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Going MADM

An Introduction to Multi-Attribute Decision-Making Tools for Aerospace Engineers

Scoring Matrix: Low Complexity

	Balloon	Airplane	Rotorcraft				Priority Vector
Balloon	1	5	9	0.763	0.806	0.600	0.723
Airplane	1/5	1	5	0.153	0.161	0.333	0.216
Rotorcraft	1/9	1/5	1	0.085	0.032	0.067	0.061
Sum	1.3	6.2	15.0	1.0	1.0	1.0	

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What is MADM?

Multi-Attribute Decision-Making

more than one

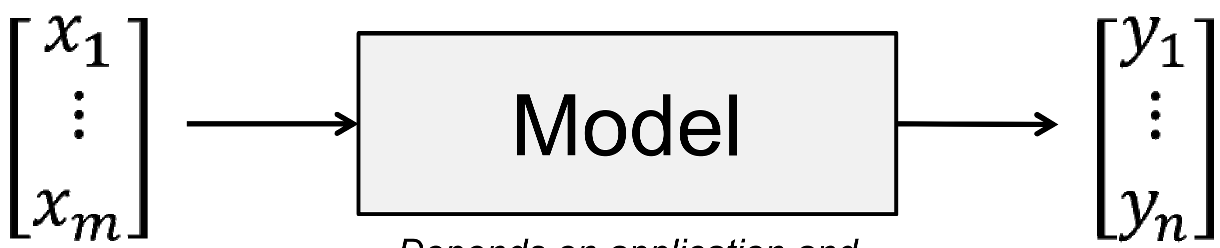
characteristic

Choosing

Term Definitions

- In the context of MADM, an **attribute** is a characteristic of a candidate solution or design. Typically it is the consequence of some setting of design variables.
- An **objective** is an attribute with direction.
- A **design variable** is a quantity or quality over which the decision-maker has direct control.
- A **candidate design** is a unique setting of the design variables describing the system of interest.

For a given candidate design:



Design Variable Settings

Depends on application and required fidelity; may be mathematical or mental

Attribute Outputs

Term Examples for an Airplane

Suppose you are an engineer charged with designing a new airplane. What might you have as design variables, and what attributes and objectives might be relevant to you?

Design Variables
Wing Area
Wing Aspect Ratio
Wing Sweep Angle
Fuselage Fineness Ratio
Engine Type
Number of Engines
Landing Gear Config.
Empennage Config.

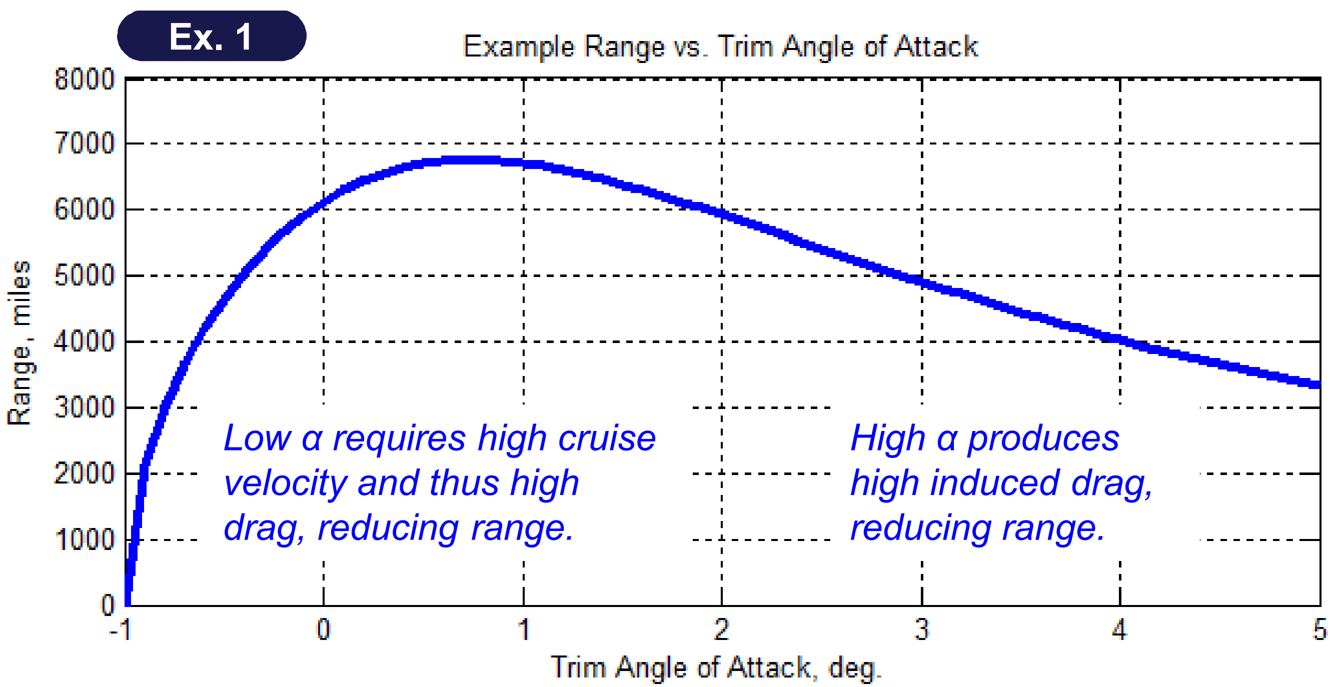
Attributes	Objectives
Payload Mass	High Payload Mass
Range	Long Range
Endurance	Long Endurance
Maneuverability	High Maneuverability
Takeoff Field Length	Short Takeoff Field Length
Landing Field Length	Short Landing Field Length
Development Time	Short Development Time
Development Cost	Low Development Cost
Operating Cost	Low Operating Cost

How can you choose the right inputs (design variables) to create a design that will fulfill all these objectives??

Single-Objective Optimization

Suppose you are planning a flight in a business jet. At what trim angle of attack should the plane cruise to maximize its range?

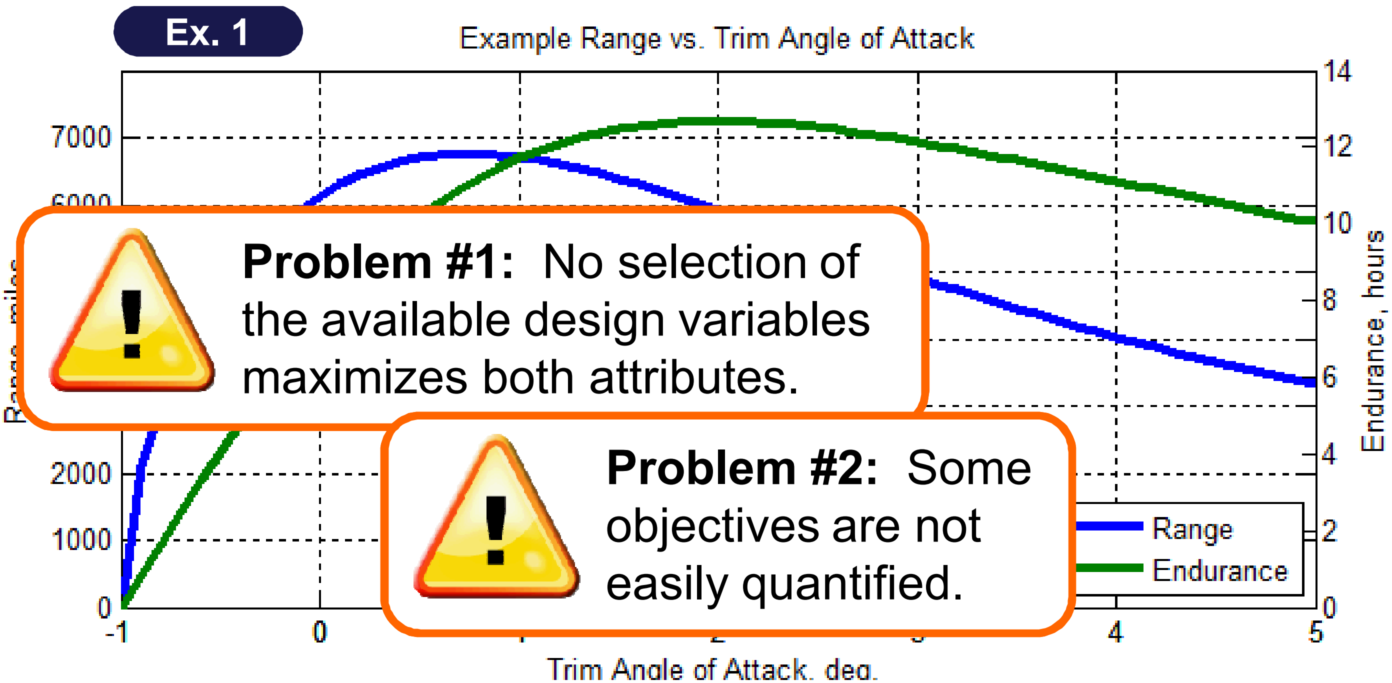
α → Aircraft Aerodynamic Coefficient Graphs/Tables → C_L, C_D → Breguet Range Equation → Range



Two-Objective Optimization?

Suppose you are planning a flight in a NOAA jet, with a mission of gathering high-resolution spatial and temporal atmospheric profiles of a hurricane. At what trim angle of attack should the plane cruise to maximize range and endurance?

What if we also want to simultaneously maximize visibility of the hurricane to the pilot, minimize aircraft loads, and maximize flight control effectiveness?



Problem #1: No selection of the available design variables maximizes both attributes.

Problem #2: Some objectives are not easily quantified.

Simple Additive Weighting Method

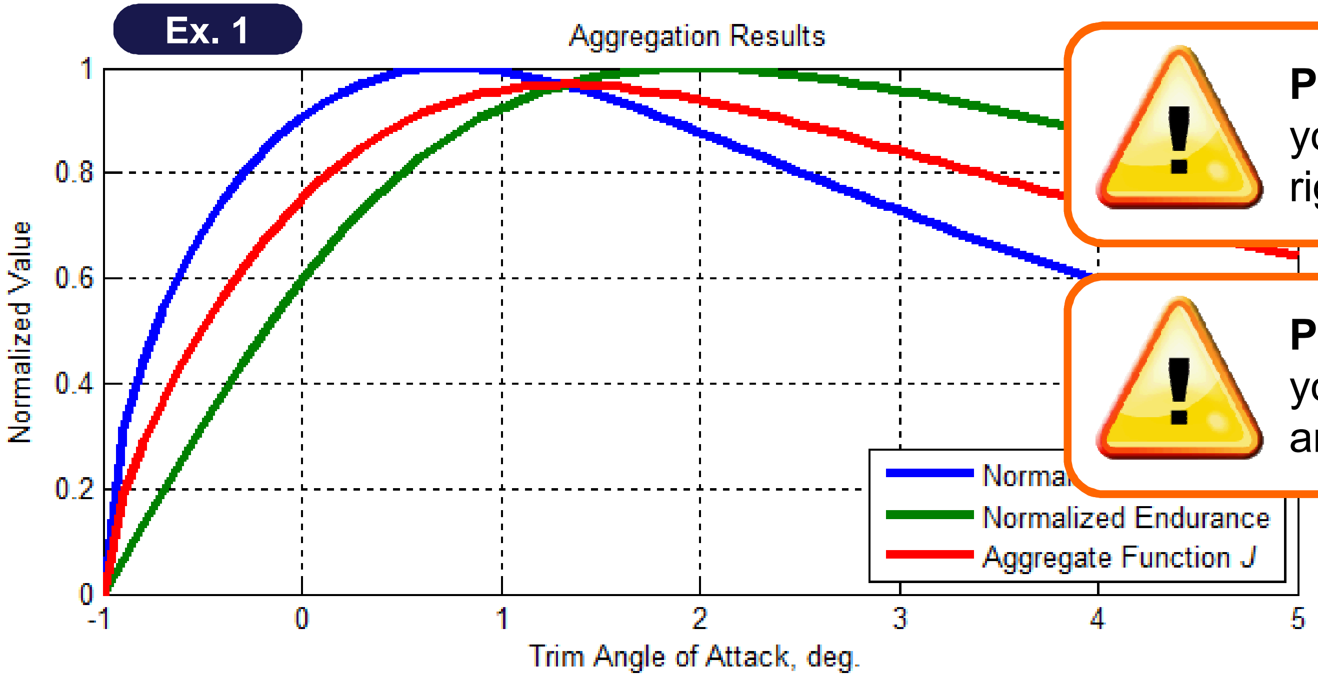
1. Define the set of candidate designs.
2. Normalize the range of each attribute to fall between 0 and 1, such that larger values are more preferred.
3. Select weights w_k on each objective k, such that the sum of all weights is unity.
4. Compute the aggregate objective function J for each candidate design. Select the design with the highest value of J.

$$J = \sum_{k=1}^{n} w_k y_k = w_1 y_1 + w_2 y_2 + \cdots + w_n y_n$$

Simple Additive Weighting Method

applied to our Endurance/Range Example

1. **Candidate designs:** α = -1°, α = 0°, α = 1°, α = 2°, etc.
2. **Normalize attributes:** Divide all endurance values by 12.7 hours (max seen in the graph) and all range values by 6,750 miles (max seen in the graph) such that both vary from 0 to 1.
3. **Select weights:** For demonstration, set $w_1 = w_2 = 0.5$.
4. **Compute *J* and select design with highest value*:***



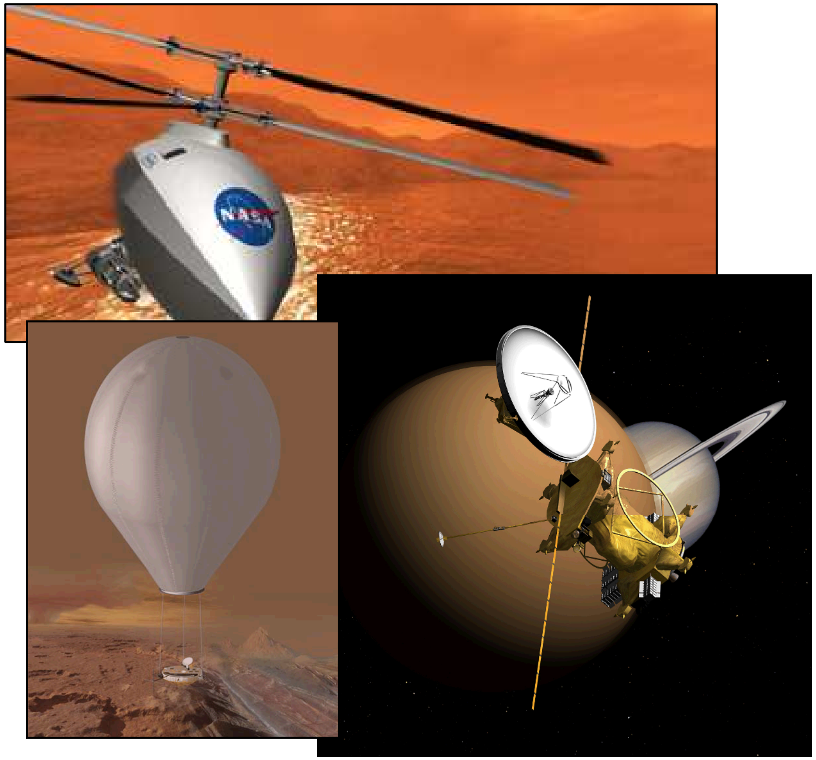
Problem #1: How do you know you chose the right weights?

Problem #2: How can you handle attributes that are difficult to quantify?

Example #2: Titan Explorer

Ex. 2

- In 2009, a joint NASA/ESA team proposed a flagship mission to the Saturn system, including aerial vehicle exploration of Titan. How could you tackle the problem of deciding among the possible aerial vehicle options?
- Candidate Designs:
  - Balloon
  - Airplane
  - Rotorcraft
- Objectives:
  - Long Endurance
  - Long Range
  - Low Complexity
  - Thorough Surface Analysis



Analytic Hierarchy Process (AHP)

1. Define the set of candidate designs and set of attributes/objectives.
2. Determine the objective priority vector from pairwise evaluations in the objective prioritization matrix.
3. Conduct pairwise evaluations of alternatives in terms of each of the *n* objectives in the form of *n* matrices. Compute the associated score vectors from each matrix.
4. For each candidate design, compute the aggregate score. Select the design with the highest overall score.

Priority of objective *k*
(from priority vector)

$$J = \sum_{k=1}^{n} w_k y_k = w_1 y_1 + w_2 y_2 + \cdots + w_n y_n$$

Score of this design in
terms of objective *k*

Analytic Hierarchy Process (AHP)

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Step 2: Objective Prioritization

Ex. 2

AHP Prioritization Matrix

	Long Endurance	Long Range	Low Complexity	Thorough Surface Analysis
Long Endurance	1	1/5	1/7	1/5
Long Range	5	1	1/5	1/3
Low Complexity	7	5	1	3
Thorough Surface Analysis	5	3	1/3	1

For each element in white, ask the question, "How much does the decision-maker prefer the item in the row over the item in the column?"

If the item in the column is preferred, use the reciprocal of the appropriate preference in the AHP weighting scale (at right).

- White elements in upper right: Use AHP weighting scale
- Light gray elements in bottom left: Reciprocals of upper right
- Dark gray elements on the diagonal: Unity by definition

AHP Weighting Scale

9	Absolutely Prefer
8	
7	Very Strongly Prefer
6	
5	Strongly Prefer
4	
3	Weakly Prefer
2	
1	Neutrally Prefer

Analytic Hierarchy Process (AHP)

Step 2: Objective Prioritization

Ex. 2

AHP Prioritization Matrix	Long Endurance	Long Range	Low Complexity	Thorough Surface Analysis	**Prioritization Matrix, Normalized by Column**				Priority Vector
Long Endurance	1	1/5	1/7	1/5	0.056	0.022	0.085	0.044	0.052
Long Range	5	1	1/5	1/3	0.278	0.109	0.119	0.074	0.145
Low Complexity	7	5	1	3	0.389	0.543	0.597	0.662	0.548
Thorough Surface Analysis	5	3	1/3	1	0.278	0.326	0.199	0.221	0.256
Sum	18.0	9.2	1.7	4.5	1.0	1.0	1.0	1.0	

Analytic Hierarchy Process (AHP)

Step 2: Objective Prioritization

Ex. 2

AHP Prioritization Matrix	Long Endurance	Long Range	Low Complexity	Thorough Surface Analysis	**Average over each row**				Priority Vector
Long Endurance	1	1/5	1/7	1/5	0.056	0.022	0.085	0.[illegible]44	0.052
Long Range	5	1	1/5	1/3	0.278	0.109	0.119	0.[illegible]74	0.145
Low Complexity	7	5	1	3	0.389	0.543	0.597	0.[illegible]62	0.548
Thorough Surface Analysis	5	3	1/3	1	0.278	0.326	0.199	0.[illegible]21	0.256
Sum	18.0	9.2	1.7	4.5	1.0	1.0	1.0	1.0	

Analytic Hierarchy Process (AHP)

Step 3: Design Evaluations

Ex. 2

Scoring Matrix: Long Endurance	Balloon	Airplane	Rotorcraft				Priority Vector
Balloon	1	9	5	0.763	0.600	0.806	0.723
Airplane	1/9	1	1/5	0.085	0.067	0.032	0.061
Rotorcraft	1/5	5	1	0.153	0.333	0.161	0.216
Sum	1.3	15.0	6.2	1.0	1.0	1.0	

Scoring Matrix: Low Complexity	Balloon	Airplane	Rotorcraft				Priority Vector
Balloon	1	5	9	0.763	0.806	0.600	0.723
Airplane	1/5	1	5	0.153	0.161	0.333	0.216
Rotorcraft	1/9	1/5	1	0.085	0.032	0.067	0.061
Sum	1.3	6.2	15.0	1.0	1.0	1.0	

Scoring Matrix: Long Range	Balloon	Airplane	Rotorcraft				Priority Vector
Balloon	1	7	9	0.797	0.840	0.692	0.777
Airplane	1/7	1	3	0.114	0.120	0.231	0.155
Rotorcraft	1/9	1/3	1	0.089	0.040	0.077	0.069
Sum	1.3	8.3	13.0	1.0	1.0	1.0	

Scoring Matrix: Thorough Surface Analysis	Balloon	Airplane	Rotorcraft				Priority Vector
Balloon	1	5	1/5	0.161	0.333	0.153	0.216
Airplane	1/5	1	1/9	0.032	0.067	0.085	0.061
Rotorcraft	5	9	1	0.806	0.600	0.763	0.723
Sum	6.2	15.0	1.3	1.0	1.0	1.0	

Analytic Hierarchy Process (AHP)

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Step 4: Aggregation and Selection

AHP Prioritization Matrix	Long Endurance	Long Range	Low Complexity	Thorough Surface Analysis					Priority Vector
Long Endurance	1	1/5	1/7	1/5	0.056	0.022	0.085	0.044	0.052
Long Range	5	1	1/5	1/3	0.278	0.109	0.119	0.074	0.145
Low Complexity	7	5	1	3	0.389	0.543	0.597	0.662	0.548
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Sum	18.0	9.2	1.7	4.5	1.0	1.0	1.0	1.0	

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Scoring Matrix: Thorough Surface Analysis	Balloon	Airplane	Rotorcraft				Priority Vector
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Rotorcraft	5	9	1	0.806	0.600	0.763	0.723
Sum	6.2	15.0	1.3	1.0	1.0	1.0	

		Evaluation Scores		
Objective	**w_k**	**Balloon**	**Airplane**	**Rotorcraft**
Long Endurance	0.05	0.72	0.06	0.22
Long Range	0.14	0.78	0.15	0.07
Low Complexity	0.55	0.72	0.22	0.06
Thorough Surface Analysis	0.26	0.22	0.06	0.72
Aggregate Scores		**0.60**	**0.16**	**0.24**

Ex. 2

$$J = \sum_{k=1}^{4} w_k y_k = w_1 y_1 + w_2 y_2 + w_3 y_3 + w_4 y_4$$

Hot Air Balloon selected for NASA/ESA Titan Saturn System Mission (TSSM) proposal

Analytic Hierarchy Process (AHP)

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Assessing Consistency

How can you be confident that your prioritizations and evaluations were self-consistent, and that your understanding of the problem did not change while filling out the matrices?

The **consistency ratio (CR)** serves as a measure of the randomness (or inconsistency) of the matrix, based on the fact that the columns of the matrix would ideally be scalar multiples of each other if the user were perfectly consistent in his pairwise priorities.

Saaty, who proposed AHP in the mid-1970s, suggested **CR ≤ 0.100** as an acceptable consistency criterion.

Ex. 2

AHP Prioritization Matrix

	ong Endurance	ong Range	ow Complexity	Thorough Surface Analysis
Long Endurance	1	1/5	1/7	1/5
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Thorough Surface Analysis	5	3	1/3	1

$$CR = \frac{CI}{RI_{avg}} = \frac{\lambda_{max} - n}{(n-1)RI_{avg}}$$

For this matrix:

n = 4 (number of objectives)

λ_{max} = 4.24 (max real eigenvalue)

RI_{avg} = 0.8824 (table at right)

→ **CR = 0.0908**

n	RI_{avg}
2	0
3	0.5406
4	0.8824
5	1.1152
6	1.2489
7	1.3449
8	1.4058
9	1.4528
10	1.4867
11	1.514
12	1.5419
13	1.5496
14	1.5723
15	1.5806
16	1.5972
17	1.6035
18	1.6173
19	1.623
20	1.6273

Recap

- Definitions of **design variable**, **candidate design, attribute**, and **objective**.
- Example #1: Jet Airplane Range vs. Endurance
  - Candidate Designs: $\alpha = -1^{o}$, $\alpha = 0^{o}$, $\alpha = 1^{o}$, etc.
  - Objectives: Long Range and Long Endurance
  - Demonstrated **Simple Additive Weighting** aggregation method (ideal when using quantitative data and weights are easily agreed upon).
- Example #2: Titan Explorer Platform Decision
  - Candidate Designs: Balloon, Airplane, Rotorcraft
  - Objectives: Long Range, Long Endurance, Low Complexity, Thorough Surface Analysis
  - Demonstrated **Analytic Hierarchy Process** aggregation method (ideal when qualitative attributes are involved and/or weights are difficult to agree upon without pairwise comparison).

More to MADM

- This class has covered two simple MADM techniques. Many others exist, and each have advantages and disadvantages.
- **Higher-order aggregate objective functions** attempt to model nonlinear human preferences (e.g., diminishing rates of marginal substitution).
- **Multi-attribute utility theory** attempts to create a "true" map of decision-maker preferences through elicitation.
- **Multi-dimensional methods** define optimality in a more general way (i.e., Pareto optimality) to allow elimination of objectively suboptimal solutions, without making assumptions about the relative importance of one objective over another.
- Whatever method you choose to make decisions, you must always understand its limitations. In many cases, the value in these methods is not the final output, but the insight gained along the way.

Questions?

Note: Both examples used in this presentation are notional.