathrm{control} = \begin{cases} H_{\mathrm{QES}} = \frac{\Omega}{2}(\sigma_x^{\otimes N} + \sigma_z^{\otimes N}) & \\ U_{\mathrm{DD}} = (\sigma_x^{\otimes N} * \sigma_z^{\otimes N}) & \text{at frequency } \Omega \end{cases}$$

- Move to interaction picture with respect to control

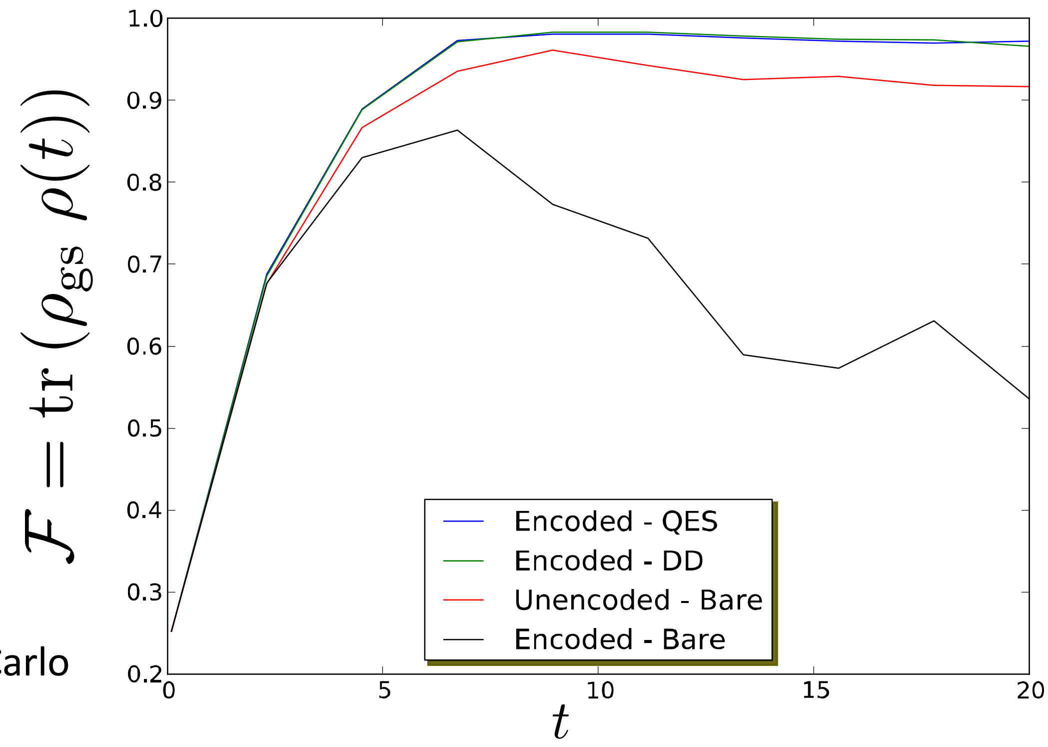
$$H_{\mathrm{rot}}(t) = H_{\mathrm{QUBO}}(t) + \sum_i \eta_i(t) \times \begin{cases} \mathrm{SqrWav}(\Omega t) Z_i & \mathrm{DD} \\ \exp\left(\sum_{\{S_j, Z_i\}=0} 2i\,\Omega S_j t\right) Z_i & \mathrm{QES} \end{cases}$$

- In rotating frame, both QES and DD result in periodic modulation of noise.
  - Sum in QES is taken over stabilizers, S, which anticommute with error, Z
- Effective noise rate is similar in both cases.
  - Fermi Golden Rule results in roughly equal leakage from the ground state.

Simulations

- Simulations of two-qubit QUBO problem showing equivalent performance of DD and QEC - Calculated by Monte Carlo with 1/f noise

  - Increase in performance with time because more adiabatic
  - Long-time decrease due to accumulation of error
  - Encoding without correction doubles likelihood of error
  - QES and DD perform nearly equally well
  - Squiggly lines because of poorly converged Monte Carlo



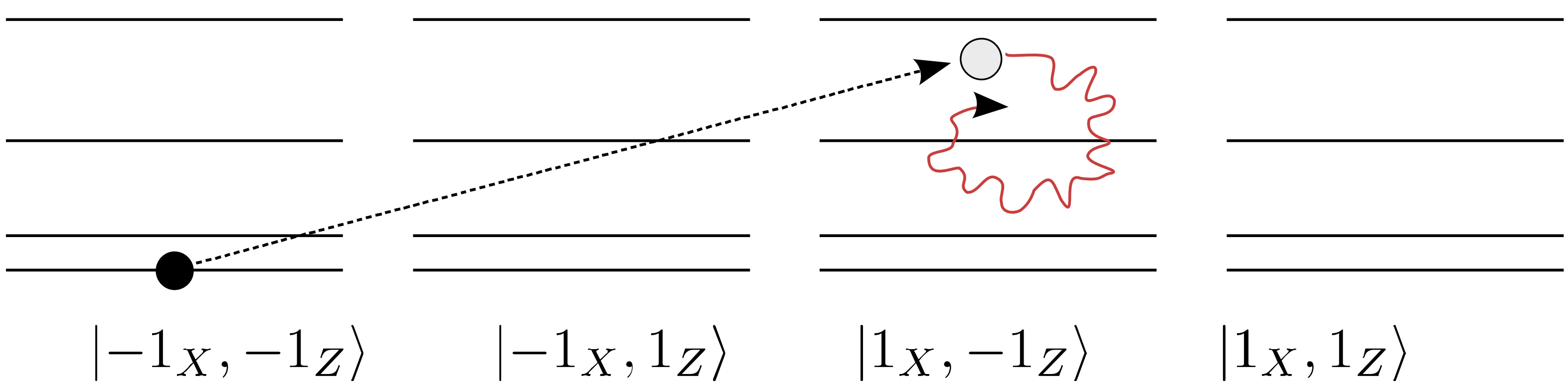
What if an error does happen?

Coherent evolution (by logical Hamiltonian) leads to mixing of states.

Hamiltonian and error operator don't commute, so $E_j |\Psi\rangle$ is not an eigenstate. This state then evolves under the action of the logical Hamiltonian before it is corrected.

$$|\Psi\rangle = E_j E_j |\Psi\rangle \qquad E_j U(t) E_j |\Psi\rangle \neq |\Psi\rangle$$

<u>Another way to see this problem:</u>
A low weight physical error gets "dressed" by the (always on) logical Hamiltonian and gets converted into a high-weight uncorrectable error



Limitations of error suppression in AQC

- Error suppression must be so strong that state is in code space (not correctible space, as in circuit model QC) with high probability.

- For EGP implementation of error suppression, require high-weight (many-body) terms to enforce energy penalties.

What about adding error correction?

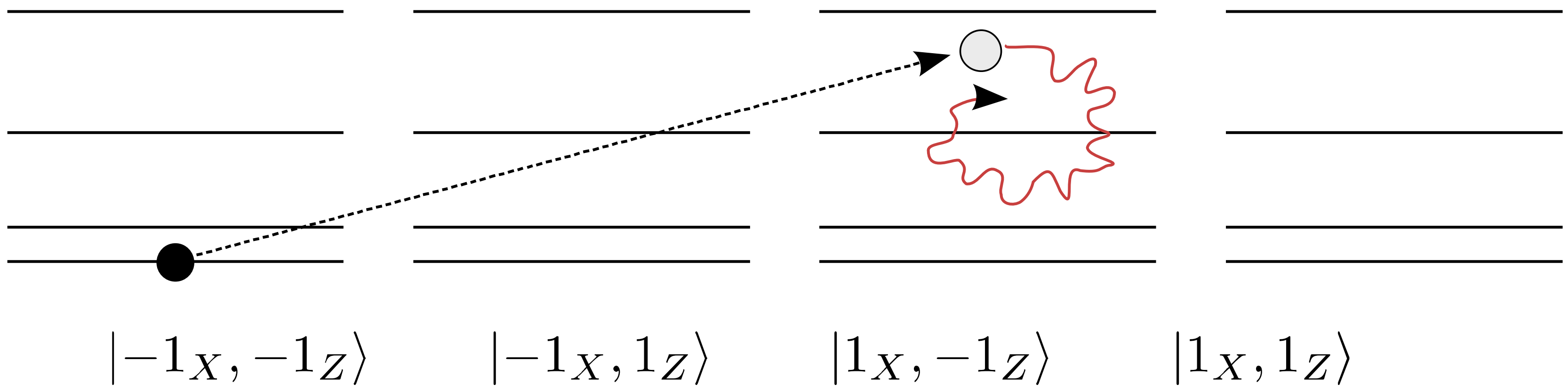
- A continuous form of error correction is appealing for AQC – meshes better with the adiabatic philosophy.

- Stabilizer Hamiltonians, Hamiltonians composed entirely of stabilizer elements, naturally fit into a continuous error correction framework because:
  - Local error operators promote the ground state to excited energy eigenstates (logical Hamiltonian = 0 in this case).
  - Hence these errors can be reversed (corrected) by another application of a local operator.
  - We can *cool* the system by embedding it in a cold reservoir that couples locally and linearly to the system.
  - As long as the temperature of this reservoir can be maintained below the noise temperature, the system will be protected (error correction will overcome noisy fluctuations).
  - NOTE: The localized cooling implementation of error correction is effective because of the local structure of stabilizer Hamiltonian excitations.

But …

- This works well for quantum memories, but when logical evolution is added to the Hamiltonian (e.g. encoded AQC), the locality of excitations is broken – the whole Hamiltonian is no longer a stabilizer Hamiltonian.
- Logical Hamiltonian also introduces extra energy splitting – may be difficult to reach resonance without large bath.
- Related to this picture:

  Coherent evolution (by logical Hamiltonian) leads to mixing of states.



"Solution" 1: Timescales

- We want the error correction procedure to be effective
  - Between error correction steps, the state must *not* evolve out of the correctable space
  - Equivalent to demanding that

$$\tau \ll \frac{\hbar}{||H_{\text{logical}}||}$$

  - But the TOTAL time that the system runs is proportional to the norm of the logical Hamiltonian (increase norm, decrease run time), implying that no matter the norm, you will always require the same number of error correction steps.
  - If the error correction steps are driven by coupling to a bath, then the coupling should be such that

$$\frac{J\tau}{\hbar} \simeq \frac{\pi}{2}$$

    - This ensures a complete step is completed in time τ
- If this is fast enough, the resonance issue may be taken care of by lifetime broadening of the resonance line.

"Solution"2 : Protected Hamiltonians

- It is possible to rewrite the logical Hamiltonian in an error protected way, so that the erred states *are* eigenstates

$$H_{\mathrm{pro}}(t) = \sum_j E_j P_0 H(t) P_0 E_j$$

  - Where the code-space projectors are defined as:

$$P_0 = \prod_j \frac{1}{2}(1 + S_j) = \sum_n |n; 0, 0, \ldots\rangle \langle n; 0, 0, \ldots|$$

  - A simple calculation shows that $[H_{\mathrm{pro}}(t), E_j] = 0$
    - Logical Hamiltonian induces no evolution on erred states.

- This is not a reasonable thing to do physically, because the projectors are many-body operators. Could be useful mathematically. Also demonstrates that there are (at least 2) Hamiltonians which can effect the same evolution on the code space. Are there more practical ones?